Special and common methods for planning and problem complexity

- Why the *C-space* use is efficient?
- Complexity of a path planning and its' relation to the *C-space*
- Complete and incomplete approaches
- Potential field-based planning
- Space decomposition approach, examples
- Using roadmaps for planning

- References

Why the *C-space* based planning stands efficient?

- Reduces complexity of the planning approach/solution for robots with physical dimensions (many constraints) in Euclidean space $\longrightarrow$ substitute of complex constrains/cases by multi-dimensional space (*C-space*) and point-like robot (simplifies implementation of the planning approach, reduces number of the planning constraints by additional *C-space* dimensions, that stand for these constraints) process.
- *C-space* stands for unified framework good for comparison and evaluation of various planning algorithms

Major drawback(s):

- The motion and path planning is *continuous* from principle (given by the *C-space* definition)

Which can be resolved via:

- Making the planning space discrete (the *C-space*)
- Making the trajectory discrete
- Discretizing both the previous items

Complexity of the path-planning

- Complete kind of path planning (rare) is computationally intensive. A „complete planner" either: (a) finds an admissible solution (path), or (b) reports, that a solution doesn't exist

  ...which needs to search through the whole state space.

- More common approaches rely on incomplete methods (approximate methods), that: (a) fetch at least „some" solution (mainly not very optimal) but being delivered in much shorter time (or in a given time, „any time algorithms")

  or

  (b) do not find any solution at all (nevertheless, any confidence, that there is no solution does not still exists in such cases)

- Essentially, the complete methods exhibit a computational complexity of:
  - Exponential order with the *C-space* dimension (corresponds to degrees of freedom)
  - Polynomial order with the complexity of C_{obst} in the *C-space* (obstacles, number of their borders, the order of their algebraic description)

2+1 basic (and complete) ways to resolve a planning problem

(1) A complete decomposition of the workspace (an exact cell-decomposition)

- Double-exponential complexity $\sim 2^{2^d}$, where *d* stands for space dimension
- Based on the principle of decomposing the C_{free} into simple unique regions (= elementary cells, pixels or other primitives) and related connectivity relation between these (i.e. a graph of neighborhood)

(2) A method of roadmaps

- Simple-exponential complexity 2^d , where *d* denotes the space dimension
- Relies on computation of a „silhouette“ of the C_{free} space. Represents connectivity in C_{free} by a graph in a form of a network of 1D curves (transitions inbetween nodes, or roads)

The previous holds for a complete planning (which is not very practical), so simplification makes the task easier to compute:

- Simplifying geometry/shape of the robot/obstacles via their approximation
- Limiting of number of DoFs $\longrightarrow$ constraining the dimension of the workspace
- Simplifying of road(s) description(s), decreasing the order of trajcetories, i.e to linear segments, etc.

Therefore the planning is typically performed as a 2-step procedure as:

(1) Determination of the connectivity of the free workspace C_{free} and representing it as a graph (or as a function)
(2) Search for the final path in the graph (or search along the function values)

Following the afore aspects, the other possible method for planning enables to approach the problem as an (objective) function optimization problem - a „potential field" approach

The potential field approach

- Takes the advantage of a potential field kind of functions (harmonic potentials), continuos and smooth functions that satisfy the additional Laplace condition:

$$\nabla^2 f\left(\vec{x}\right) = 0$$

denoting the $f\left(\vec{x}\right)$ as a conservative field function, which is differentiable at any point and exhibits monotonic and steady sinking (or rising) behavior and has only a single and global extreme at the loci of the target configuration (position).

- As to the afore mentioned approach, the optimal path can be determined by performing a steepest descent (or ascent) search along the function values.
- Computational complexity of the solution is proportional (linear) to the path length (or number of transitions/steps if the case of discrete representation of the path)
- The idea of the potential field approach is bases on creating vector (gradient) field (i.e. a force-field of virtual forces) using the aforementioned potential (differentiable) function U

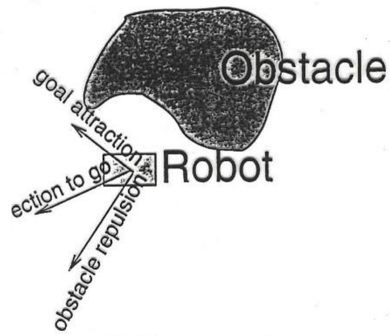
$$U: \; \mathcal{C}_{free} \longrightarrow \Re, \text{ so that } \quad \vec{F}(q) = -\vec{\nabla} U(q)$$

- The afore force field $\vec{F}(x)$ then attracts the robot to the goal position, whilst it does not guarantee, that the robot will move right along an obstacle border - a problem (!).

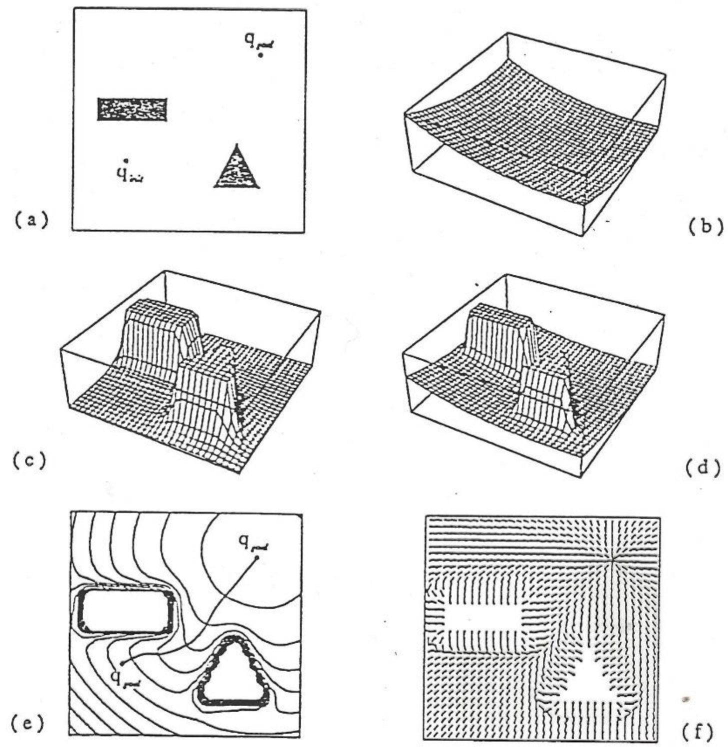
Can be resolved by using yet another potential field, that repulses the robot from the obstacle border, so that:

$$\vec{F}(x) = \vec{F}_{att} + \vec{F}_{rep} = -\vec{\nabla} U_{att} - \vec{\nabla} U_{rep}$$

- Goal



The basic situation for the potential field planning.



(a) The scene setup, (b) Attractive potential, (c) Repulsive potential, (d) Superposition of the repulsiveand attractive potentials, (e) Equivalent potentials lines, (f) Force vector field

Building the potentials

Example 1:

(a) The electric field in homogenous conductive environment (i.e. rezistive foil (2D), or liquid (3D)) The obstacles represented by insulated regions.

(b) A liquid flowwing through an environment. The obstacles represented physically by themselves.

Example 2: The „harmonic“ potential function is denoted by the additional condition $\nabla^2 f(x) = 0$ (a conservative field) does not exhibit any local extremes and assures finding of the solution always. The harmonic field is far more costly to be computed. Electric or gravitation field are conservative and generate harmonic potentials. Magnetic field is not conservative and denotes simple potential field.

The potential filed for path planning usage is normally generated in an arificial way, an example of possible buildup:

The attractive field: $U_{att}(q) = \frac{1}{2}\xi \left\| q - q_{goal} \right\|^2$, where ξ denotes scale $\xi \in \mathbb{R}^+$

and the term $\| \|$ stands for Euclidean distance

Besides, always $U_{att}(q) \geq 0$ and $U_{att}(q_{goal}) = 0$ as well as U_{att} is continuously differentiable $q \in C_{free}$, so that always exists: $\vec{F}_{att}(q) = -\vec{\nabla} U_{att}(q) = -\xi(q - q_{goal})$

The repulsive field:

- Creates a barrier in a vicinity of the obstacle to prevent the robot to get too close to, and collide with the obstacle
- Frequent common requirement is, that a robot in sufficiently large distance is not influsenced by the repulsive field at all:

$$U_{rep}(q) = \frac{1}{2}\eta\left(\frac{1}{r(q)} - \frac{1}{r_0}\right)^2, \forall\ r(q) \le r(q_o)$$

$$U_{rep}(q) = 0, \text{ othewise}$$

Wherereis r_0 stands for the distance of influence and the squared term above denotes the inverted distance between the robot and the obstacle such that: $r(q) = \min_{q' \in obstacle} \left\| q - q' \right\|$

and featuring: $U_{rep}(q) = 0, \forall\ r(q) \ge r_0$ as for the distance of influence

and $U_{rep}(q) \to \infty; \forall\ r(q) \to 0$ as for the obstacle

Since the boundary of the obstacle is at least piecewise continuously differentiable, the repulsive force stands:

$$\vec{F}_{rep}(q) = -\vec{\nabla} U_{rep}(q) = \eta\left(\frac{1}{r(q)} - \frac{1}{r_0}\right)\frac{1}{r^2(q)}\vec{\nabla} r(q); \forall\ r(q) \le r_0$$

$$\vec{F}_{rep}(q) = -\vec{\nabla} U_{rep}(q) = 0, \text{ otherwise}$$

Computation of the robot path using the potential field approach

The main steps:

(1) Discretization of the robot workspace C_{free}

(2) Computation of the potentila function over the robot workspace with the minimal value posed at q_{goal}

(3) Search for the optimal (steepest descent) path from teh current standpoint to the goal q_{goal} (gradient driven optimization, best-first search, greedy approach, etc.)

Further remarks

- In the case of using regular (non-harmonic) potential function the search procedure may stuck in local extreme of the force-field. This can be resolved in multiple ways: restart of the search with modified initial conditions, simulated anealing, etc.
- Application of a „randomized potential – a combination of a „potential-based“ method and a „random walk“ method as: