#### **Data Structures for Computer Graphics**

#### INTRODUCTION

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#### **Lectures Overview**

- 1. Rules of the game, lectures overview, review of sorting and searching, intro to computer graphics algorithms, questions to the course.
- 2. Introduction to hierarchical and regular data structures used in CG
- 3. Incidence operations used in computer graphics
- 4. Point based representations and data structures
- 5. Object based and imaged based representations in 2D and 3D
- 6. Proximity search and its applications I
- 7. Proximity search and its applications II
- 8. Proximity search in high-dimensional spaces, data structures for sampling

#### **Lectures Overview**

9. Ray shooting and its applications I
10. Ray shooting and its applications II
11. Visibility culling for large scale scenes
12. Static and advanced collision detection
13 and 14. Reserved + Easter Holidays

#### **Data Structures & Algorithms**

- Data structures inherent part of algorithms
- Large data, many queries
   Efficiency is crucial!
- Example:
  - Ray tracing ~10M rays per frame
  - Real time (50FPS): 2ns per ray
  - Scene with 10M triangles
    - naïve alg. 2.5x10<sup>15</sup> intersections/second

#### **Organizing Data Gives Advantage**

- Improve searching performance
  - Naïve search: O(n) time
  - With sorting: O(log n)!
  - In special cases even O(1)

#### **Basic Data Structures**

- 1D arrays
- Multidimensional arrays
   (1D array X 1D array X ....)
- Linked lists (unidirectional, bidirectional, circular)
- Stacks and Queues
- Hash tables

#### **Data Structures Properties (Complexity)**

- Preprocessing (construction) time T<sub>C</sub>
- Space (storage) S
- Running time T<sub>R</sub>
- **Properties**
- Asymptotic bounds  $(O, \Omega, \Theta)$

(worst case complexity and average case complexity)

 Instance based measurements (running time in seconds, used memory in Bytes, ...)

# At the Core of Algorithms we have:

# Sorting and Searching

#### **Recall Sorting and Searching Methods**

- Sequential / Binary search
- Quicksort
- Merge sort
- Insertion sort
- Radix sort
- Asymptotic bounds

#### **Searching in 1D Arrays**

1D Sorted array: O(N) space, O(N log N) time

- Sequential search ... in 1D
- Binary search ... in 1D sorted arrays O(log N)
- Interpolation search (using tangent) O(log log N) for some instances
- Binary-Interpolation search one step binary search one step interpolation search
- Binary-Sequential search at first binary search, at the end sequential search

#### Searching in (balanced) binary trees

- Balanced/unbalanced trees
- Huffman encoding (digital trees)
- Construction O(N log N)
- Space O(N)
- Search O(log N)
- Inorder, preorder, postorder traversal



#### **Basic Sorting Algorithms**

| Algorithm      | Method       | Best               | Average            | Worst              |
|----------------|--------------|--------------------|--------------------|--------------------|
| Heapsort       | Selection    | O(n log n)         | O(n log n)         | O(n log n)         |
| Selection sort | Selection    | O(n <sup>2</sup> ) | O(n²)              | O(n²)              |
| Quicksort      | Partitioning | O(n log n)         | O(n log n)         | O(n²)              |
| Bucket sort    | Distribution | O(n)               | O(n)               | O(n²)              |
| Merge sort     | Merging      | O(n log n)         | O(n log n)         | O(n log n)         |
| Bubble sort    | Exchanging   | O(n)               | O(n <sup>2</sup> ) | O(n²)              |
| Insertion sort | Insertion    | O(n)               | O(n <sup>2</sup> ) | O(n <sup>2</sup> ) |

Space complexity: O(n)

#### Quicksort

- Pick up a pivot Q
- Reorder the data into two subarrays
  - left part  $\leq Q$
  - right part > Q
- Recurse in both subarrays
- O(N<sup>2</sup>), but with high probability O(N log N)
- Simple & usually fast commonly

Relevant to partitioning techniques

#### Merge Sort

- From single numbers to tuples, from tuples to 4-tuples (1->2->4->8->16->32->64 ... )
- Auxiliary array needed
- O(N) space
- O(N log N) running time

Relevant to clustering techniques

#### **Insertion Sort**

- Keep partially sorted array
- Sorting by insertion
- O(N) space
- O(N<sup>2</sup>) complexity

Relevant to incremental DS construction

### Other Sorting Techniques for Limited Precision and/or Data Distribution

- Bucket sort only for unique keys with limited distribution
- Radix sort
  - bottom up (from LSB to MSB)
  - top-down (distribution sort to many equivalence classes and recurse)

Relevant to rasterization

#### Sorting & Searching in Computer Graphics Algorithms

#### **Data Entities in Computer Graphics**

- Not just numbers !
- Multidimensional data or objects (at least 2D)
- Examples: points, lines, oriented half-lines, disks, oriented hemispheres, triangles, polygons, spheres, general objects in 2D, 3D etc., images, contours, multi-layer data structures, extension to temporal domain
- **Consequence:** Instead of 1D problem we solve problems in multidimensional spaces!

#### **Geometric Search Problems**



**Nearest Neighbors** 



Intersection detection



#### **Search Problems in Rendering**

 $Q \ge S \rightarrow A$ , Q query domain, S search space, A domain of answers

| Problem                   | Q      | S         | А         |
|---------------------------|--------|-----------|-----------|
| Ray shooting              | ray    | {objects} | point     |
| Hidden Surface<br>Removal | {rays} | {objects} | {points}  |
| Visibility culling        | {rays} | {objects} | {objects} |
| Photon maps               | point  | {points}  | {points}  |
| Ray maps                  | point  | {rays}    | {rays}    |
| Irradiance<br>caching     | point  | {spheres} | {spheres} |

# 90% of Computer Graphics is about Sorting and Searching...

#### **Example 1: Ray Shooting**

**Task:** Given a ray, find out the first object intersected.



Input: a *scene* and a *ray* 

Output: the *object* C

#### **Example 2: Ray Casting for Image**

- Cast ray for each pixel (primary ray)
- Step 1: spatial data structure (XYZ)
  - Preprocess
  - Trees ~ quick sort
  - Grid ~ distribution sort
- Step 2: search for nearest intersection
  - Min selection with early termination



#### **Example 3: Recursive Ray Tracing**



#### **Sample Ray Traced Image**



#### **Example 4: Photon Mapping**

Phase I: photon shooting

Phase II: density estimation



#### Sample Image by Photon Mapping



#### **Example 5: Density Estimation in 1D**



### **Density Estimation**: given the samples, estimate probability density function p(x)

#### **Example 6: Z-buffer based Rendering**

- Rasterize polygons in arbitrary order
- Maintain per pixel depths
- Step 1: rasterization (YX)
  - Bucket sort like
- Step 2: per pixel depth<sup>1</sup> comparison (Z)
  - Min selection



#### **Example 7: Collision Detection**

- Incidence among simple/complex objects
- Real-time demands
- Application: games, motion planning for industry (car assembly), etc.

#### **Example I: Ray Tracing**



#### **Example II: Photon Density Estimation**



## Sorting and Searching – Typical usage in CG



#### Thank you for your attention!