

Game theory - lab 2

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Illuminating Orthogonal Polygons

Proof the following theorem

Theorem

$\lfloor \frac{n}{4} \rfloor$ stationary guards are always sufficient and occasionally necessary to illuminate a orthogonal polygon with n vertices.

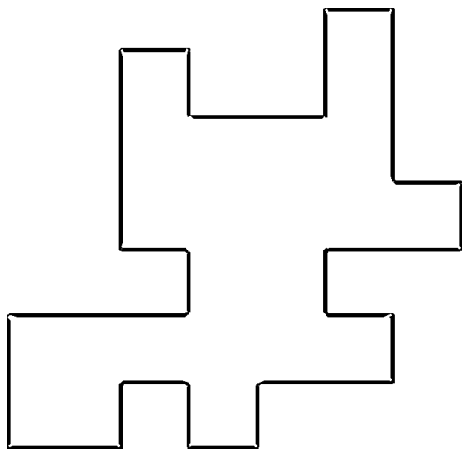


Figure: Orthogonal polygon with 24 vertices

First part of the proof

Theorem

Any orthogonal polygon is convex quadrilaterizable.

Proof.

Showing that every orthogonal polygon can be split to smaller polygons eventually resulting in convex quadrilaterals. \square

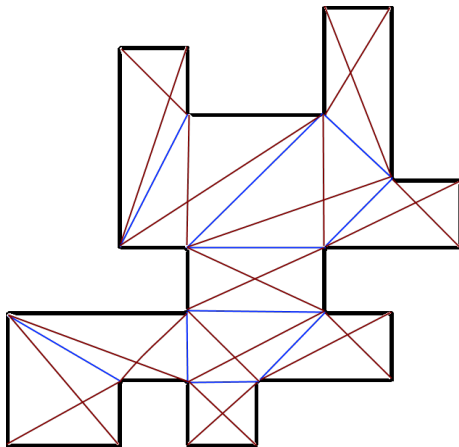


Figure: Quadrilateralization of the polygon (blue lines) and diagonals of resulting quadrilaterals (red)

End of the proof

Theorem

Quadrilateralized polygon with diagonal edges in quadrilaterals forms a 4-colorable graph.

Proof.

Dual graph Q is clearly a tree. When Q is one quadrilateral it is 4-colorable. By induction show that Q with added leaf quadrilateral is still 4-colorable. \square

By the construction of the graph each quadrilateral has all four colors. Therefore, placing guards to one color illuminates the polygon. Selecting color with the fewest vertices gives the result.

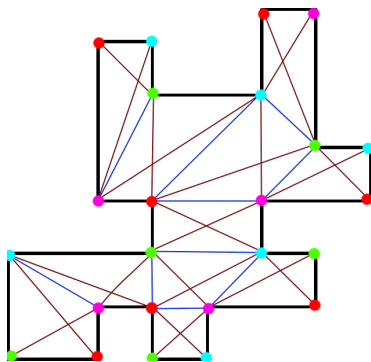


Figure: 4-colored polygon.

Necessary

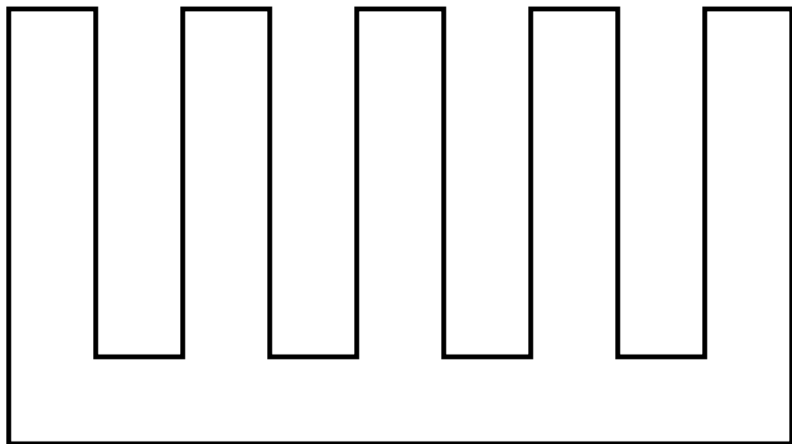


Figure: Class of polygons where $\lfloor \frac{n}{4} \rfloor$ guards is necessary.

Linear number of recontamination necessary

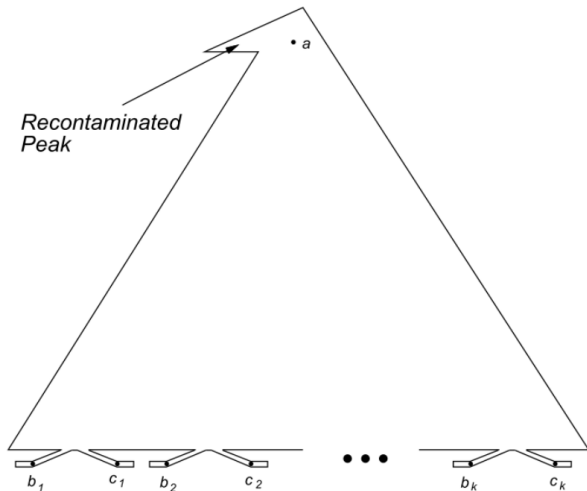


Figure: Class of polygons where peak is recontaminated linearly to n .

Gap edges

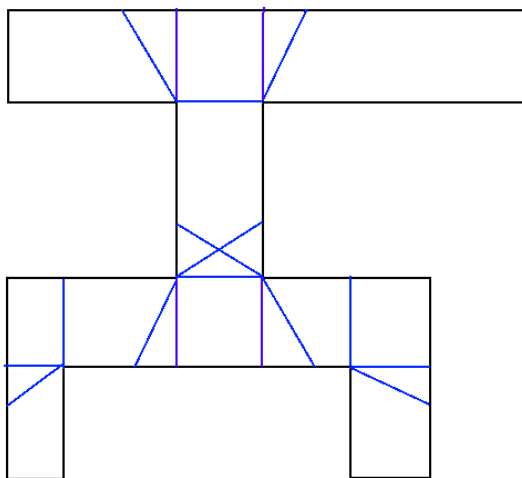


Figure: Gap edges in the example polygon.

Double Oracle

Try Double Oracle algorithm on the following matrix game

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Player 1 selects $W \rightarrow A = 1, V = 0, W = 1$

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Player 2 selects D \rightarrow solve matrix game

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Strategy for Player 2 $A = \frac{1}{8}, D = \frac{7}{8}$

	A	B	C	D	E	
V	-8	9	0	7	-6	5.125
W	6	9	6	5	6	5.125
X	1	-8	3	8	7	7.125
Y	5	2	6	-5	2	-3.75
Z	4	3	3	0	8	0.5

Double Oracle

Player 1 selects X \rightarrow solve matrix game

	A	B	C	D	E
V	-8	9	0	7	-6
W	6	9	6	5	6
X	1	-8	3	8	7
Y	5	2	6	-5	2
Z	4	3	3	0	8

Double Oracle

Strategy for Player 2 $A = \frac{3}{8}, D = \frac{5}{8}$
Strategy for Player 1 $V = 0, W = \frac{7}{8}, X = \frac{1}{8}$

	A	B	C	D	E	
V	-8	9	0	7	-6	1.375
W	6	9	6	5	6	5.375
X	1	-8	3	8	7	5.375
Y	5	2	6	-5	2	-1.25
Z	4	3	3	0	8	1.5
	$5\frac{3}{8}$	$6\frac{7}{8}$	$5\frac{5}{8}$	$5\frac{3}{8}$	$6\frac{1}{8}$	

Proof the following theorem

Theorem

Each polygon with $n \geq 6$ vertices has a diagonal splitting it to two and both have $\leq \frac{2n}{3}$ vertices

The End