Game theory - lab 3

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Overview

1. ERASER-C
2. Perimeter patrol with larger turns
3. Area patrol
4. Perimeter patrolling as area patrolling

ERASER

ERASER algorithm, $C = \{c_t | \forall t \in T\}$ is the defender's coverage vector over targets and $A = \{a_t | \forall t \in T\}$ is attacker's vector one-hot encoding attacked target and d and k are unbounded continuous variables. $U_{\{\Theta,\Psi\}}(t, C)$ is value of target t given coverage vector C for defender and attacker respectively. Z is sufficiently large number.

$$
\begin{aligned}
max \quad & d & \\
a_t \in \quad & \{0, 1\} & \forall t \in T \\
\sum_{t \in T} a_t = \quad & 1 & \\
c_t \in \quad & [0, 1] & \forall t \in T \\
\sum_{t \in T} c_t = \quad & m & \\
d - U_\Theta(t, C) \leq \quad & (1 - a_t) \cdot Z & \forall t \in T \\
0 \leq k - U_\Psi(t, C) \leq \quad & (1 - a_t) \cdot Z & \forall t \in T
\end{aligned}
$$

Constraint the resources

Modify the ERASER algorithm to consider following constraints

Schedules

Resources are assigned to *schedules* covering multiple targets. Set of legal schedules $S = \{s_1, \cdots s_l\}$ is a subset of power set of the targets. Restrictions on this set represent scheduling constraints. Relationship is given by function $M : S \times T \rightarrow \{0, 1\}$, which evaluates to 1 if and only if t is covered in s. The strategy of the defender is an assignment of resources to schedules rather than targets.

Resource types

$\Omega = \{\omega_1, \cdots, \omega_v\}$ is set of *resource types*, each has a capability to cover different subset of S. The number of available resources of each type is given by the function $R(\omega)$. Coverage capabilities for each type are given by the function $Ca : S \times \Omega \rightarrow \{0, 1\}$, which is 1 if the type can cover the given schedule and 0 otherwise.

ERASER-C

$$
\begin{array}{rll}
\max & d & \\
a_t \in & \{0,1\} & \forall t \in T \\
c_t, q_s, h_{s,\omega} \in & [0,1] & \forall t \in T, \forall s \in S, \forall \omega \in \Omega \\
\sum_{t \in T} a_t = & 1 & \\
\sum_{\omega \in \Omega} h_{s,\omega} = & q_s & \forall s \in S \\
\sum_{s \in S} q_s M(s,t) = & c_t & \forall t \in T \\
\sum_{s \in S} h_{s,\omega} Ca(s,\omega) \leq & R(\omega) & \forall \omega \in \Omega \\
h_{s,\omega} \leq & Ca(s,\omega) & \forall s,\omega \in S \times \Omega \\
d - U_\Theta(t,C) \leq & (1-a_t) \cdot Z & \forall t \in T \\
0 \leq k - U_\Psi(t,C) \leq & (1-a_t) \cdot Z & \forall t \in T
\end{array}
$$

Basic algorithm

For turning time $\tau = 1$ we have the following algorithm

Create matrix M of size [2d + 1, 2d + 1], initialized with 0s

Fill out all entries in M as follows:

M[2d, 2d] = 1

for *i = 0, i < 2d, i++* **do**

    M[*i*,(2(*i* mod 2==0 ? 1 : -1)+*i*) mod (2d+1)] = p

    M[*i*,((*i* mod 2==0 ? 1 : -1)+*i*) mod (2d+1)] = 1-p

end

MT = M^t

Res = vector of size d initialized with 0s

for *loc = 0, loc* $\leq$ *2d, loc++* **do**

    V = vector of size 2d+1 initialized with 0s

    V[2*loc*]=1

    Res[*loc*] = V $\times$ MT[2d]

end

return Res

Larger turn time

Modify the algorithm to work with $\tau > 1$.

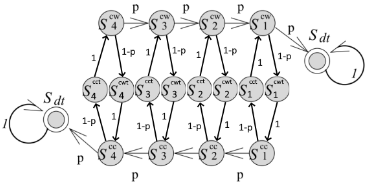
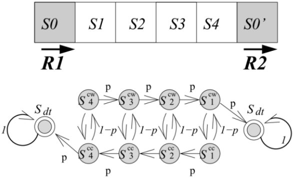


Figure: Problem with larger τ



	S_1^{cc}	S_1^{cw}	S_2^{cc}	S_2^{cw}	S_3^{cc}	S_3^{cw}	S_4^{cc}	S_4^{cw}	S_{dt}
S_1^{cc}	0	$1-p$	p	0	0	0	0	0	0
S_1^{cw}	$1-p$	0	0	0	0	0	0	0	p
S_2^{cc}	0	0	0	$1-p$	p	0	0	0	0
S_2^{cw}	0	p	$1-p$	0	0	0	0	0	0
S_3^{cc}	0	0	0	0	0	$1-p$	p	0	0
S_3^{cw}	0	0	0	p	$1-p$	0	0	0	0
S_4^{cc}	0	0	0	0	0	0	0	$1-p$	p
S_4^{cw}	0	0	0	0	0	p	$1-p$	0	0
S_{dt}	0	0	0	0	0	0	0	0	1

Figure: Original problem

Reducing the size of the patrolling graph

Given following patrolling problem, remove unnecessary nodes and edges from the graph.

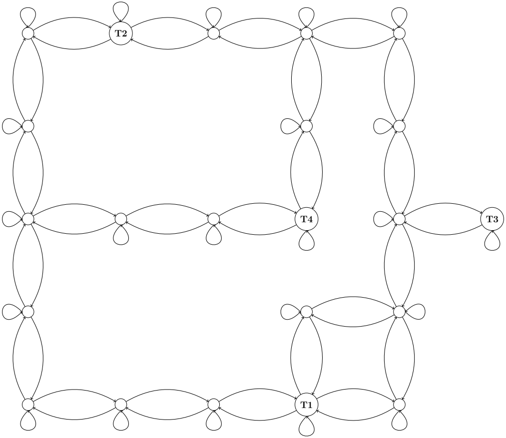
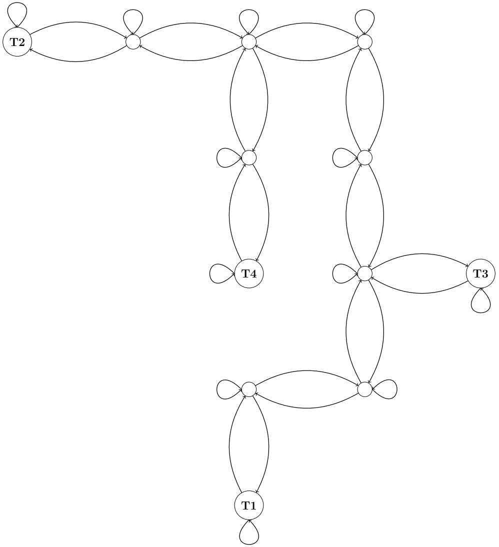


Figure: Given patrolling problem.

Example of reduced graph



Area patrol program

We assume $\forall t \in T: v_a(t) = v_d(t)$

$a(i,j) = 1$ if the patrol can move form i to j in one step; else 0

$P_c(t,h)$ is the probability of stopping an attack at target t started when the patrol was at node h

$\gamma_{i,j}^{w,t}$ is the probability that the patrol reaches node j from i in w steps without visiting target t

$$\max u$$

$$\alpha_{i,j} \geqslant 0 \quad \forall i, j \in V$$

$\alpha_{i,j}$ is the probability of moving from i to j

$$\sum_{j \in V} \alpha_{i,j} = 1 \quad \forall i \in V$$

$$u_{\mathbf{d}}(x) = \begin{cases} \sum_{i \in T} v_{\mathbf{d}}(i), & x = \textit{intruder-capture} \text{ or } \textit{no-attack} \\ \sum_{i \in T \setminus \{t\}} v_{\mathbf{d}}(i), & x = \textit{penetration-t} \end{cases}$$

$$\alpha_{i,j} \leqslant a(i,j) \quad \forall i, j \in V$$

$$\gamma_{i,j}^{1,t} = \alpha_{i,j} \quad \forall t \in T, \ i, j \in V \setminus \{t\}$$

$$\gamma_{i,j}^{w,t} = \sum_{x \in V \setminus \{t\}} \left(\gamma_{i,x}^{w-1,t} \alpha_{x,j} \right) \quad \forall w \in \{2, \ldots, d(t)\}, \ t \in T, \ i, j \in V \setminus \{t\}$$

$$P_c(t,h) = 1 - \sum_{j \in V \setminus \{t\}} \gamma_{h,j}^{d(t),t} \quad \forall t \in T, \ h \in V$$

$$u \leqslant u_{\mathbf{d}}(\textit{intruder-capture}) P_c(t,h) + u_{\mathbf{d}}(\textit{penetration-t}) \big(1 - P_c(t,h)\big)$$

Resulting program

$V = \{s_i^{cc}, s_i^{cw}\} \forall i \in \{0, \cdots, d-1\}$ is set of all possible states and V^k refers to $V \setminus \{s_k^{cc}, s_k^{cw}\}$

$$
\begin{aligned}
max \quad & u & \\
p \in \quad & [0,1] & \\
\alpha_{s_i^{cc}, s_{i+1 \pmod d}^{cc}} = \quad & p & \forall i \in \{0, \cdots, d-1\} \\
\alpha_{s_i^{cw}, s_{i-1 \pmod d}^{cw}} = \quad & p & \forall i \in \{0, \cdots, d-1\} \\
\alpha_{s_i^{cw}, s_i^{cc}} = \quad & 1 - p & \forall i \in \{0, \cdots, d-1\} \\
\alpha_{s_i^{cc}, s_i^{cw}} = \quad & 1 - p & \forall i \in \{0, \cdots, d-1\} \\
\gamma_{i,j}^{1,k} = \quad & \alpha_{i,j} & \forall k \in \{0, \cdots, d-1\}, \forall i, j \in V^k \\
\gamma_{i,j}^{w,k} = \quad & \sum_{x \in V^k} \gamma_{i,x}^{w-1,k} \alpha_{x,j} & \forall k \in \{0, \cdots, d-1\}, \forall i, j \in V^k, \forall w \in \{2, \cdots, t\} \\
P_k = \quad & 1 - \sum_{x \in V^k} \gamma_{s_0^{cc}, x}^{t,k} & \forall k \in \{0, \cdots, d-1\} \\
u \leq \quad & 1 - P_k & \forall k \in \{0, \cdots, d-1\}
\end{aligned}
$$

The End