Learning for vision IV training & layers

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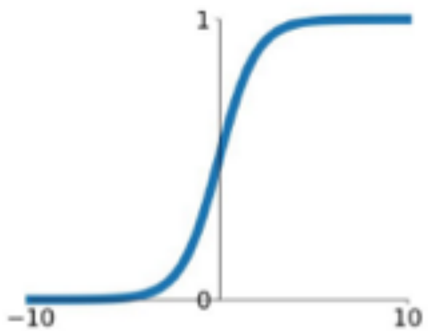
Outline

- layers:
  - activation function (i.e. non-linearities)
  - batch normalization layer
  - max-pooling layer
  - loss-layers
- summary of the learning procedure
  - train, test, val data,
  - hyper-parameters,
  - regularizations

Activation functions

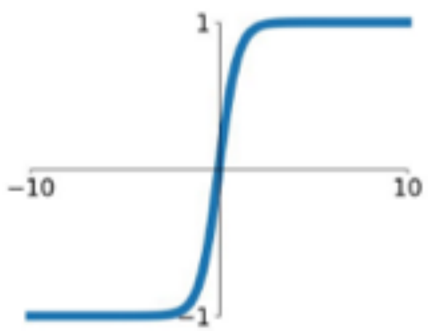
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



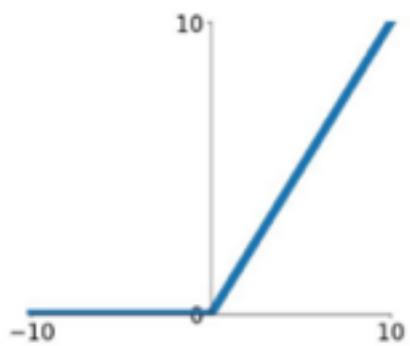
tanh

$\tanh(x)$



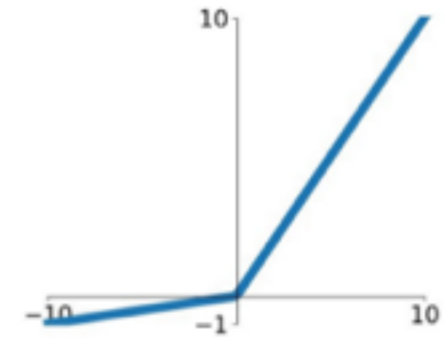
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

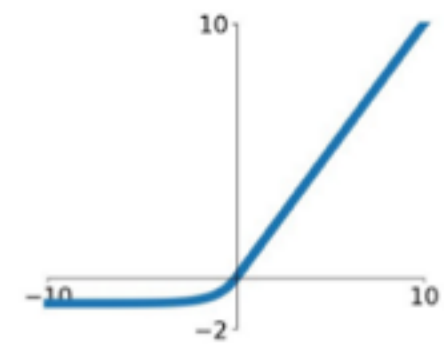


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

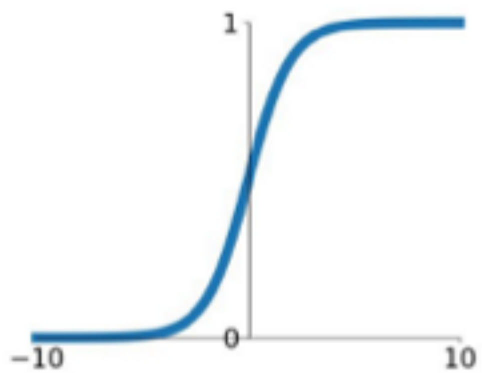
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



- what happen to backprop gradient when weights are **huge**?

Sigmoid

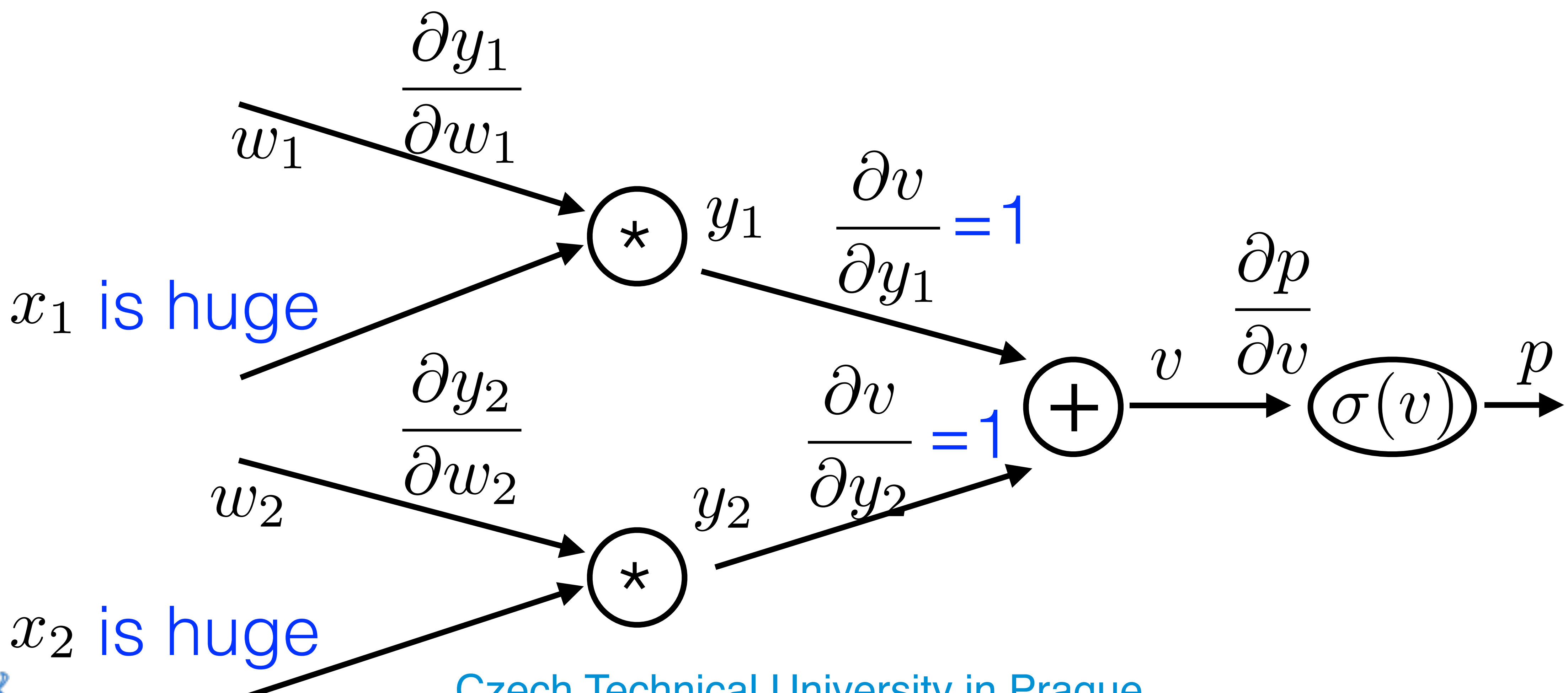
$\sigma(x) = \frac{1}{1+e^{-x}}$



- zero gradient when saturated
- not zero-centered (pos. output)
- computationally expensive

$$\frac{\partial p}{\partial w_1} = \frac{\partial y_1}{\partial w_1}\frac{\partial v}{\partial y_1}\frac{\partial p}{\partial v} = 0$$

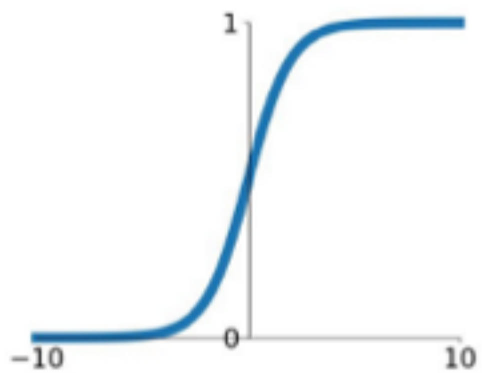
$$\frac{\partial p}{\partial w_2} = \frac{\partial y_2}{\partial w_2}\frac{\partial v}{\partial y_1}\frac{\partial p}{\partial v} = 0$$



Activation functions

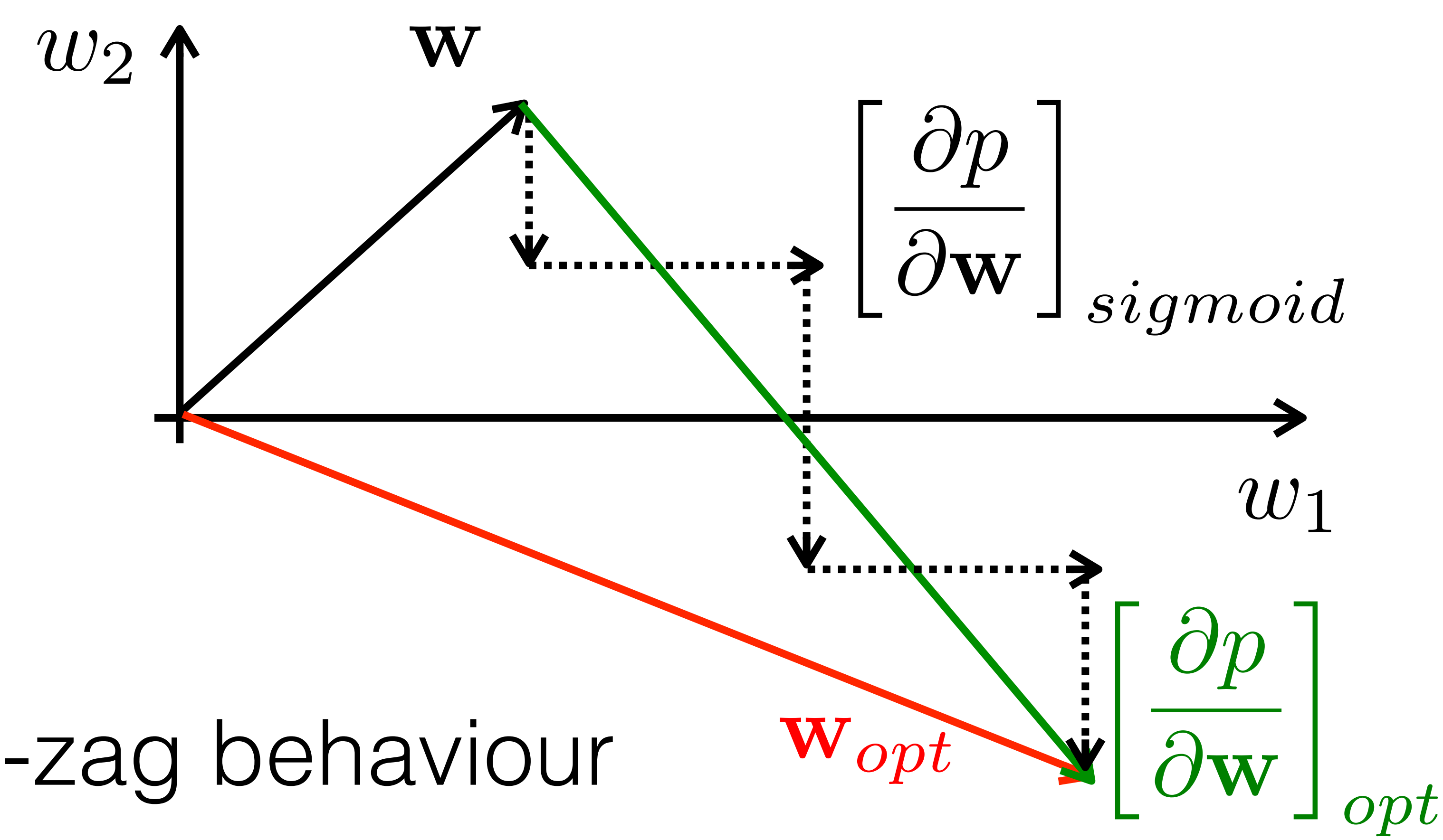
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



- zero gradient when saturated
- not zero-centered (pos. output)
- computationally expensive

$$\frac{\partial p}{\partial w_1} = x_1 \cdot 1 \cdot \frac{\partial p}{\partial v} \quad \begin{matrix} >0 \\ <0 \end{matrix} \qquad \frac{\partial p}{\partial w_2} = x_2 \cdot 1 \cdot \frac{\partial p}{\partial v} \quad \begin{matrix} >0 \\ <0 \end{matrix}$$

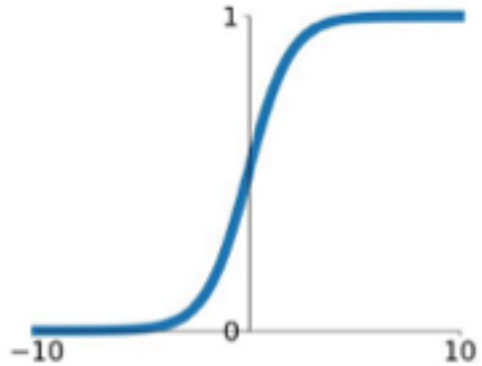


Undesired zig-zag behaviour

Activation functions

Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



- zero gradient when saturated
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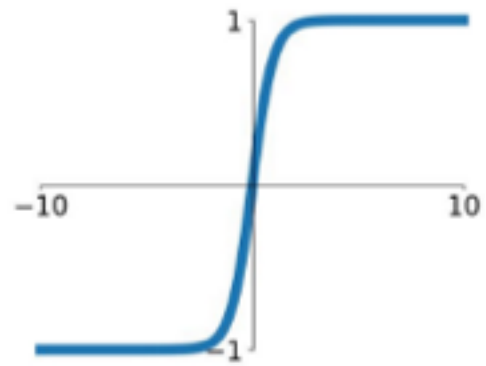
Activation functions

tanh

$\tanh(x)$

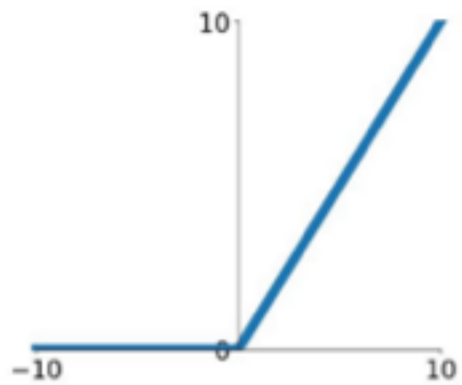


- zero gradient when saturated
- ~~not zero-centered (only positive ouputs)~~
- computationally expensive

Activation functions

ReLU

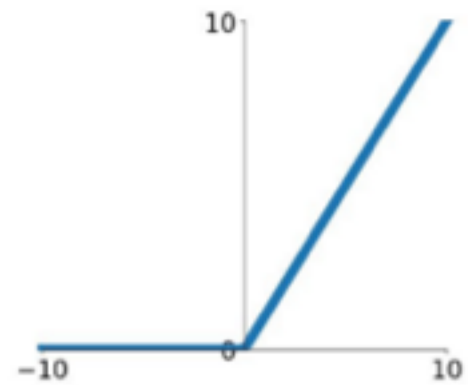
$\max(0, x)$



- ~~zero gradient when saturated~~ *(partially => dead ReLU!)*
- not zero-centered (only positive ouputs)
- ~~computationally expensive~~

ReLU

$\max(0, x)$



- ~~zero gradient when saturated~~ *(partially => dead ReLU!)*
- not zero-centered (only positive ouputs)
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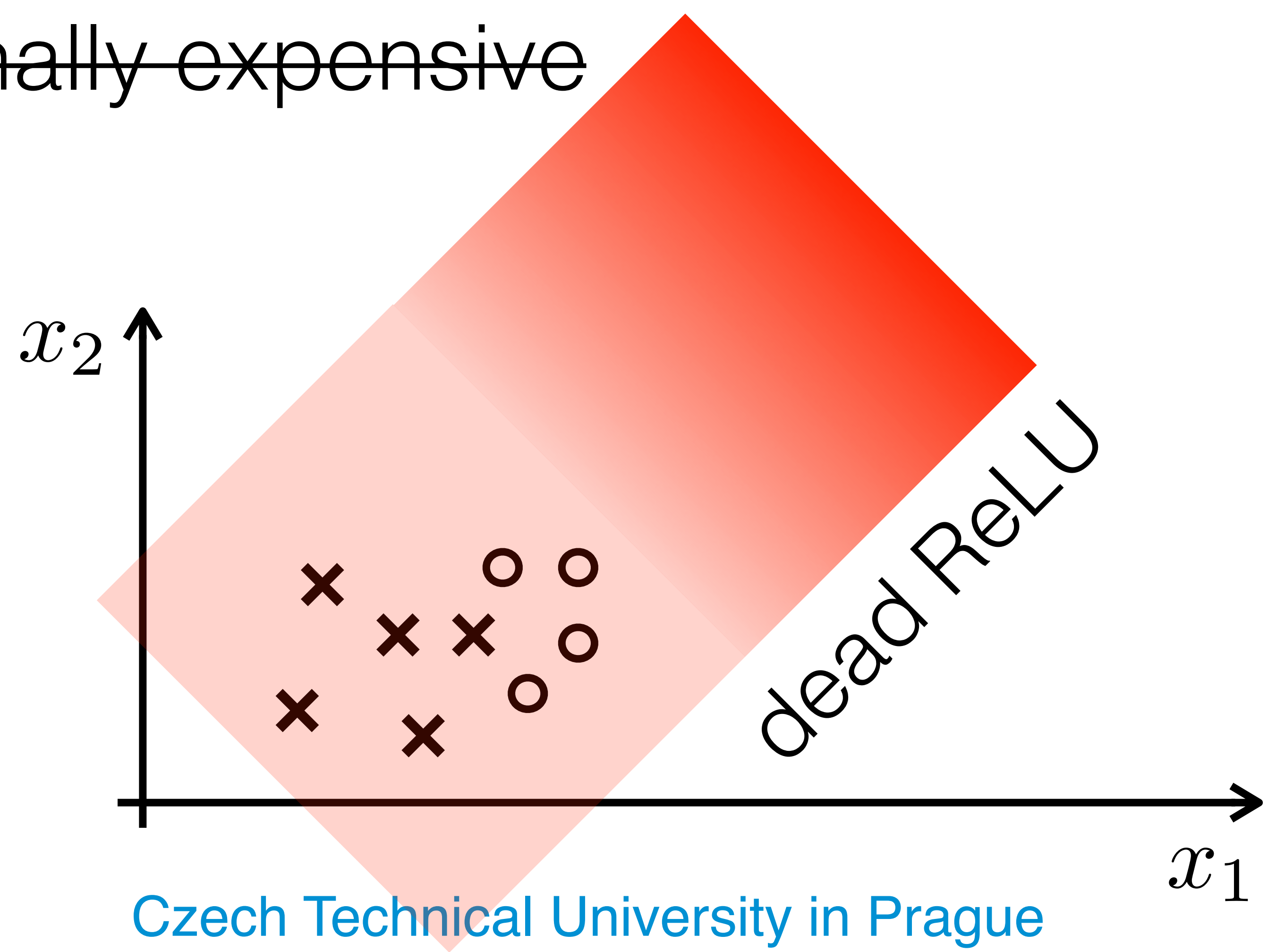

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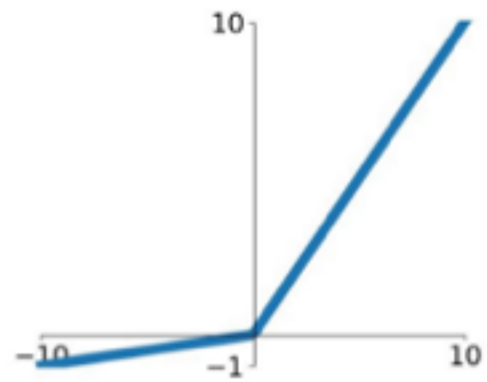


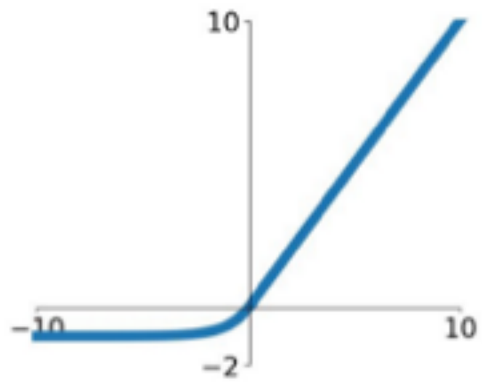
- ~~zero gradient when saturated~~
- ~~not zero-centered (only positive ouputs)~~
- ~~computationally expensive~~

Small gradient for negative values give tiny chance to recover

Activation functions

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



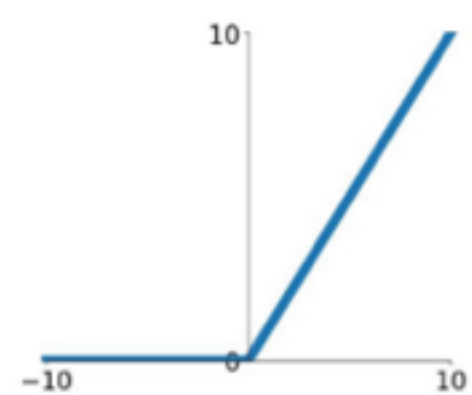
- ~~zero gradient when saturated~~ *(partially)*
- ~~not zero-centered (only positive ouputs)~~
- computationally expensive

Summary

- Use ReLU and avoid undesired properties by
  - good weight initialization
  - data preprocessing
  - batch normalization
- Still you want to keep “reasonable values”
(i.e. small but not too much and distributed around zero)

ReLU

$\max(0, x)$



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Data preprocessing & initializations

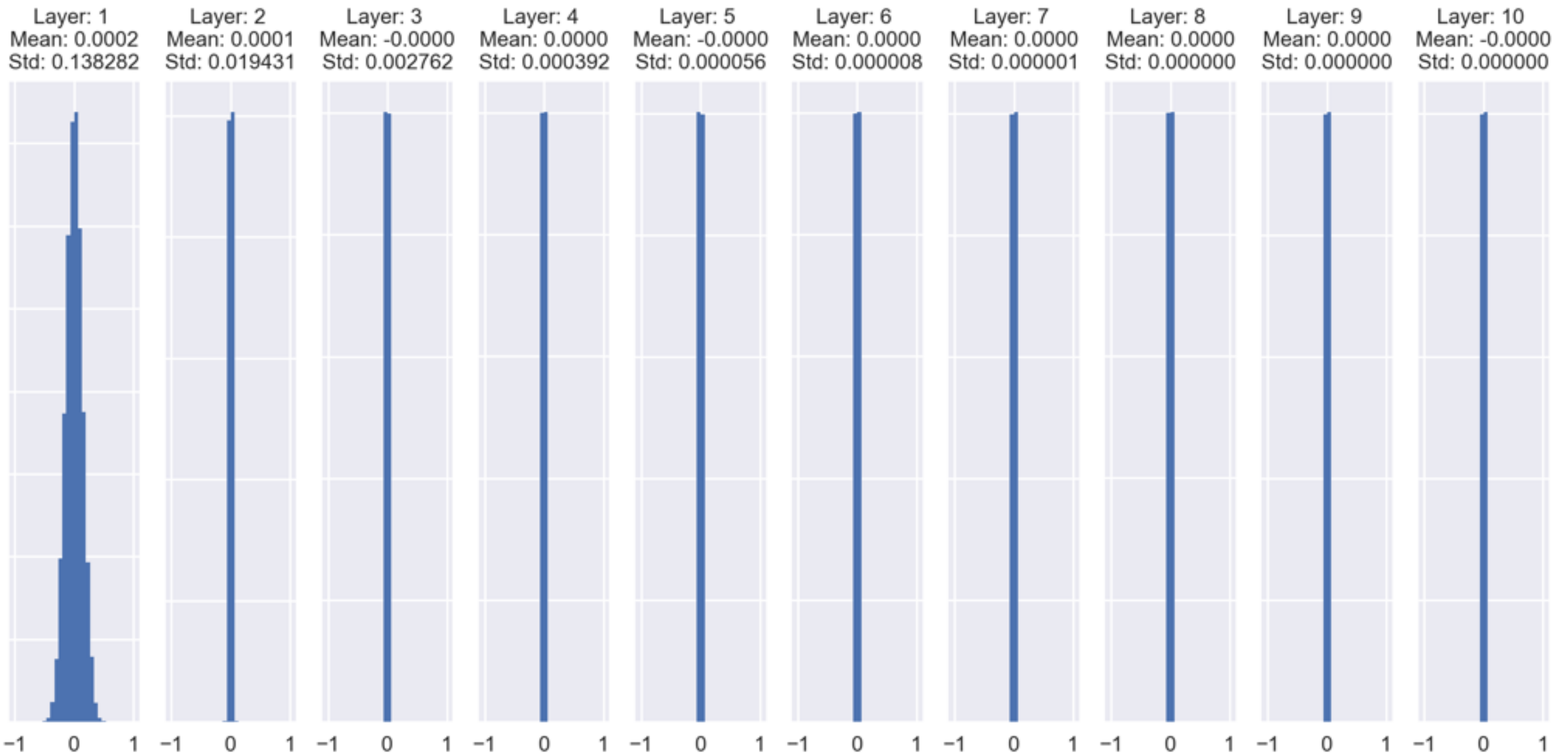
- Pixels values shifted zero mean to avoid only positive inputs and the unwanted “zig-zag” behaviour

Data preprocessing & initializations

- Pixels values shifted zero mean to avoid only positive inputs and the unwanted “zig-zag” behaviour
- Weight initialization:
  - $\mathbf{w} = 0$ all gradients the same
  - $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma)$ diminishing gradients in backprop
  - $\mathbf{w}^{(i)} \sim \mathcal{N}(\mathbf{0}, \sigma * 1/N^{(i)})$ preserves variance of signal among layers (Xavier init [Glorot 2010])

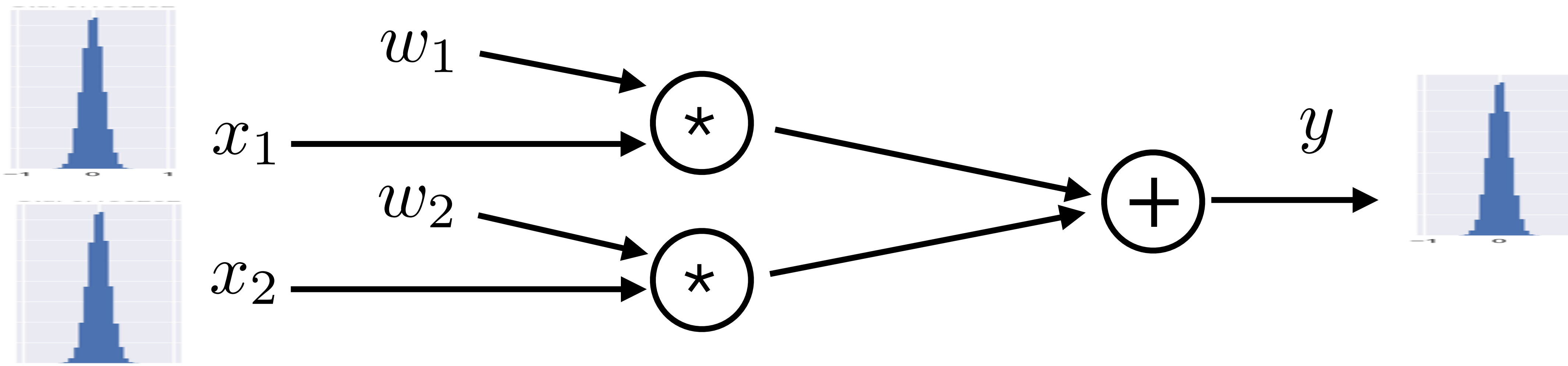
Xavier initialization [Glorot 2010]

Signal in randomly initialized weights $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma)$ forward (and backward) pass



Xavier initialization [Glorot 2010]

- We want to preserve variance of signal among layers (i.e. $\text{var}(y) = \text{var}(x_i)$)



$$\text{var}(y) = \text{var}(w_1 x_1 + w_2 x_2 + \cdots + w_N x_N) =$$

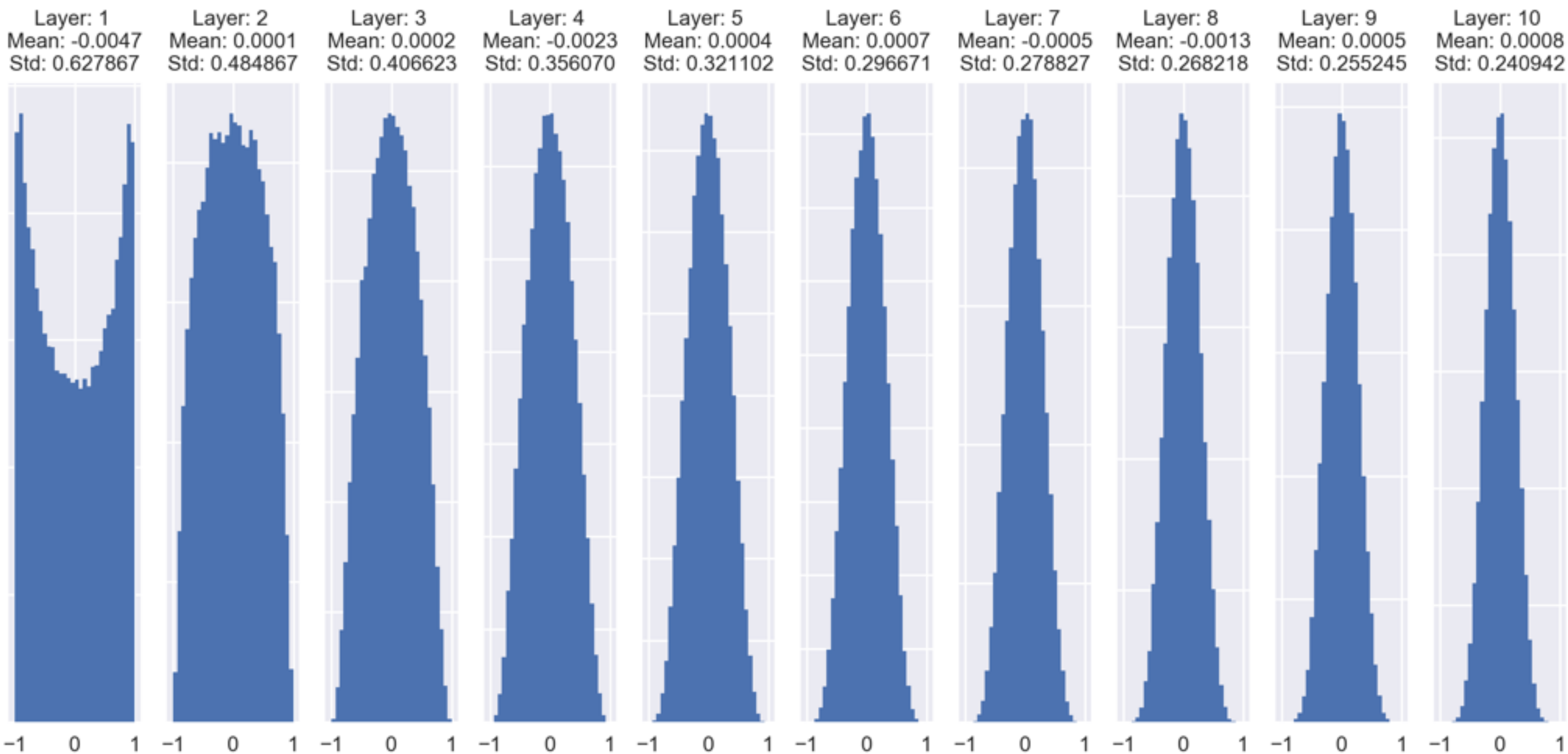
$$= \sum_{i=1}^{N} \mathbb{E}(x_i)^2 \text{var}(w_i) + \mathbb{E}(w_i)^2 \text{var}(x_i) + \text{var}(w_i x_i) =$$

$$= \sum_{i=1}^{N} \text{var}(w_i)\text{var}(x_i) = N * \text{var}(w_i)\text{var}(x_i)$$

$$\Rightarrow N * \text{var}(w_i) = 1$$

Xavier initialization [Glorot 2010]

Signal in Xavier initialized weights $\mathbf{w}^{(i)} \sim \mathcal{N}(\mathbf{0}, \sigma * 1/N^{(i)})$ forward (and backward) pass (better but not ideal)



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Batch normalization layer [Ioffe and Szegedy 2015]
https://arxiv.org/pdf/1502.03167.pdf (over 6k citation)

- **Learning:**
- Normalize each dimension of input feature map in each layer
- Learn parameters $\gamma^{(k)},\ \beta^{(k)}$ for each dimension k

$x_1 \dots x_m$ →

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\dots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

→ $y_1 \dots y_m$

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$$\sigma^2_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} (x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

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$$\widehat{x}_i \leftarrow \frac{x_i - \mu_\mathcal{B}}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \qquad \text{// normalize}$$

→ $y_1 \dots y_m$

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$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

→ $y_1 \dots y_m$

Batch normalization layer [Ioffe and Szegedy 2015]
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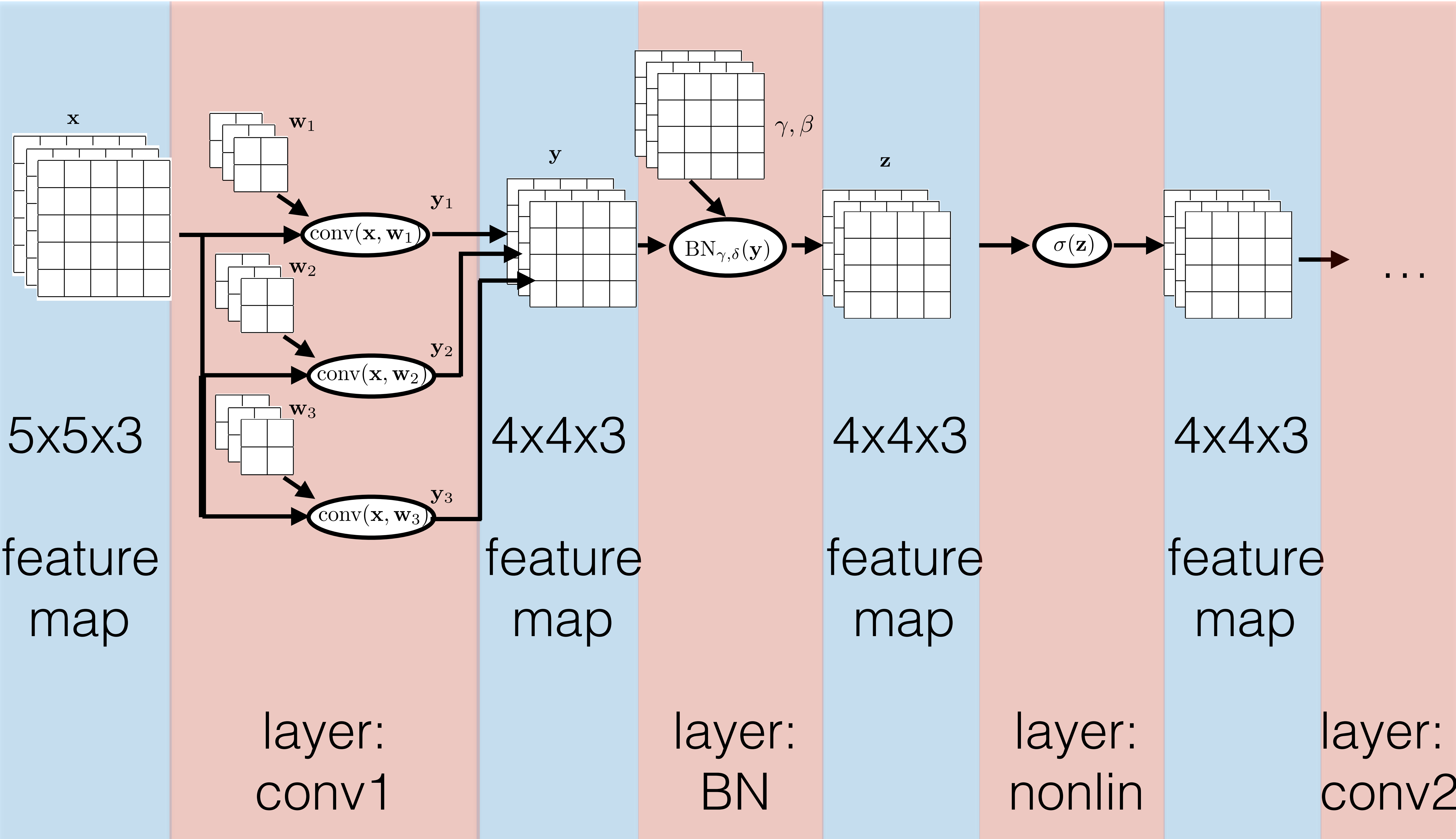
- **Inference:** estimate $\mathbb{E}(x^{(k)})$ and $\mathrm{var}(x^{(k)})$
- Use learned parameters $\gamma^{(k)}$, $\beta^{(k)}$ and $\mathbb{E}(x^{(k)})$, $\mathrm{var}(x^{(k)})$

$$\mathbf{x} = [x^{(1)} \dots x^{(n)}]^\top \qquad \mathbf{y} = [y^{(1)} \dots y^{(n)}]^\top$$

$$y^{(k)} = \gamma^{(k)} \left(\frac{x^{(k)} - \mathbb{E}(x^{(k)})}{\mathrm{var}(x^{(k)})} \right) + \beta^{(k)}$$

Batch normalization layer [Ioffe and Szegedy 2015] https://arxiv.org/pdf/1502.03167.pdf (over 6k citation)



Batch normalization layer [Ioffe and Szegedy 2015]
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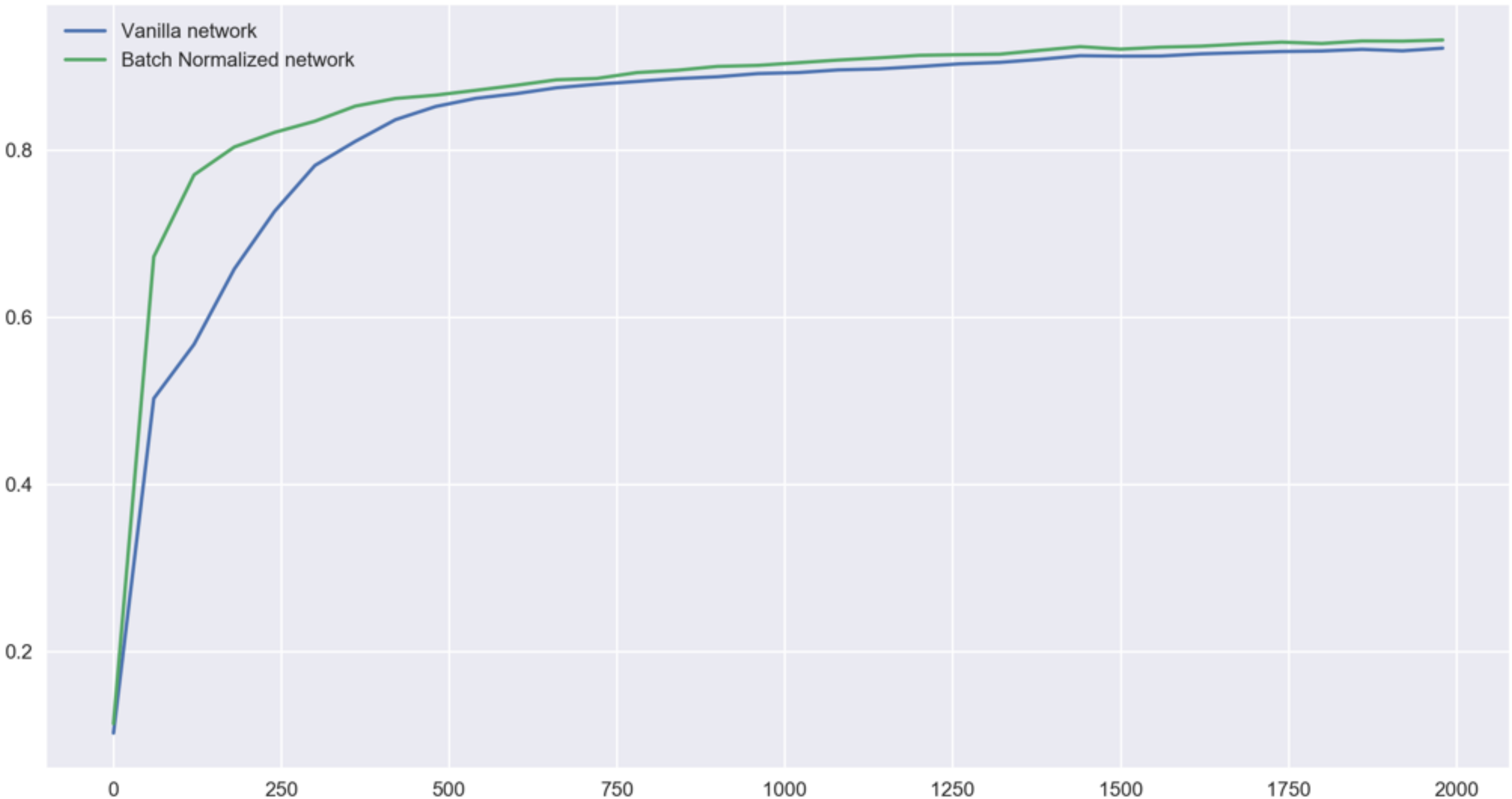
Good weight initialization



Batch normalization layer [Ioffe and Szegedy 2015]

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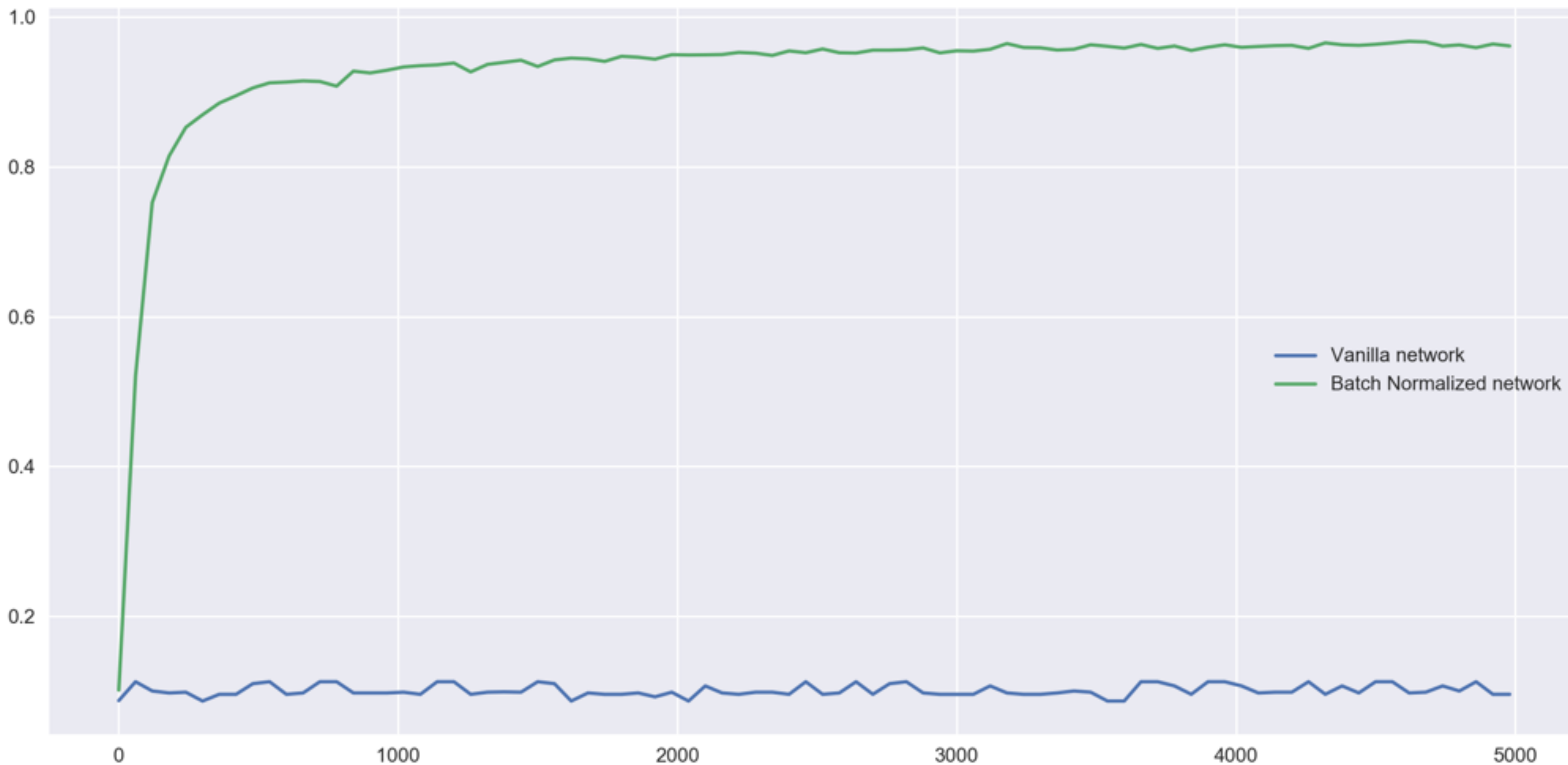
Bad weight initialization



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Summary

- Normalize each dimension of input feature map in each layer independently.
- Different behaviour for learning and inference
- BN yields
  - Reduced learning time
  - Model regularizer (one training example always normalized differently => small jittering of each sample)
  - Reduce dependency on good weight initialization

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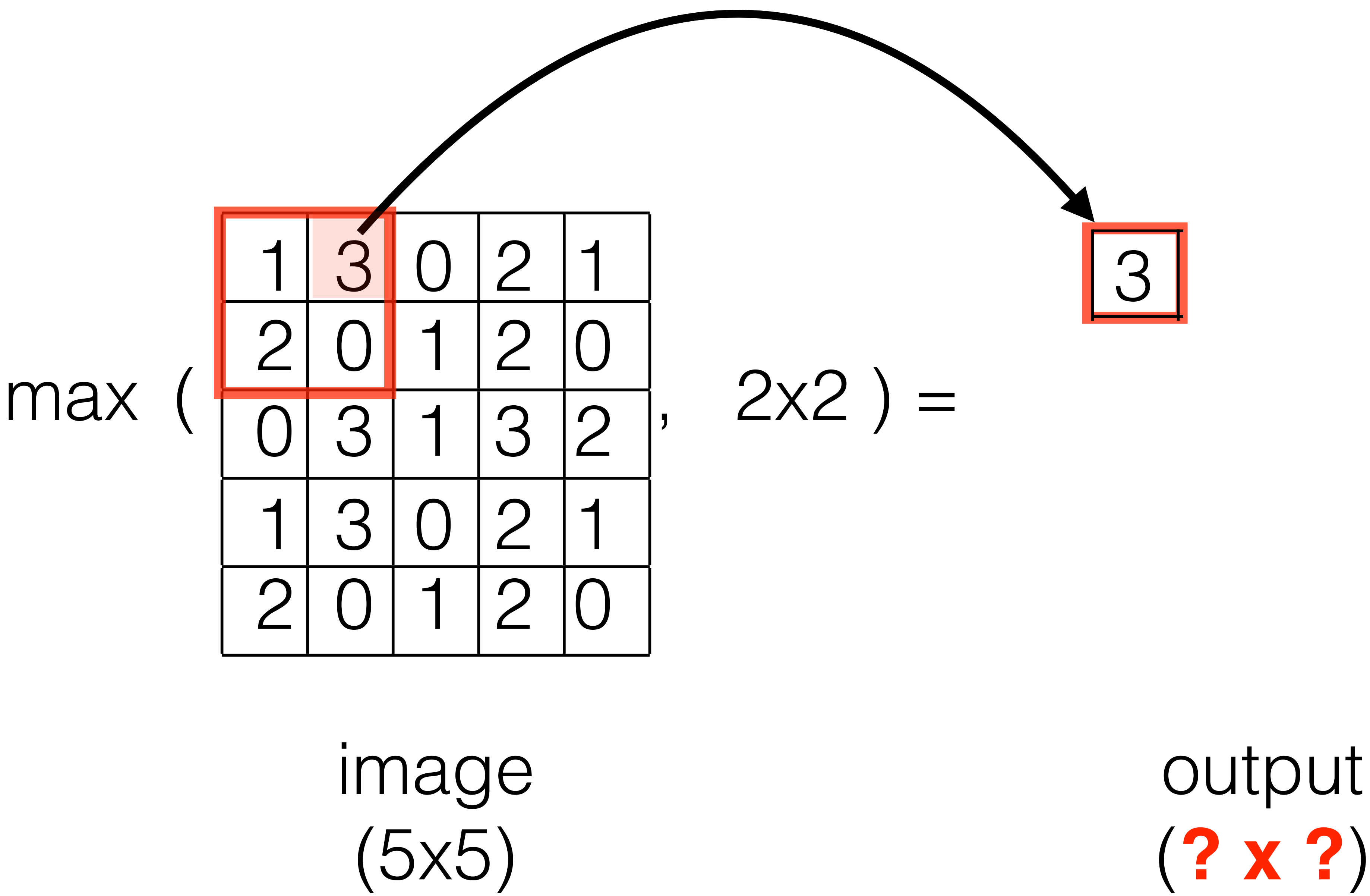
Max-pooling



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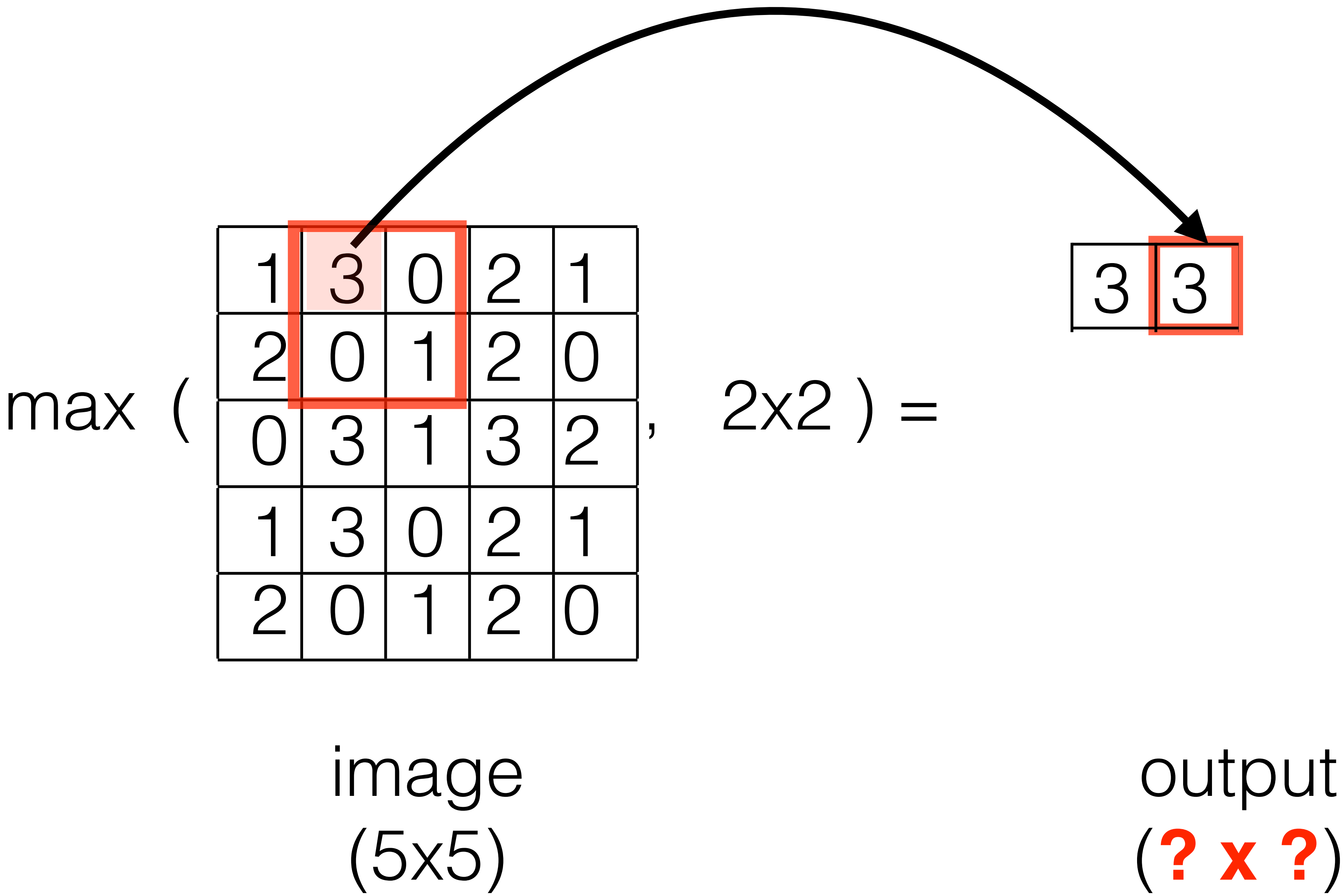

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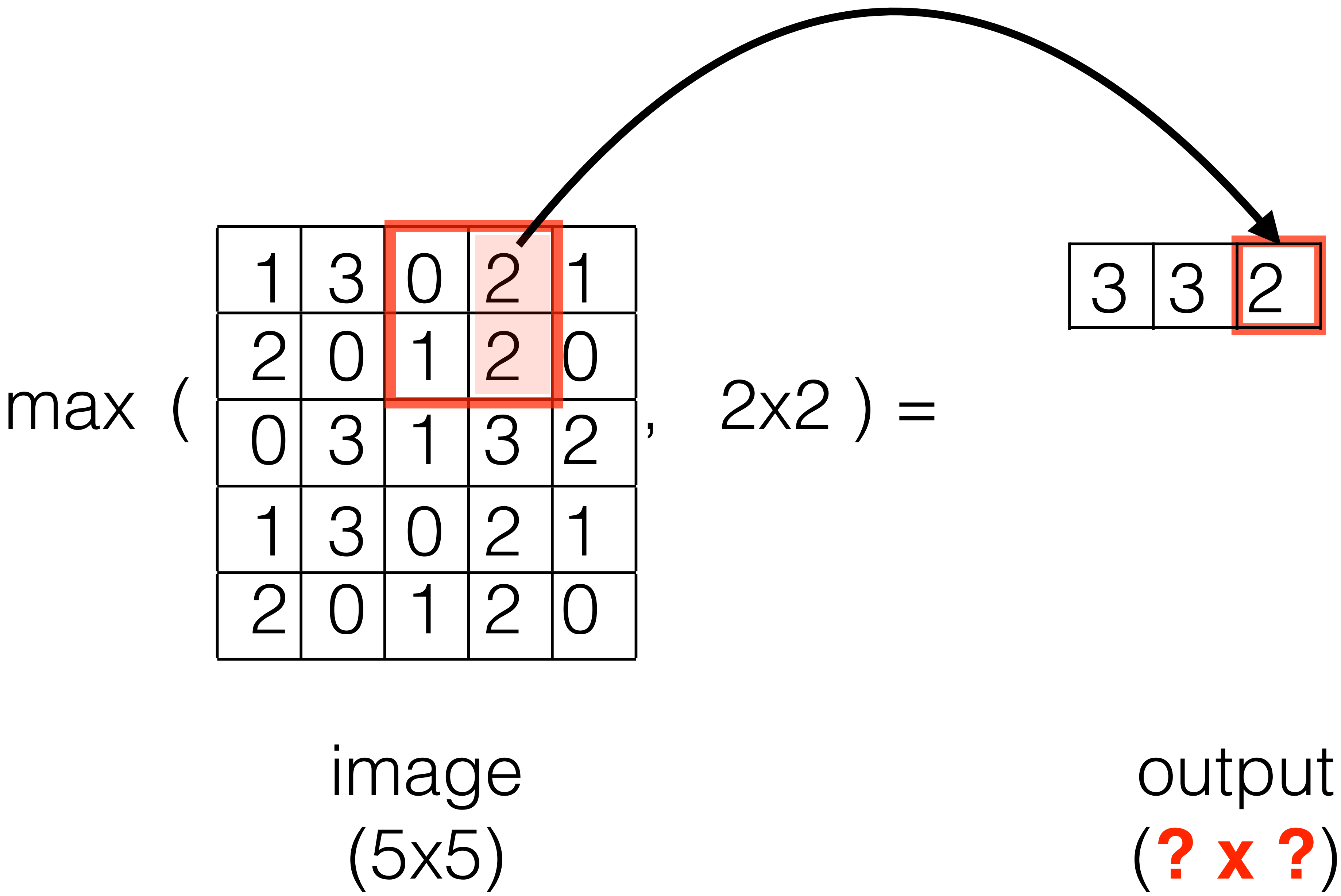


Max-pooling

max (image, 2x2) =

image (5x5):

1	3	0	2	1
2	0	1	2	0
0	3	1	3	2
1	3	0	2	1
2	0	1	2	0

output (**4 x 4**):

3	3	2	2
3	3	3	3
3	3	3	3
3	3	2	2

Max-pooling

$$M = (N+2*pad-K) / stride + 1$$

The same as for convolution

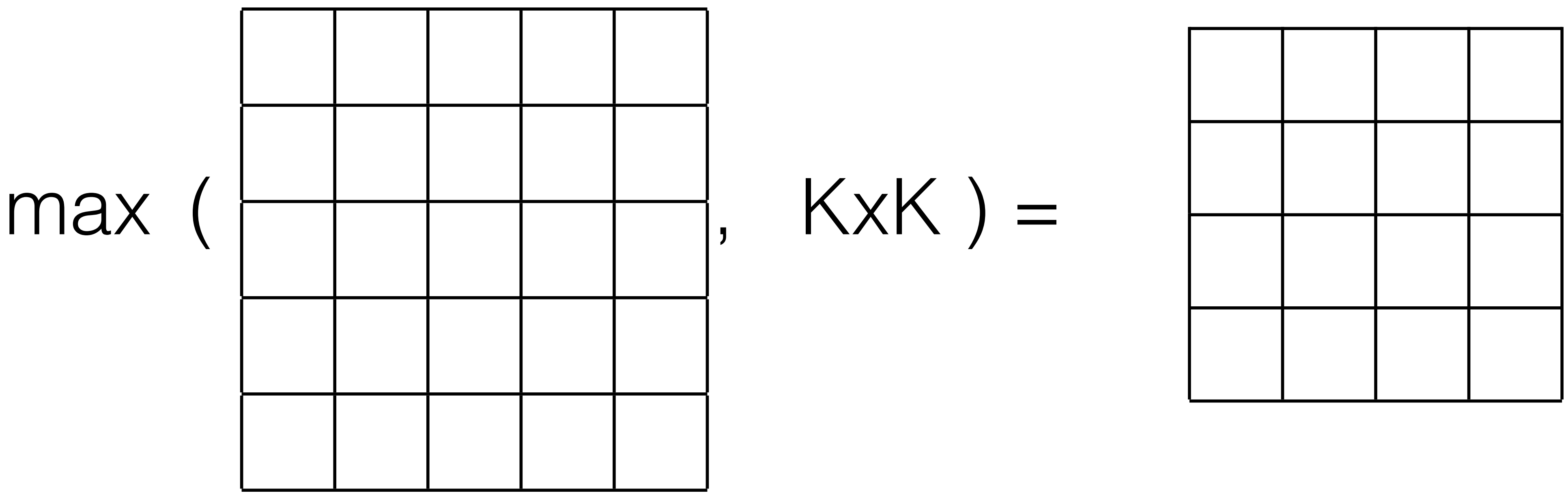
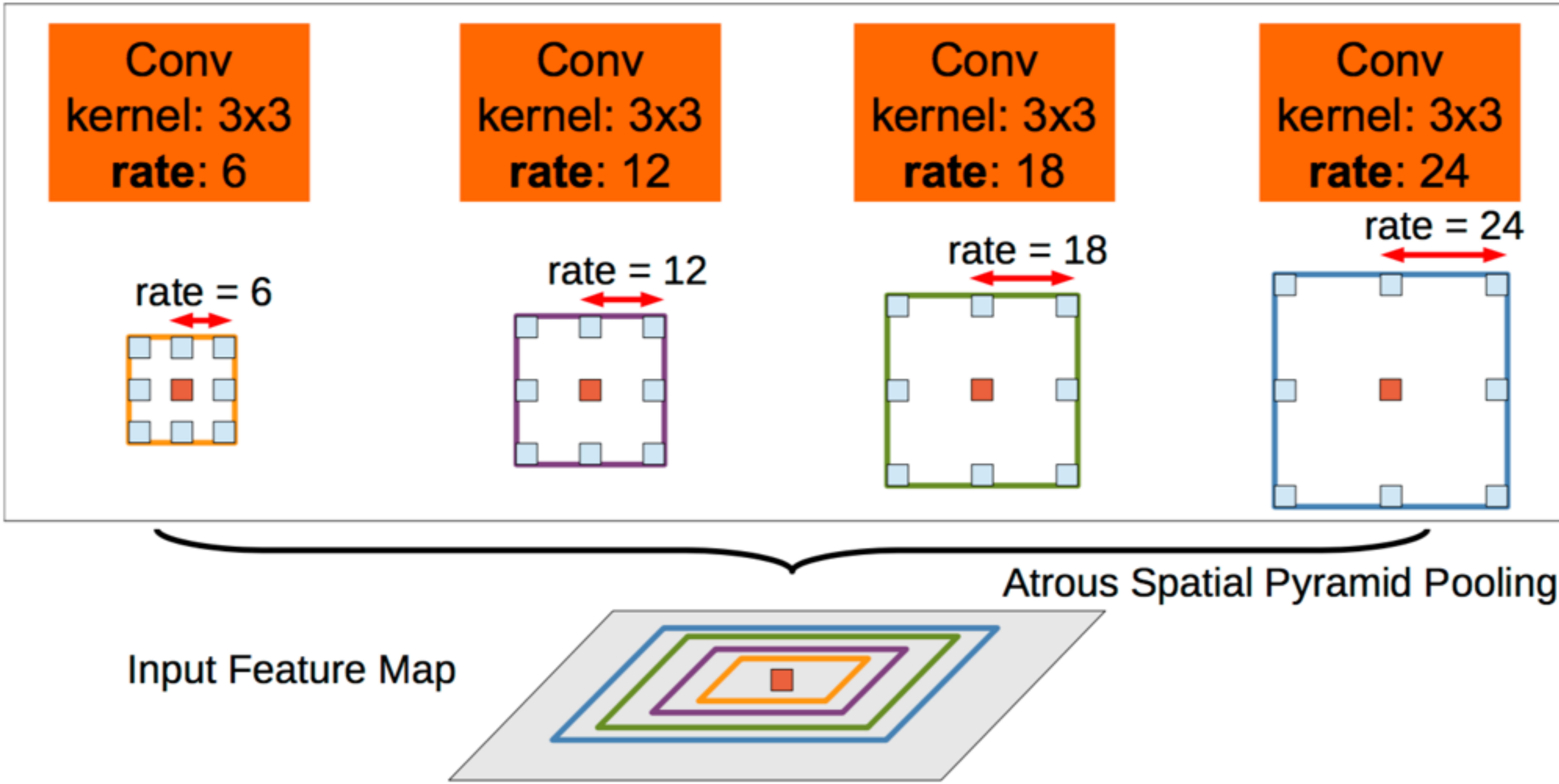


image
(NxN)

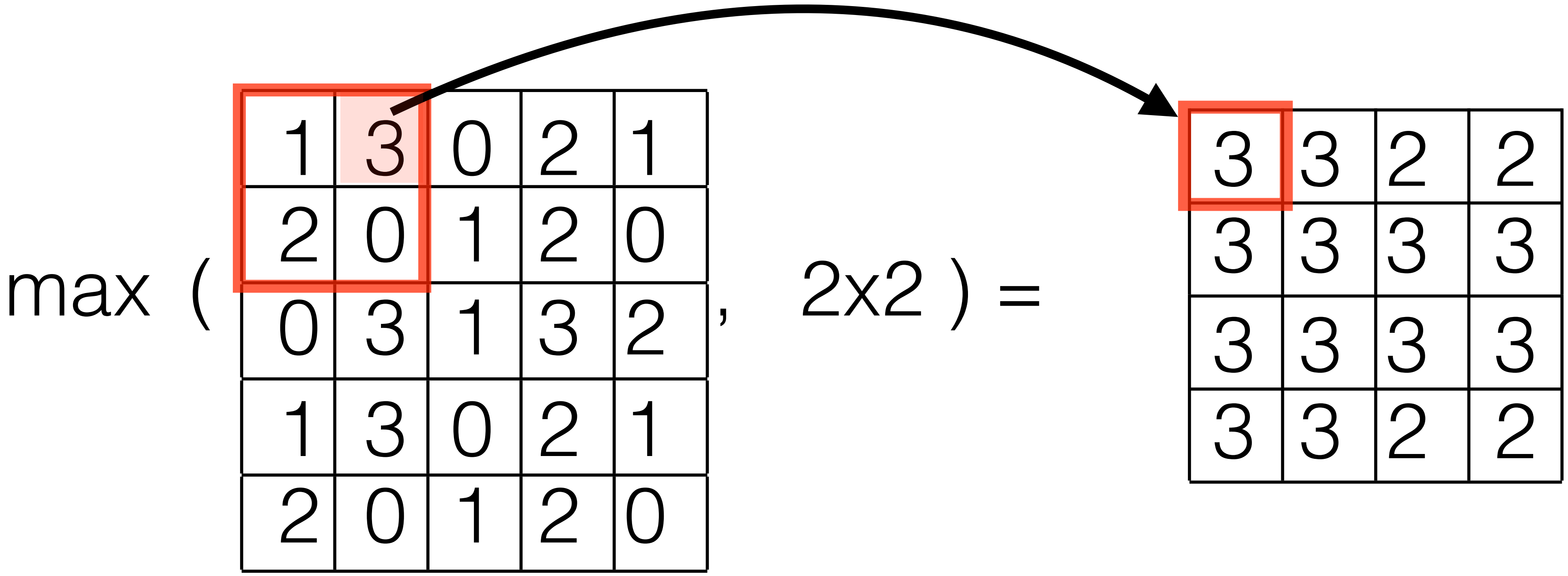
output
(M x M)

Atrous Spatial Pyramid Pooling (ASPP)

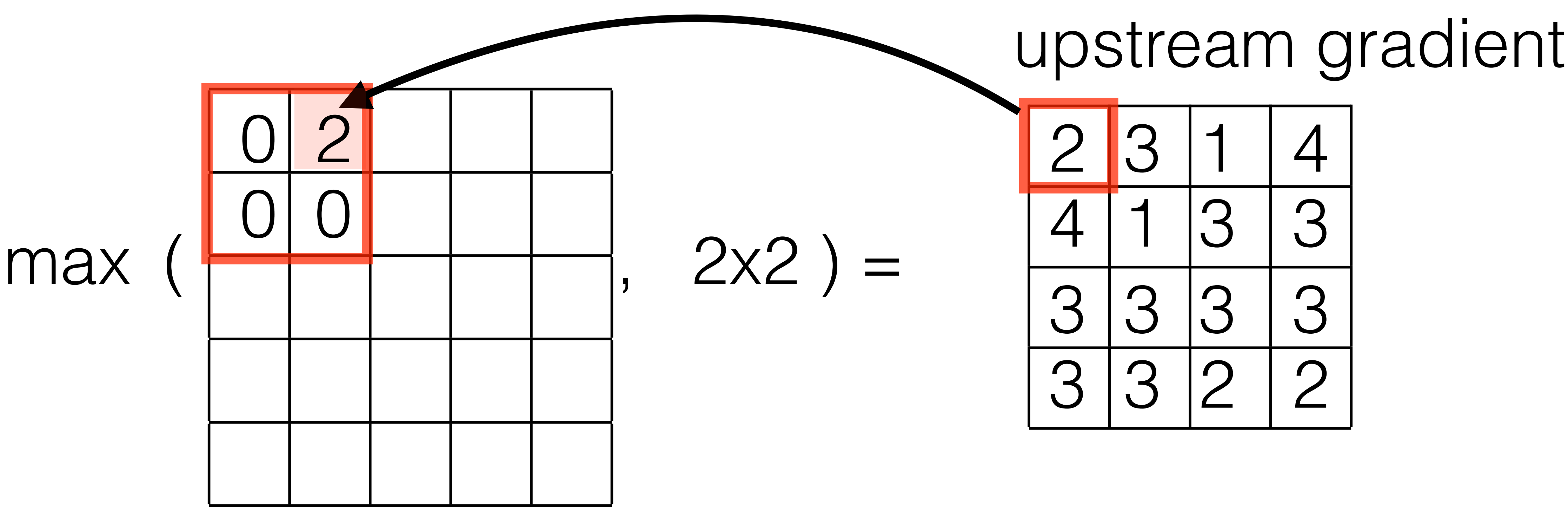


[Chen et al. TPAMI 2018] https://arxiv.org/pdf/1606.00915.pdf

Max-pooling feed-forward



Max-pooling Backprop



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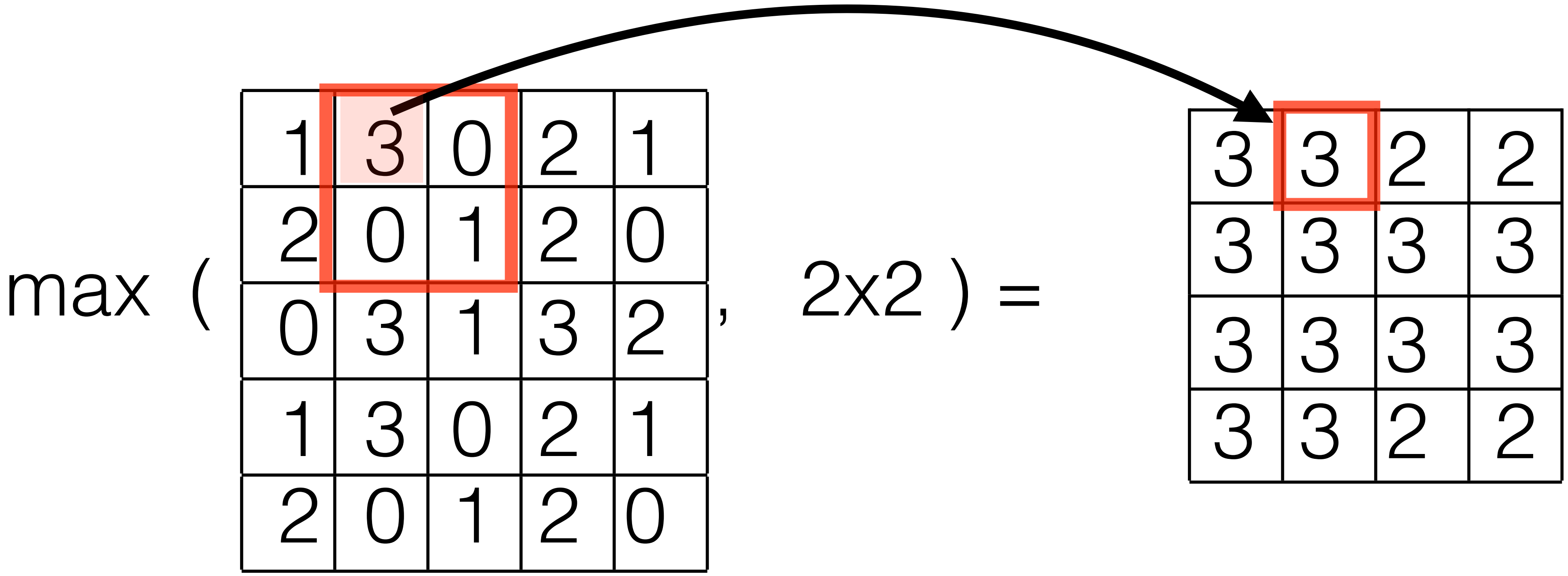
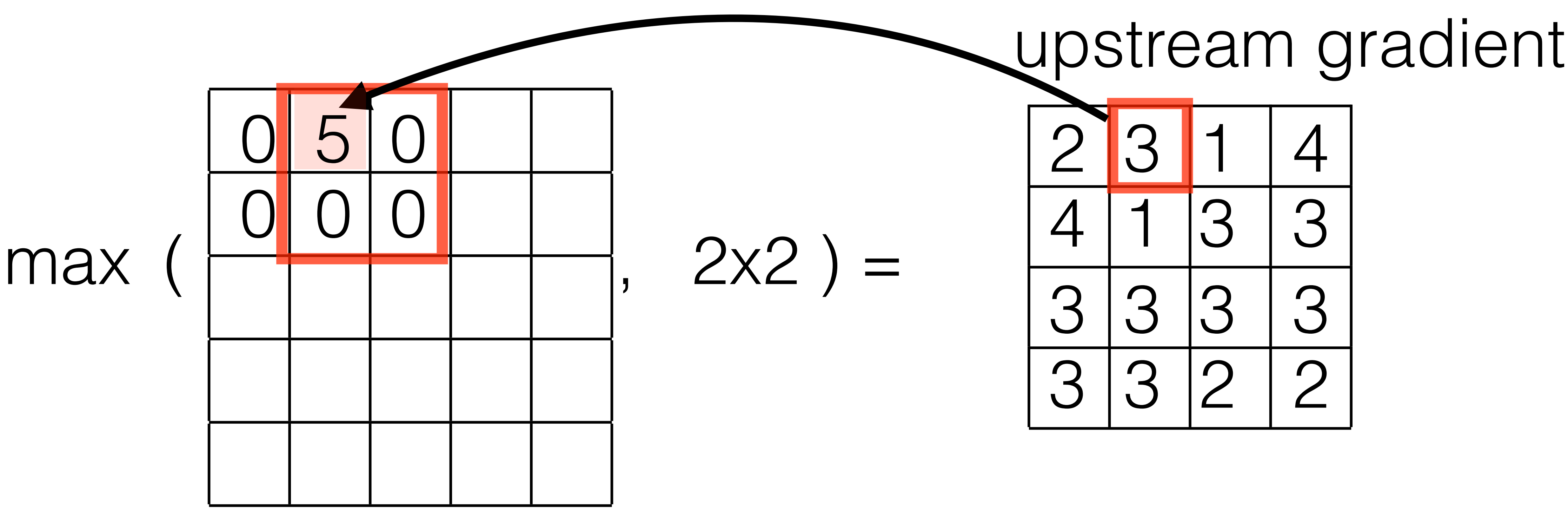


Max-pooling Backprop



Max-pooling feed-forward

max (

1	3	0	2	1
2	0	1	2	0
0	3	1	3	2
1	3	0	2	1
2	0	1	2	0

, 2x2) =

3	3	2	2
3	3	3	3
3	3	3	3
3	3	2	2

Max-pooling Backprop

max (

0	5	0	1/2	
0	0	0	1/2	

, 2x2) =

upstream gradient

2	3	1	4
4	1	3	3
3	3	3	3
3	3	2	2

Max-pooling summary

- Forward pass
  - similar to convolution but takes maximum over kernel
  - it has no parameters to be learnt!
- Backprop
  - propagate gradient only to active connections
- Main purpose is to reduce dimensionality and overfitting
- It seems that max pooling layers will disappear in future
  - should be avoided in generative models (GAN, VAE)
  - they can be replaced by conv-layers with larger stride in discriminative models https://arxiv.org/abs/1412.6806
  - Geoffrey Hinton: "*The pooling operation used in convolutional neural networks is a big mistake and the fact that it works so well is a disaster.*" (Reddit AMA)

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Loss functions

- Regression:
  - L2 loss
  - L1 loss
- Classification:
  - cross entropy loss (N-output classifier $\mathbf{f}(\mathbf{x}, \mathbf{w})$)
  - logistic loss (single output dichotomy classifier $f(\mathbf{x}, \mathbf{w})$)

$$L_2(\mathbf{w}) = \sum_i \|\mathbf{f}(\mathbf{x}_i, \mathbf{w}) - \mathbf{y}_i\|_2^2$$

$$L_1(\mathbf{w}) = \sum_i |\mathbf{f}(\mathbf{x}_i, \mathbf{w}) - \mathbf{y}_i|$$

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(1) convert output to probability (softmax function)

$$\mathbf{s}(\mathbf{f}(\mathbf{x}, \mathbf{w})) = \begin{bmatrix} \exp(f_1(\mathbf{x}, \mathbf{w})) \\ \exp(f_2(\mathbf{x}, \mathbf{w})) \\ \vdots \\ \exp(f_N(\mathbf{x}, \mathbf{w})) \end{bmatrix} / \sum_{k=1}^{N} \exp(f_k(\mathbf{x}, \mathbf{w}))$$

(2) compute cross entropy

$$H(\mathbf{w}) = \sum_i -\log \mathbf{s}_{y_i}(\mathbf{f}(\mathbf{x}_i, \mathbf{w}))$$

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$$L(\mathbf{w}) = \sum_i \log\left[1 + \exp(-y_i\, f(\mathbf{x}_i, \mathbf{w}))\right]$$

Derivative can be found here:
https://deepnotes.io/softmax-crossentropy

Loss functions

- Regression:
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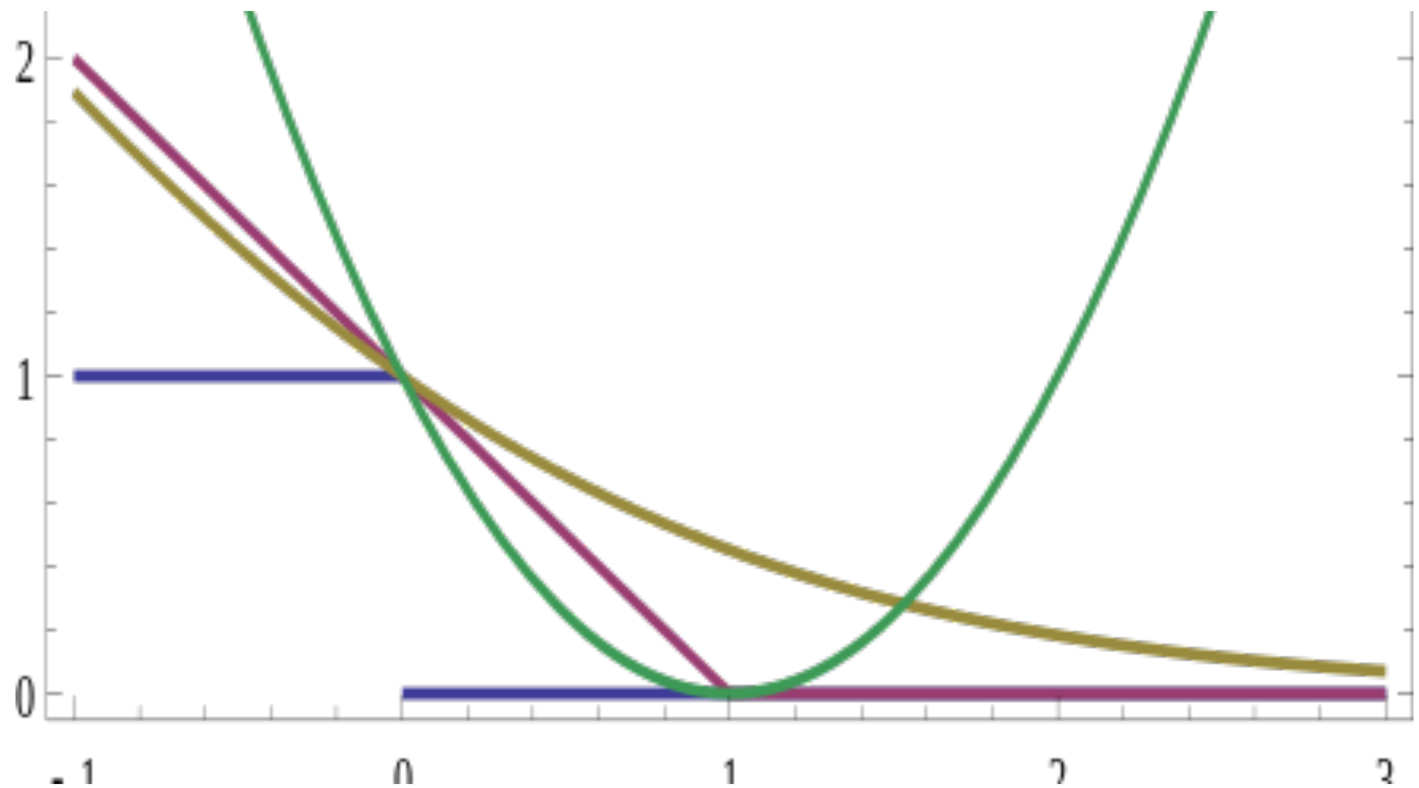
  - L1 loss
- Classification:
  - cross entropy loss (N-output classifier $\mathbf{f}(\mathbf{x}, \mathbf{w})$)
  - logistic loss (single output dichotomy classifier $f(\mathbf{x}, \mathbf{w})$)
  - other loss functions



https://en.wikipedia.org/wiki/Loss_functions_for_classification

Outline

- SGD vs deterministic gradient
- what makes learning to fail
- layers:
  - activation function (i.e. non-linearities)
  - batch normalization layer
  - max-pooling layer
  - loss-layers
- regularizations
- summary of the learning procedure
  - train, test, val data,
  - hyper-parameters,

Regularization

- L2, L1 norms on weights are simple regularizations
- Batch norm is regularization
- Drop out is regularization (it trains committee of experts)
- Jittering of training data is regularization

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Training procedure

- Choose:
  - Weight initialization
  - Network architecture (ideally re-use pre-trained net)
  - Learning rate and other hyper-parameters.
  - Loss + regularization
- Divide data on three represenatative subsets:
  - Training data (the set on which the backprop is used to estimate weights)
  - Validation data (the set on which hyper-param are tuned)
  - Testing data (the set on which the error is only observed)

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