Multiagent Systems (BE4M36MAS)

Distributed Constraint Programming

Branislav Bošanský and Michal Pěchouček

Artificial Intelligence Center,
Department of Computer Science,
Faculty of Electrical Engineering,
Czech Technical University in Prague

branislav.bosansky@agents.fel.cvut.cz

December 6, 2016

Previously ... on multi-agent systems.

1. Distributed Constraint Satisfaction Programming

Constraint Network

Definition

A constraint network $\mathcal{N}$ is formally defined as a triple $\langle X, D, C \rangle$, where:

- $X = x_1, \ldots, x_n$ is a set of variables;
- $D = \{D_1, \ldots, D_n\}$ is a set of variable domains, which enumerate all possible values of the corresponding variables; and
- $C = \{C_1, \ldots, C_m\}$ is a set of constraints; where a constraint C_i is defined on a subset of variables $S_i \subseteq X$ which comprise the scope of the constraint ($r_i = |S_i|$ is the arity of constraint i)

Hard vs. Soft Constraints

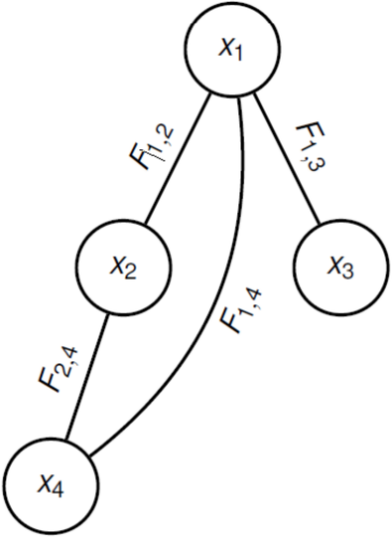
Hard constraint C_i^h is a Boolean predicate P_i that defines valid joint assignments of variables in the scope

$$P_i : D_1^i \times \ldots \times D_{r_i}^i \to \{F, T\}$$

Soft constraint C_i^s is a function F_i that maps every possible joint assignment of all variables in the scope to a real value

$$F_i : D_1^i \times \ldots \times D_{r_i}^i \to \mathbb{R}$$

Binary Constraint Networks



Synchronous Branch-and-Bound

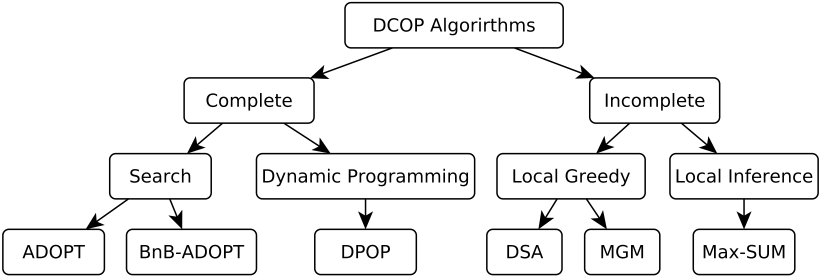
Agents agree on an variable order and repeat:

1. send partial solution up to X_{k-1} to k-th agent.
2. k-th agent generates the next extension to this partial solution whose partial cost (i.e. lower bound) is not greater than the upper bound.
3. if the solution cannot be extended: $k \leftarrow k-1$ (backtrack control to previous agent).
4. if solution can be extended consistently: update lower bound, $k \leftarrow k+1$ (pass control to the next agent)
5. if $k > n$: stop $\rightarrow$ if lower-bound (now the total cost) $<$ upper bound, then upper bound $=$ lower bound; remember best so far assignment
6. if $k < 1$: stop $\rightarrow$ return best so far assignment.

Asynchronous Backtracking Algorithm (ABT) – assumptions

- Agents communicate by sending messages
- An agent can send messages to others, iff it knows their identifiers (directed communication / no broadcasting)
- The delay transmitting a message is finite but random
- For any pair of agents, messages are delivered in the order they were sent
- Agents know the constraints in which they are involved, but not the other constraints
- Each agent owns a single variable (agents = variables)
- Constraints are binary (2 variables involved)

Types of Asynchronous Algorithms for DCOPs



ADOPT: Asynchronous Distributed OPTimization[1]

First *asynchronous complete* algorithm for optimally solving DCOP.

Distributed backtrack search using a "opportunistic" best-first strategy

- agents keep on choosing the best value based on the current available information

Backtrack thresholds used to speed up the search of previously explored solutions.

Termination conditions that check if the bound interval is less than a given valid error bound (0 if optimal).

Theorem (Modi et. al 2005)

For finite DCOPs with binary non-negative constraints, ADOPT is guaranteed to terminate with the globally optimal solution.

[1] Modi et al. "Adopt: asynchronous distributed constraint optimization with quality guarantees" AIJ 2005

ADOPT Overview

Opportunistic best-first search strategy, i.e., each agent keeps on choosing the value with minimum lower bound.

- Lower bounds are more suitable for asynchronous search—a lower bound can be computed without necessarily having accumulated global cost information.

Each agent keeps a lower and upper bound on the cost for the sub-problem below it (given assignments from above) and on the sub-problems for each one of its children.

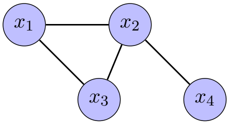
It then tells the children to look for a solution but ignore any partial solution whose cost is above the lower bound because it already knows that it can get that lower cost.

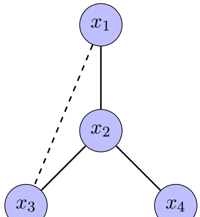
ADOPT: DFS Tree

ADOPT assumes that agents are arranged in a depth-first search (DFS) tree:

- split constraint graph into a spanning tree and backedges
- two constrained nodes must be in the same path to the root by tree links (same branch), i.e., backedges from a node go to the ancestors of the node
- also termed pseudochildren and pseudoparent of a node

Every graph admits a DFS tree. A DFS can be constructed in polynomial time using a distributed algorithm.





ADOPT: Messages

- **value**($\texttt{parent} \rightarrow \texttt{children} \cup \texttt{pseudochildren}, a$): parent informs its descendants that it has taken value a;
- **cost**($\texttt{child} \rightarrow \texttt{parent}$, $\texttt{lower\_bound}$, $\texttt{upper\_bound}$, $\texttt{context}$): a child informs a parent of the best cost of its assignment; attached context to detect obsolescence;
- **threshold**($\texttt{parent} \rightarrow \texttt{children}$, $\texttt{threshold}$): minimum cost of solution in child is at least $\texttt{threshold}$;
- **termination**($\texttt{parent} \rightarrow \texttt{children}$): solution found, terminate

ADOPT: Data Structures

Each agent x_j stores the following data:

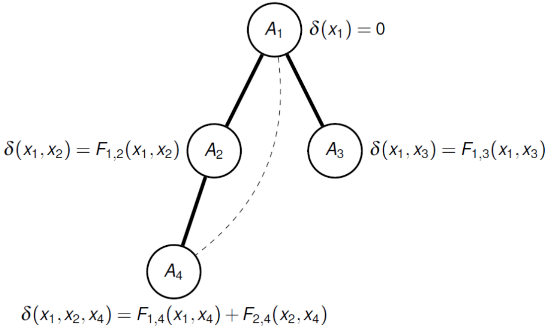
1. **Current context** (agent view): list (x_i, v) of values v of higher-level agents x_i sharing a constrain with x_j
2. **Bounds**: for each value d and each child x_k
   - lower bounds $lb(d, x_k)$
   - upper bounds $ub(d, x_k)$
   - thresholds $t(d, x_k)$
   - contexts $c(d, x_k)$
3. **Threshold**

Stored contexts must be active $\rightarrow$ left-hand side is satisfied in the current context.

If a child's x_k context becomes obsolete, it is reset, i.e., $lb(., x_k), t(., x_k) \leftarrow 0, ub(., x_k) \leftarrow \infty$.

Local Cost Function

The local cost function $\delta(x_i)$ for an agent A_i is the sum of the values of constraints involving only higher-level neighbors in the DFS.



Key Idea: Best First Search

$$OPT_{x_j}(\mathcal{C}) = \min_{d \in d_j}(\delta_j(d) + \sum_{x_k \in \text{children}(x_j)} OPT_{x_k}(\mathcal{C} \cup \{x_j, d\})$$

i.e. the best value for x_j is a value minimizing the sum of x_j's local cost and the lowest cost of children under the context extended with the assignment.

OPT_{x_k} values are incrementally bounded using $[lb_k, ub_k]$ intervals propagated in cost messages.

Bound Computation

Lower bound computation:

- Each agent evaluates for each possible value of its variable: its local cost function with respect to the current context adding all the compatible lower bound messages received from its children
- $LB_j(d) = \delta_j(d) + \sum_{x_k \in \text{children}(x_j)} lb(d, x_k)$
- $LB_j = \min_{d \in d_j} LB_j(d)$

Similarly for upper bound:

- $UB_j(d) = \delta_j(d) + \sum_{x_k \in \text{children}(x_j)} ub(d, x_k)$
- $UB_j = \min_{d \in d_j} UB_j(d)$

ADOPT Steps

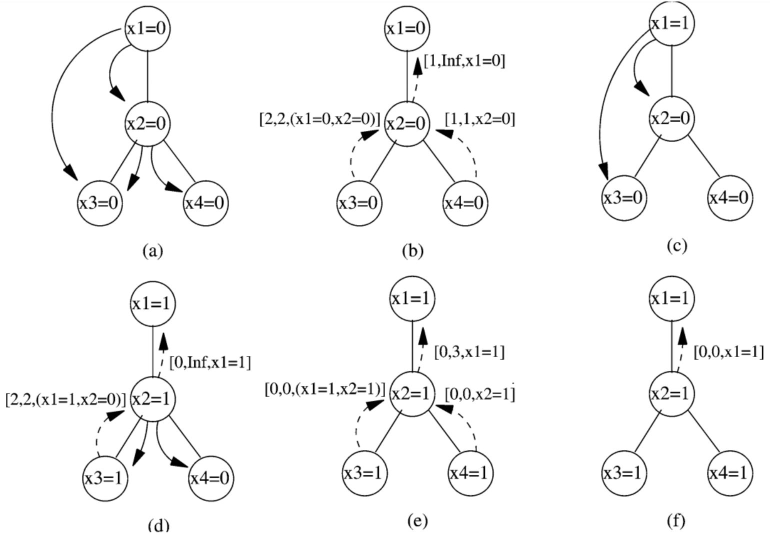
Each time an agent receives a message:

1. process the message
   - the message can invalidate the current context
   - may take a new value minimizing its lower bound
2. Sends value messages to its children and pseudochildren
3. Sends a cost message to its parent
4. Eventually sends threshold messages

The search strategy is based on lower bounds.

- Each agent adopts the value with minimal lower bound.
- Lower/upper bounds only stored for the current context.
- Values abandoned before proven to be suboptimal.

ADOPT Example



Threshold messages

The algorithm can re-visit previously abandoned partial solutions.

However:

- Reconstructing from scratch is inefficient
- Remembering solutions is expensive (in terms of memory)

Detailed cost information lost but stored at parents node in an aggregated form.

Can be used for effective reconstruction of abandoned solutions.

Thresholds

Backtrack thresholds: used to speed up the search of previously explored solutions.

- lower bound previously determined by children
- polynomial space

Send by parents to a child as allowance on solution cost:

- child then heuristically re-subdivides, or allocates, the threshold among its own children.
- can be incorrect: correct for over-estimates over time as cost feedback is (re)received from the children.

Control backtracking to efficiently search:

- Key point: do not change value until LB(current_value) $>$ threshold, i.e., there is a strong reason to believe that current value is not the best (wait until having accumulated enough cost messages)

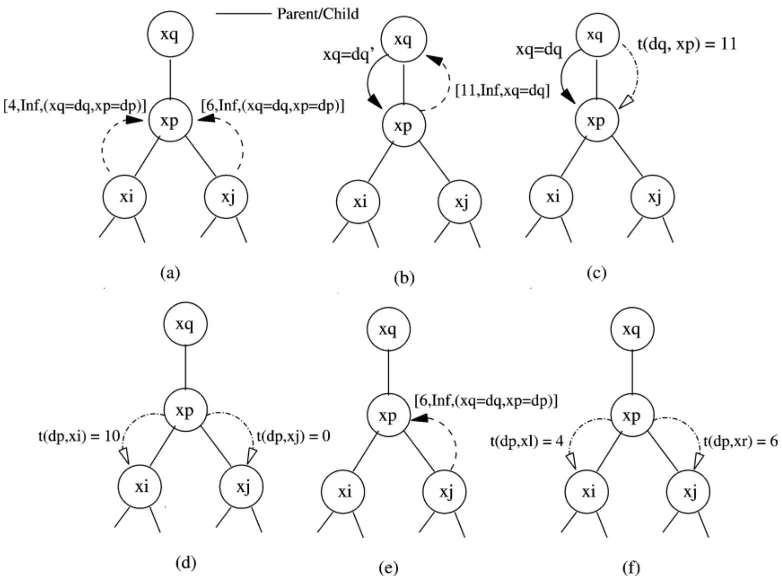
Threshold Re-balancing

Parent distributes the accumulated bound among children and corrects subdivision as feedback is received from children.

ADOPT maintain invariants:

- **allocation invariant**: the threshold on cost for x_j must equal the local cost of choosing d plus the sum of the thresholds allocated to x_j's children.
- **child threshold invariant**: The threshold allocated to child x_k by parent x_j cannot be less than the lower bound or greater than the upper bound reported by x_k to x_j.

Backend Threshold: Example



Approximate Algorithms

Optimality in practical applications often not achievable.

Approximate algorithms:

- sacrifice optimality in favor of computational and communication efficiency
- well-suited for large-scale distributed applications

NOTE: In the following, we assume the maximization version of DCOPs.

Local Search Approaches

Start from a random assignment for all the variables

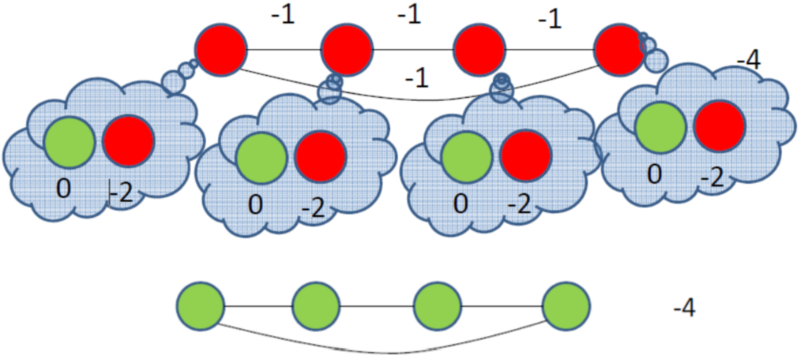
Do local moves if the new assignment improves the value (local gain)

Local: changing the value of a small set of variables (in most case just one)

The search stops when there is no local move that provides a positive gain, i.e., when the process reaches a local maximum.

Local Search Approaches

We need a coordination among agents:



Local Search Approaches

- *Randomize to decide whether an agent is going to act.*
  → DSA-1 algorithm
  → Generates a random number and executes only if it is less than *an activation probability*.

- *Negotiate with neighbors.*
  → MGM-1 algorithm
  → Agents compute and exchange possible gains and only the with maximum (positive) gain executes the action.

FRODO: a FRamework for Open/Distributed Optimization

Framework for experimental evaluation of DCSP/DCOP algorithms.

Input:

- files defining optimization problems to be solved
- configuration files defining the algorithm to be used to solve them

Many implemented algorithms:

- SynchBB, MGM and MGM-2, ADOPT, DSA, DPOP, SDPOP, MPC-Dis(W)CSP4, O-DPOP, AFB, MB-DPOP, Max-Sum, ASO-DPOP, P-DPOP, P^2-DPOP, E[DPOP], Param-DPOP, and $P^{\frac{3}{2}}$-DPOP

Supports various performance metrics:

- numbers and sizes of messages sent
- Non-Concurrent Constraint Checks
- simulated time

`https://sourceforge.net/projects/frodo2/`