1 Game theory [15 pts.]

i) Consider following two player card game. The game begins with a mandatory bet of 1 chip for both players. After the bet, both players obtain 1 card randomly chosen from the deck containing J and 2 Qs. The players know their own card but do not observe the card of the opponent. After the cards are dealt, player 1 decides if he wants to continue playing by betting another chip or end the game by folding. If he decides to bet, player 2 also decides between betting 1 chip or folding. If both players decide to bet, the cards are revealed and the player with higher card wins all the chips currently in game. If they hold cards with equal value, the chips are split evenly between them. If either of players decides to fold, he immediately loses and all the chips currently in game are won by his opponent.

(a) Formalize this scenario as an extensive-form game and draw the complete game tree.

(b) Convert this game to a normal-form representation (create the whole matrix with corresponding labels of columns and rows, compute at least half of the entries of the matrix).

ii) Consider the following game:

	X	Y	Z
A	-3, 1	1, 3	-2, -4
B	3, -1	2, -2	-1, 1
C	2, -1	-1, 1	0, 0

(a) Remove all strictly dominated pure strategies.

(b) Compute a (possibly mixed) Nash equilibrium.

2 Social choice [15 pts.]

$$15 : b > a > d > c$$
$$30 : a > c > b > d$$
$$6 : c > a > b > d$$
$$20 : b > a > c > d$$

i) Formally define a voting rule.

ii) Find a winner according to Plurality voting, Single transferable vote, and Borda's rule for the preference profiles above.

iii) Create an elimination order under which b wins in Pairwise elimination on the preference profile above, if there is no such order explain why.

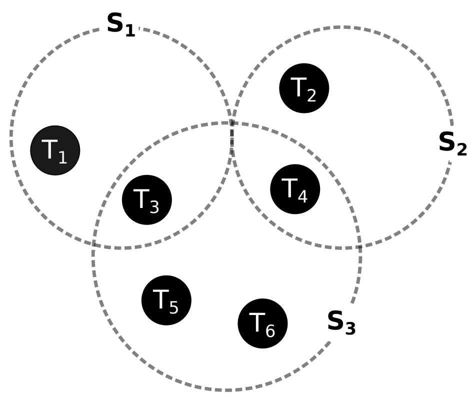
iv) Add an alternative e to the preference profiles above in such a way that b becomes the winner when applying Borda's rule.

v) Define Condorcet winner and Condorcet loser, and find them in the example.

3 Distributed Constraint Optimization [15 pts.]

Consider an airport with the following security system. Inside the main airport corridor, there are specialized sensors capable of detecting whether a weapon is carried by a selected traveller within the reach of the sensor. To detect that a weapon is carried by a traveller, a sensor needs to target the selected traveller (within its detection radius); furthermore, a sensor can only scan one person within its detection range at a time.

Consider the specific situation depicted in the figure below. The individual travellers are denoted as $T_1, \ldots, T_6$ and the range of each senors is depicted as a dashed circle.



The security personnel estimated the prior probability that a traveller carries a weapon as follows:

$$p_1 = 0.03, \quad p_2 = 0.04, \quad p_3 = 0.05, \quad p_4 = 0.06, \quad p_5 = 0.01, \quad p_6 = 0.02$$

The goal is to select travellers for scanning by the three sensors so that the probability of an undetected weapon being carried through is minimized. Assume that only one scan can be performed by each sensor while passengers $T_1, \ldots, T_6$ are within reach.

1. Formalize the problem as a DCOP (i.e. write down variables, domains, soft constraints and agents involved).
2. Find an optimal solution of the problem (by hand – not necessarily running any formal algorithm).
3. Describe the core ideas of the ADOPT algorithm.
4. Name at least one incomplete algorithm for DCOPs. Discuss the advantages and disadvantages of complete versus incomplete algorithms for DCOPs.

4 Auctions [15 pts.]

Consider a second-price, sealed-bid auction. Assume that there are two bidders who have independent, private values v_i which are either 1 or 2. For each bidder, the probabilities of $v_i = 1$ and $v_i = 2$ are each 0.5. Assume that if there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x. We also assume that the value of the object to the seller is 0 and if the bidders do not win the auction, their utility is zero.

(a) Write down a normal-form game for each observation of the first player (Hint: there are two possible observations for this player – either his private value of the item is 1 or 2. Moreover, this player does not know the private value of the opponent, but knows the probability with which the opponents of different private values appear.)

(b) Calculate the expected revenue of the seller assuming both bidders act rationally.

(c) Let's assume that the buyers can coordinate before the auction. What is their optimal strategy in this setting? Justify your answer.

Now let's suppose that the seller sets a reserve price of R with $1 < R < 2$: that is, the object is sold to the highest bidder if her bid is at least R, and the price this bidder pays is the maximum of the second highest bid and R. If no bid is at least R, then the object is not sold, and the seller receives 0 revenue. Suppose that all bidders know R.

(c) What is the seller's expected revenue, if $R = 1.5$?