# Artificial Intelligence in Robotics

## Lecture 13: Patrolling

**Viliam Lisý**

Artificial Intelligence Center
Department of Computer Science, Faculty of Electrical Eng.
Czech Technical University in Prague

# Mathematical programming

LP

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^{\mathrm{T}}\mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \\ \text{and} & \mathbf{x} \geq \mathbf{0} \end{array}$$

MILP

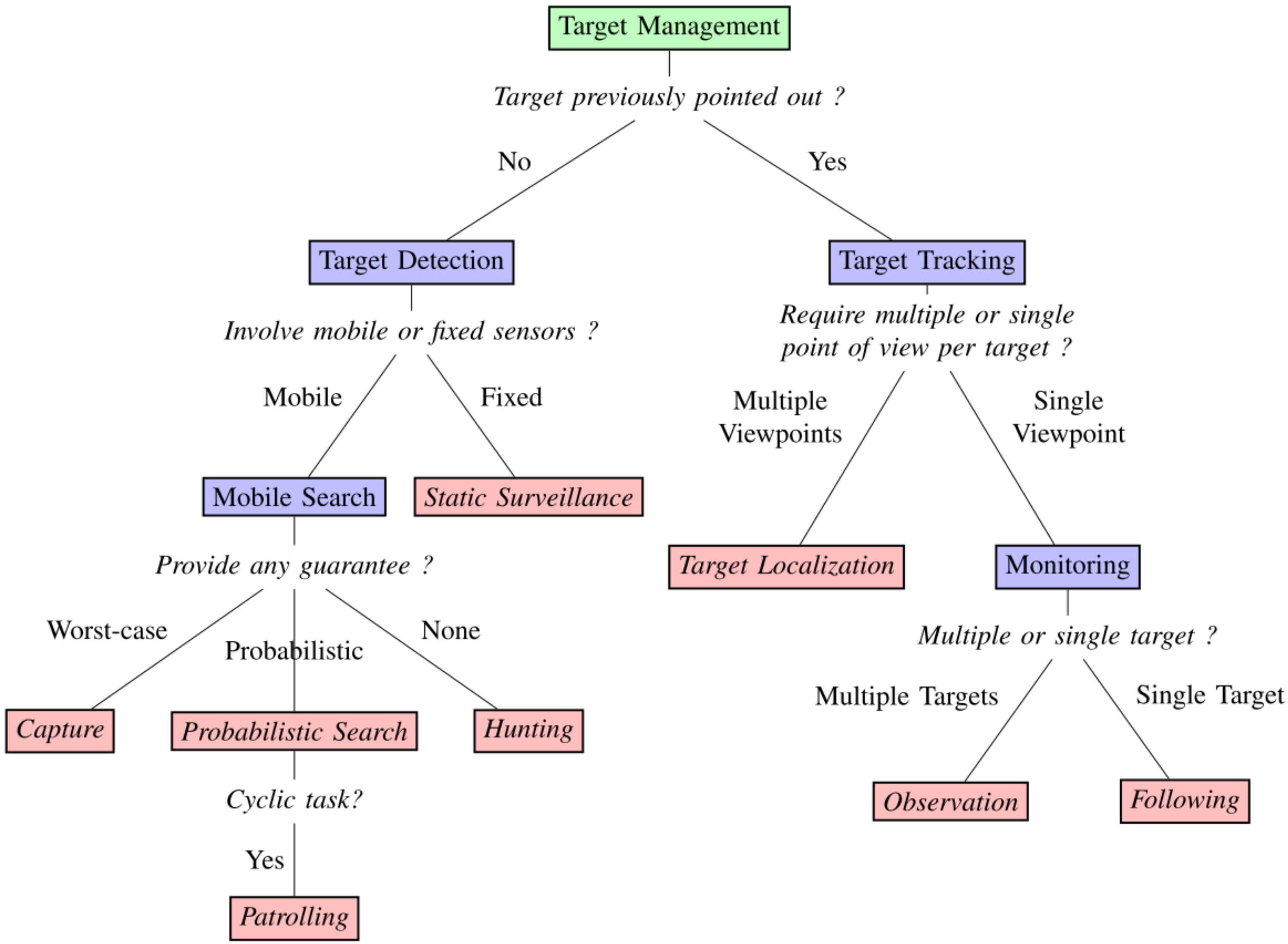
- Some of the variables are integer
- Objective and constraints are still linear

Convex program

- Optimize a convex function over a convex set

Non-convex program

# Task Taxonomy



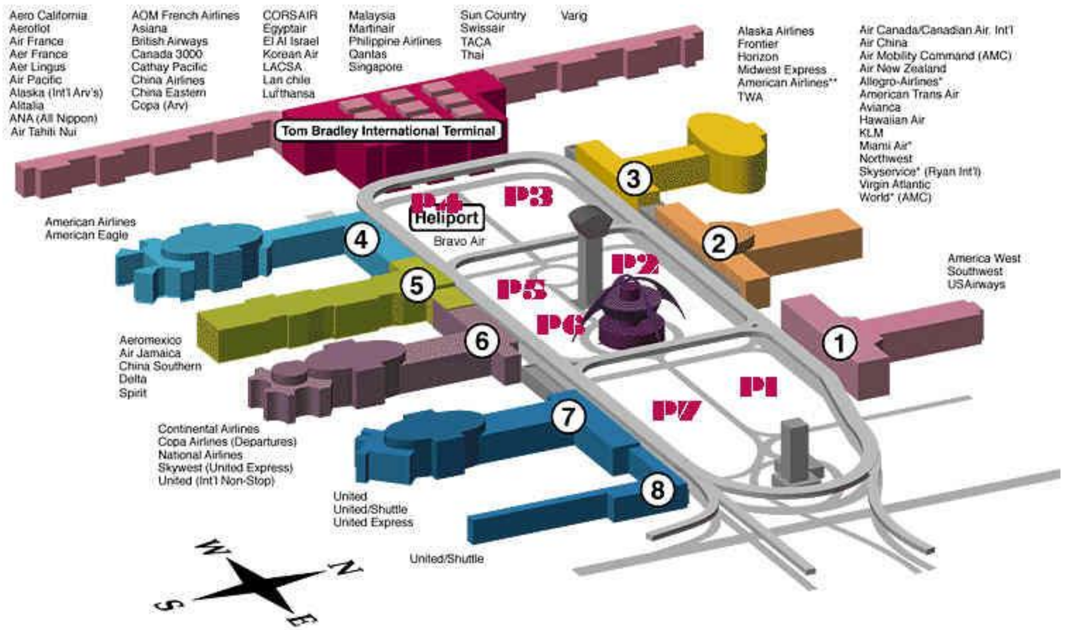

Robin, C., & Lacroix, S. (2016). Multi-robot target detection and tracking: taxonomy and survey. Autonomous Robots, 40(4), 729–760.

# Resource allocation games

Developed by team of prof. M. Tambe at USC (2008-now)

In daily use by various organizations and security agencies

# Resource allocation games






| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | -15 |
| Unprotected | 10 | 11 | 9 | 15 | 11 | 15 | 14 | 6 | -14 |
| Protected | 5 | 4 | 5 | 7 | 6 | 5 | 7 | 3 | -11 |
| Optimal strategy | 0 | 0.14 | 0 | 0.62 | 0.2 | 0.49 | 0.56 | 0 | -10 |

# Resource allocation games

Set of targets: $T = t_1, \dots, t_n$

Limited (homogeneous) security resources $r \in \mathbb{N}$

- Each resource can fully protect (cover) a single target

The attacker attacks a single target

Attacker's utility for covered/uncovered attack: $U_a^c(t) < U_a^u(t)$

Defender's utility for covered/uncovered attack: $U_d^c(t) > U_d^u(t)$

# Stackelberg equilibrium

the leader $(l)$ – publicly commits to a strategy

the follower $(f)$ – plays a best response to leader

$$\arg \max_{\sigma_l \in \Delta(A_l);\, \sigma_f \in BR_f(\sigma_l)} r_l(\sigma_l, \sigma_f)$$

Example

| | L | R |
|---|---|---|
| U | (4,2) | (6,1) |
| D | (3,1) | (5,2) |

Why?

- The defender needs to commit in practice (laws, regulations, etc.)
- It may lead to better expected utility

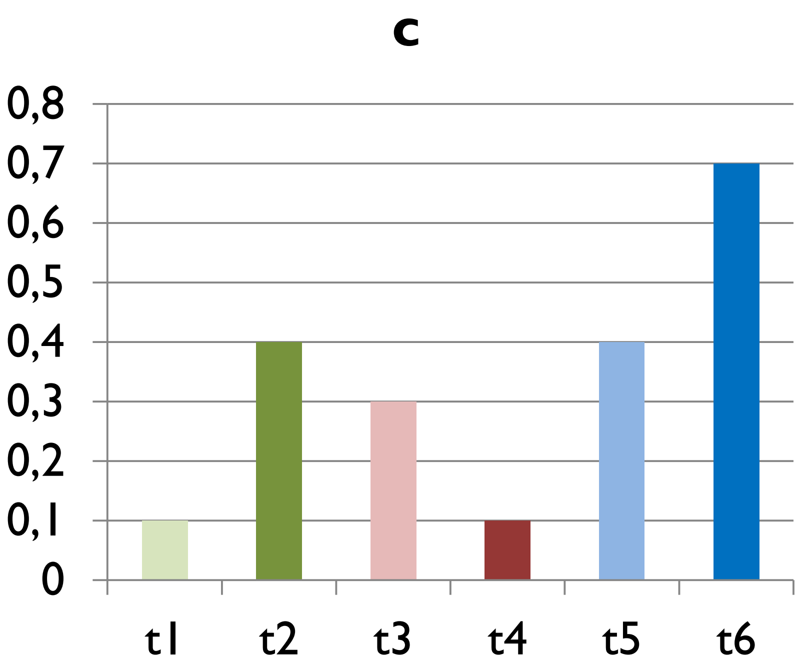
# Solving resource allocation games

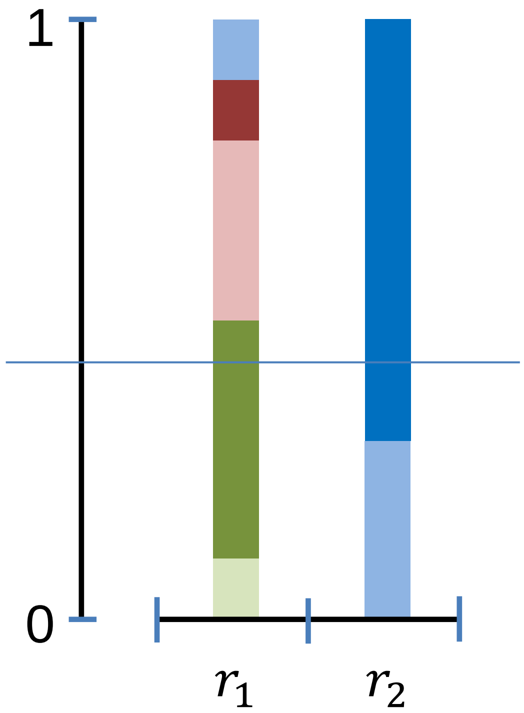
Kiekintveld, et al.: Computing Optimal Randomized Resource Allocations for Massive Security Games, AAMAS 2009

Only coverage vector $c_t$ matters, $Z$ is a sufficiently large number

$$
\begin{aligned}
\max \quad & d & \\
a_t \in \quad & \{0,1\} & \forall t \in T \\
\sum_{t \in T} a_t = \quad & 1 & \\
c_t \in \quad & [0,1] & \forall t \in T \\
\sum_{t \in T} c_t \leq \quad & m & \\
d - U_\Theta(t,C) \leq \quad & (1 - a_t) \cdot Z & \forall t \in T \\
0 \leq k - U_\Psi(t,C) \leq \quad & (1 - a_t) \cdot Z & \forall t \in T
\end{aligned}
$$

# Sampling the coverage vector



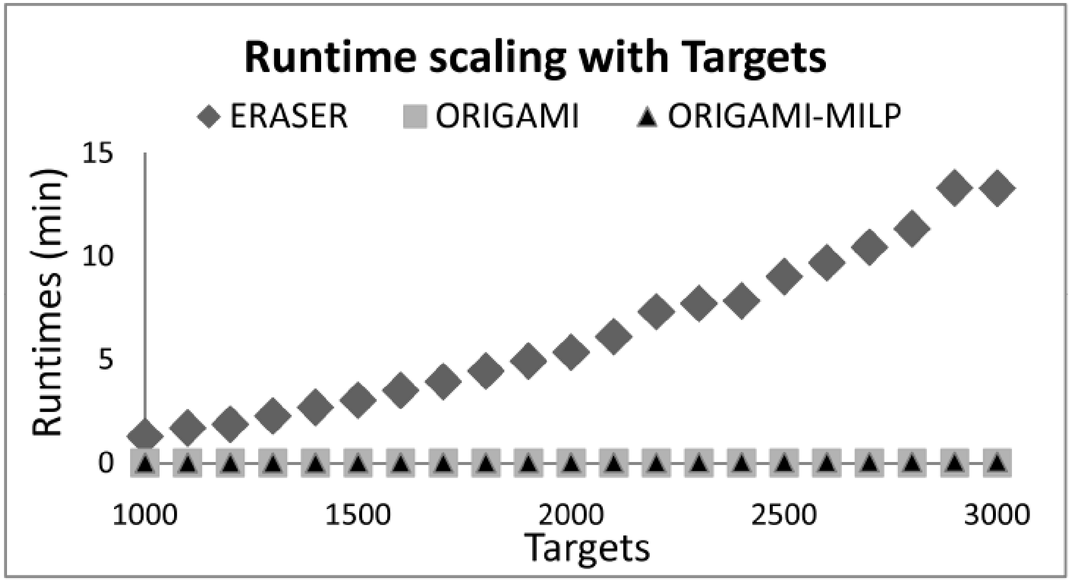



# Scalability

25 resources, 3000 targets => $5 \times 10^{61}$ defender's actions

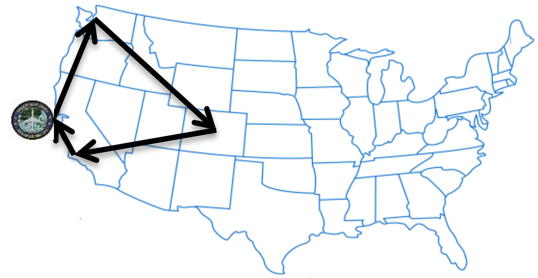
no chance for matrix game representation

The algorithm explained above is ERASER



# Studied extensions

Complex structured defender strategies



Probabilistically failing actions


Attacker’s types




Resource types and teams





Bounded rational attackers

# Resource allocation (security) games

Advantages

- Wide existing literature (many variations)
- Good scalability
- Real world deployments

Limitation

- The attacker cannot react to observations (e.g., defender’s position)

# Perimeter patrolling

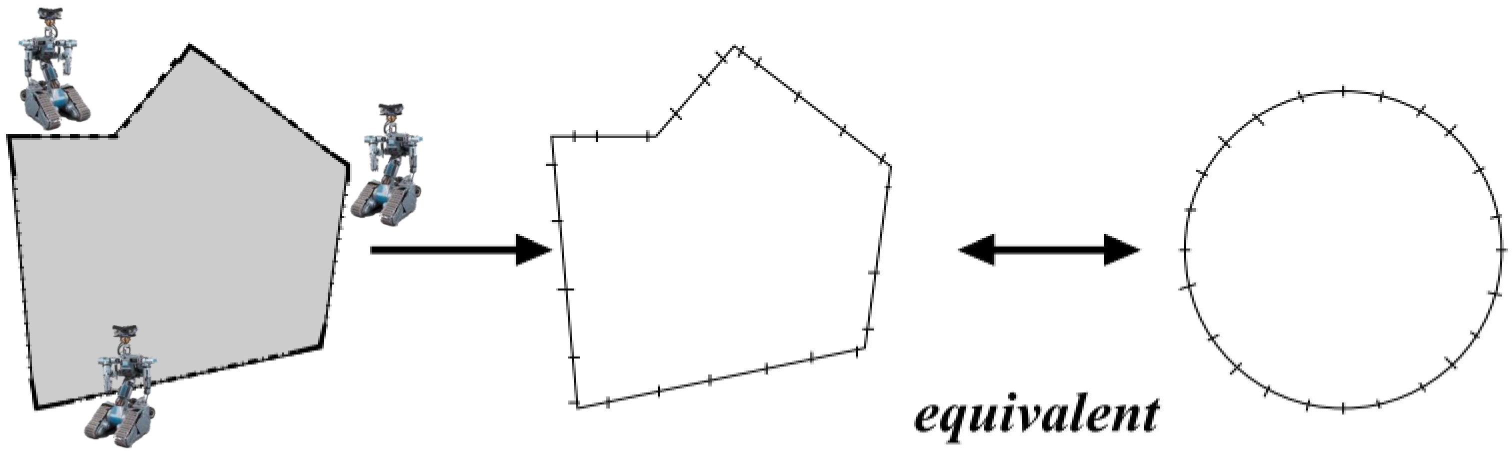
Agmon et al.: Multi-Robot Adversarial Patrolling: Facing a Full-Knowledge Opponent. JAIR 2011.



The attacker can see the patrol!

# Perimeter patrolling

Polygon $P$, perimeter split to $N$ segments




Defender has homogenous resources $k > 1$

- move 1 segment per time step
- turn to the opposite direction in $\tau$ time steps

Attacker can wait infinitely long and sees everything

- chooses a segment where to attack
- requires $t$ time steps to penetrate

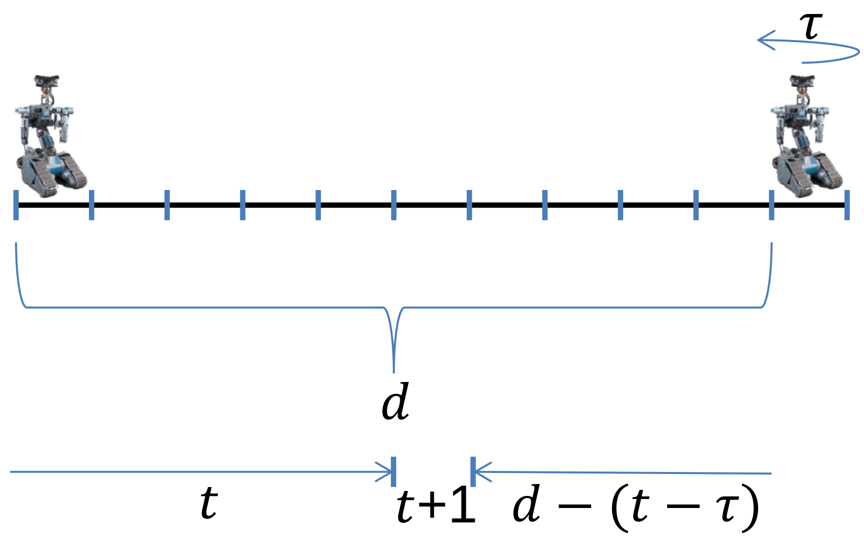
# Interesting parameter settings

Let $d = \frac{N}{k}$ be the distance between equidistant robots

There is a perfect deterministic patrol strategy if $t \geq d$

the robots can just continue in one direction

What about $t = \frac{4}{5} d$ ?




The attacker can guarantee success if $t + 1 < d - (t - \tau) \Rightarrow t < \frac{d + \tau - 1}{2}$

# Optimal patrolling strategy

Class of strategies: continue with probability $p$, else turn around

**Theorem:** In the optimal strategy, all robots are equidistant and face in the same direction.

Proof sketch:

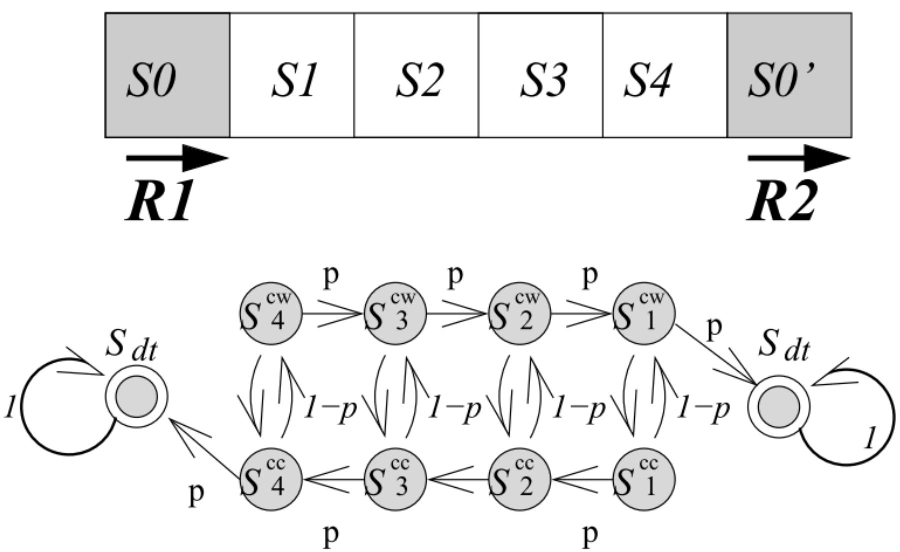
1. the probability of visiting the worst case segment between robots increases with increasing distance between the robots
2. making a move in different directions increases the distance

# Probability of penetration

For simplicity assume $\tau = 1$

Probability of visiting $s_i$ at least once in next $t$ steps

- = probability of visiting the absorbing end state from $s_i$
- sum of each direction visited separately




| | $S_1^{cc}$ | $S_1^{cw}$ | $S_2^{cc}$ | $S_2^{cw}$ | $S_3^{cc}$ | $S_3^{cw}$ | $S_4^{cc}$ | $S_4^{cw}$ | $S_{dt}$ |
|---|---|---|---|---|---|---|---|---|---|
| $S_1^{cc}$ | 0 | $1-p$ | $p$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_1^{cw}$ | $1-p$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $p$ |
| $S_2^{cc}$ | 0 | 0 | 0 | $1-p$ | $p$ | 0 | 0 | 0 | 0 |
| $S_2^{cw}$ | 0 | $p$ | $1-p$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_3^{cc}$ | 0 | 0 | 0 | 0 | 0 | $1-p$ | $p$ | 0 | 0 |
| $S_3^{cw}$ | 0 | 0 | 0 | $p$ | $1-p$ | 0 | 0 | 0 | 0 |
| $S_4^{cc}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $1-p$ | $p$ |
| $S_4^{cw}$ | 0 | 0 | 0 | 0 | 0 | $p$ | $1-p$ | 0 | 0 |
| $S_{dt}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# Probability of penetration

**Algorithm 1** Algorithm FindFunc$(d, t)$

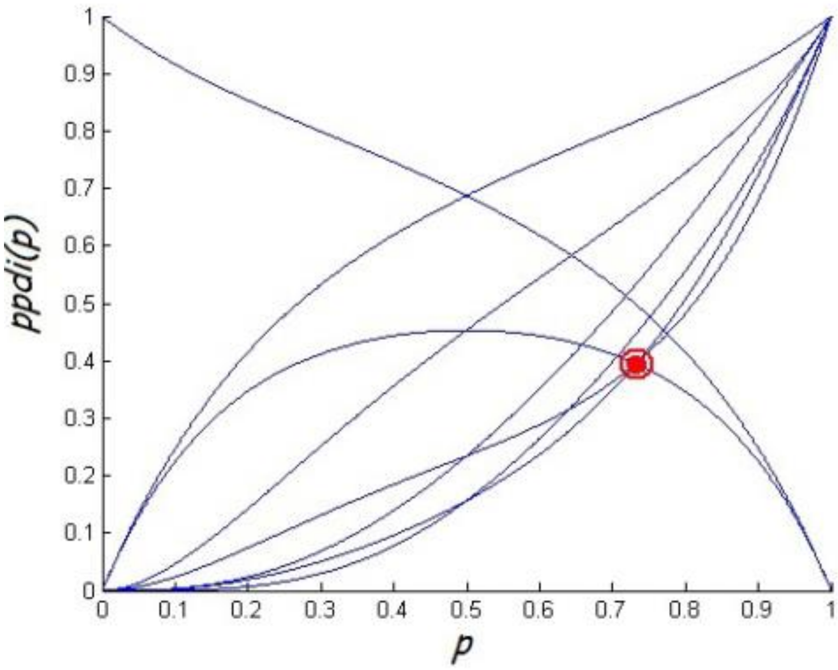
Create matrix $M$ of size $(2d+1)(2d+1)$, initialized with 0s
Fill out all entries in $M$ as follows:
$M[2d+1, 2d+1] = 1$
**for** $i \leftarrow 1$ to $2d$ **do**
    $M[i, \max\{i+1, 2d+1\}] = p$
    $M[i, \min\{1, i-2\}] = 1-p$
Compute $MT = M^t$
$Res$ = vector of size $d$ initialized with 0s
**for** $1 \leq loc \leq d$ **do**
    $V$ = vector of size $2d+1$ initialized with 0s.
    $V[2loc] \leftarrow 1$
    $Res[loc] = V \times MT[2d+1]$
Return $Res$

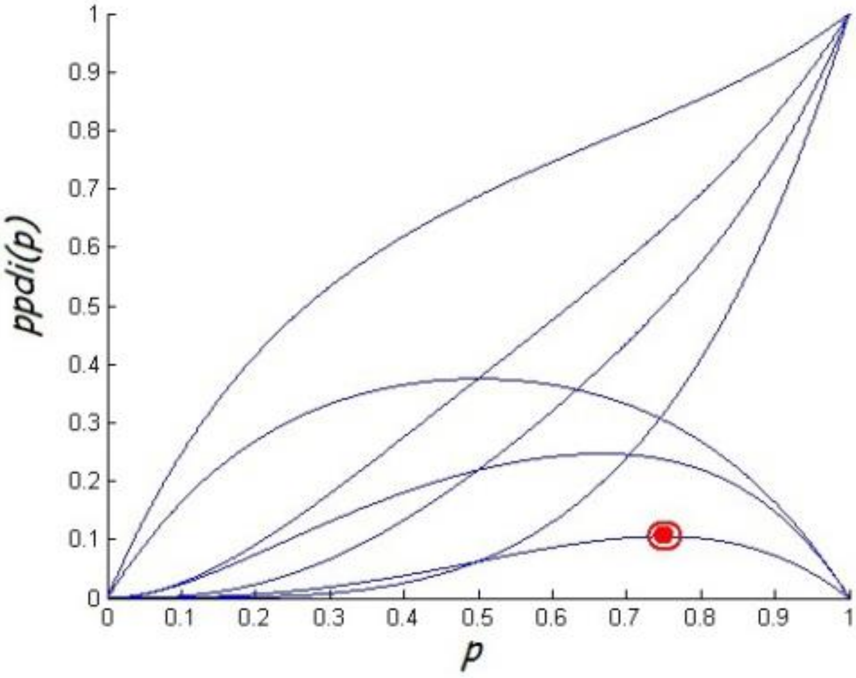
All computations are symbolic. The result are functions $ppd_i\colon [0,1] \rightarrow [0,1]$.

# Optimal turn probability

Maximin value for $p$

Each line represents one segment ($ppd_i$)




Iterate all pairs of intersection and maximal points to find solution

- it is all polynomials

# Perimeter patrol – summary

Split the perimeter to segments traversable in unit time

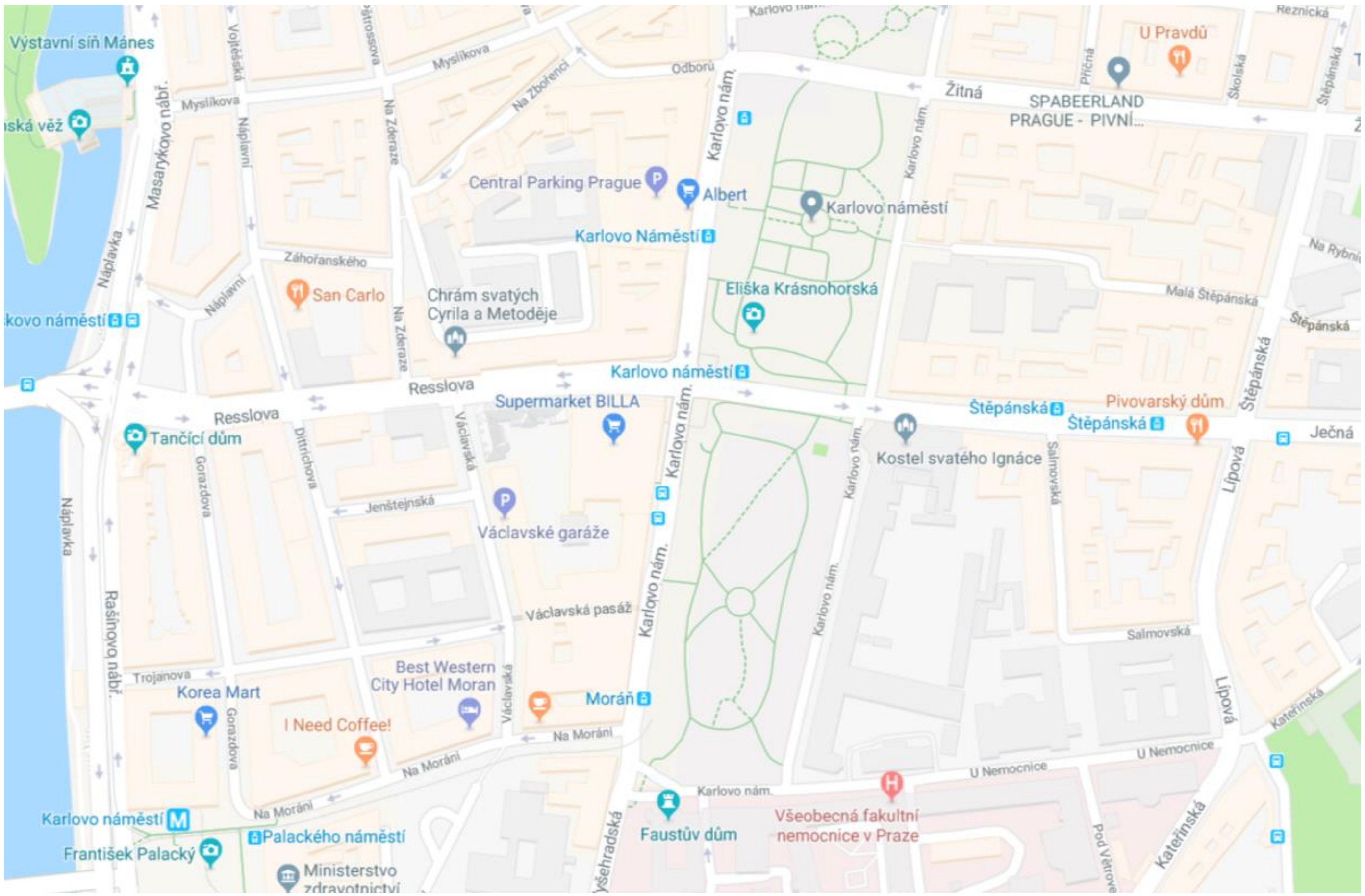
Distribute patrollers uniformly along the perimeter

Coordinate them to always face the same way

Continue with probability $p$ turn around with probability $(1 - p)$

# Area patrolling

Basilico et al.: Patrolling security games: Definition and algorithms for solving large instances with single patroller and single intruder. AIJ 2012.



# Area patrolling - Formal model

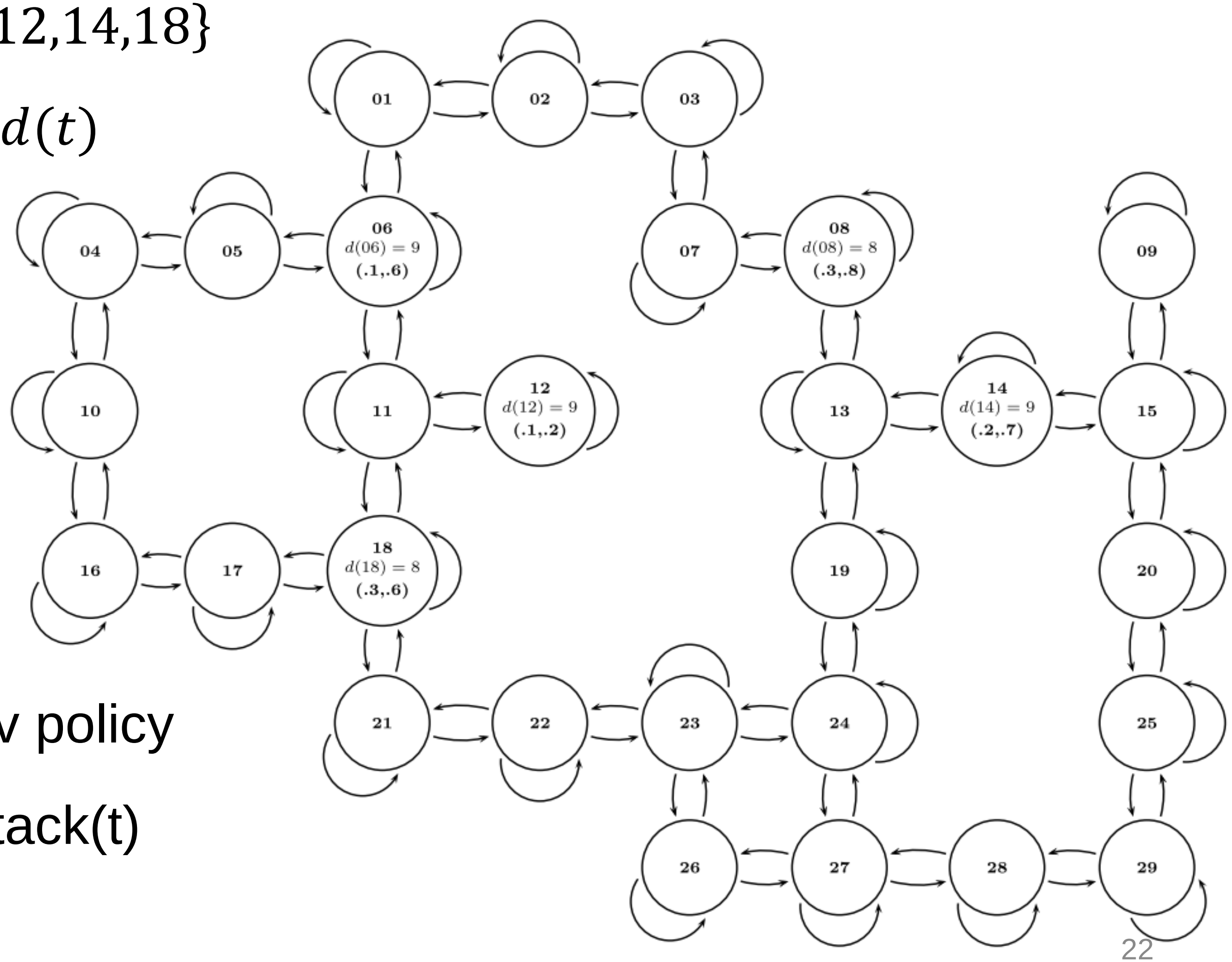
Environment represented as a graph

Targets $T = \{6,8,12,14,18\}$

Penetration time $d(t)$

Target values

$(v_d(t), v_a(t))$




Defender: Markov policy

Attacker: wait, attack(t)

# Solving zero-sum patrolling game

We assume $\forall t \in T: \; v_a(t) = v_d(t)$

$a(i,j) = 1$ if the patrol can move form $i$ to $j$ in one step; else 0

$P_c(t,h)$ is the probability of stopping an attack at target $t$ started when the patrol was at node $h$

$\gamma_{i,j}^{w,t}$ is the probability that the patrol reaches node $j$ from $i$ in $w$ steps without visiting target $t$

$$
\begin{aligned}
&\max u \\
&\quad \alpha_{i,j} \geqslant 0 \quad \forall i,j \in V \\
&\quad \sum_{j \in V} \alpha_{i,j} = 1 \quad \forall i \in V \\
&\quad \alpha_{i,j} \leqslant a(i,j) \quad \forall i,j \in V \\
&\quad \gamma_{i,j}^{1,t} = \alpha_{i,j} \quad \forall t \in T,\; i,j \in V \setminus \{t\} \\
&\quad \gamma_{i,j}^{w,t} = \sum_{x \in V \setminus \{t\}} \left(\gamma_{i,x}^{w-1,t} \alpha_{x,j}\right) \quad \forall w \in \{2,\dots,d(t)\},\; t \in T,\; i,j \in V \setminus \{t\} \\
&\quad P_c(t,h) = 1 - \sum_{j \in V \setminus \{t\}} \gamma_{h,j}^{d(t),t} \quad \forall t \in T,\; h \in V \\
&\quad u \leqslant u_{\mathbf{d}}(\textit{intruder-capture}) P_c(t,h) + u_{\mathbf{d}}(\textit{penetration-t})\left(1 - P_c(t,h)\right)
\end{aligned}
$$

$\alpha_{i,j}$ is a probability of moving from $i$ to $j$

$$
u_{\mathbf{d}}(x) = \begin{cases} \sum_{i \in T} v_{\mathbf{d}}(i), & x = \textit{intruder-capture} \text{ or } \textit{no-attack} \\ \sum_{i \in T \setminus \{t\}} v_{\mathbf{d}}(i), & x = \textit{penetration-t} \end{cases}
$$

AI (GT) problems can often be solved by transformation to MP