Artificial Intelligence in Robotics
Lecture 12: Visibility-based pursuit evasion

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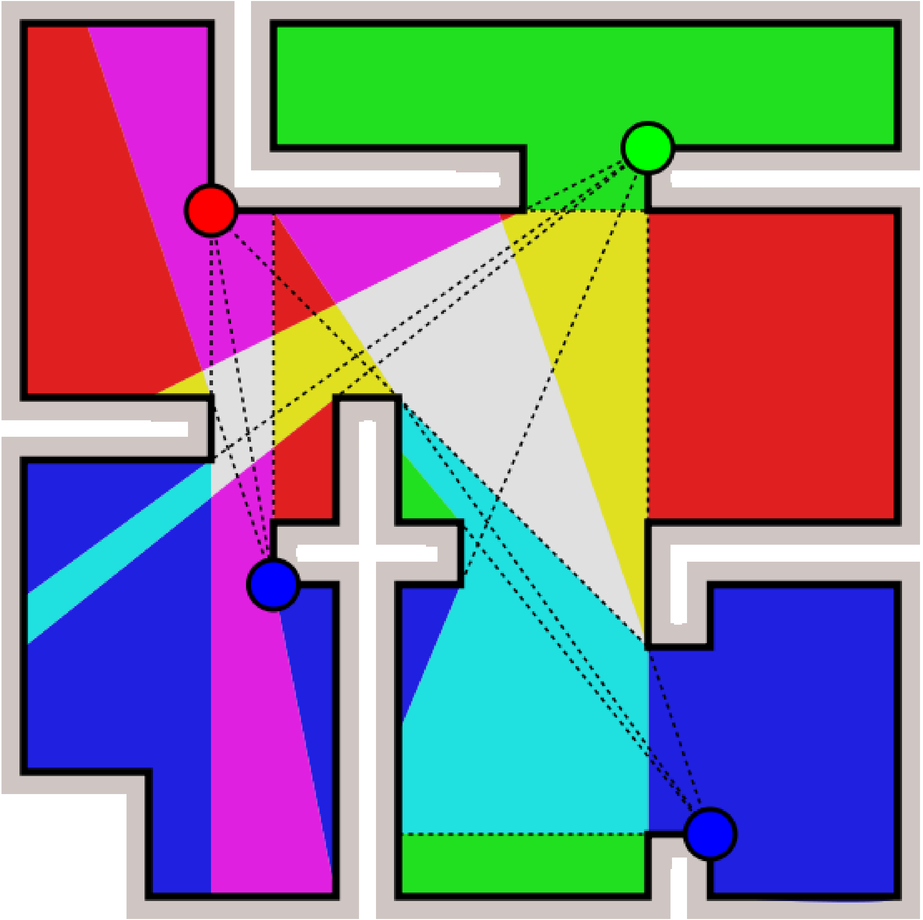
STATIC "PURSUER"
NO EVADER

Art gallery problem

By Victor Klee in 1973

- simple polygon $\mathrm{P}: v_1, \ldots v_n$
- $x \in P$ covers $y \in P$ iff $xy \subseteq P$
- minimal number of “guards” to cover the whole space?



Picture by Claudio Rocchini

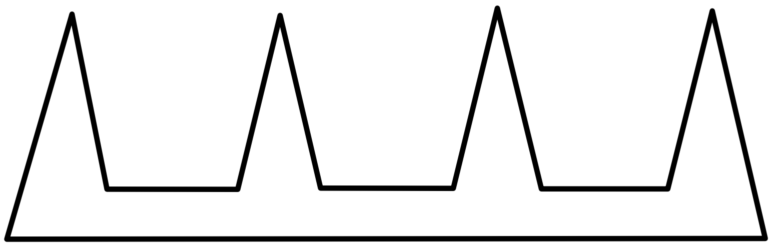
Art gallery problem

Theorem (Václav Chvátal 1975):

$\lfloor n/3 \rfloor$ guard is sometimes necessary and always sufficient to solve the art gallery problem.

Necessary

comb



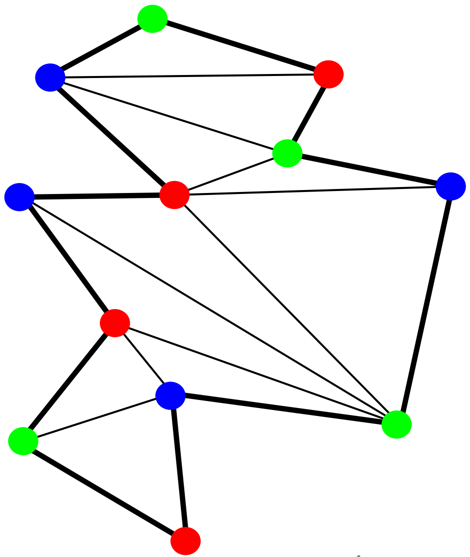
Sufficient (Fisk 1978)

simple polygons always have triangulation

triangulated polygon can be 3-colored

least used color is used no more than $\lfloor n/3 \rfloor$ times

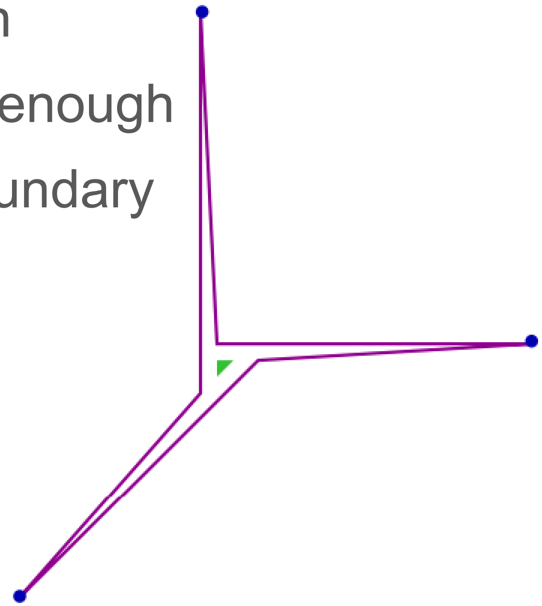
vertices of each color cover the whole polygon

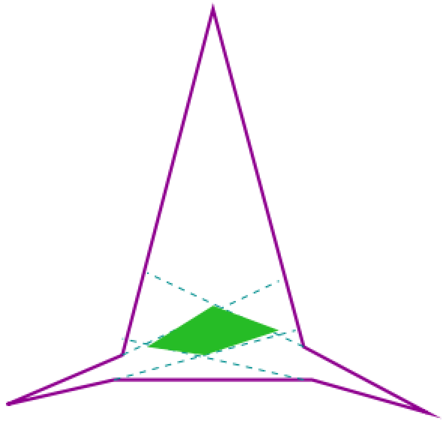


Art gallery problem

Pathological cases (from Subhash Suri's slides):

- less guards may be enough
- seeing the boundary is not enough
- optimal positions not on boundary





Fun facts:

- For orthogonal polygons, only $\lfloor n/4 \rfloor$ guards are needed.
- Computing minimal number of guards for a polygon is NP-hard.
- The problem is closely connected to the set cover problem.

STATIC PURSUER
MOBILE EVADER

More realistic art gallery problem

There are m cameras (angles)

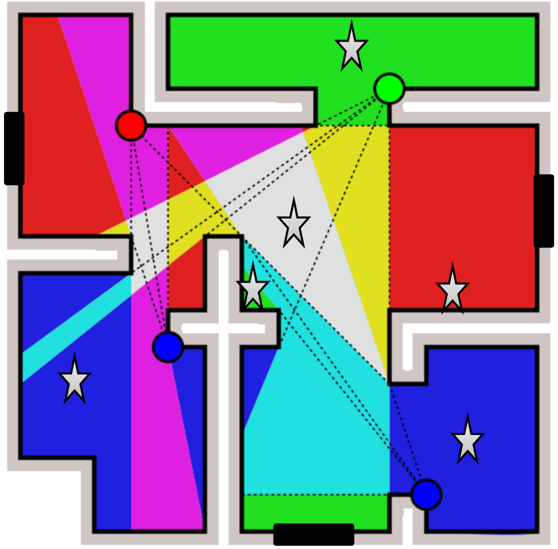
A guard can watch k cameras

What cameras to show?

Thief has to enter, steal, exit

Penalty for each seen second/meter

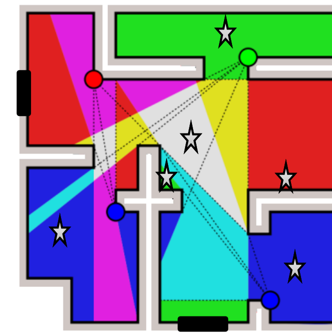
Inspired by: McMahan, Gordon, Blum: Planning in the presence of cost functions controlled by an adversary. ICML 2003.



Matrix game representation

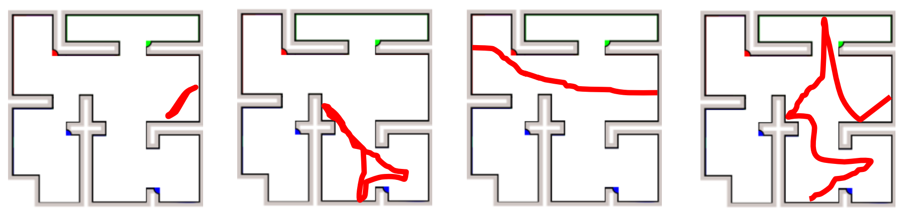
Defender's action: watch *k* of *m* cameras

Attacker's action: path door-target-door

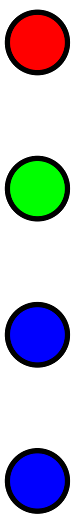


p – prob. of not being detected when seen

∞

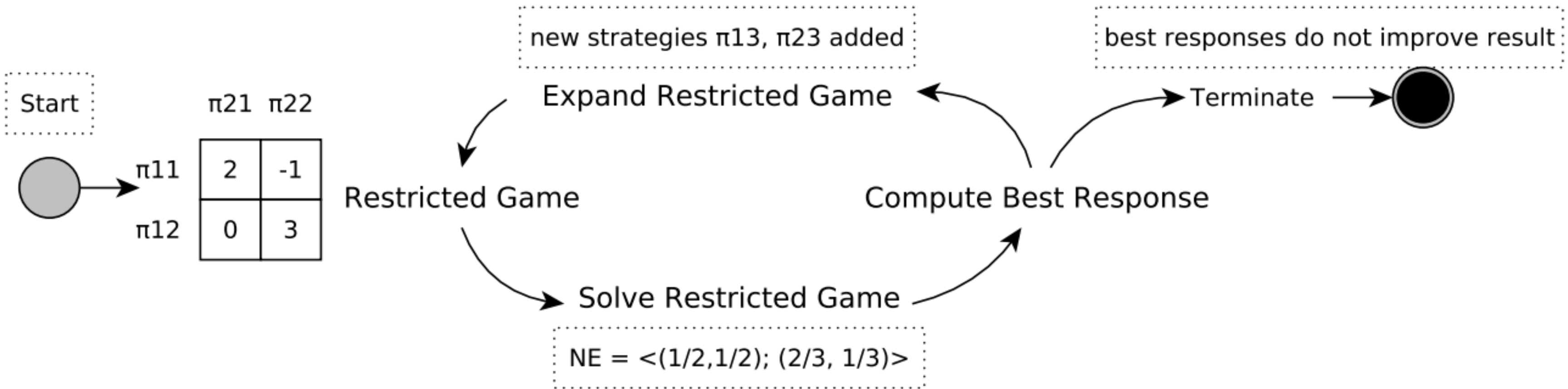


$\binom{n}{k}$



$p^{d_1} * v_1$	$v_2 + v_3$	…	…
v_1	$p^{d_2} * (v_2 + v_3)$	…	…
v_1	$v_2 + v_3$	…	…
v_1	$p^{d_3} * (v_2 + v_3)$	…	…

Double oracle framework



McMahan, Gordon, Blum: Planning in the presence of cost functions controlled by an adversary. ICML 2003.

Double-oracle in Matrix game

1	2	4	3	3
	3	3	7	2
	2	4	5	5
	2	1	4	1

Double-oracle in Matrix game

	1			
0	2	4	3	3
1	3	3	7	2
	2	4	5	5
	2	1	4	1

Double-oracle in Matrix game

	0.5			0.5
0.5	2	4	3	3
0.5	3	3	7	2
	2	4	5	5
	2	1	4	1

Double-oracle in Matrix game

	0.75			0.25
0	2	4	3	3
0.75	3	3	7	2
0.25	2	4	5	5
	2	1	4	1

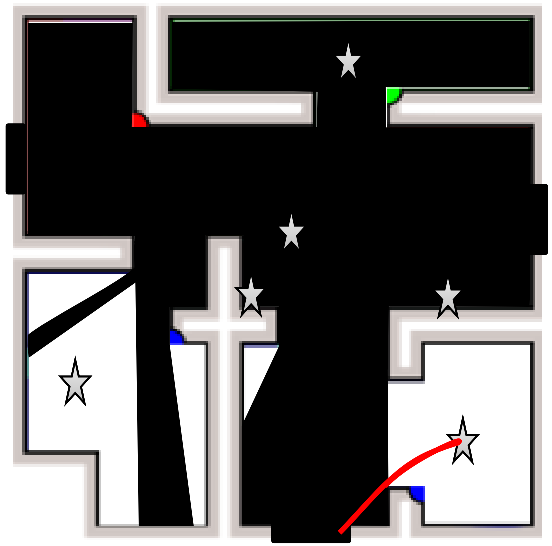
Always converges and finds NE.

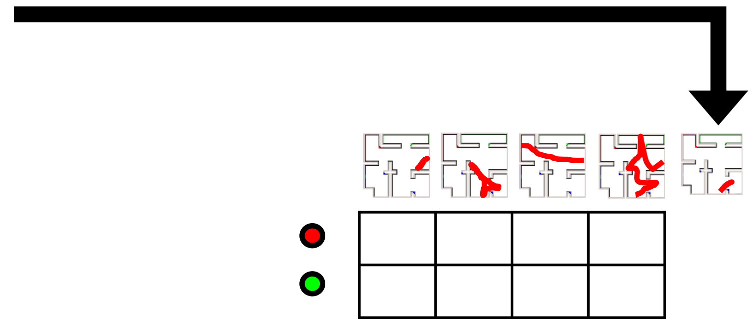
Attacker’s best response oracle

Defender’s current strategy

$\frac{1}{3}$ (red)

$\frac{2}{3}$ (green)

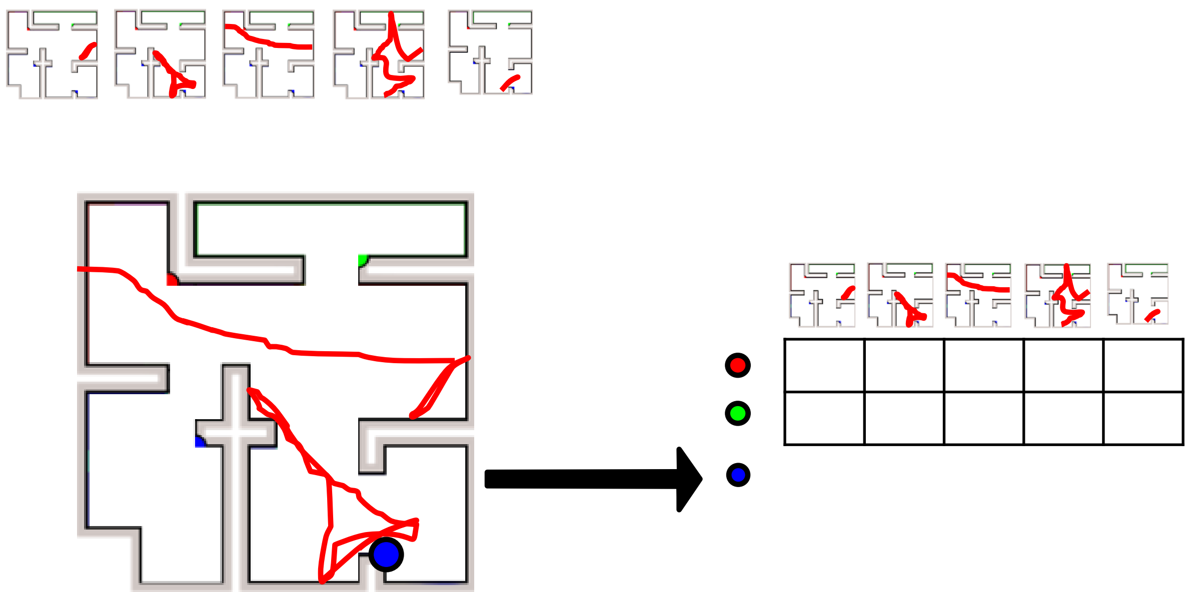




Path planning with costs defined by cameras in use (A*, TSP, etc.)

Defender’s best response oracle

$$\frac{1}{4} \quad \frac{1}{6} \quad \frac{1}{4} \quad 0 \quad \frac{1}{3}$$



Greedy / combinatorial search for best k camera positions

MOBILE PURSUER
INFINITELY FAST EVADER

Clearing polygonal environment

Hunters and prey problem

- simple polygon $\mathrm{P}: v_1, \dots v_n$
- k hunters with bounded speed
- prey with unbounded speed
- can hunters spot the prey?

Definitions

- $h^i: [0, \infty) \to P$ is the pursuer i's strategy
- $e: [0, \infty) \to P$ is the evader's strategy
- $V(q) \subseteq P$ are the points visible from $q \in P$

Solution

- Strategy $h = h^1, \dots, h^k$ is a solution if for every continuous $e: [0, \infty) \to P$ there exists $t \in [0, \infty), i \in \{1, \dots, k\}$, such that $e(t) \in V(h^i(t))$.

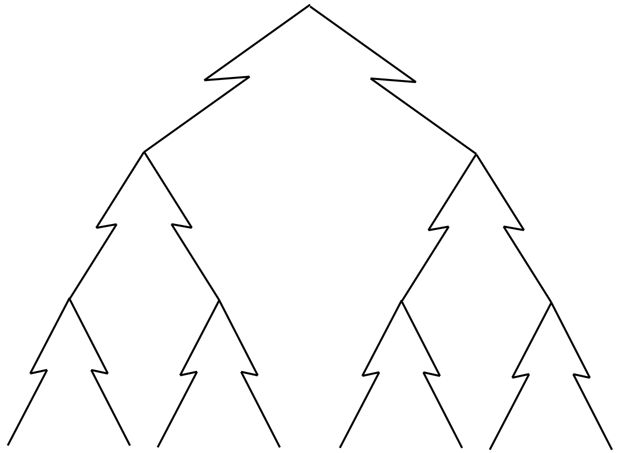
Clearing polygonal environment

Theorem (Urrutia, 1997): $O(\log n)$ hunters are always sufficient and occasionally necessary to spot a prey in polygon with n vertices.

Sufficient

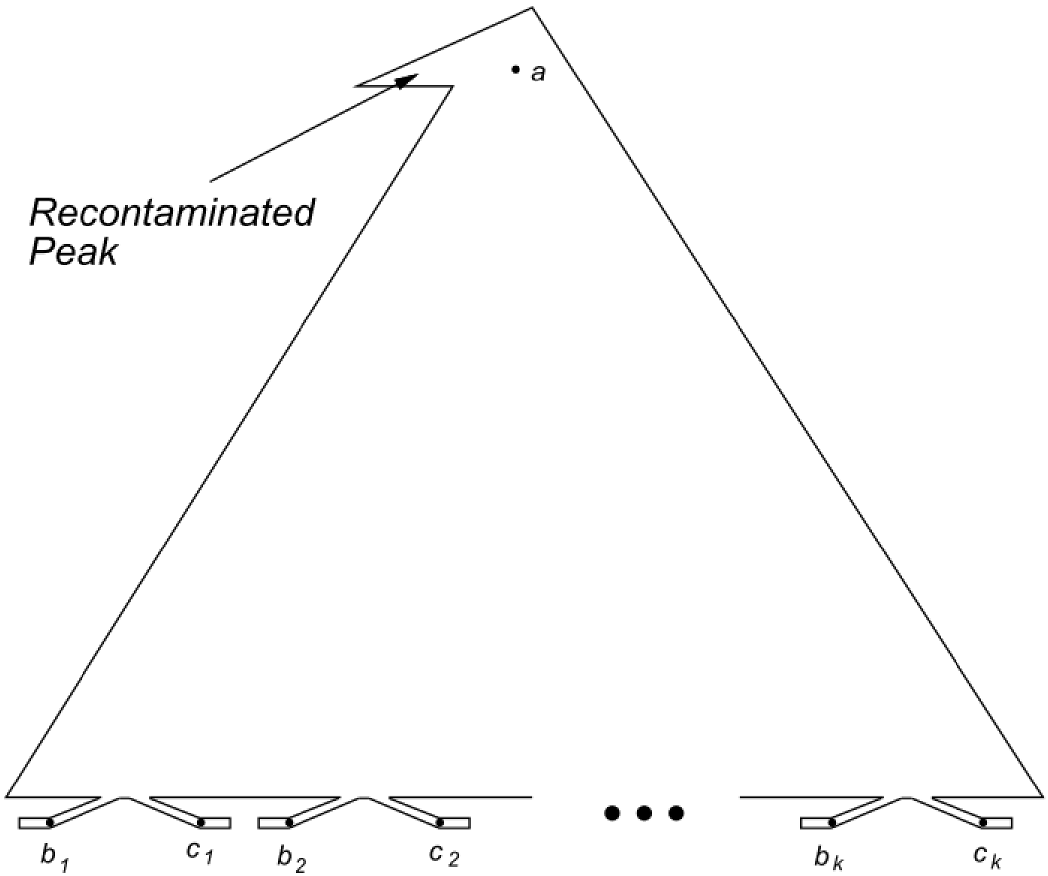
- let $f(n)$ be the required number of hunters
- each polygon has a diagonal splitting it to two with $\leq \frac{2n}{3}$ vertices
- if one guard guards the diagonal, $f(n) \leq f\left(\frac{2n}{3}\right) + 1$
- from master theorem, $f(n) \in O(\log n)$

Necessary



Clearing polygonal environment

Theorem (Guibas et al. 1997): There exists a sequence of simply-connected free spaces clearable by single pursuer, such that O(n) recontaminations are required for n edges.



Clearing polygonal environment

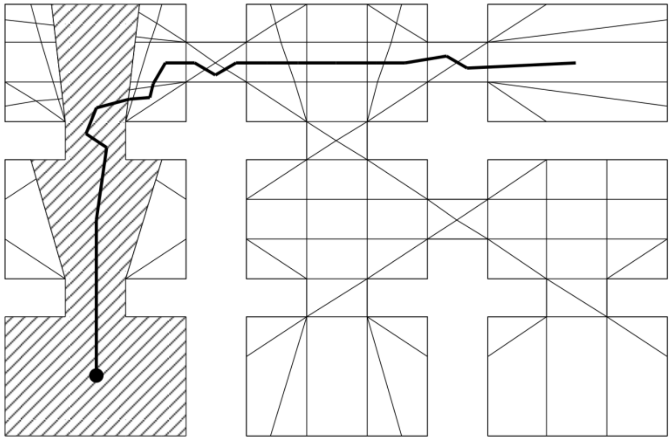
Guibas, L. J., Latombe, J.-C., Lavalle, et al.: Visibility-Based Pursuit-Evasion in a Polygonal Environment. WADS, 1997

- hunter and prey setting - we assume a **single hunter**
- critical event analysis (similar to event-based simulation)

Definitions

- information state $\eta = (x, S); x \in P, S \subseteq P$ are pursuer/evader positions
- $\Psi(\eta, h, t_0, t_1)$ is the inf. state after executing h from η during $[t_0, t_1]$
- region $D \subseteq P$ is conservative, if for all continuous $h_1, h_2 : [t_0, t_1] \rightarrow D$
$h_1(t_0) = h_2(t_0)$ & $h_1(t_1) = h_2(t_1) \Rightarrow \Psi(\eta, h_1, t_0, t_1) = \Psi(\eta, h_2, t_0, t_1)$

Clearing polygonal environment



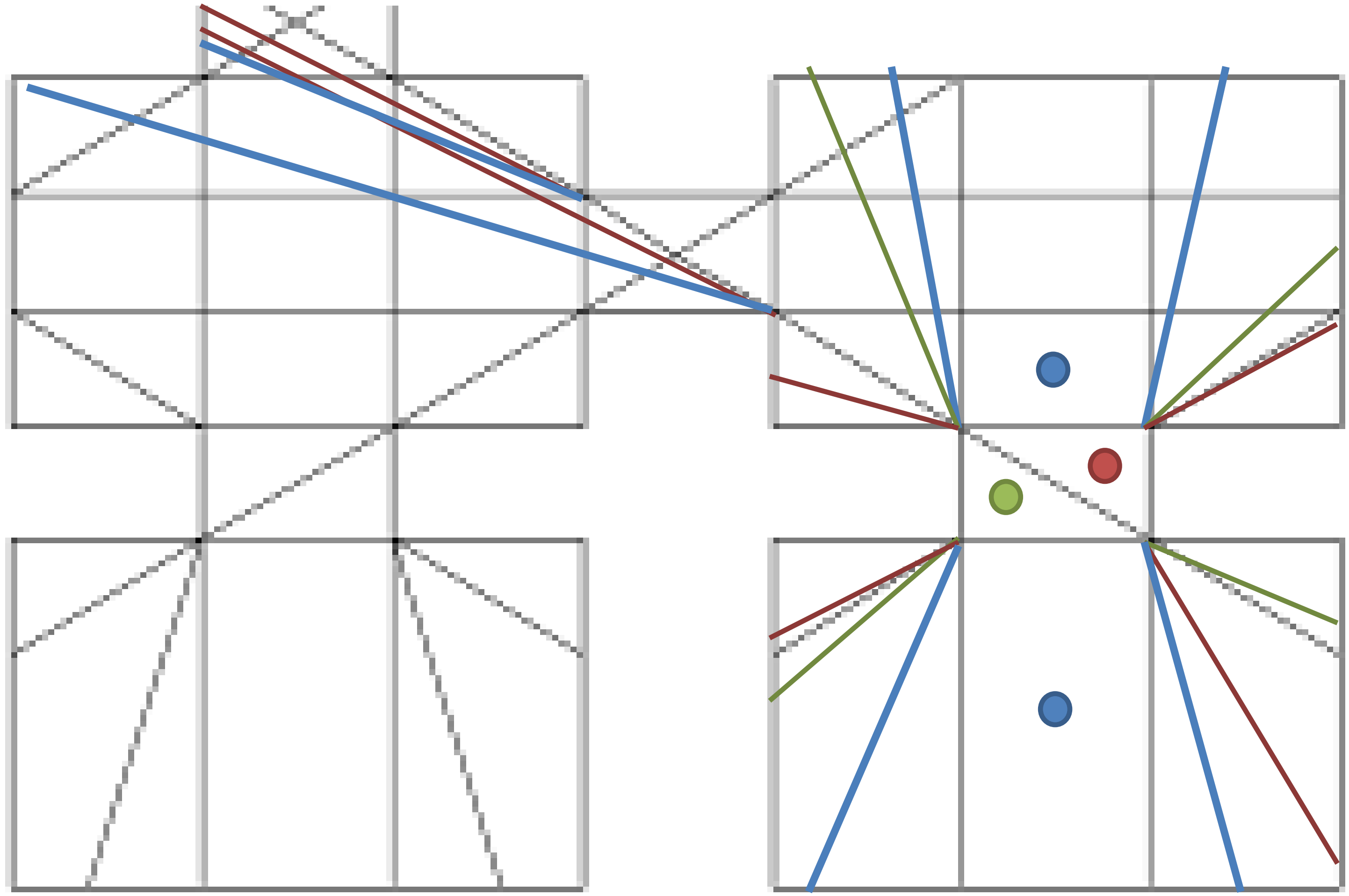
Extend the edges

- obstacle edges in both directions
- pairs of vertices outwards

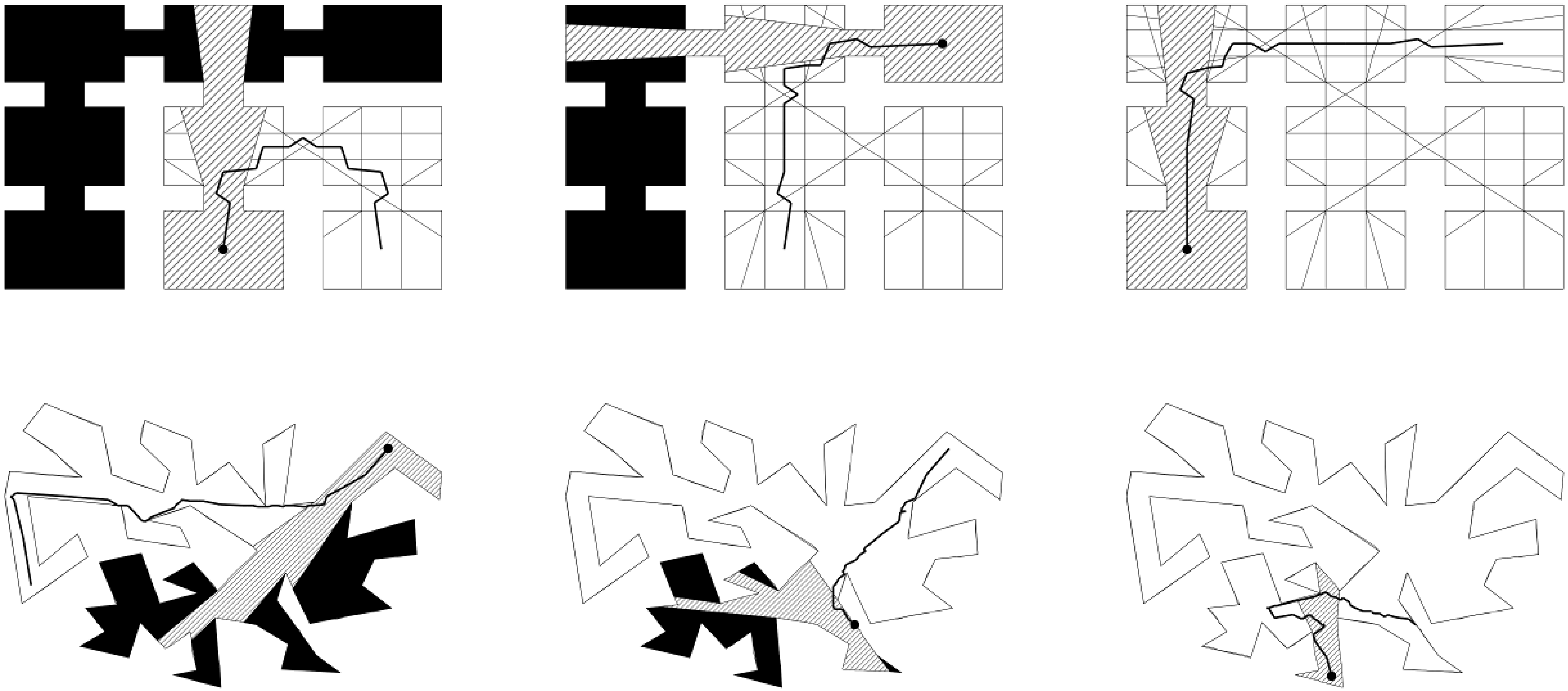
Search graph

- adjacent cell graph
- gap edge labeling: “1” contaminated, “0” clear
- corresponding gap edges determine change in labeling

Gap edge labeling



Clearing polygonal environment



Quiz: goo.gl/3S8nHh

MOBILE PURSUER
MOBILE EVADER

Visibility-based tracking

graph of locations (V, E)

visibility relation $Sees(v_1, v_2)$

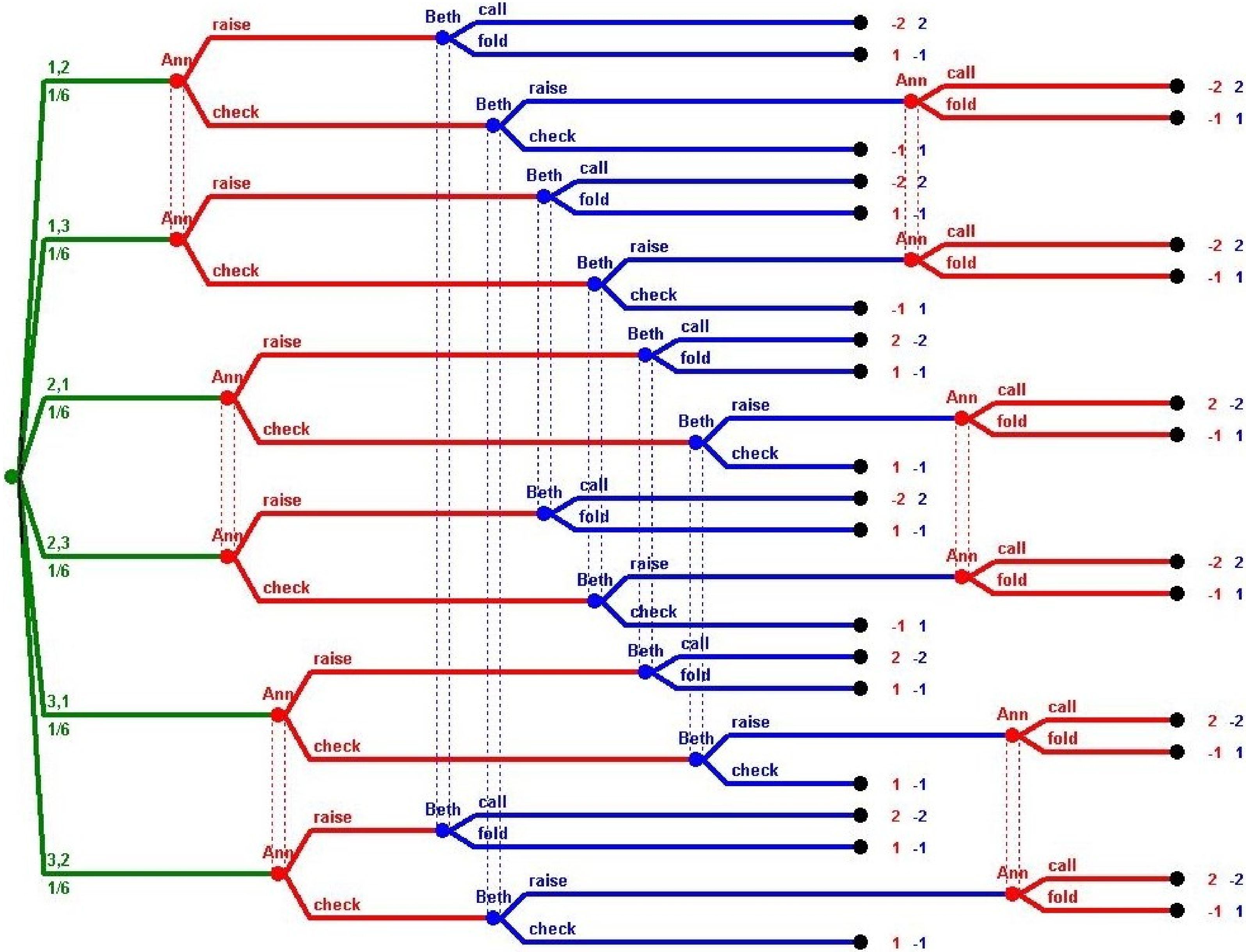
k pursuers, 1 evader

- both move on the graph
- both unit speed

Goal

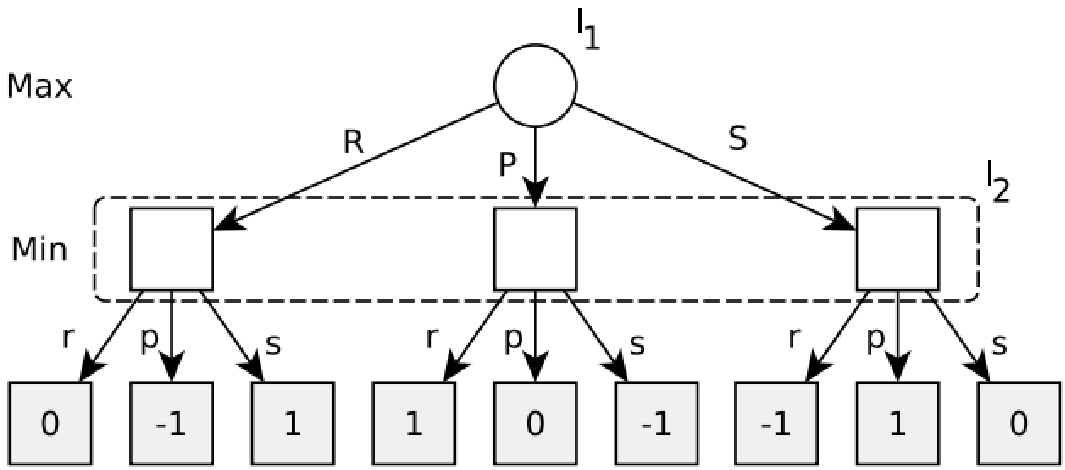
- See as often as possible
- Minimize the set of possible positions

Extensive form game

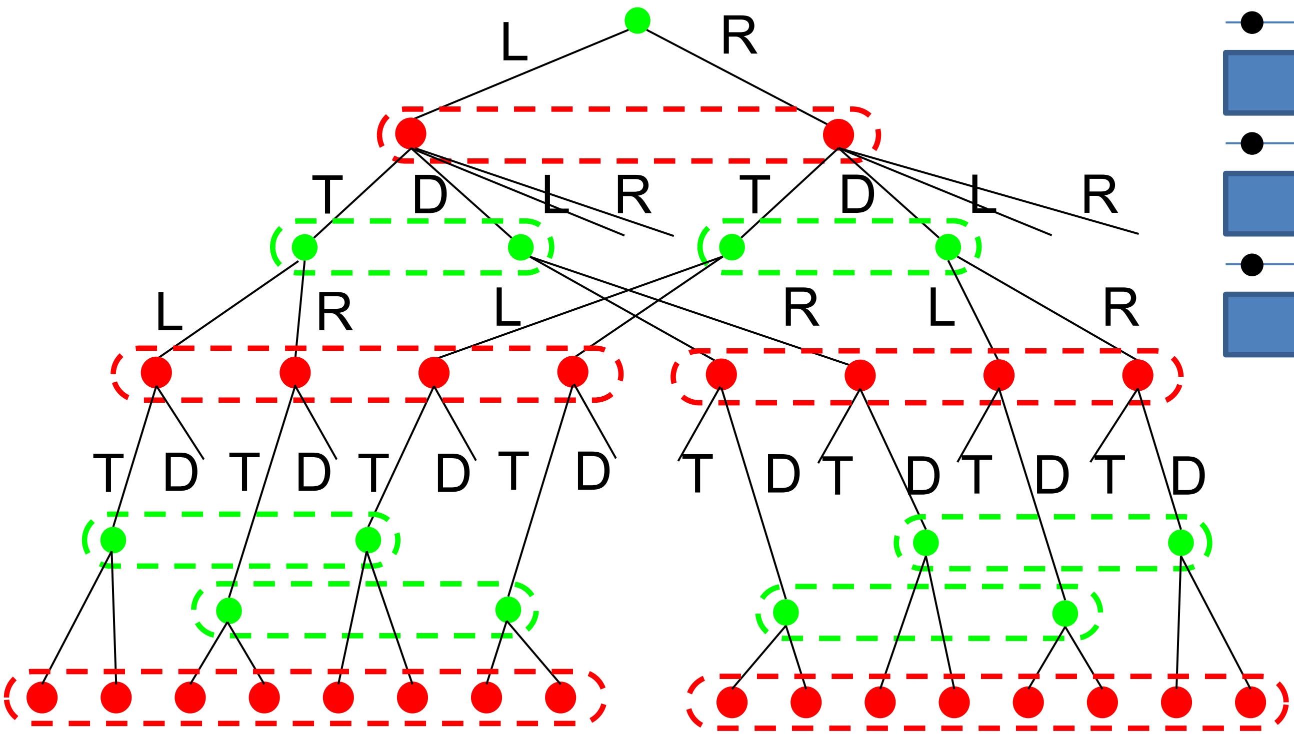


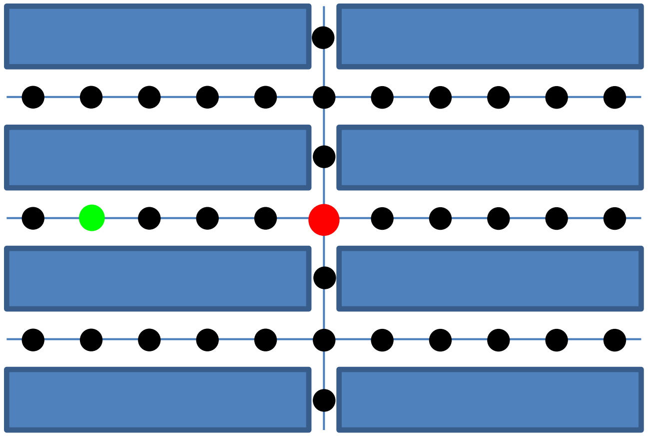
Simultaneous moves in EFG

	r	p	s
R	0	−1	1
P	1	0	−1
S	−1	1	0

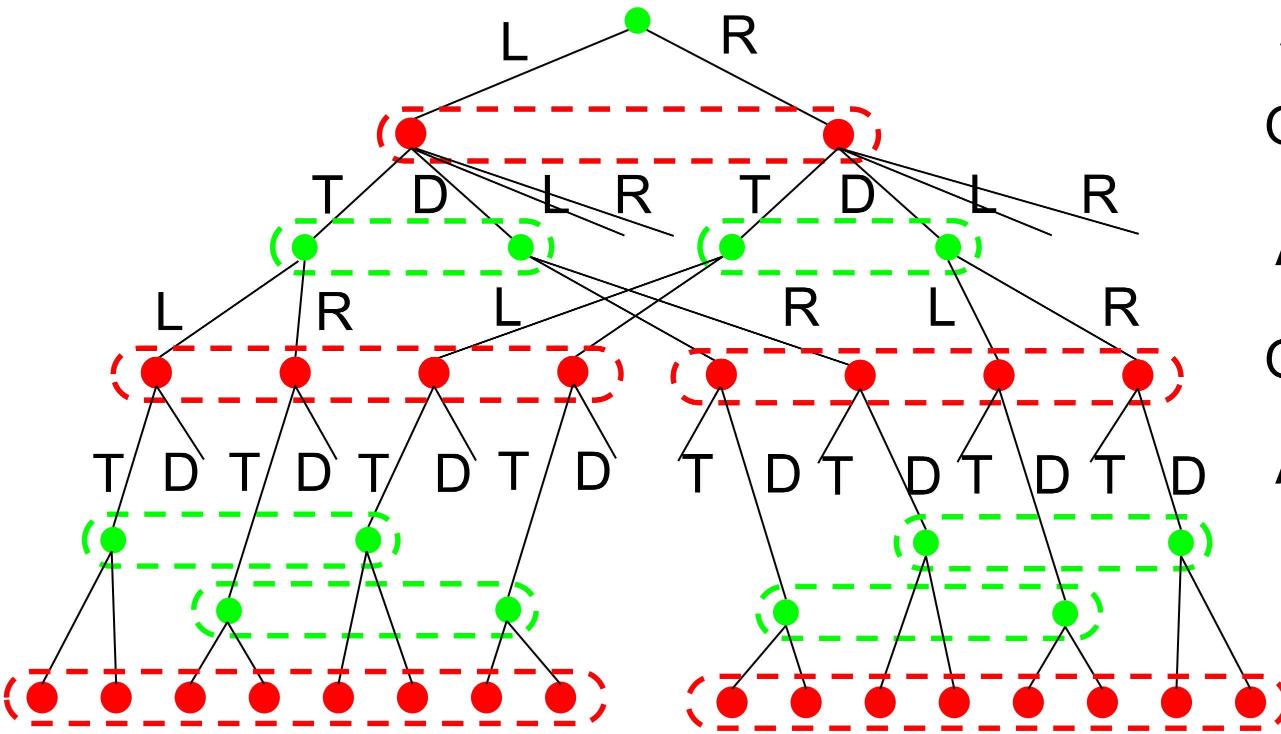


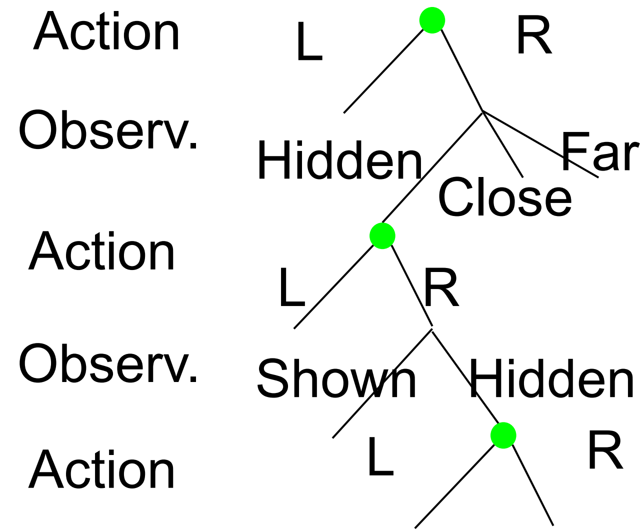
Pursuit evasion as EFG





EFG vs. Information Set Tree

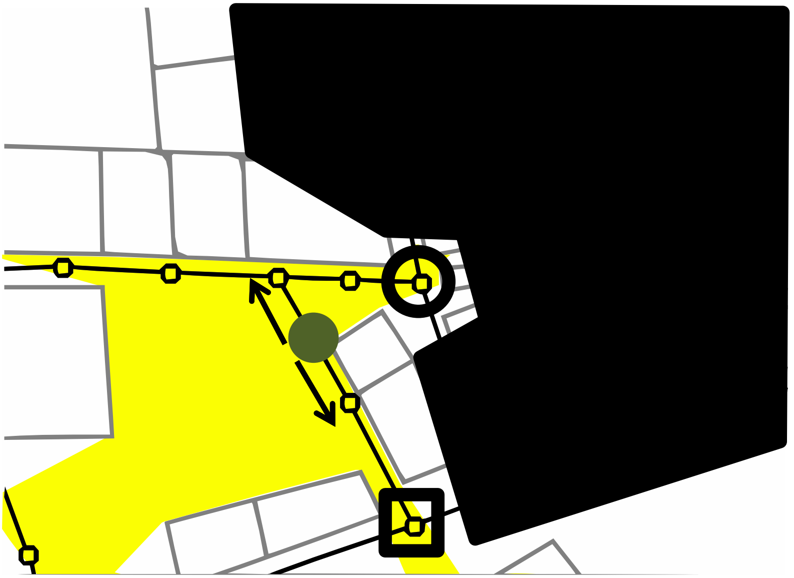


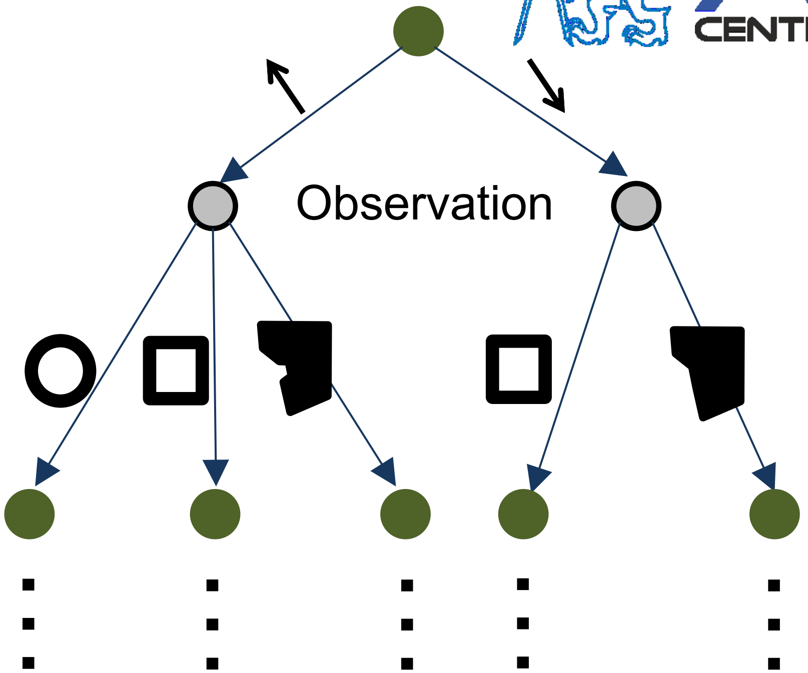


Nodes in IST are Info. Sets in EFG

+ IST is much smaller
+ solved as perfect information

- overly pessimistic (worst possible observation)



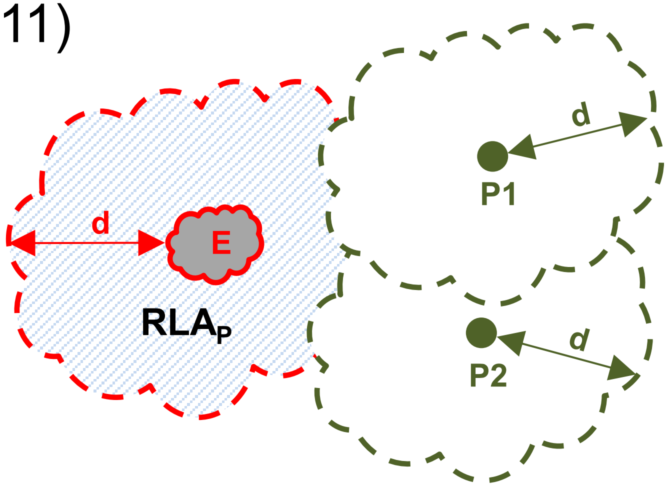


Relaxed look-ahead heuristic (Raboin at al. 2011)

|positions reachable by evader
- positions that can be possibly seen|

evader can be on worst possible position
pursuers can be everywhere at once

usable in iterative deepening minimax or MCTS



Summary

Static camera position

Camera switching

Capturing spotting fast evader

Tracking realistic evader

Resources

Urrutia, J. (1997). Art Gallery and Illumination Problems. Handbook of Computational Geometry, 973–1027.

Guibas, L. J., Latombe, J. C., LaValle, S. M., Lin, D., & Motwani, R. (1997, August). Visibility-based pursuit-evasion in a polygonal environment. In Workshop on Algorithms and Data Structures (pp. 17-30).

McMahan, Gordon, Blum (2003): Planning in the presence of cost functions controlled by an adversary. ICML.

Raboin, E., Nau, D., Kuter, U., Gupta, S. K., & Svec, P. (2010). Strategy generation in multi-agent imperfect-information pursuit games. AAMAS, pp. 947-954.