Cooperative Game Theory

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Motivating Example: Car Pooling

People drive to work and would like to form car pools.

- Some can pick up others on their way to work. Others have to go out of their way to pick up others.
- A car can only hold 5 people.

Assume people care about (1) **money** and (2) **time** and it is possible to convert between the two.

***Who** should carpool **together**?*

***How much** should they **pay** each other?*

Introduction

Cooperative Game Theory

Non Cooperative vs. Cooperative GT

Non-cooperative GT	Cooperative GT
Payoffs go **directly to individual** agents	Payoffs go **to coalitions** which **redistribute** them to their members*
Players choose an **action**	Players choose a **coalition** to join and agree on **payoff distribution**
Model of **strategic confrontation**	Model of **team / cooperation formation**
Players are **self-interested**	

*transferable utility games

Cooperative Game Theory

Model of **coalition (team) formation**

- friends agreeing on a trip
- entrepreneurs trying to form companies
- companies cooperating to handle a large contract

Assumes a **coalition** can **achieve more** than (the sum of) individual agents

- Better to team up and split the payoff than receive payoff individually

Also called **coalitional game theory**

Called cooperative but agents still **pursue their own interests!**

Example: Voting Game

The parliament of Micronesia is made up of **four political parties**, A, B, C, and D, which have **45, 25, 15, and 15 representatives**, respectively.

They are to vote on whether to pass a $100 million **spending bill** and how much of this amount should be controlled by each of the parties.

A **majority vote**, that is, a **minimum of 51 votes**, is required in order to pass any legislation, and if the bill does not pass then every party gets zero to spend.

Example: Buying Ice-cream

$\boldsymbol{n}$ **children**, each has some amount of money

- the i-th child has b_i dollars

Three types of ice-cream tubs are for sale:

- Type 1 costs $7, contains 500g
- Type 2 costs $9, contains 750g
- Type 3 costs $11, contains 1kg

Children have **utility for ice-cream**,
and do not care about **money**

The **payoff of each group**: the maximum **quantity** of **ice-cream** the members of the group can buy by pooling their money

The ice-cream can be **shared arbitrarily** within the group

How Is a Cooperative Game Played?

1. Knowing the payoffs for different coalitions, agents **analyze** which coalitions and which payoff distributions would be **beneficial** for them.
2. Agents agree on **coalitions** and **payoff** distributions
   - requires contracts – infrastructure for cooperation
3. Task is executed and **payoff** distributed.

We will now see how to formalize these ideas.

Basic Definitions

Cooperative Game Theory

Coalitional Games

TRANSFERABLE UTILITY GAMES

Payoffs are given **to the group** and then divided among its members.

Satisfied whenever there is a **universal currency** that is used for exchange in the system.

NON-TRANSFERABLE UTILITY GAMES

Group actions result in **payoffs to individual** group members.

There is no universal currency.

Coalitional Game

Transferable utility assumption: the payoff to a coalition may be freely redistributed among its members.

Definition (Coalitional game with transferable utility)

A **coalitional game with transferable utility** is a pair $(\boldsymbol{N}, \boldsymbol{v})$ where

- N is a finite **set of players** (also termed **grand coalition**), indexed by i; and
- $v: 2^N \mapsto \mathbb{R}$ is a **characteristic function** (also termed **valuation function**) that associates with each coalition $S \subseteq N$ a real-valued **payoff** $\boldsymbol{v}(\boldsymbol{S})$ that the coalition's members can distribute among themselves. We assume $v(\emptyset) = 0$.

Simple Example

$$N = \{1,2,3\}$$

S	$v(S)$
(1)	2
(2)	2
(3)	4
(12)	5
(13)	7
(23)	8
(123)	9

Illustrative Example

Chralie: $4  Marcie: $3  Pattie: $3 

 w = 500, p = $7

 w = 750, p = $9

 w = 1000, p = $11

Characteristic function $v(C)$

- $v(\emptyset) = v(\{C\}) = v(\{M\}) = v(\{P\}) = 0$
- $v(\{C, M\}) = 500, v(\{C, P\}) = 500, v(\{M, P\}) = 0$
- $v(\{C, M, P\}) = 750$

Outcome and Payoff Vector

Definition (Outcome and Payoff)

An **outcome** of a game (N, v) is a pair $(CS, \vec{x})$ where

- $CS = (C_1, \dots, C_k)$, $\bigcup_i C_i = N$, $C_i \cap C_j = \emptyset$ for $i \neq j$, is a **coalition structure**, i.e., a partition of N into coalitions.
- $\vec{x} = (x_1, \dots, x_n)$, $x_i \geq 0$ for all $i \in N$, $\sum_{i \in C} x_i = v(C)$ for each $C \in CS$, is a **payoff (distribution) vector** which distributes the value of each coalition in CS to the coalition's members.

Payoff is **individually rational** (also called **imputation**)
if $x_i \geq v(\{a_i\})$

Note: When the coalition structure is not explicitly mentioned, a grand coalition (all agents) is assumed

Example

S	$v(S)$
(1)	2
(2)	2
(3)	4
(1 2)	5
(1 3)	7
(2 3)	8
(1 2 3)	9

Outcome examples

(1)(2)(3)
2 + 2 + 4 = 8

(1)(2 3)
2 + 8 = 10

(2) (1 3)
2 + 7 = 9

(3) (1 2)
4 + 5 = 9

(1 2 3)
9

$$\vec{x} = (2, 3, 4)$$

Distributing Payments

How should we *fairly* distribute a coalition's payoff?

S	$v(S)$
()	0
(1)	1
(2)	3
(12)	6

? If the agents form (12), how much should each get paid?

Fairness: Axiomatic Approach

What is fair?

Axiomatic approach – a fair payoff distribution should satisfy:

- **Symmetry:** if two agents *contribute the same*, they should receive the same pay-off (they are interchangeable)
- **Dummy player:** agents that *do not add value* to any coalition should get what they earn on their own
- **Additivity:** if two *games are combined*, the value a player gets should be the sum of the values it gets in individual games

Axiomatizing Fairness: Symmetry

Agents i and j **are interchangeable** if they always contribute the same amount to every coalition of the other agents.

- for all S that contains neither i nor j, $v(S \cup \{i\}) = v(S \cup \{j\})$.

The symmetry axiom states that such agents should receive the same payments.

Axiom (Symmetry)

If i and j are interchangeable, then $x_i = x_j$.

Axiomatizing Fairness: Dummy Player

Agent i is a **dummy player** if the amount that i contributes to any coalition is exactly the amount that i is able to achieve alone.

- for all S such that $i \notin S$: $v(S \cup \{i\}) - v(S) = v(\{i\})$.

The dummy player axiom states that dummy players should receive a payment equal to exactly the amount that they achieve on their own.

Axiom (Dummy player)

If i is a dummy player, then $x_i = v(\{i\})$.

Axiomatizing fairness: Additivity

Consider two different coalitional game theory problems, defined by two different characteristic functions v' and v'', involving the same set of agents.

The **additivity axiom** states that if we re-model the setting as a single game in which each coalition S achieves a payoff of $v'(S) + v''(S)$, the agents' payments in each coalition should be *the sum of the payments* they would have achieved for that coalition under *the two separate games*.

Axiom (Additivity)

If $\vec{x}'$ and $\vec{x}''$ are payment distributions in the game (N, v') and (N, v''), respectively, then $x_i^+ = x_i' + x_i''$ where $\vec{x}^+$is the payment distribution in a game $(N, v' + v'')$.

Shapley Value

Theorem

Given a coalitional game (N, v), there is a **unique payoff division** $\vec{\phi}(N, v)$ that divides the full payoff of the grand coalition and that satisfies the Symmetry, Dummy player and Additivity axioms.

This payoff division is called **Shapley value.**

Lloyd F. Shapley. 1923–. Responsible for the core and Shapley value solution concepts.

Shapley Value

Definition (Shapley value)

Given a coalitional game (N, v), the **Shapley value** of player i is given by

$$\phi_i(N, v) = \frac{1}{N!} \sum_{S \subseteq N \setminus \{i\}} |S|!\,(|N| - |S| - 1)!\,[v(S \cup \{i\}) - v(S)]$$

This captures the "**average marginal contribution**" of agent i, **averaging** over all the **different sequences** according to which the grand coalition could be built up from the empty coalition.

Shapley Value: Example

If they form (12), how much should each get paid?

S	$v(S)$
()	0
(1)	1
(2)	3
(12)	6

$$\phi_1 = \frac{1}{2}\left(v(1) - v(\,) + v(21) - v(2)\right)$$
$$= \frac{1}{2}(1 - 0 + 6 - 3) = 2$$

$$\phi_2 = \frac{1}{2}\left(v(2) - v(\,) + v(12) - v(1)\right)$$
$$= \frac{1}{2}(3 - 0 + 6 - 1) = 4$$

Does Shapley value always exist? Yes.

Classes of Coalition Games

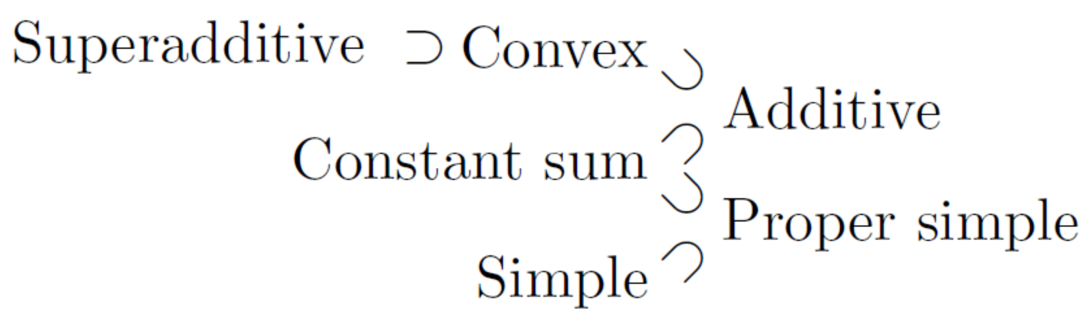
Superadditive game

Additive game

Constant-sum game

Convex game

Simple game



Superadditive Games

Definition (Superadditive game)

A **coalitional game** (N, v) is called **superadditive** if $v(C \cup D) \geq v(C) + v(D)$ for every pair of disjoint coalitions $C, D \subseteq N$.

In superadditive games, two coalitions can always **merge** without losing money (i.e. their members can work **without interference**); hence, we can assume that players form the **grand coalition.**

? Is the icecream game superadditive?

Yes.

Convex Games

An important subclass of superadditive games

Definition (Convex game)

A **coalitional game** (N, v) is termed **convex** if $v(C \cup D) \geq v(C) + v(D) - v(C \cap D)$ for every pair of coalitions $C, D \subseteq N$.

Convexity is a **stronger condition** than superadditivity.

- "a player is more useful when he joins a bigger coalition"

Simple Games

Definition (Simple game)

A **coalitional game** (N, v) is termed **simple** if $v(C) \in \{0,1\}$ for any $C \subseteq N$ and v is **monotone**, i.e., if $v(C) = 1$ and $C \subseteq D$, then $v(D) = 1$.

Model of yes/no voting systems.

A coalition C in a simple game is said to be **winning** if $v(C) = 1$ and **losing** if $v(C) = 0$.

A player i in a simple game is a **veto player** if $v(C) = 0$ for any $C \subseteq N \setminus \{i\}$

- equivalently, by monotonicity, v(N\{i}) = 0.

Traditionally, in simple games an outcome is identified with a payoff vector for N.

Solution Concepts

Cooperative Games

Solution Concepts

What are the **outcomes** that are likely to arise in cooperative games?

Rewards from cooperation need to be divided in a **motivating** way.

Fairness: How well payoffs reflect each agent’s contribution?

Stability: What are the incentives for agents to stay in a coalition structure?

What Is a Good Outcome?

C: $4  M: $3  P: $3 

Characteristic function

$v(\emptyset) = v(\{C\}) = v(\{M\}) = v(\{P\}) = v(\{M, P\}) = 0, v(\{C, M\}) = v(\{C, P\}) = 500, v(\{C, M, P\}) = 750$

How should the players share the ice-cream?

- What about sharing as (200, 200, 350) ?
- The outcome (200, 200, 350) is **not stable** (Charlie and Marcie can get more ice-cream by buying a 500g tub on their own, and splitting it equally)

The Core

Under **what payment** distributions is the **outcome** of a game **stable**?

- As long as each subcoalition earns at least as much as it can make on its own.
- This is the case if and only if the payoff vector is drawn from a set called the core.

Definition (Core)

A payoff vector $\vec{x}$ is in the **core** of a coalitional game (N, v) iff

$$\forall S \subseteq N, \sum_{i \in S} x_i \geq v(S)$$

The **core** of a game is the set of **all stable outcomes**, i.e., outcomes that no coalition wants to deviate from.

- analogue to strong Nash equilibrium (allows deviations by groups of agent)

Ice-Cream Game: Core

C: \$4  M: \$3  P: \$3 

$v(\emptyset) = v(\{C\}) = v(\{M\}) = v(\{P\}) = v(\{M, P\}) = 0, v(\{C, M\}) = v(\{C, P\}) = 500, v(\{C, M, P\}) = 750$

(200, 200, 350) **is not** in the core:

- $v(\{C, M\}) > x_C + x_M$

(250, 250, 250) **is** in the core:

- no subgroup of players can deviate so that each member of the subgroup gets more

(750, 0, 0) **is** also in the core:

- Marcie and Pattie cannot get more on their own! → *fairness*?

Core: Example

S	$v(S)$
(1)	1
(2)	2
(3)	2
(12)	4
(13)	3
(23)	4
(123)	6

$\sum_{i\in S} x_i$	$\sum_{i\in S} x'_i$	$\sum_{i\in S} x''_i$
2	2	1
1	2	3
2	2	2
3	4	3
4	4	3
3	4	5
5	6	6

? In the core, i.e., $\forall S \subseteq N, \sum_{i\in S} x_i \geq v(S)$?

$\vec{x} = (2, 1, 2)$ No

$\vec{x}' = (2, 2, 2)$ Yes

$\vec{x}'' = (1, 2, 3)$ No

Core: Existence

? Is the core **always non-empty**?

No. Core existence guaranteed only for certain special subclasses of games.

Core is also **not unique** (there might be infinitely many payoff divisions in the core).

It can be different in specific subclasses

- **convex games** always have non-empty core (and Shapley value is in the core)
- a **simple game** has a non-empty core iff it has a veto player.

ε-Core

If the core is empty, we may want to find **approximately stable** outcomes

Need to relax the notion of the core:

- core: $x(C) \geq v(C)$ for all $C \subseteq N$
- **ε-core**: $x(C) \geq v(C) - \varepsilon$ for all $C \subseteq N$

Example:

$N = \{1, 2, 3\}$, $v(C) = 1$ if $|C| > 1$, $v(C) = 0$ otherwise

- 1/3-core is non-empty: (1/3, 1/3, 1/3)$\in$ 1/3-core
- ε-core is empty for any ε < 1/3:
  <= $x_i \geq 1/3$ for some $i = 1, 2, 3$, so $x(N \setminus \{i\}) \leq 2/3$, $v(N\{i\}) = 1$

Least Core

If an outcome $\vec{x}$ is in ε-core, the deficit $v(C) - \vec{x}(C)$ of any coalition is at most ε.

We are interested in outcomes that **minimize** the **worst-case deficit**.

Let $\varepsilon^*(G) = \inf\{\varepsilon | \varepsilon\text{−core of } G \text{ is not empty}\}$

- it can be shown that $\varepsilon^*(G)$-core is not empty

Definition: $\varepsilon^*(G)$-core is called the **least core** of G

- $\varepsilon^*(G)$ is called the value of the least core

Example (previous slide): least core = 1/3-core

Further Solution Concepts

Nucleolus

Bargaining set

Kernel

} more complicated stability considerations

Representation Aspects

Cooperative Game Theory

Need for Compact Representations

A **naive representation** of a coalition game is infeasible (**exponential** in the number of agents):

- e.g. for three agents $\{1, 2, 3\}$:

$(1) = 5$ $\quad$ $(1,3) = 10$

$(2) = 5$ $\quad$ $(2,3) = 20$

$(3) = 5$ $\quad$ $(1,2,3) = 25$

$(1,2) = 10$

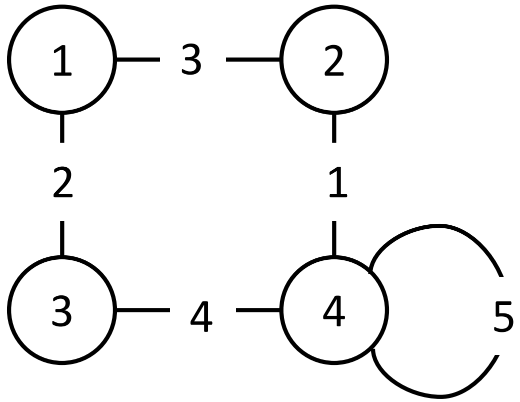
We need a **succinct/compact** representations.

Completeness vs. succinctness

- **Complete**: can represent any game but not necessarily succinct.
- **Succinct:** small-size but incomplete **–** can only represent an (important) subclass.

Induced Subgraph (Weighted Graph) Games

Characteristic function defined by an **undirected weighted graph**.
Value of a coalition $S \subseteq N$: $v(S) = \sum_{\{i,j\} \subseteq S} w_{i,j}$



$v(\{1, 2, 3\}) = 3 + 2 = 5$
$v(\{4\}) = 5$
$v(\{2,4\}) = 1 + 5 = 6$
$v(\{1,3\}) = 2$

Incomplete representation (not all characteristic functions can be represented)

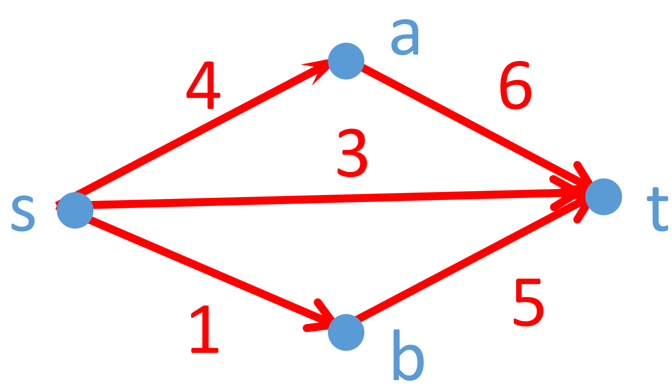
If all edge weights are **non-negative**, the game is **convex** (=> non-empty core.)

Easy to compute the **Shapley value** for a given agent in polynomial time: $sh_i = \frac{1}{2} \sum_{j \neq i} w_{i,j}$

Other Combinatorial Representations

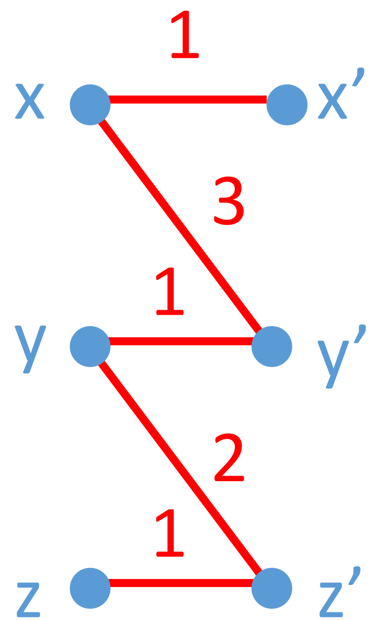
Network flow games

- agents are edges in a network with source s and sink t
- value of a coalition = amount of s–t flow it can carry



Assignment games

- Players are vertices of a bipartite graph
- Value of a coalition = weight of the max-weight induced matching



Matching games

- generalization of assignment games to other than bipartite graphs

Weighted Voting Games

Defined by (1) overall **quota** q and (2) **weight** w_i for each agent i

Coalition is winning if the sum of their weights **exceeds the quota** $v(C) = \begin{cases} 1 & \text{if } \sum_{\{i \in C\}} w_i \geq q \\ 0 & \text{otherwise} \end{cases}$

Example: Simple **majority voting**: $w_i = 1$ and $q = \lceil |N + 1|/2 \rceil$

Succinct (but **incomplete** representation): $\langle q, w_1, \ldots, w_n \rangle$

Coalition Structure Generation

How do we **partition the set of agents** into coalitions to maximize the overall profit?

Finding Optimal Coalition Structure

We assume utilitarian solution, i.e., **maximizing** the **total payoff** of all coalitions.

Trivial if **superadditive** ➔ **grand coalition**.

Otherwise: **search** for the best coalition **structure**.

The Coalition Structure Generation Problem

Example: given 3 agents, the possible **coalitions** are:

$\{a_1\}$ $\{a_2\}$ $\{a_3\}$ $\{a_1,a_2\}$ $\{a_1,a_3\}$ $\{a_2,a_3\}$ $\{a_1,a_2,a_3\}$

The possible **coalition structures** are:

$\{\{a_1\},\{a_2\},\{a_3\}\}$ $\{\{a_1,a_2\},\{a_3\}\}$ $\{\{a_2\},\{a_1,a_3\}\}$ $\{\{a_1\},\{a_2,a_3\}\}$ $\{\{a_1,a_2,a_3\}\}$

The **input** is the characteristic function

$v(\{a_1\}) = 20$

$v(\{a_2\}) = 40$

$v(\{a_3\}) = 30$

$v(\{a_1,a_2\}) = 70$

$v(\{a_1,a_3\}) = 40$

$v(\{a_2,a_3\}) = 65$

$v(\{a_1,a_2,a_3\}) = 95$

What we want as **output** is a coalition structure in which the **sum of values is maximized**

$V(\ \{\{a_1\},\{a_2\},\{a_3\}\}\ ) = 20+40+30 = 90$

$V(\ \{\{a_1,a_2\},\{a_3\}\}\ ) = 70+30 =$ **100**

$V(\ \{\{a_2\},\{a_1,a_3\}\}\ ) = 40+40 = 80$

$V(\ \{\{a_1\},\{a_2,a_3\}\}\ ) = 20+65 = 85$

$V(\ \{\{a_1,a_2,a_3\}\} = 95$

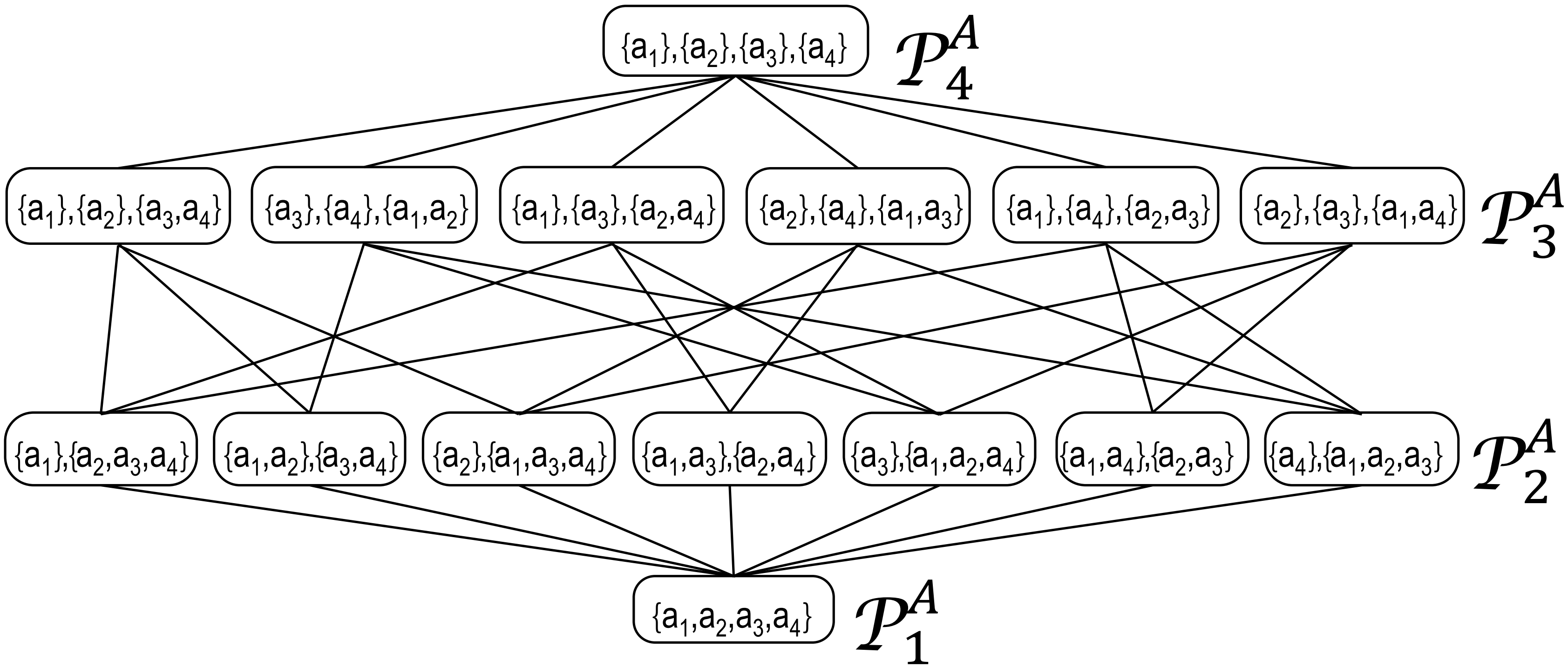
optimal coalition structure

Search Space Representation

1. Coalition structure graph
2. Integer partition graph

Coalition Structure Graph (for 4 agents)

$\mathcal{P}_i^A \subseteq \mathcal{P}^A$ contains all coalition structures that consist of exactly i coalitions

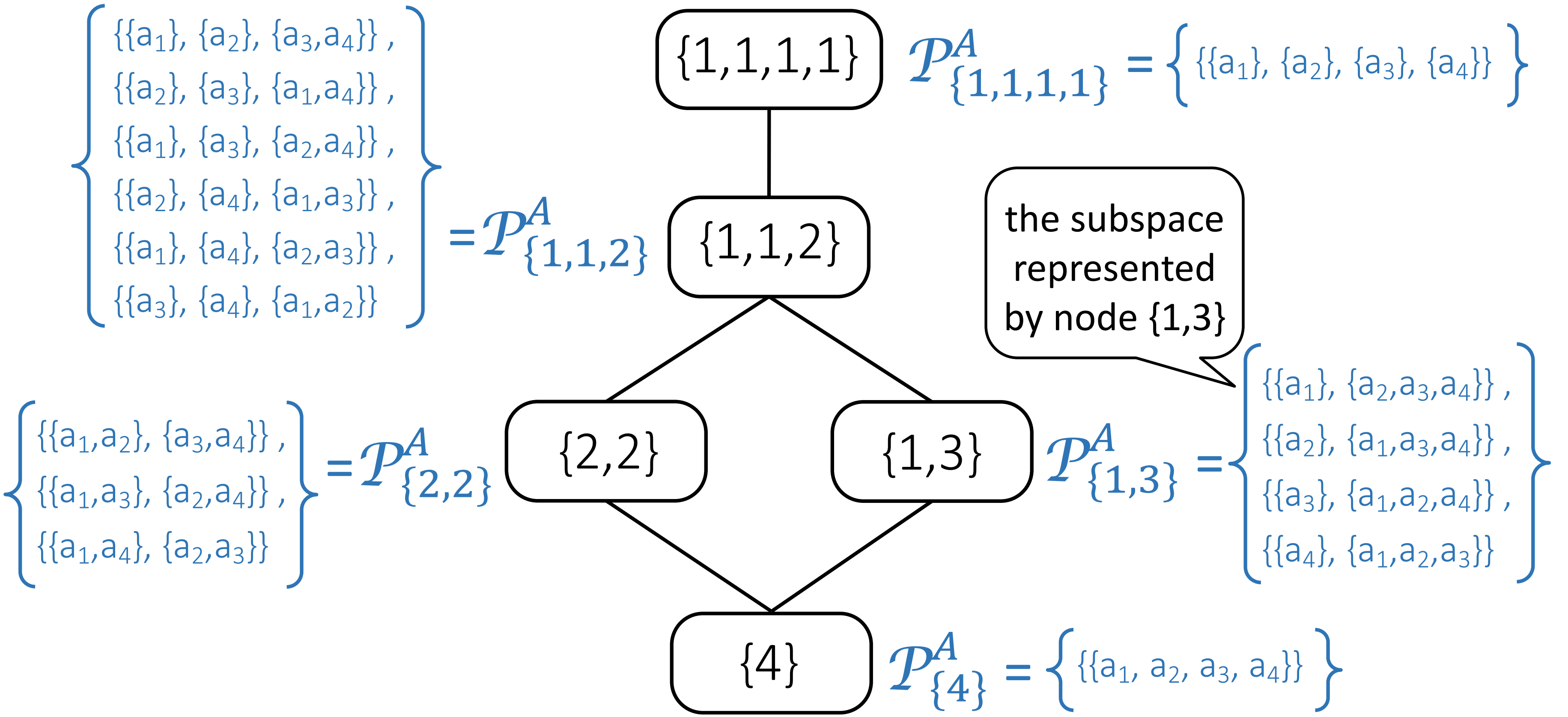


Edge connects two coalition structures iff:

1. they belong to two consecutive levels $\mathcal{P}_i^A$ and $\mathcal{P}_{i-1}^A$
2. the coalition structure in $\mathcal{P}_{i-1}^A$ can be obtained from the one in $\mathcal{P}_i^A$ by merging two coalitions into one

Integer Partition Graph (example of 4 agents)

Every node represents a subspace (coalition sizes match the integers in that node)



Two nodes representing partitions $I, I' \in \mathcal{I}^n$ are connected iff there exists two parts $i, j \in I$ such that $I' = (I \setminus \{i,j\}) \uplus \{i+j\}$

Challenge

Challenge: the number of coalitions for n players:

$$\alpha n^{n/2} \leq B_n \leq n^n$$

for some positive constant α (B_n is a Bell number)

Algorithms for Coalition Formation

Optimal: Dynamic programming

Anytime (suboptimal) algorithms with guaranteed bounds

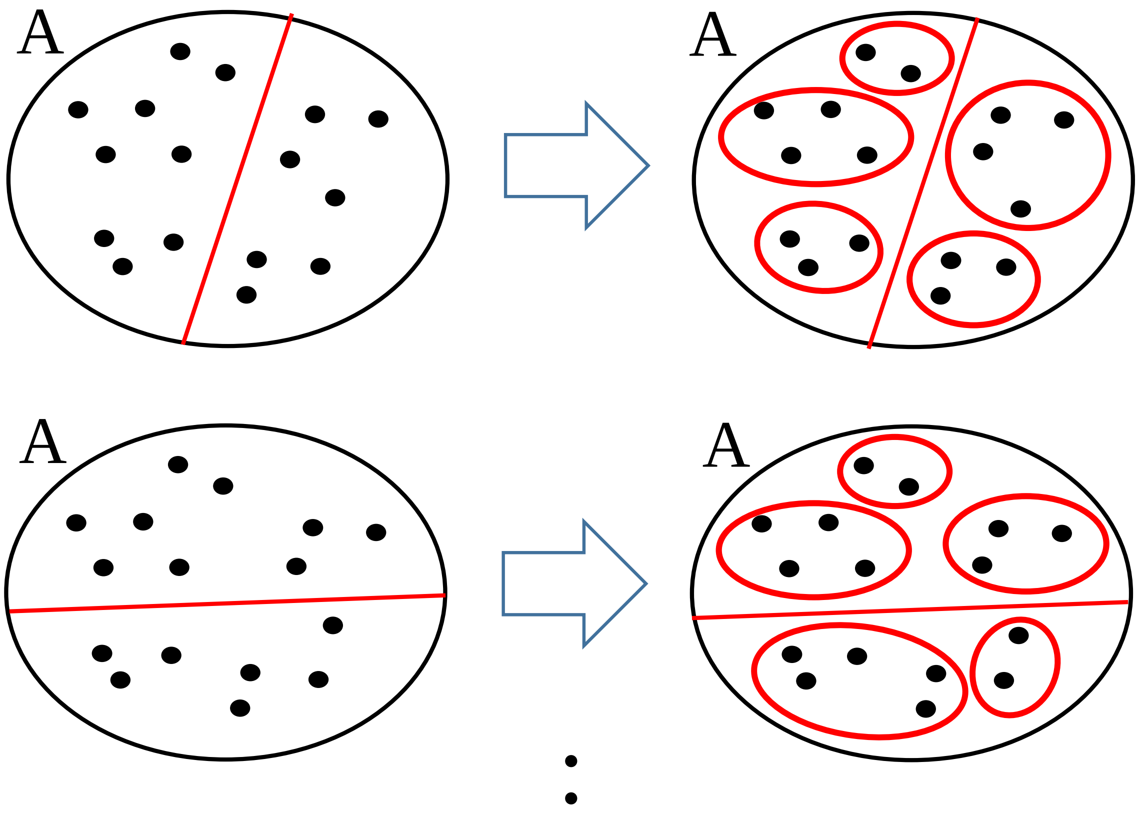
Heuristics algorithms

Algorithms for **compact representation** games

Dynamic Programming (DP) Algorithm

Main observation: To examine all coalition structure CS: $|CS2$, it is sufficient to:

- try the possible ways to split the **set of agents into two sets**, and
- for every half, find the **optimal partition** of that half.

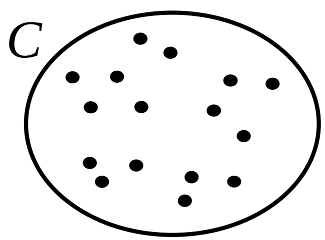


Dynamic Programming (DP) Algorithm

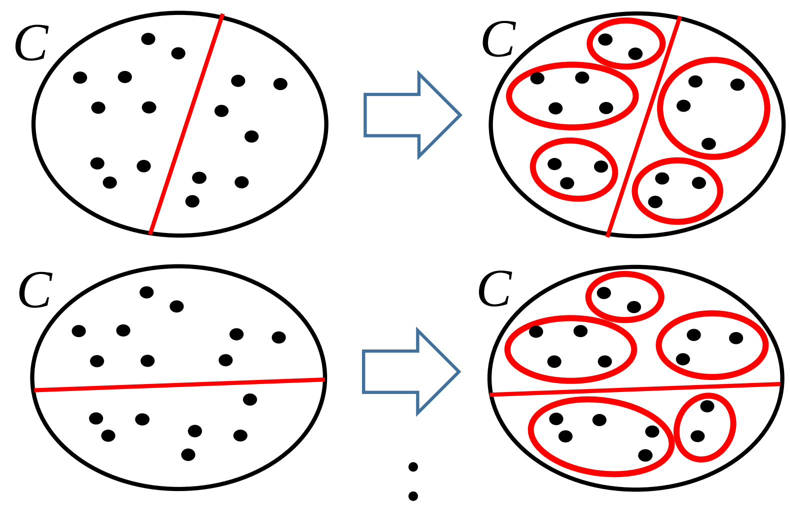
Main theorem: Given a coalition $C \in A$, let $\mathcal{P}^C$ be the set of partitions of C, and let $f(C)$ be the value of an optimal partition of C, i.e., $f(C) = \max_{P \in \mathcal{P}^C} V(P)$. Then,

$$f(C) = \begin{cases} v(C) & \text{if } |C| = 1 \\ \max\left\{v(C), \max_{\{C',C''\} \in \mathcal{P}^C} f(C') + f(C'')\right\} & \text{otherwise} \end{cases}$$

the value of the coalition itself (without partitioning)



the maximum value for all such partitions



Dynamic Programming (DP) Algorithm

Algorithm:

- Iterate over all coalitions $C: |C| = 1$, then over all $C: |C| = 2$, then all $C: |C| = 3$, etc.
- For every coalition, C, compute $f(C)$ using the above equation
- While computing $f(C)$:
  - the algorithm stores in $t(C)$ the best way to split C in two
  - unless it is more beneficial to keep C as it is (i.e., without splitting)
- By the end of this process, $f(A)$ will be computed, which is by definition the value of the optimal coalition structure
- It remains to compute the optimal coalition structure itself, by using $t(A)$

input:

v({1}) = 30
v({2}) = 40
v({3}) = 25
v({4}) = 45
v({1,2}) = 50
v({1,3}) = 60
v({1,4}) = 80
v({2,3}) = 55
v({2,4}) = 70
v({3,4}) = 80
v({1,2,3}) = 90
v({1,2,4}) = 120
v({1,3,4}) = 100
v({2,3,4}) = 115
v({1,2,3,4}) = 140

	coalition	evaluations performed before setting f	t	f
step 1	{1}	v({1})=30	{1}	30
	{2}	v({2})=40	{2}	40
	{3}	v({3})=25	{3}	25
	{4}	v({4})=45	{4}	45
step 2	{1,2}	v({1,2})=50 f({1})+f({2})=70	{1} {2}	70
	{1,3}	v({1,3})=60 f({1})+f({3})=55	{1,3}	60
	{1,4}	v({1,4})=80 f({1})+f({4})=75	{1,4}	80
	{2,3}	v({2,3})=55 f({2})+f({3})=65	{2} {3}	65
	{2,4}	v({2,4})=70 f({2})+f({4})=85	{2} {4}	85
	{3,4}	v({3,4})=80 f({3})+f({4})=70	{3,4}	80
step 3	{1,2,3}	v({1,2,3})=90 f({1})+f({2,3})=95 f({2})+f({1,3})=100 f({3})+f({1,2})=95	{2} {1,3}	100
	{1,2,4}	v({1,2,4})=120 f({1})+f({2,4})=115 f({2})+f({1,4})=110 f({4})+f({1,2})=115	{1,2,4}	120
	{1,3,4}	v({1,3,4})=100 f({1})+f({3,4})=110 f({3})+f({1,4})=105 f({4})+f({1,3})=105	{1} {3,4}	110
	{2,3,4}	v({2,3,4})=115 f({2})+f({3,4})=120 f({3})+f({2,4})=110 f({4})+f({2,3})=110	{2} {3,4}	120
step 4	{1,2,3,4}	v({1,2,3,4})=140 f({1})+f({2,3,4})=150 f({2})+f({1,3,4})=150 f({3})+f({1,2,4})=145 f({4})+f({1,2,3})=145 f({1,2})+f({3,4})=150 f({1,3})+f({2,4})=145 f({1,4})+f({2,3})=145	{1,2} {3,4} step 5	150

Dynamic Programming (DP) Algorithm

Note:

- While DP is guaranteed to find an **optimal coalition structure**, many of its operations were shown to be redundant
- An improved dynamic programming algorithm (called IDP) was developed that avoids all redundant operations

Advantage:

- IDP is the **fastest** algorithm that finds an **optimal** coalition structure in $O(3^n)$

Disadvantage:

- IDP provides **no interim solutions** before completion, meaning that it is not possible to trade computation time for solution quality.

Anytime Algorithms

Anytime algorithm is one whose solution quality improves gradually as computation time increases.

- This way, an **interim** solution is always **available** in case the algorithm run to completion.

Advantages:

- agents might not have time to run the algorithm to completion
- being anytime makes the algorithm more robust against failure.

Categories of algorithms

- algorithms based on Identifying **Subspaces with Worst-Case Guarantees**
- algorithms based on the **integer-partition based** representation.

Conclusions

Cooperative game theory models the formation of **teams of selfish agents.**

- **coalitional game** formalizes the concept
- **core** solution concept address the issue of coalition stability
- **Shapley value** solution concept represents a fair distribution of payments

For practical computation, **compact representations** of coalition games are required.

For non-superadditive games, (optimal) **coalition structure** needs to be found.

Reading:

- **[Weiss]: Chapter 8**
- [Shoham]: 12.1-12.2
- [Vidal]: Chapter 4