# Data Structures for Computer Graphics 

## INTRODUCTION

Vlastimil Havran

## Czech Technical University in Prague

## Lectures Overview

1. Rules of the game, lectures overview, review of sorting and searching, intro to computer graphics algorithms, questions to the course.
2. Introduction to hierarchical and regular data structures used in CG
3. Incidence operations used in computer graphics
4. Point based representations and data structures
5. Object based and imaged based representations in 2D and 3D
6. Proximity search and its applications I
7. Proximity search and its applications II
8. Proximity search in high-dimensional spaces, data structures for sampling

## Lectures Overview

9. Ray shooting and its applications I
10. Ray shooting and its applications II
11. Visibility culling for large scale scenes
12. Static and advanced collision detection

13 and 14. Reserved + Easter Holidays

## Data Structures \& Algorithms

- Data structures - inherent part of algorithms
- Large data, many queries

Efficiency is crucial!

- Example:
- Ray tracing $\sim 10 \mathrm{M}$ rays per frame
- Real time (50FPS): 2ns per ray
- Scene with 10 M triangles
- naïve alg. $2.5 \times 10^{15}$ intersections/second


## Organizing Data Gives Advantage

- Improve searching performance
- Naïve search: O(n) time
- With sorting: $\mathrm{O}(\log \mathrm{n})$ !
- In special cases even O(1)


## Basic Data Structures

- 1D arrays
- Multidimensional arrays (1D array X 1D array X ....)
- Linked lists (unidirectional, bidirectional, circular)
- Stacks and Queues
- Hash tables


## Data Structures Properties (Complexity)

- Preprocessing (construction) time $\mathrm{T}_{\mathrm{C}}$
- Space (storage) S
- Running time $\mathrm{T}_{\mathrm{R}}$

Properties

- Asymptotic bounds $(\mathrm{O}, \Omega, \Theta)$ (worst case complexity and average case complexity)
- Instance based measurements (running time in seconds, used memory in Bytes, ...)


## At the Core of Algorithms we have:

## Sorting and Searching

## Recall Sorting and Searching Methods

- Sequential / Binary search
- Quicksort
- Merge sort
- Insertion sort
- Radix sort
- Asymptotic bounds


## Searching in 1D Arrays

1D Sorted array: $\mathrm{O}(\mathrm{N})$ space, $\mathrm{O}(\mathrm{N}$ log N$)$ time

- Sequential search ... in 1D
- Binary search ... in 1D sorted arrays $\mathrm{O}(\log \mathrm{N})$
- Interpolation search (using tangent) $\mathrm{O}(\log \log \mathrm{N})$ for some instances
- Binary-Interpolation search - one step binary search one step interpolation search
- Binary-Sequential search - at first binary search, at the end sequential search


## Searching in (balanced) binary trees

- Balanced/unbalanced trees
- Huffman encoding (digital trees)
- Construction $\mathrm{O}(\mathrm{N} \log \mathrm{N})$
- Space O(N)
- Search O(log N)
- Inorder, preorder, postorder traversal



## Basic Sorting Algorithms

| Algorithm | Method | Best | Average | Worst |
| :---: | :---: | :---: | :---: | :---: |
| Heapsort | Selection | $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ |
| Selection sort | Selection | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ |
| Quicksort | Partitioning | $\mathrm{O}(\mathrm{n}$ log n$)$ | $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ |
| Bucket sort | Distribution | $\mathrm{O}(\mathrm{n})$ | $\mathrm{O}(\mathrm{n})$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ |
| Merge sort | Merging | $\mathrm{O}(\mathrm{n}$ log n$)$ | $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ | $\mathrm{O}(\mathrm{n} \log \mathrm{n})$ |
| Bubble sort | Exchanging | $\mathrm{O}(\mathrm{n})$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ |
| Insertion sort | Insertion | $\mathrm{O}(\mathrm{n})$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ | $\mathrm{O}\left(\mathrm{n}^{2}\right)$ |

Space complexity: O(n)

## Quicksort

- Pick up a pivot Q
- Reorder the data into two subarrays
- left part $\leq \mathrm{Q}$
- right part > Q
- Recurse in both subarrays
- $\mathrm{O}\left(\mathrm{N}^{2}\right)$, but with high probability $\mathrm{O}(\mathrm{N} \log \mathrm{N})$
- Simple \& usually fast - commonly

Relevant to partitioning techniques

## Merge Sort

- From single numbers to tuples, from tuples to 4-tuples (1->2->4->8->16->32->64 ... )
- Auxiliary array needed
- $\mathrm{O}(\mathrm{N})$ space
- $\mathrm{O}(\mathrm{N} \log \mathrm{N})$ running time

Relevant to clustering techniques

## Insertion Sort

- Keep partially sorted array
- Sorting by insertion
- $\mathrm{O}(\mathrm{N})$ space
- $\mathrm{O}\left(\mathrm{N}^{2}\right)$ complexity

Relevant to incremental DS construction

## Other Sorting Techniques for Limited Precision and/or Data Distribution

- Bucket sort - only for unique keys with limited distribution
- Radix sort
- bottom up (from LSB to MSB)
- top-down (distribution sort to many equivalence classes and recurse)

Relevant to rasterization

## Sorting \& Searching in Computer Graphics Algorithms

## Data Entities in Computer Graphics

- Not just numbers !
- Multidimensional data or objects (at least 2D)
- Examples: points, lines, oriented half-lines, disks, oriented hemispheres, triangles, polygons, spheres, general objects in 2D, 3D etc., images, contours, multi-layer data structures, extension to temporal domain
- Consequence: Instead of 1D problem we solve problems in multidimensional spaces!


## Geometric Search Problems

Range search


Point location

Nearest Neighbors


Intersection detection


## Search Problems in Rendering

$Q \times S \rightarrow A, \quad Q$ query domain, $S$ search space, $A$ domain of answers

| Problem | Q | S | A |
| :---: | :---: | :---: | :---: |
| Ray shooting | ray | \{objects $\}$ | point |
| Hidden Surface <br> Removal | \{rays\} | \{objects | \{points |
| Visibility culling | \{rays\} | \{objects\} | \{objects\} |
| Photon maps | point | \{points\} | \{points\} |
| Ray maps | point | \{rays\} | \{rays\} |
| Irradiance <br> caching | point | \{spheres\} | \{spheres\} |

## 90\% of Computer Graphics is about Sorting and Searching...

## Example 1: Ray Shooting

Task: Given a ray, find out the first object intersected.



Input: a scene and a ray
Output: the object $C$

## Example 2: Ray Casting for Image

- Cast ray for each pixel (primary ray)
- Step 1: spatial data structure (XYZ)
- Preprocess
- Trees ~ quick sort
- Grid ~ distribution sort
- Step 2: search for nearest intersection
- Min selection with early termination



## Example 3: Recursive Ray Tracing



## Sample Ray Traced Image



## Example 4: Photon Mapping

Phase I: photon shooting
Phase II: density estimation


## Sample Image by Photon Mapping



## Example 5: Density Estimation in 1D



Density Estimation: given the samples, estimate probability density function $p(x)$

## Example 6: Z-buffer based Rendering

- Rasterize polygons in arbitrary order
- Maintain per pixel depths
- Step 1: rasterization (YX)
- Bucket sort like
- Step 2: per pixel depth ${ }^{\text {'comparison }}(Z)$
- Min selection



## Example 7: Collision Detection

- Incidence among simple/complex objects
- Real-time demands
- Application: games, motion planning for industry (car assembly), etc.


## Example I: Ray Tracing



## Example II: Photon Density Estimation



## Sorting and Searching - Typical usage in CG



## Thank you for your attention!

