AE0B17MTB – Matlab

Part #13

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Learning how to …

Basics of symbolic math

$$I = \iint_S f(x,y)\,\mathrm{d}S \qquad f(x,y) = x + y$$

$$x \in (0,2),$$

$$y \geq 0 \wedge y \leq 2 - x$$

Higher math

- two different attitudes are distinguished
  - symbolic math
  - numeric math
    - numerical errors

  - possible classification: analytical result in principle enables to get result in infinite number of decimals

- there exist wide range of techniques in Matlab (symbolical as well as numerical)
  - only selected techniques will be covered

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Handle functions – revision

- enables indirect function invoking
- reference to the function is stored in `handle`

```
handle1 = @function_name
handle2 = @(args) function_name
```

- it is quite powerful tool though a bit more complicated
  - enables to invoke a function from locations where it is not visible to Matlab
  - function handle is a data type in Matlab (see `whos`)

```
>> clear,clc;
>> doc function_handle

>> fxy = @(x, y) x^2 + y^2 - 5
>> fxy(2, -2)

>> fcos = @(alpha) cos(alpha)
>> fcos(pi)
```


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Polynomials #1

- representation of polynomials in Matlab

$$P = C_n x^n + C_{n-1} x^{x-1} + \ldots + C_1 x + C_0 = \begin{bmatrix} C_n & C_{n-1} & \ldots & C_1 & C_0 \end{bmatrix}$$

```
>> x = roots([1 0 -1]);
>> x1 = x(1)
>> x2 = x(2)
```

- function `roots` finds roots of a polynomial
- polynomial evaluation: `polyval`

```
>> x = 2
>> p1 = 3*x^5 - 7*x^3 + 1/2*x^2 - 5
>> polyval([3 0 -7 1/2 0 -5], 2)
```

- polynomial multiplication: `conv`

$$A_1 = x - 1$$

$$A_2 = x + 1$$

$$A_1 \cdot A_2 = (x-1)\cdot(x+1) = x^2 - 1$$

```
>> A1 = [1 -1]
>> A2 = [1 1]
>> conv(A1, A2)
% = [1 0 -1]
```

Polynomials #2

- polynomial division: `deconv`

```
>> deconv([1 0 -1], [1 1]) % = [1 -1]
```

$$\frac{x^2-1}{x+1}=\frac{(x-1)\cdot(x+1)}{x+1}=x-1$$

- other polynomial related functions (selection of some):
  - `residue`: residue of ratio of two polynomials
  - `polyfit`: approximation of data with polynomial of order `n`
  - `polyint`: polynomial integration
  - `polyder`: polynomial derivative

```
>> S = [1 1];
>> T = polyint(S) % = [0.5 1 0]
>> U = polyder(T) % = S = [1 1]
>> polyder(U)     % = 1
```

$$\int(x+1)\mathrm{d}x=\frac{1}{2}x^2+x$$

$$\frac{\mathrm{d}\left(\frac{1}{2}x^2+x\right)}{\mathrm{d}x}=x+1$$

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Polynomials #3

- polynomial multiplication

$$P1 = A + Bx \qquad P2 = 4x^2 + 2x - 4$$

```
>> syms A B x
>> P1 = A + B*x;            % entering 1. polynomial
>> P2 = 4*x^2 + 2*x - 4;    % 2. polynomial
>> P0 = P1*P2;              % multiplication
>> P  = expand(P0)          % expansion
```

- note: function `expand` requires Symbolic Math Toolbox

x = ?: f(x) == g(x)

- two functions are given, we want to analytically find out points where these functions are equal to each other

$$f(x)=x^2 \qquad g(x)=x+2 \qquad x=?:\{f(x)=g(x)\}$$

enter

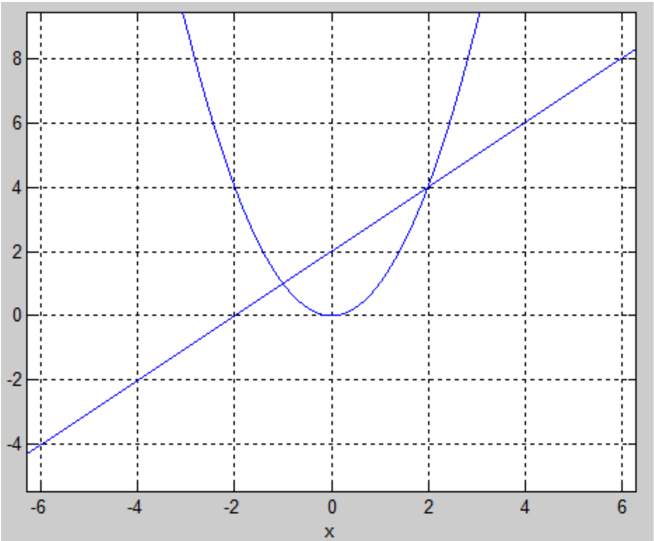
```
>> clear,clc;
>> syms x;
>> f = x^2;
>> g = x + 2;
```

solve

```
>> x0 = solve(f - g) % = 2; -1
```

check

```
>> ezplot(f);
>> hold on;
>> grid on;
>> ezplot(g);
```



Function limit

- find out function limit

$$f(x)=\frac{3x^3}{2\pi x^3+4\pi x} \qquad f(x)=\frac{3}{2\pi}\left(\frac{x^2}{x^2+2}\right) \qquad \lim_{x\to-\infty} f(x)=\lim_{x\to\infty} f(x)\overset{L'H.P.}{=}\frac{3}{2\pi}=0.4775$$

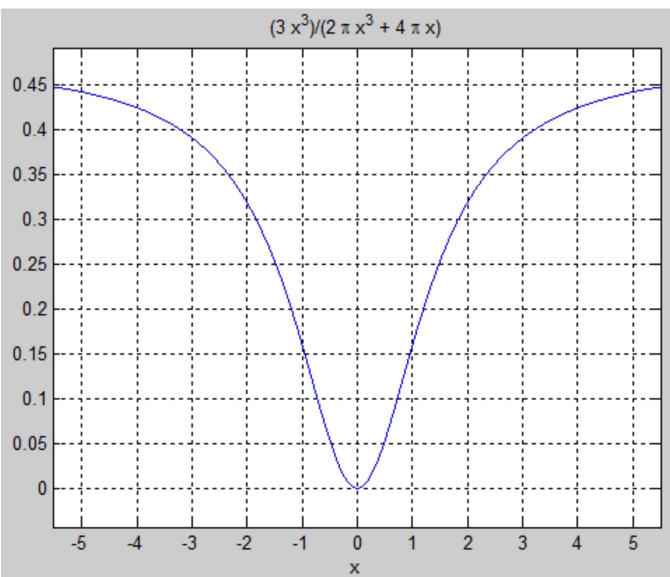
enter

```
>> clear, clc, close all;
>> syms x real;
>> f = 3*x^3/(2*pi*x^3 + 4*pi*x)
```

solve

```
>> lim1 = limit(f, x, -inf)
>> lim2 = limit(f, x, inf)

>> double(lim1) % = 0.4775
>> double(lim2) % = 0.4775
```



check

```
>> figure;
>> ezplot(f);
>> grid on;
```

Function derivative #1

- apply L‘Hospital’s rule to previous function
  - function $f(x)$ contains 3rd power of x; carry out 3rd derivative (of numerator and denominator separately) in x

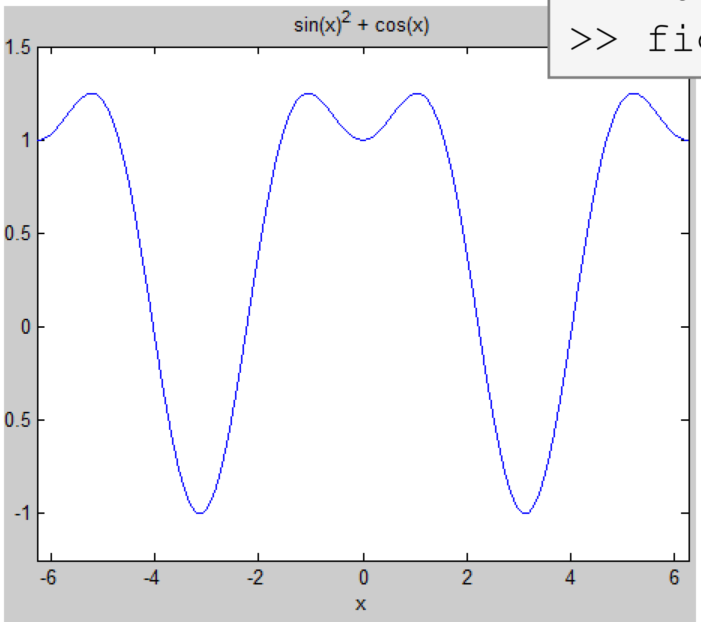
$$f(x) = \frac{3x^3}{2\pi x^3 + 4\pi x} \qquad f_1(x) = 3x^3 \qquad f_2(x) = 2\pi x^3 + 4\pi x$$

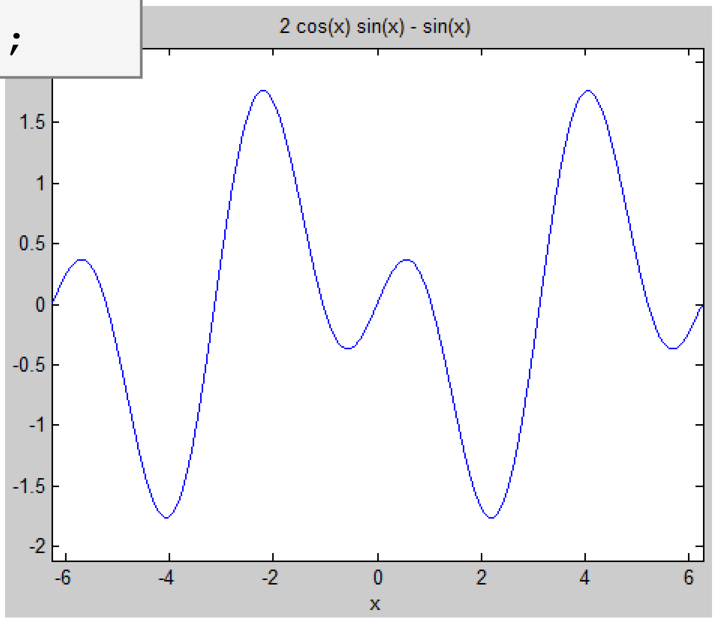
```
>> f1 = 3*x^3;
>> f2 = 2*pi*x^3 + 4*pi*x;
>> A1 = diff(f1,3)
>> A2 = diff(f2,3)
>> double(A1/A2) % = 0.4775
```

Function derivative #2

- carry out derivative of the following function in x $f(x) = \sin^2(x) + \cos(x)$
  - compare results and plot them

```
>> clear, clc;
>> syms x;
>> f = sin(x)^2 + cos(x);
>> figure; ezplot(f);
>> fd = diff(f);
>> figure; ezplot(fd);
```





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Integration #1

- let's first symbolically carry out derivative of function $f(x) = \sin(x) + 2$
- save the second derivative of *f* and call it *g*, compare results
- now integrate function *g* (1×, 2×), do we get the original function *f* ?
  - ignore integration constants

```
>> clear, clc;
>> x = sym('x');

>> f = sin(x) + 2
>> figure; ezplot(f);

>> fd = diff(f)
>> figure; ezplot(fd);

>> fdd = diff(f, 2)
>> figure; ezplot(fdd);
```

```
>> g = fdd;
>> gi = int(g)
>> figure; ezplot(gi);

>> gii = int(gi);
>> err = f - gii

figure;
subplot(1, 2, 1);
ezplot(f);
subplot(1, 2, 2);
ezplot(gii);
```


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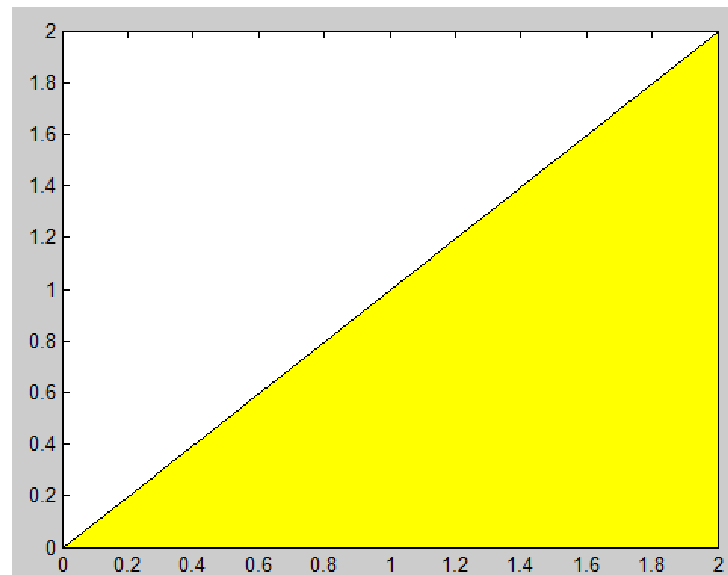
Integration #2

- integral of a function
  - calculate following integral
  - do the calculation manually, plot the function
  - calculate indefinite integral in Matlab
  - calculate definite integral on interval (0, 2), use e.g. function `int`

$$I = \int_0^2 f(x)\,\mathrm{d}x = \int_0^2 x\,\mathrm{d}x = \left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} - 0 = 2$$

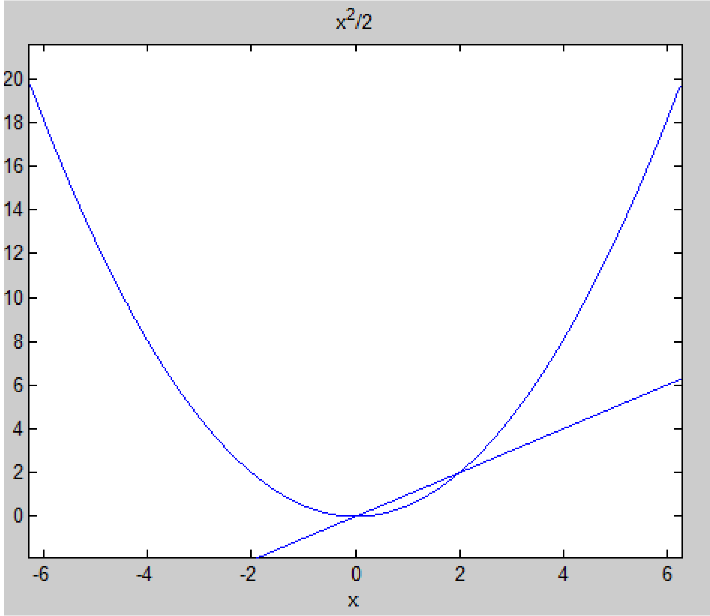
$$I = \frac{2 \cdot 2}{2} = 2$$

```
>> fill([0 2 2], [0 0 2], 'y')
```



Integration #3

- integral of a function




```
>> clear, clc;
>> syms x;
>> f = x;
>> g = int(x);

>> figure;
>> ezplot(f);
>> hold on;
>> ezplot(g);

>> int(f, x, 0, 2)              % = 2
>> polyarea([0 2 2], [0 0 2]) % = 2

% BUT!:
>> f = @(x) x   % function_handle!
>> I = quad(f, 0, 2)            % = 2
```

Numerical integration #1

- numerical approach is used whenever the closed-form (analytical) solution is not known which happens quite often in technical sciences (almost always)

- it is possible to use various numerical integration methods, see literature
- alternatively, Matlab functions can be utilized
  - `quad`, `dblquad`, `triplequad` and others
    - `integral`, `integral2`, `integral3` functions in new versions of Matlab
  - define function to be integrated (write your own function or use *function handle*)

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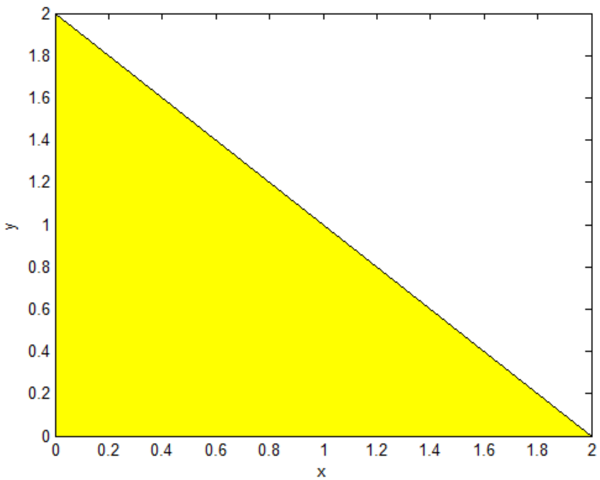
Numerical integration #2

- solve the following integral on the interval

$$x \in (0,2),$$
$$y \geq 0 \wedge y \leq 2 - x$$

$$I = \iint_S f(x,y)\,\mathrm{d}S \qquad f(x,y) = x + y$$



$$I = \int_0^2 \int_0^{y_{\max}} f(x,y)\,\mathrm{d}x\mathrm{d}y = \int_0^2 \int_0^{2-x} (\mathrm{x}+\mathrm{y})\,\mathrm{d}x\mathrm{d}y = \int_0^2 \left(x\left[\mathrm{y}\right]_0^{2\text{-x}} + \left[\frac{y^2}{2}\right]_0^{2\text{-x}} \right)\mathrm{d}x$$

$$= \int_0^2 \left(x(2-x) + \frac{(2-x)^2}{2} \right)\mathrm{d}x = \int_0^2 \left(2x - x^2 + 2 - 2x + \frac{x^2}{2} \right)\mathrm{d}x$$

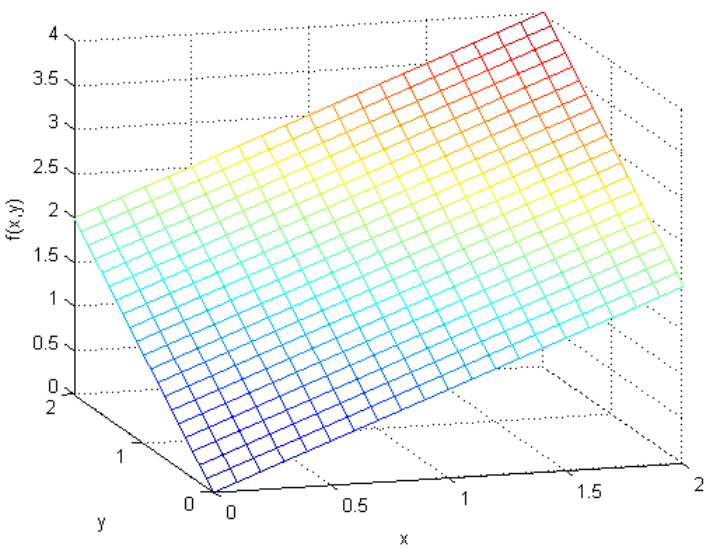
$$= \int_0^2 \left(2 - \frac{x^2}{2} \right)\mathrm{d}x = 2\left[x\right]_0^2 - \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = 4 - 8\cdot\frac{1}{6} = \frac{12-4}{3} = \frac{8}{3} = 2.\overline{666}$$

Numerical integration #3

- solve the following integral on the interval

$$x \in (0,2),$$
$$y \geq 0 \wedge y \leq 2 - x$$

$$I = \iint\limits_S f(x, y)\,\mathrm{d}S \qquad f(x, y) = x + y$$



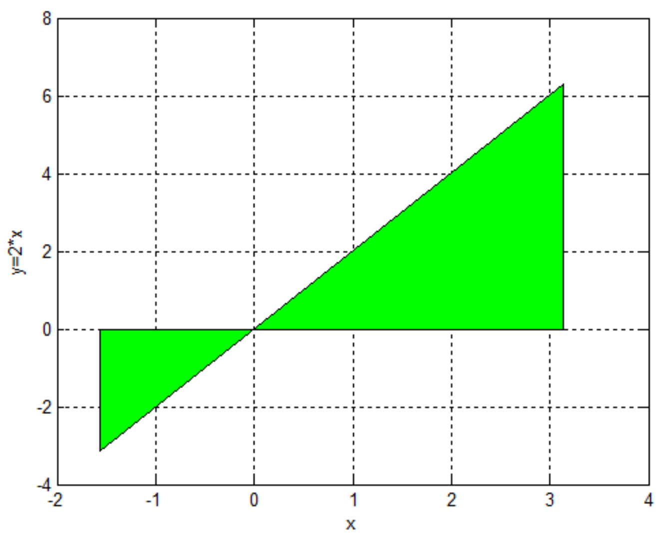

```
>> clear, clc;
% solution:
>> f = @(x, y) x + y
>> ymax = @(x) 2 - x
>> integral2(f, 0, 2, 0, ymax)

% plotting
>> t = 0:1/10:2
>> [x, y] = meshgrid(t);
>> z = x + y;
>> figure('color', 'w');
>> mesh(x, y, z);
```

Numerical integration #4

- it is possible to work with external scripts as well; i.e. having „complex“ expression that we don’t want to process as handle:

$$I = \int_x f(x)\mathrm{d}x = \int_{-\frac{\pi}{2}}^{\pi} 2x\mathrm{d}x = 2\int_{-\frac{\pi}{2}}^{\pi} x\mathrm{d}x = 2\left[\frac{x^2}{2}\right]_{-\frac{\pi}{2}}^{\pi} = \pi^2 - \frac{\pi^2}{4} = \underline{\underline{\frac{3}{4}\pi^2}}$$




```
function fx = myIntFcn(x)
% function to calculate integral:
% int{2*x}

c  = 2;
fx = c*x;
```

```
>> quad(@myIntFcn, -pi/2, pi)
```

Numerical integration #1

- general problem of derivative (it is not possible to approach zero)

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

- various sophisticated numerical methods of various complexity are used

- web pages to solve this problem in a complex way :
  - `http://www.matrixlab-examples.com/derivative.html`

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Closing notes

- in the case there is a lot of symbolic calculations or when approaching Matlab limits, try another mathematical tool (for analytical solution especially Maple, Mathematica)

- nevertheless Matlab is a perfect choice for numerical computing (although both Mathematica's symbolic and numerical kernels are excellent)

Higher math

- polynomials
  - `http://www.matrixlab-examples.com/polynomials.html`
- single and double integration (symbolic)
  - `http://www.matrixlab-examples.com/definite-integrals.html`
- derivative (numerical)
  - analytic input:
    - `http://www.matrixlab-examples.com/derivative.html`
  - numeric input
    - manual derivative

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Discussed functions

`sym, syms`	create symbolic variable(s)	
`roots, polyval, conv, deconv`	polynomial-related functions 1	
`residue, polyfit, polyder, polyint, expand`	polynomial-related functions 2	
`solve`	equations and systems solver	●
`limit, diff, int`	function limit, derivative, function integration	
`ezplot`	symbolic function plotter	
`quad (integral), quad2d (integral2)`	numeric integration	●

Thank you!

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