Statistical Machine Learning (BE4M33SSU)
Seminar 1. Machine Learning Examples

Czech Technical University in Prague

Example 1: ImageNet (visual object classification)

ImageNet Challenge

IMAGENET

- 1,000 object classes (categories).
- Images:
  - 1.2 M train
  - 100k test.

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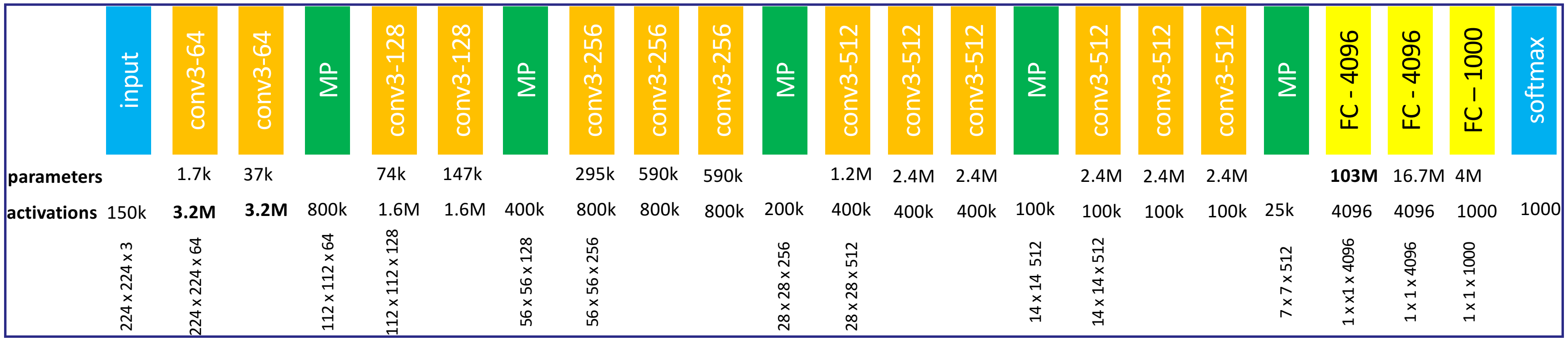
- **Training set:** $\mathcal{T}^m = \{(x^i, y^i) \in \mathcal{X} \times \mathcal{Y} \mid i = 1, \ldots, m\}$, where:
  - $\mathcal{X}$ are images from ImageNet,
  - $\mathcal{Y}$ is a set of output classes (ILSVRC 2012 defines $|\mathcal{Y}| = 1000$ of them).

Example 1: ImageNet (visual object classification)



- **Class of prediction strategies:** VGGNet (Zisserman, et al., 2014), i.e. a convolutional neural network with fixed structure. Note that a convolutional neural network with p layers is a function composition $h_\theta(x) = (f^p_{\theta_p} \circ f^{p-1}_{\theta_{p-1}} \circ \ldots \circ f^1_{\theta_1})(x)$. Its outputs are interpreted as class probabilities.
- **Loss function:** negative log-likelihood of class probabilities (a.k.a. cross entropy)

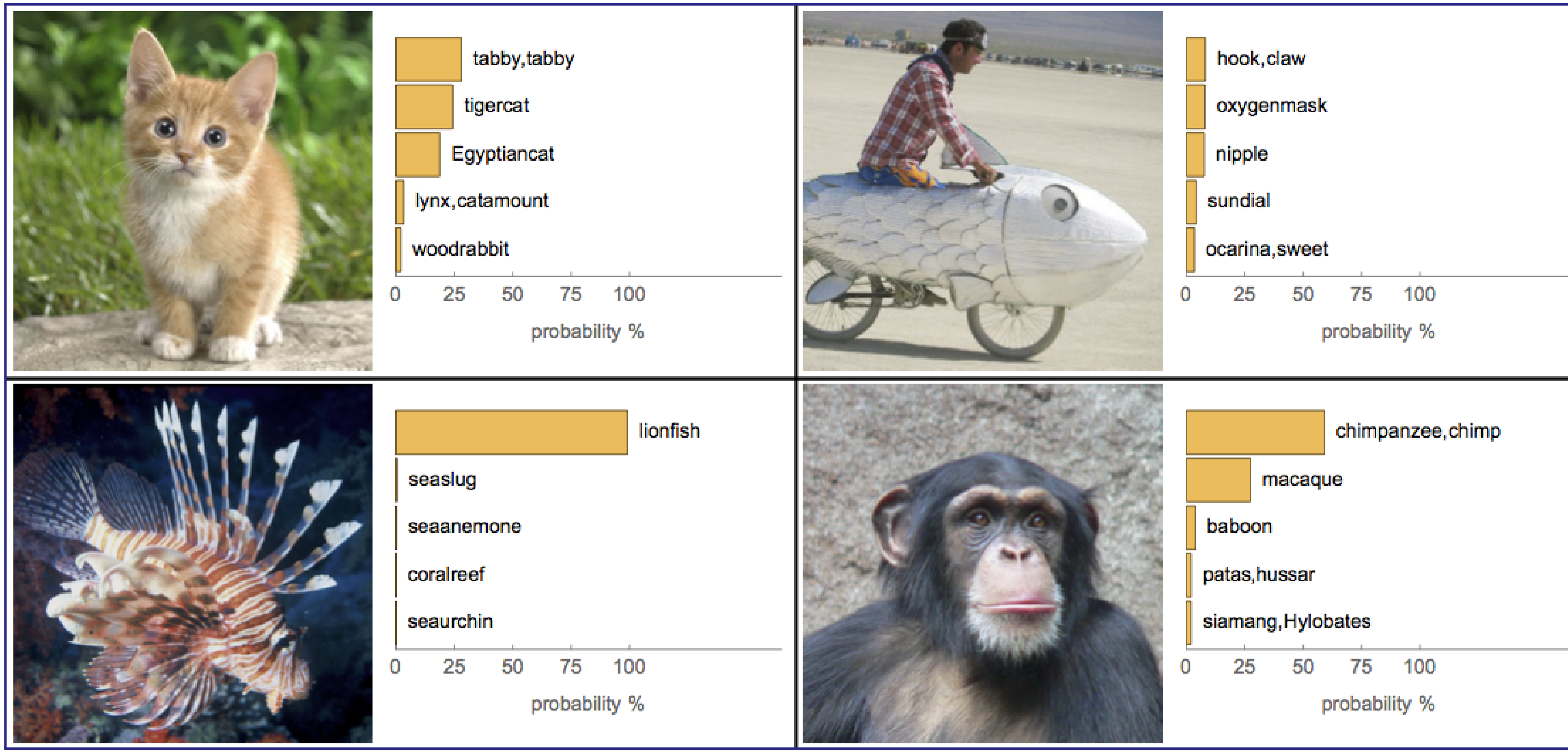
$$\ell(y^i, h(x^i)) = -\sum_{c \in \mathcal{Y}} \mathbb{I}\left\{y^i = c\right\} \log(h_c(x^i)).$$

- **Learning approach:** empirical risk minimisation, gradient descent

$$R_{\mathcal{T}^m}(\theta) = \frac{1}{m} \sum_{i=1}^{m} \ell(y^i, h_\theta(x^i)) \to \min_\theta$$

Example 1: ImageNet (visual object classification)

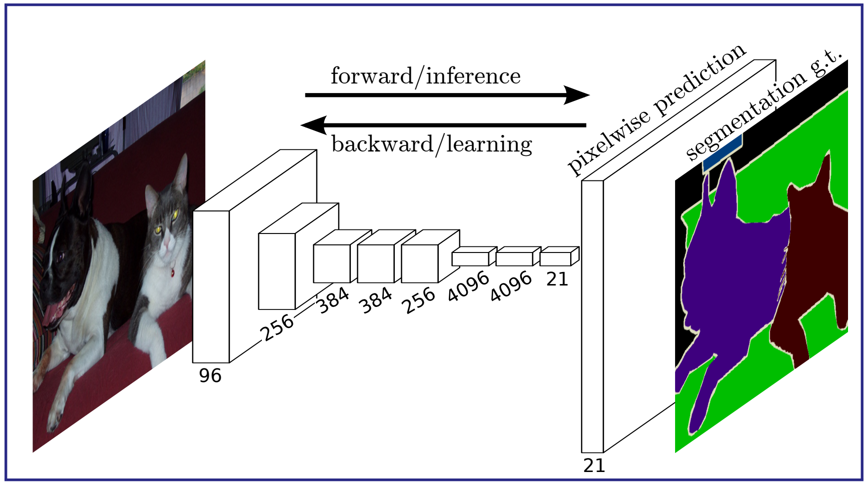
- Results by VGGNet

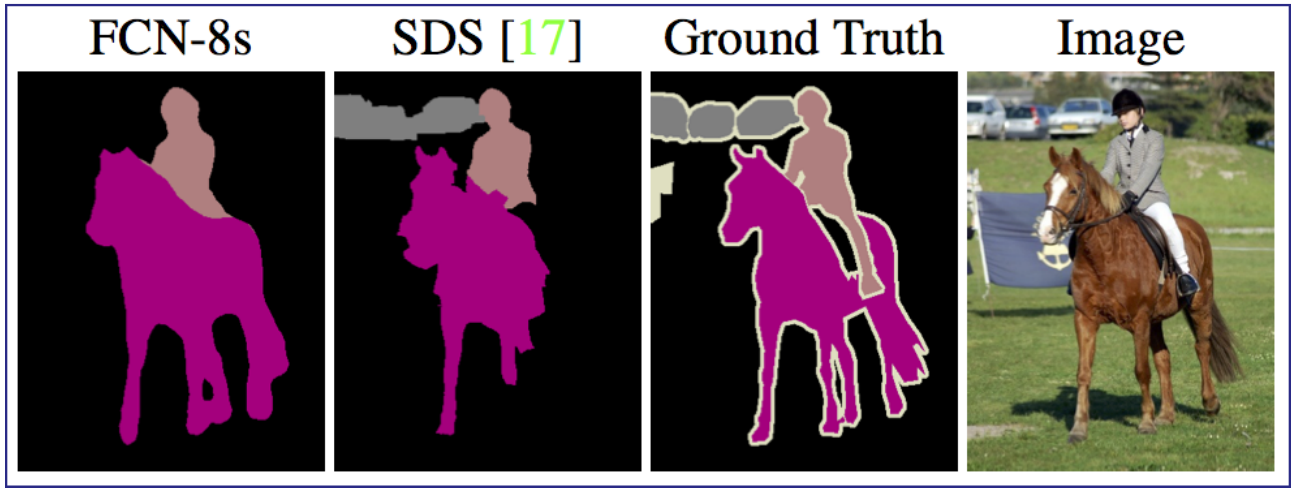


- More details in lectures on **deep learning**

Example 1: ImageNet (visual object classification)

A “spin-off”: a fully convolutional network for semantic segmentation





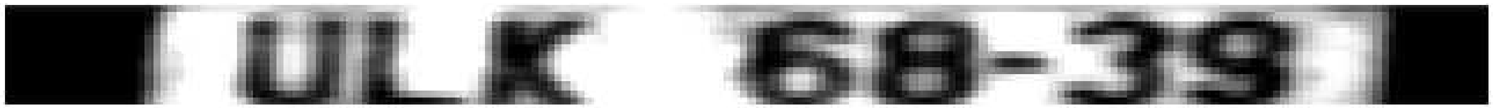
- uses transposed convolutions for up-sampling
- uses “transfer learning” from VGGNet

Example 2: licence plate recognition

Online app estimating the Travel Time for cars in Prague based on the number plate recognition: `https://unicam.camea.cz/Discoverer/TravelTime3/map`

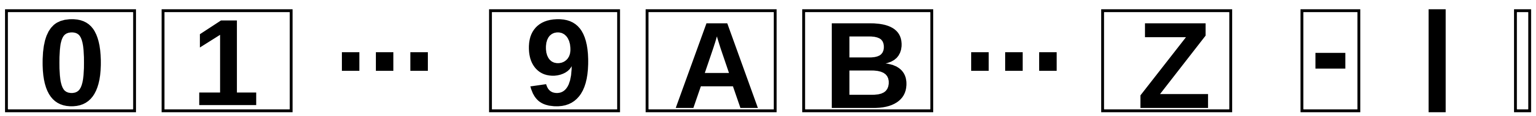
Example 2: licence plate recognition

Input image $x \in \mathcal{X}$ of size $[H \times W]$

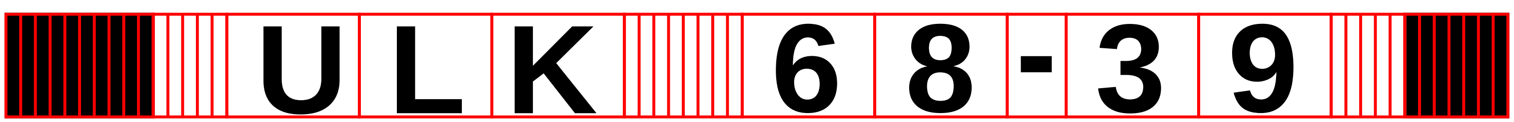


Model of synthetic license plate images

A set of templates $w = (w_a | a \in \mathcal{A})$ for each character from $\mathcal{A}$



A **segmentation** $y = (s_1, \ldots, s_L) \in \mathcal{Y}(x)$, where $s = (a, k)$, $a \in \mathcal{A}$ is a character code and $k \in \{1, \ldots, W\}$ is a character position, together with templates w defines a synthetic image:



An admissible segmentation $y \in \mathcal{Y}(x)$ ensures that the templates are not overlapping and that the synthetic image has the same width as the input image x:

$$k(s_1) = 1\,,\; W = k(s_L) + \omega(s_L) - 1\,,\text{ and } k(s_i) = k(s_{i-1}) + \omega(s_{i-1})\,,\; \forall i > 1$$

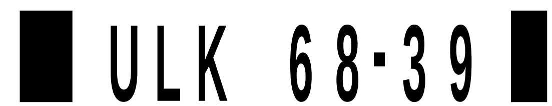
where $\omega \colon \mathcal{A} \to \mathcal{N}$ are widths of the templates.

Example 2: licence plate recognition

- We want a classifier which outputs the segmentations $y \in \mathcal{Y}(x)$ defining a synthetic image most similar (measured by correlation) to the input image x:

$$\hat{y} = h(x;w) = \operatorname*{arg\,max}_{(s_1,\dots,s_L)\in\mathcal{Y}(x)} \underbrace{\sum_{i=1}^{L(y)} \sum_{j=1}^{\omega(a(s_i))} \Big\langle \mathrm{col}\big(x, j+k(s_i)-1\big), \mathrm{col}\big(w_{a(s_i)}, j\big) \Big\rangle}$$

$\Big\langle$ 

, 

$\Big\rangle$

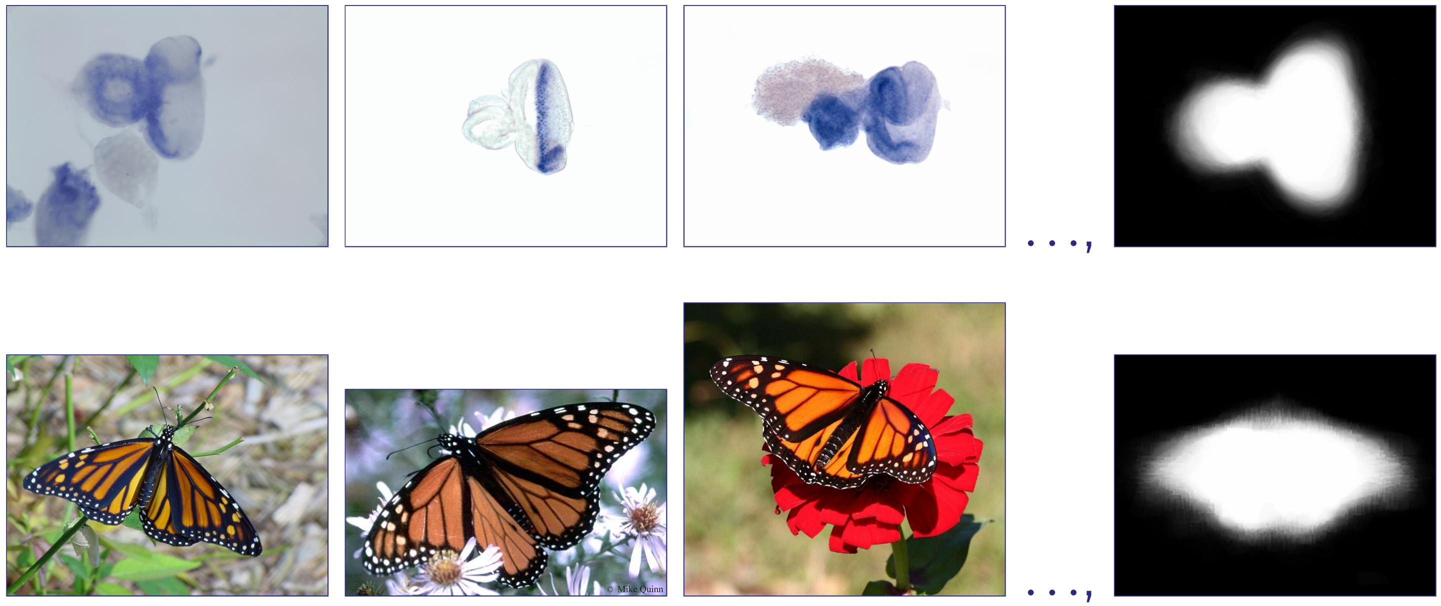
- **Problem:** How to construct the templates $w = \{w_a | a \in \mathcal{A}\}$ so that the classifier $h(x;w)$ predicts a segmentation with small Hamming distance to the correct one ?

- **Solution:** Select the templates w so that the classifier $h(x;w)$ performs well on a training set $\{(x^1, y^1), \dots, (x^m, y^m)\}$ and simultaneously control the over-fitting.

- More details during the lecture on the **Structured Output Support Vector Machines**.

Example 3: Joint segmentation & registration

Given: set of images, each containing an object instance, and a shape model



Task: segment & register each image to the reference frame (shape model)

- image $\boldsymbol{x} = \{x_i \in \mathbb{R}^3 \mid i \in D'\}$, binary segmentation $\boldsymbol{y} = \{y_i \in \{0,1\} \mid i \in D\}$
- shape model $p(\boldsymbol{y}) = \prod_{i \in D} p_i(y_i)$, with binomial distributions $p_i(y_i = 0, 1)$.
- appearance model $p_\theta\big(x_j \mid (T\boldsymbol{y})_j\big)$, $j \in D'$, where
  - T is an affine transformation,
  - $p_{\theta_0}(x_j \mid y'_j = 0)$, $p_{\theta_1}(x_j \mid y'_j = 1)$ are two mixtures of Gaussians.

Example 3: Joint segmentation & registration

- loss function $\ell(\boldsymbol{y}, \boldsymbol{y}') = \sum_{i \in D} \mathbb{I}\{y_i \neq y_i'\}$, i.e. Hamming distance

(1) Segmentation for **known** T and θ: minimise expected Hamming distance between true and estimated segmentation $\Rightarrow$

$$\boldsymbol{y} = h_{T,\theta}(\boldsymbol{x}) = \{h_i(\boldsymbol{x}) \mid i \in D\}$$

$$h_i(\boldsymbol{x}) = \arg\max_{y_i=0,1} p_\theta\big((T^{-1}\boldsymbol{x})_i \mid y_i\big) \cdot p_i(y_i)$$

(2) How to estimate unknown T and θ? See lecture on the EM-Algorithm.