Learning for vision II
Neural networks

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CTU

CZECH TECHNICAL UNIVERSITY IN PRAGUE

Department for Cybernetics

Faculty of Electrical Engineering

Czech Technical University in Prague

Outline

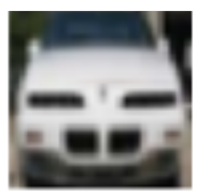
- Neuron+ computational graph
- Fully connected neural network

Linear classifier and neuron

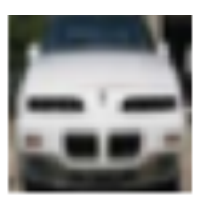
Labels | RGB images

$+1$

-1

def classify():

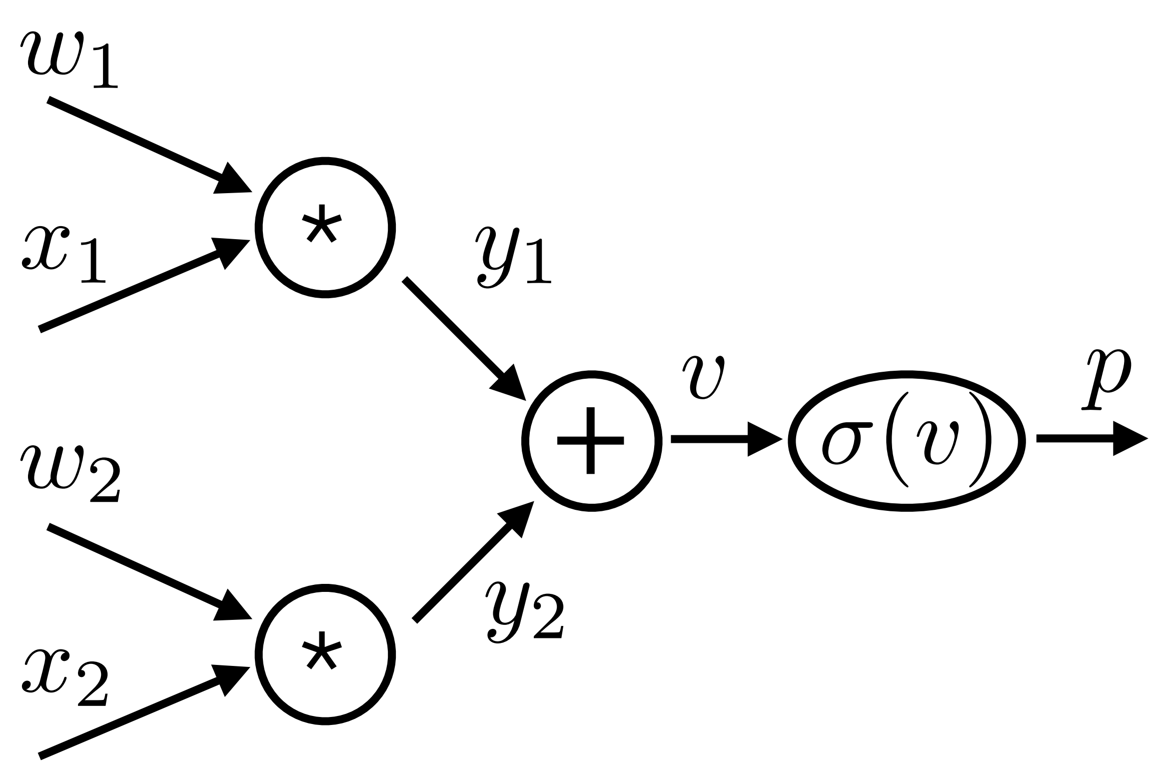
 # Linear classifier

$\mathbf{x} = \mathrm{vec}($  $)$

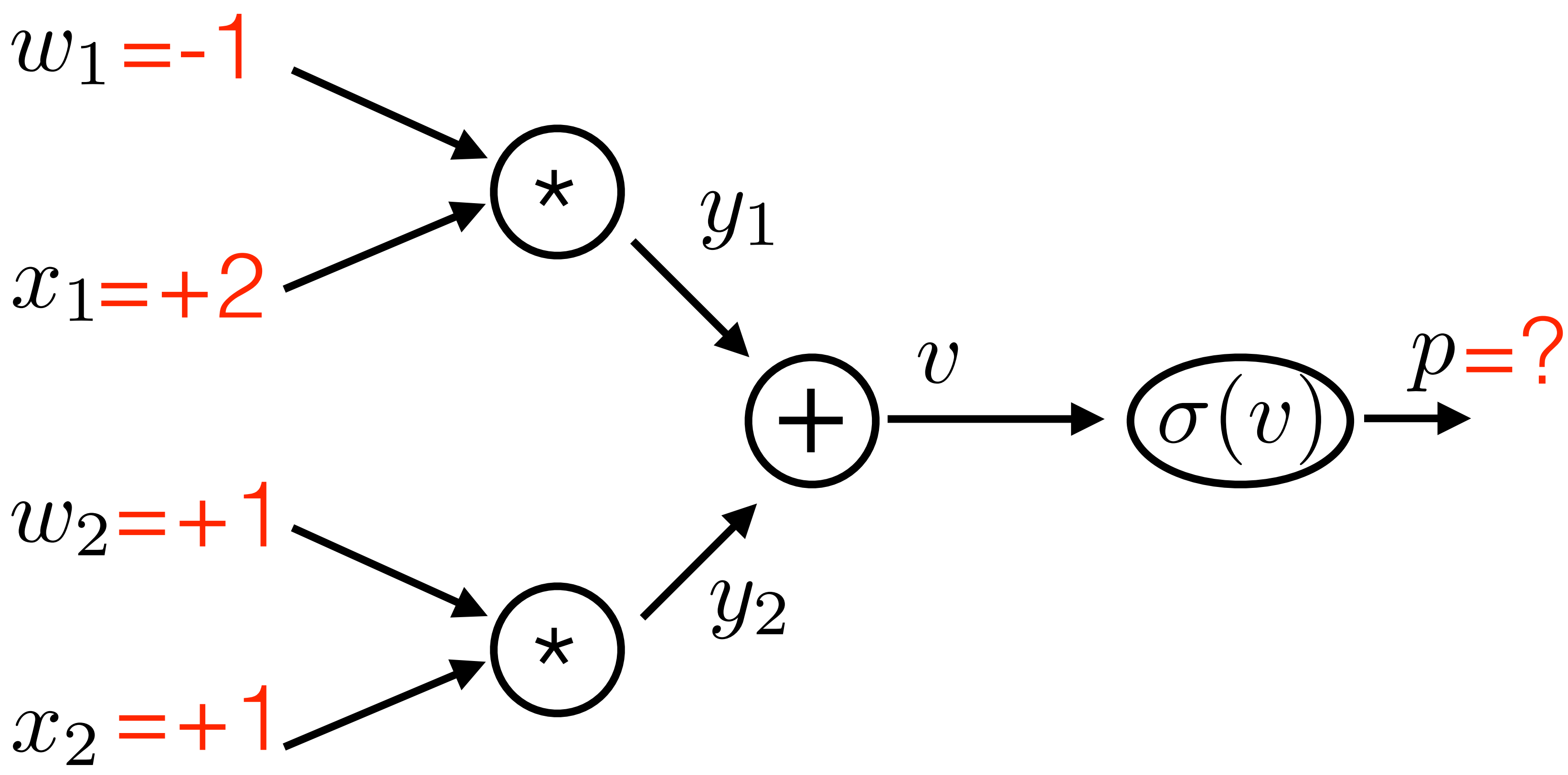
$p = \sigma\left(\mathbf{w}^\top \mathbf{x}\right)$

return p

Computational graph of linear classifier



Example I: given trained neuron, and input, what is output?



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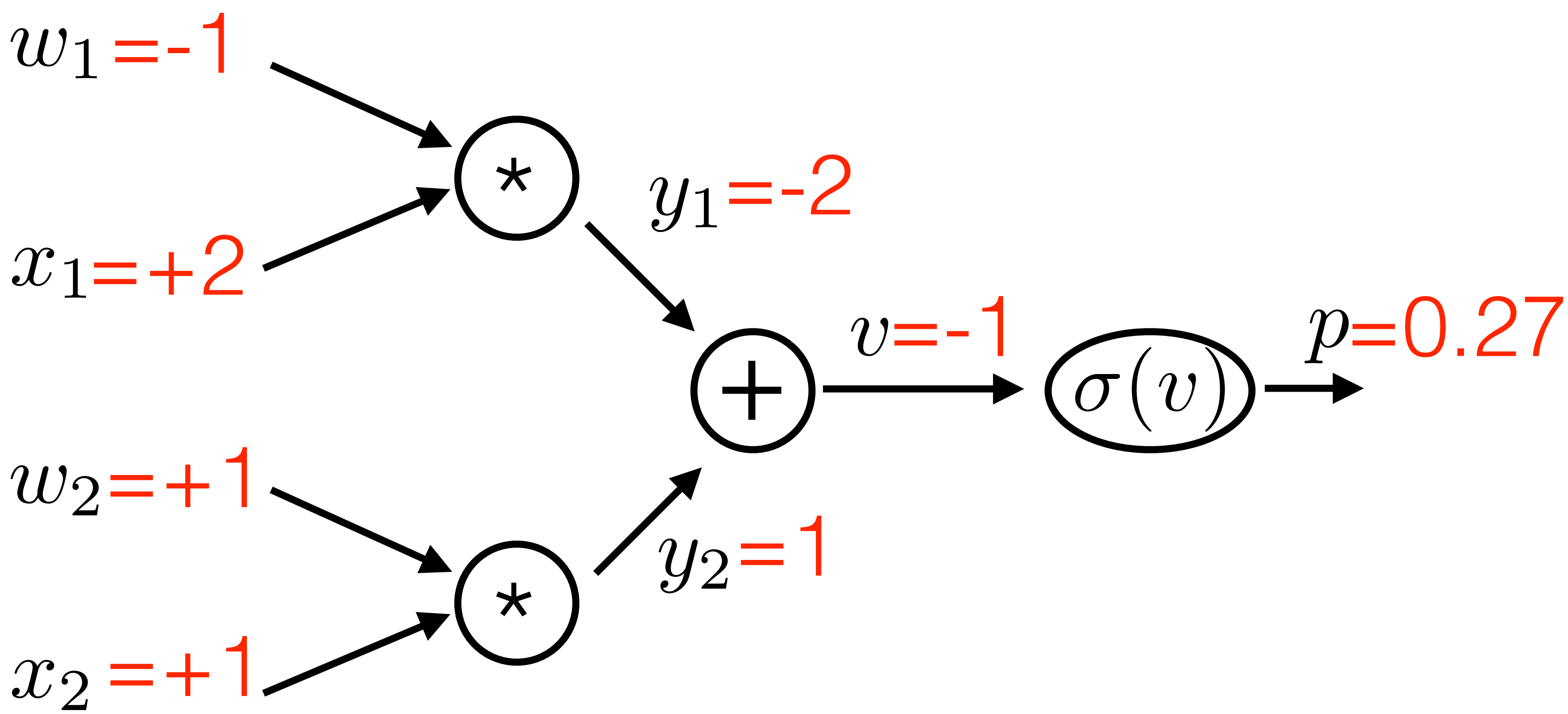
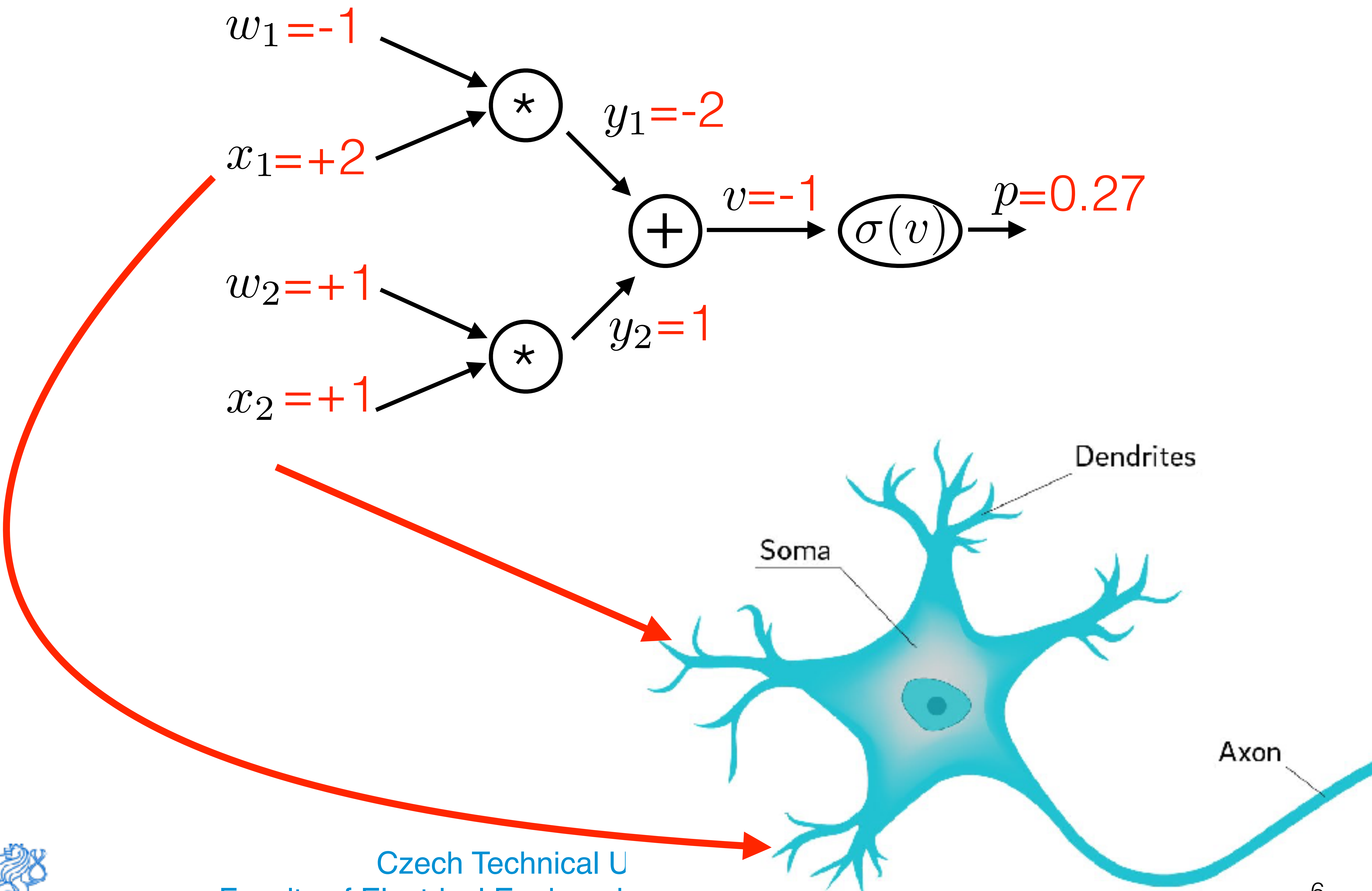


Relation to biological neuron



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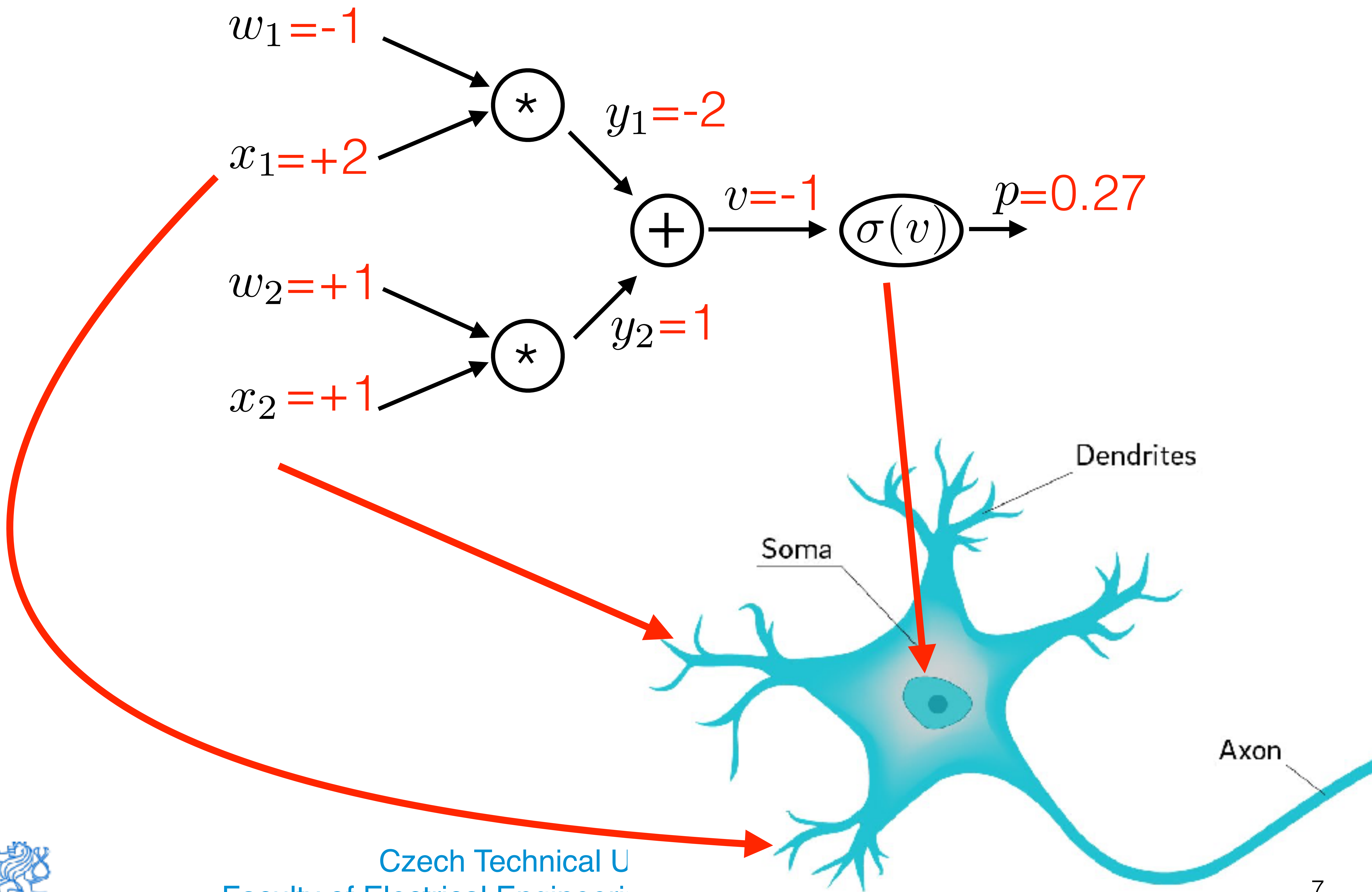
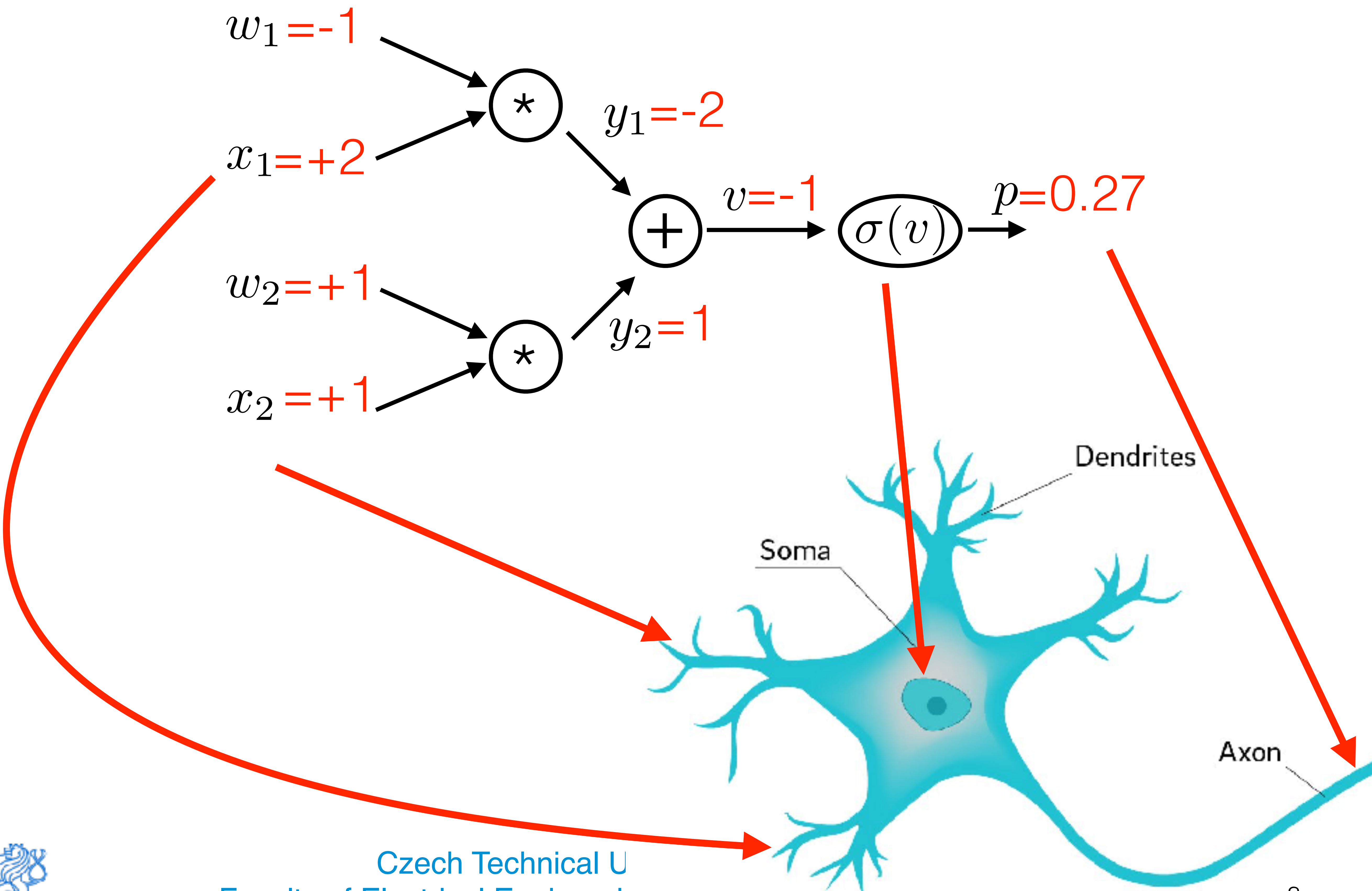
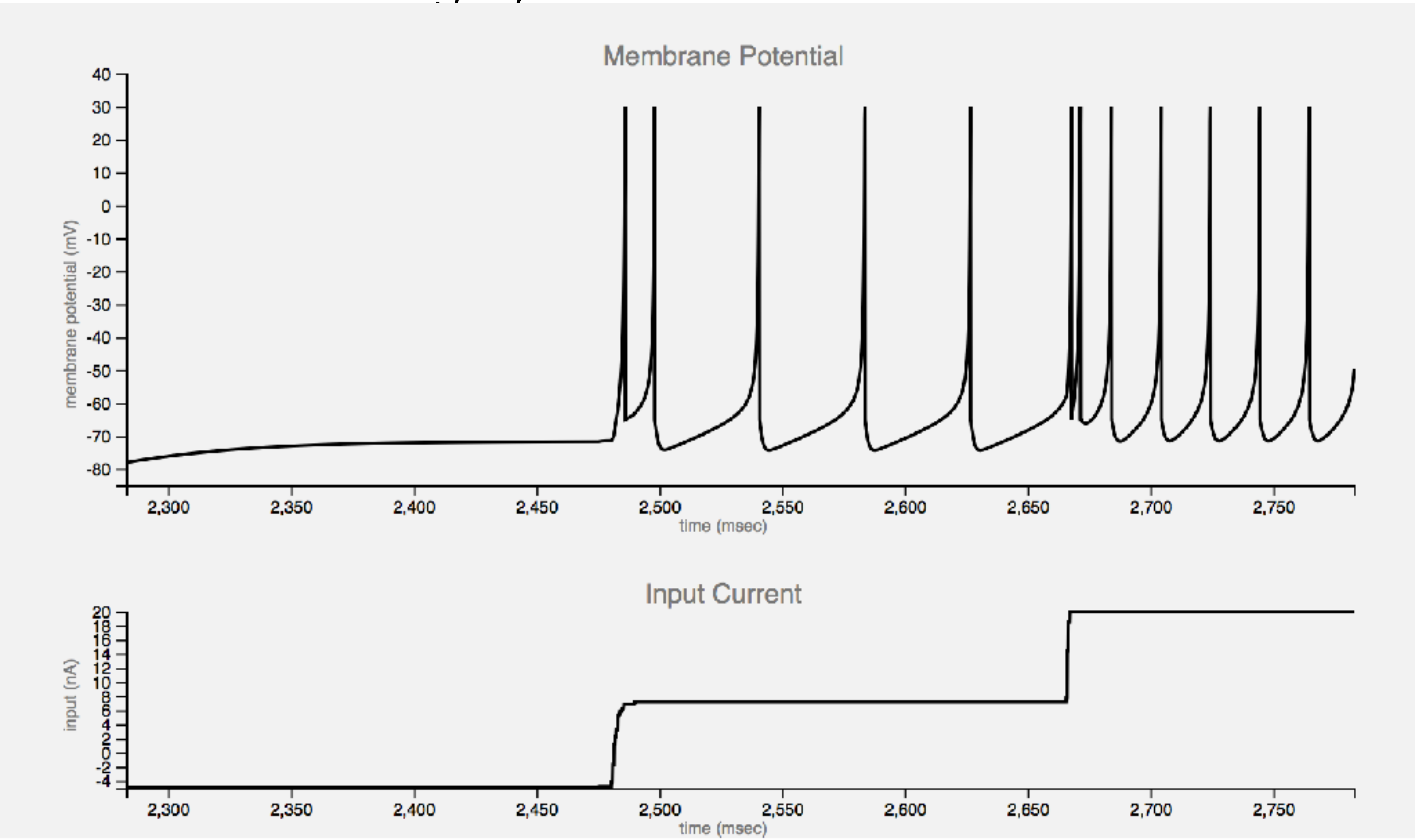


Relation to biological neuron



Modeling dynamic neuron behaviour



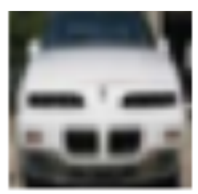
http://jackterwilliger.com/biological-neural-networks-part-i-spiking-neurons/

Linear classifier and neuron

Labels | RGB images

$+1$

-1

def classify():

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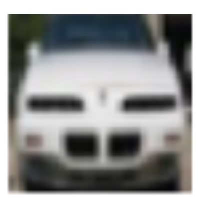
$\mathbf{x} = \text{vec}($  $)$

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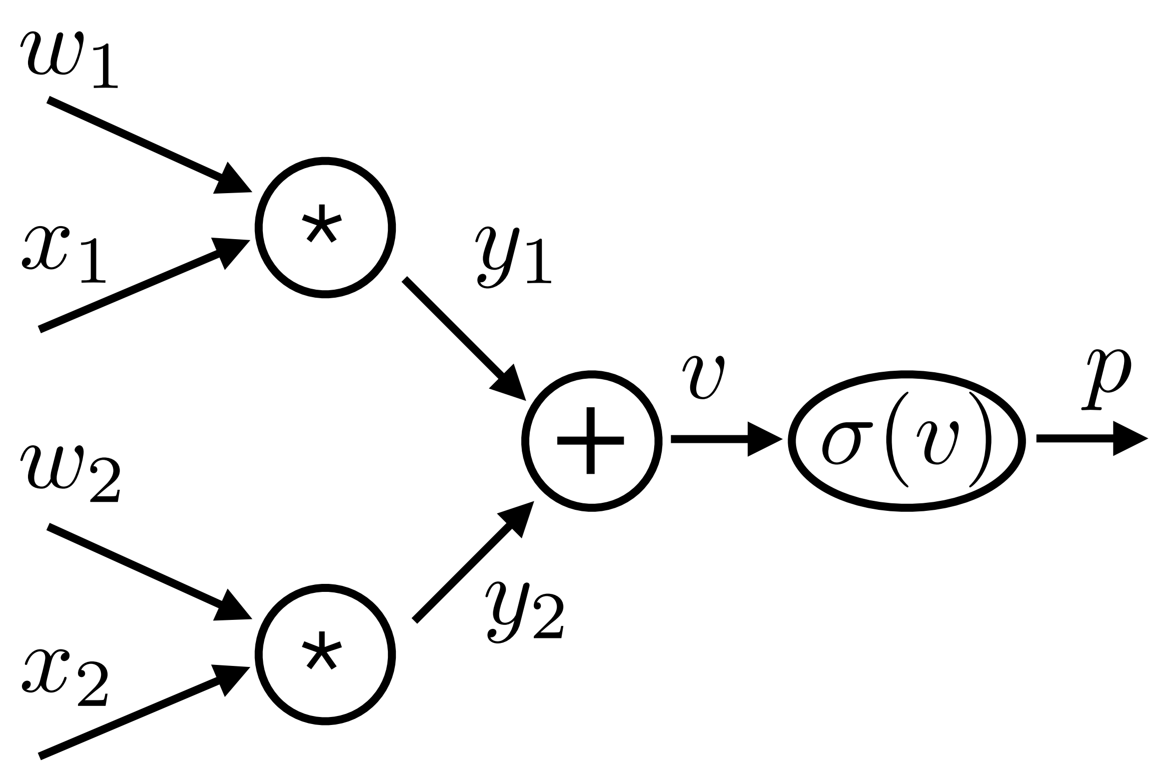

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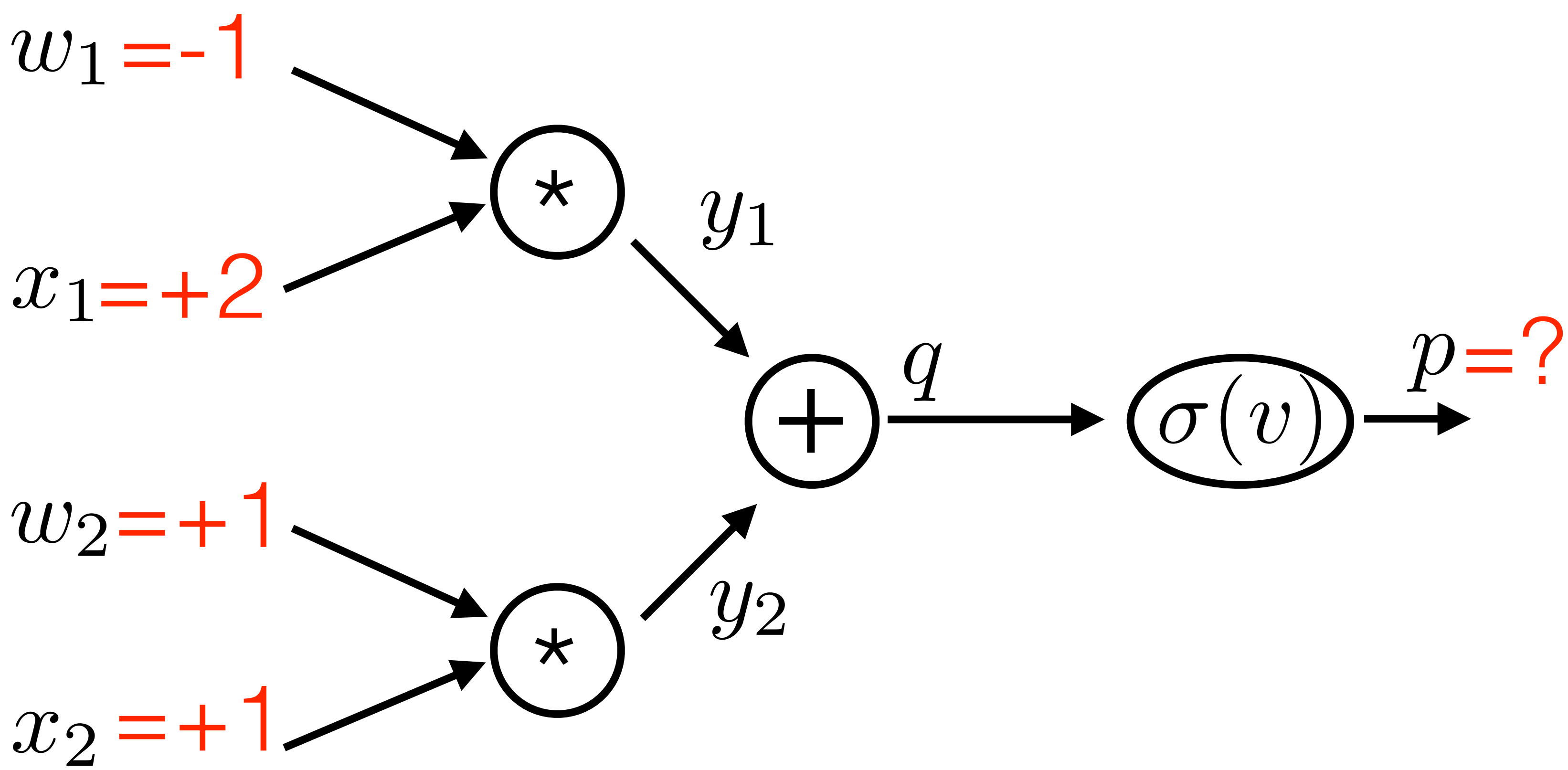
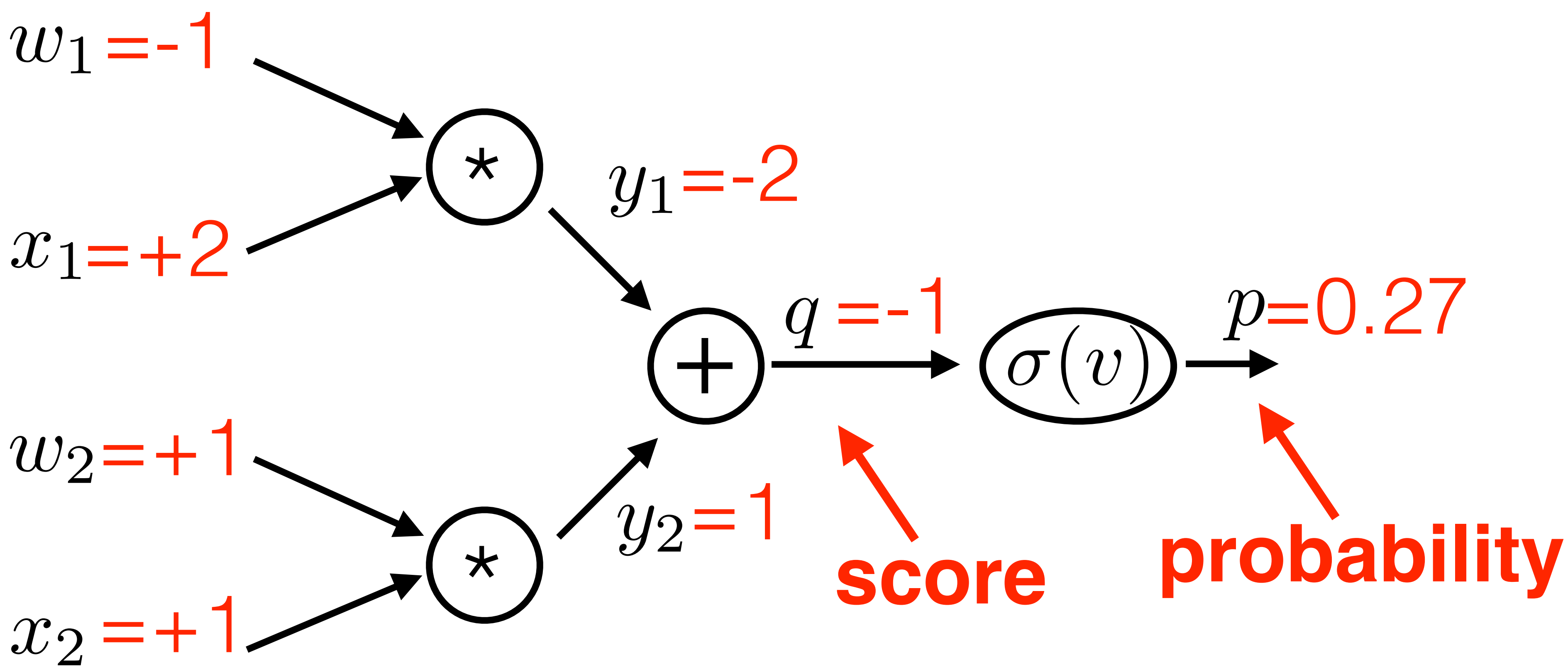
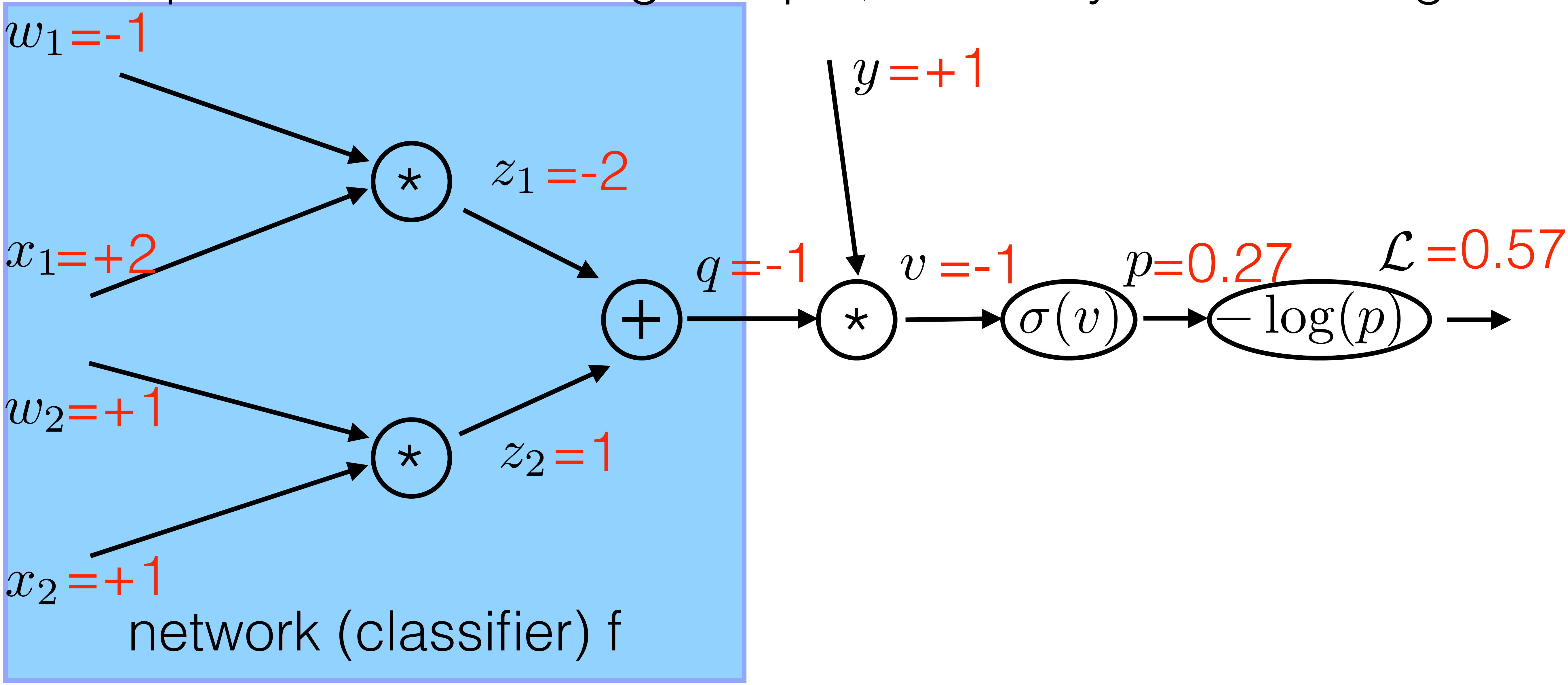


Example I: given trained classifier, and input, what is output?



Example II: Given training sample, how do you learn weights?



$$\arg\min_{\mathbf{w}} \Big(-\log\big[\sigma\big(y_i\, f(\mathbf{x}_i, \mathbf{w})\big)\big] \Big)$$

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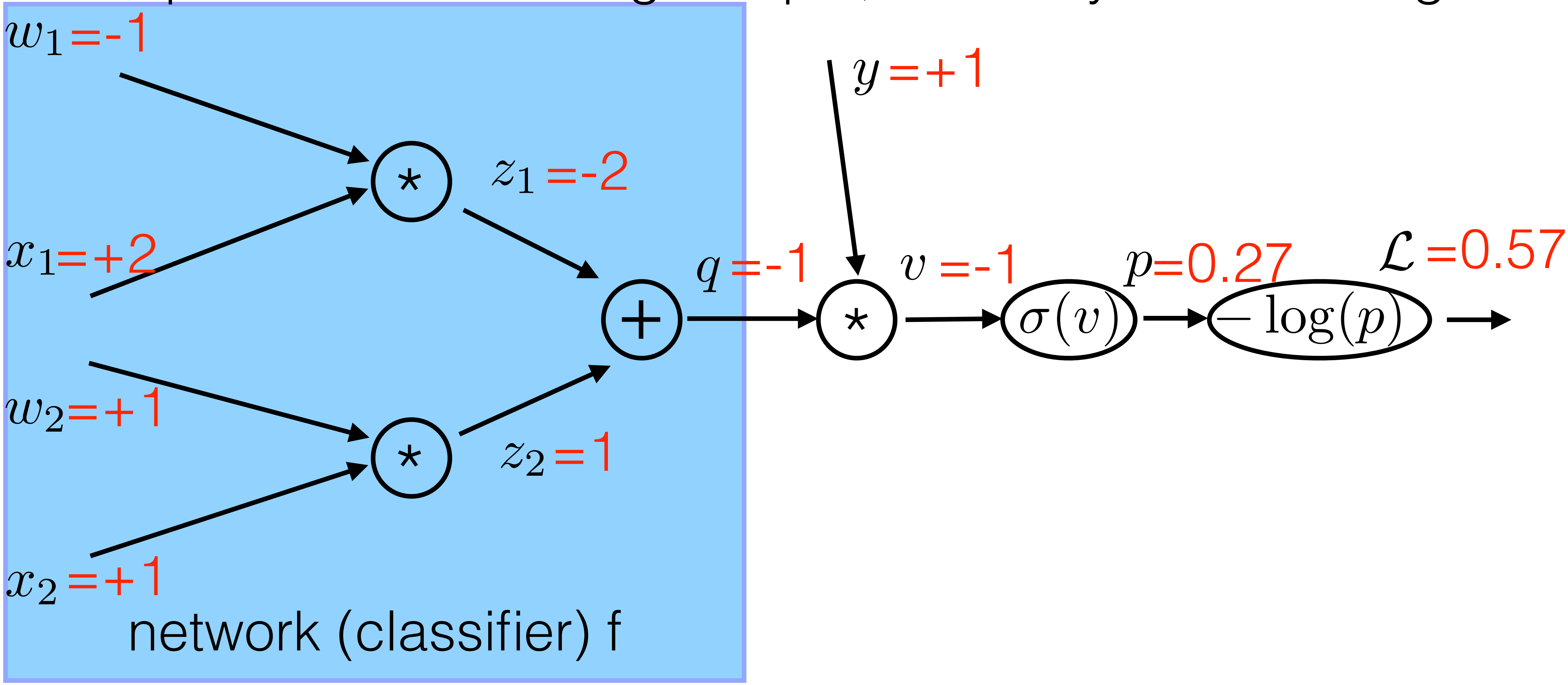

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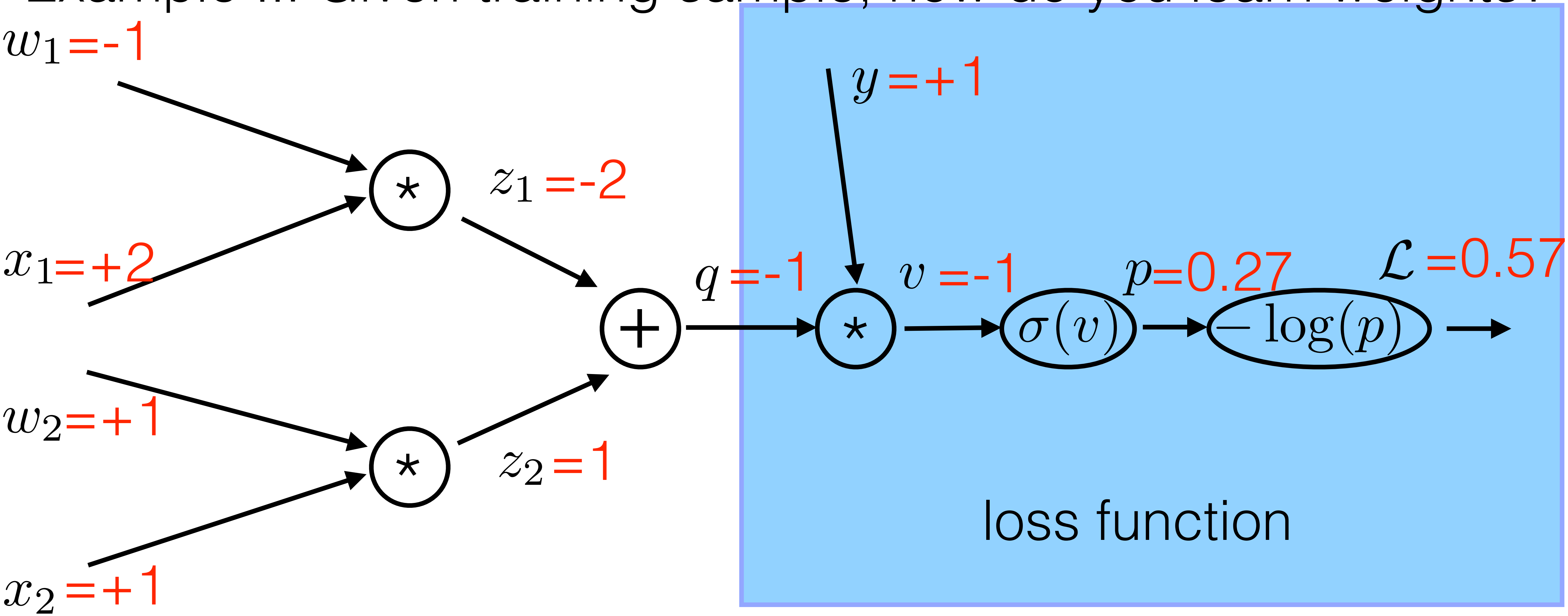

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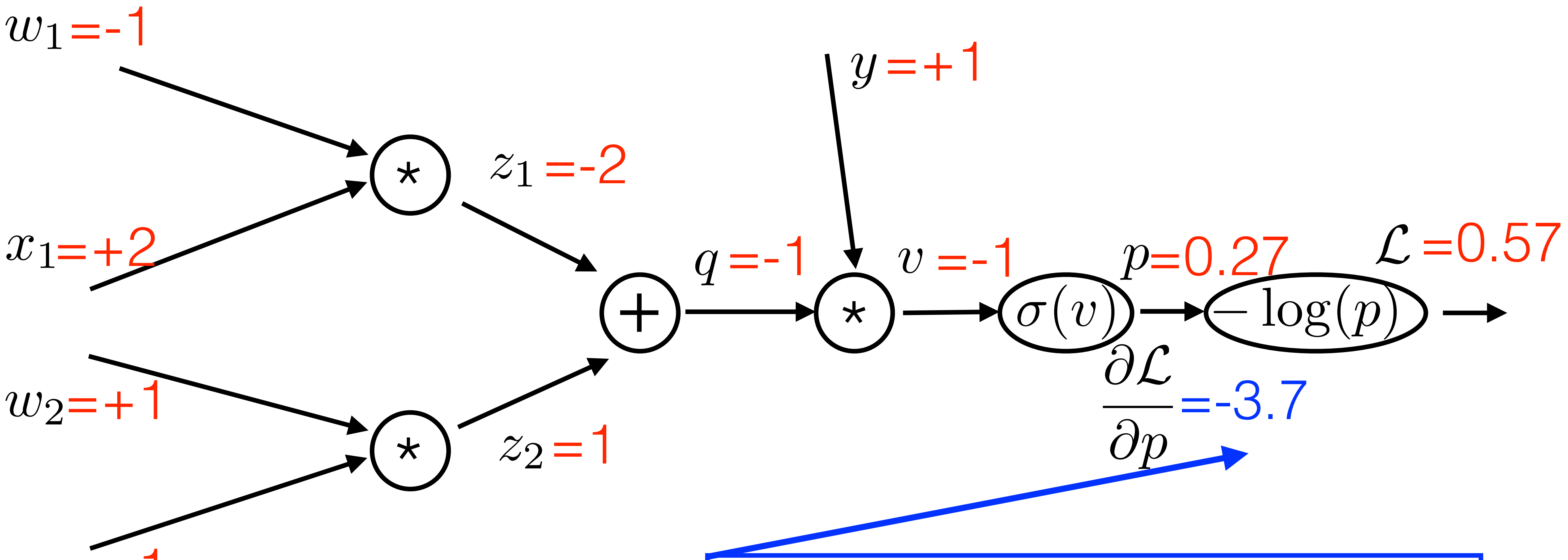

Local gradient:

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{\partial(-\log(p))}{\partial p} = -\frac{1}{p}$$

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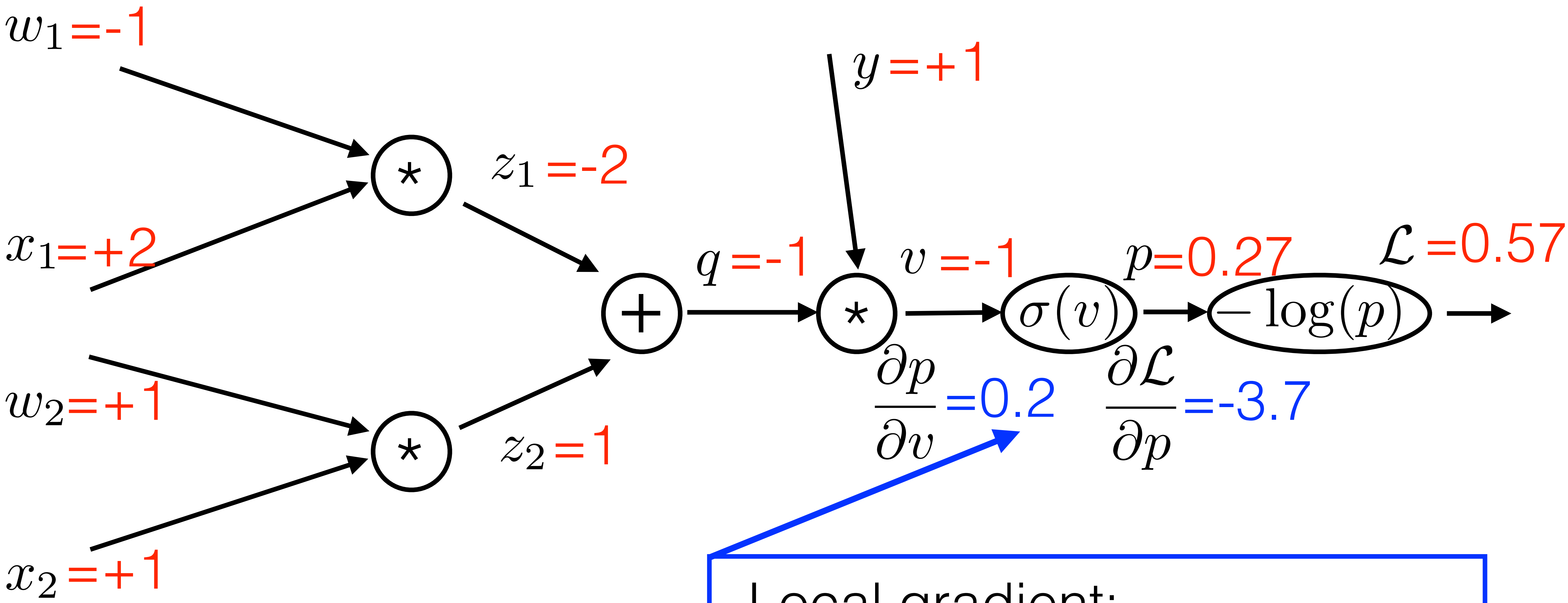

Local gradient:

$$\frac{\partial p}{\partial v} = \frac{\partial \sigma(v)}{\partial v} = \sigma(v)(1 - \sigma(v))$$

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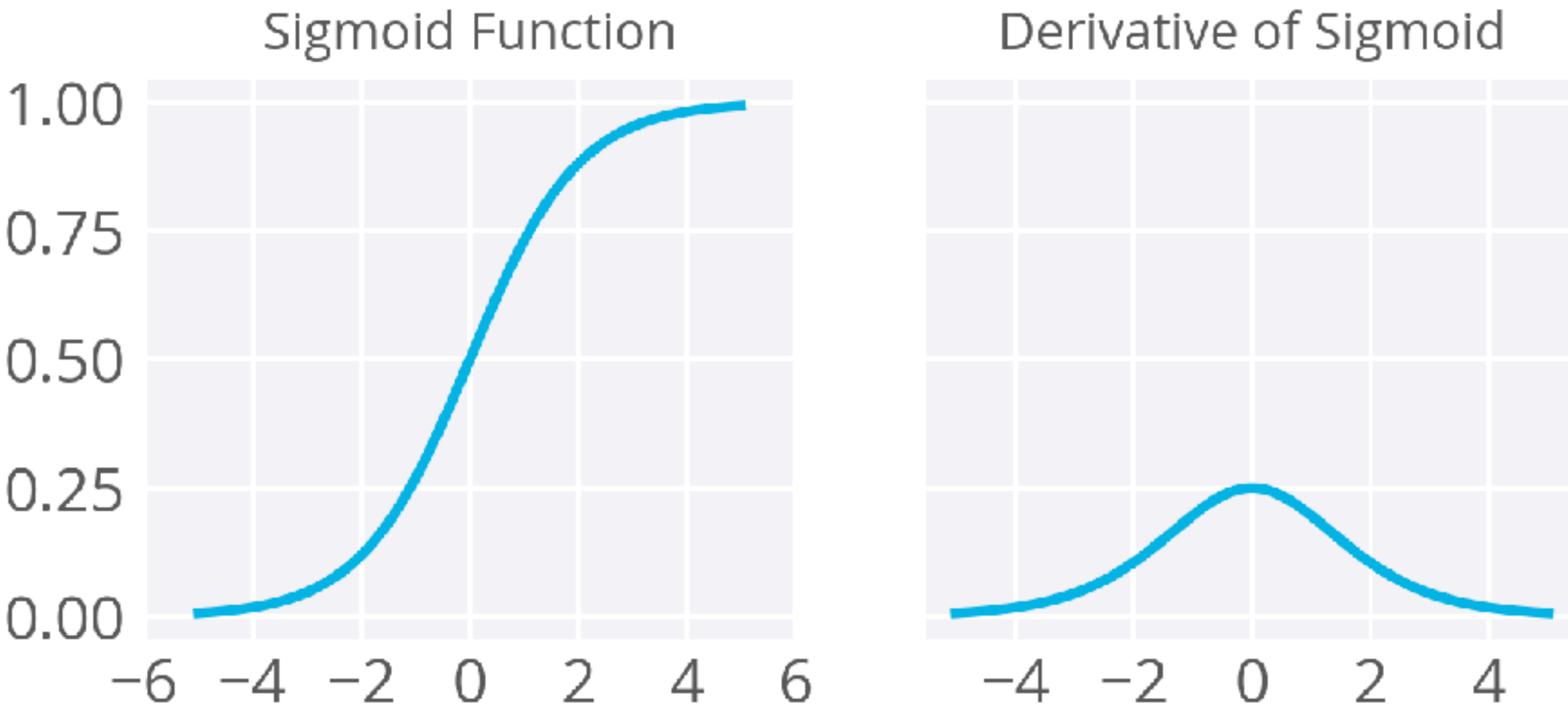

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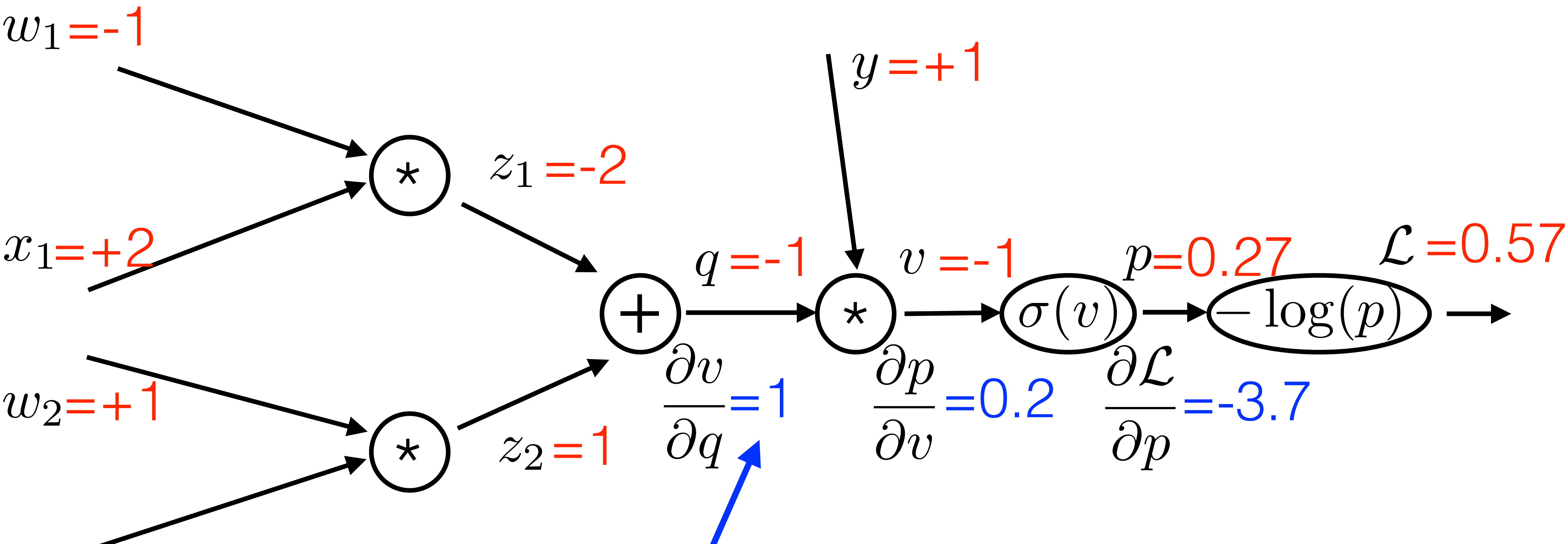

Local gradient:

$$\frac{\partial v}{\partial q} = \frac{\partial (yq)}{\partial q} = y$$

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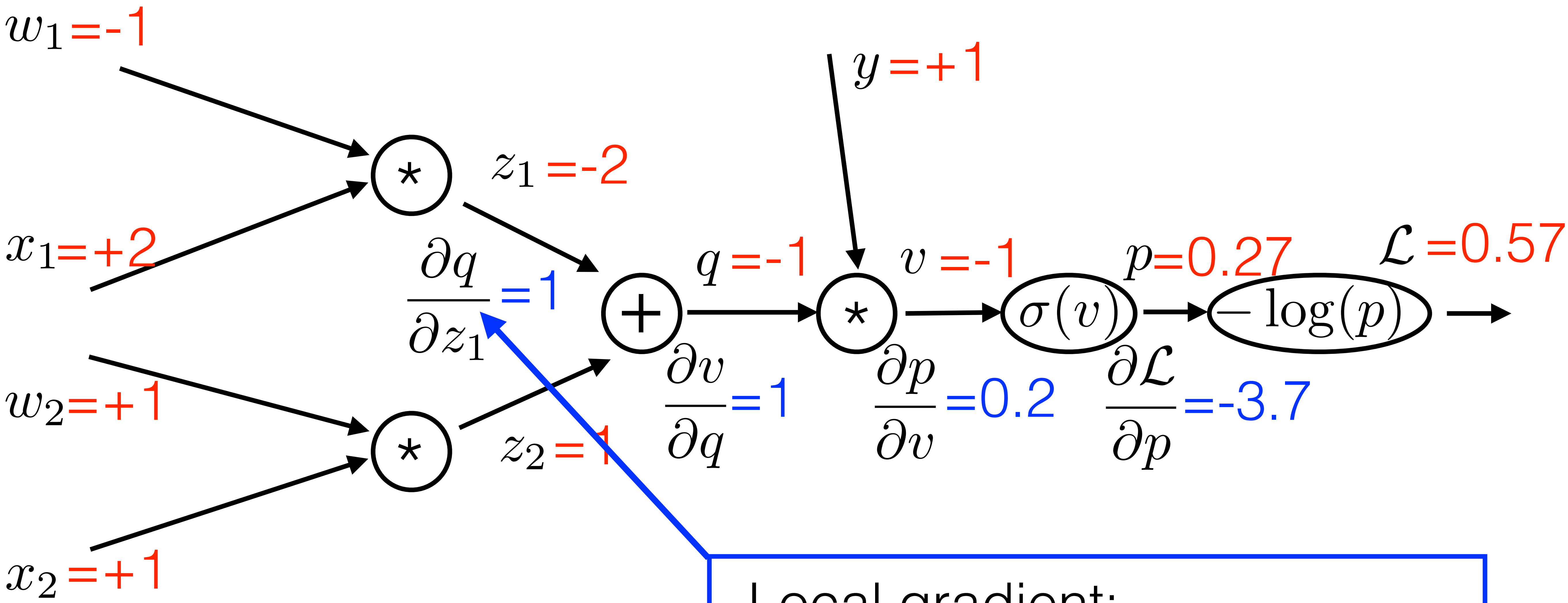
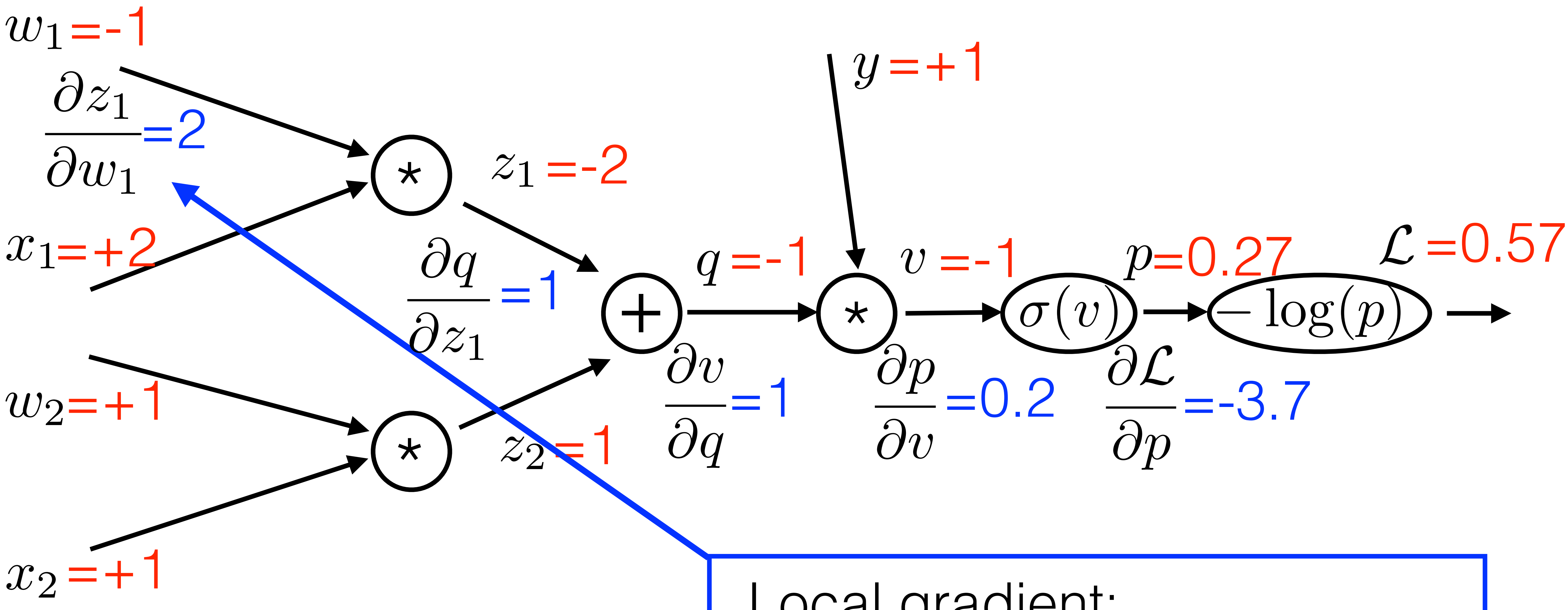


Local gradient:

$$\frac{\partial q}{\partial z_1} = \frac{\partial(z_1 + z_2)}{\partial z_1} = 1$$

Example II: Given training sample, how do you learn weights?



Local gradient:

$$\frac{\partial z_1}{\partial w_1} = \frac{\partial(w_1 x_1)}{\partial w_1} = x_1$$

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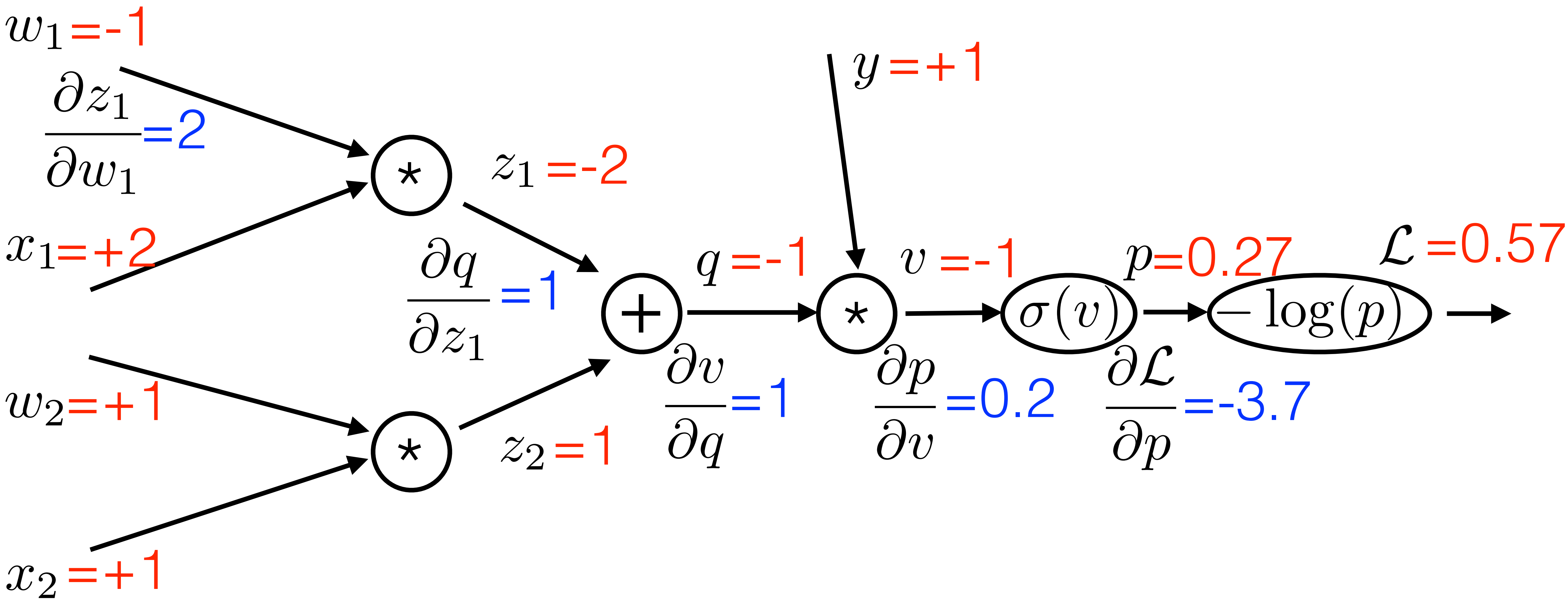
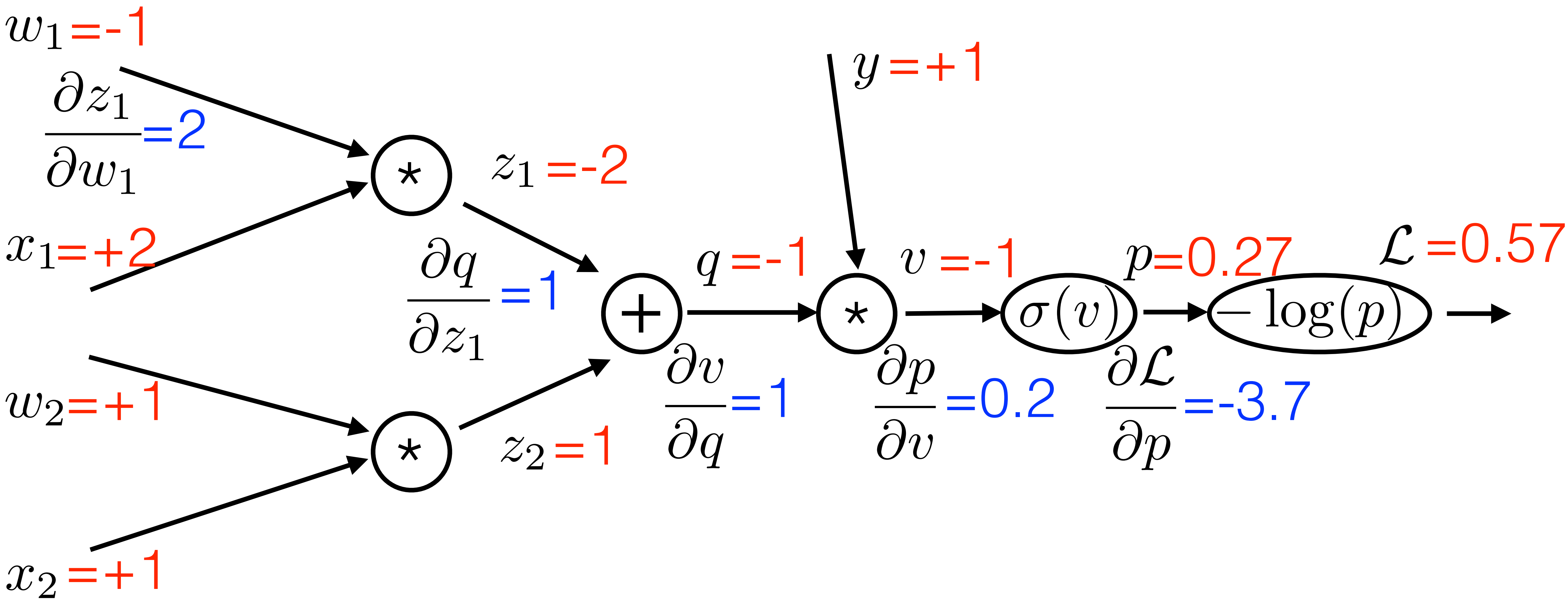


$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial p}\frac{\partial p}{\partial v}\frac{\partial v}{\partial q}\frac{\partial q}{\partial z_1}\frac{\partial z_1}{\partial w_1} = -3.7*0.2*1*1*2 = -1.48$$

Example II: Given training sample, how do you learn weights?



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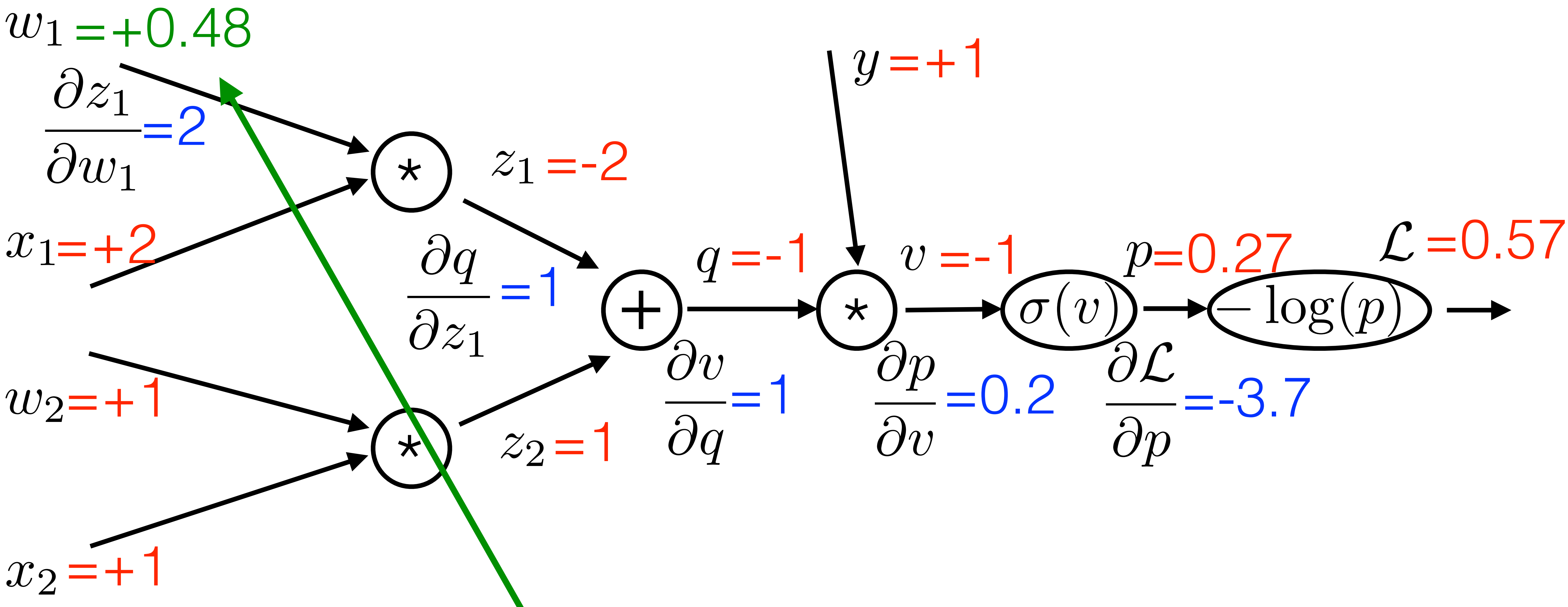

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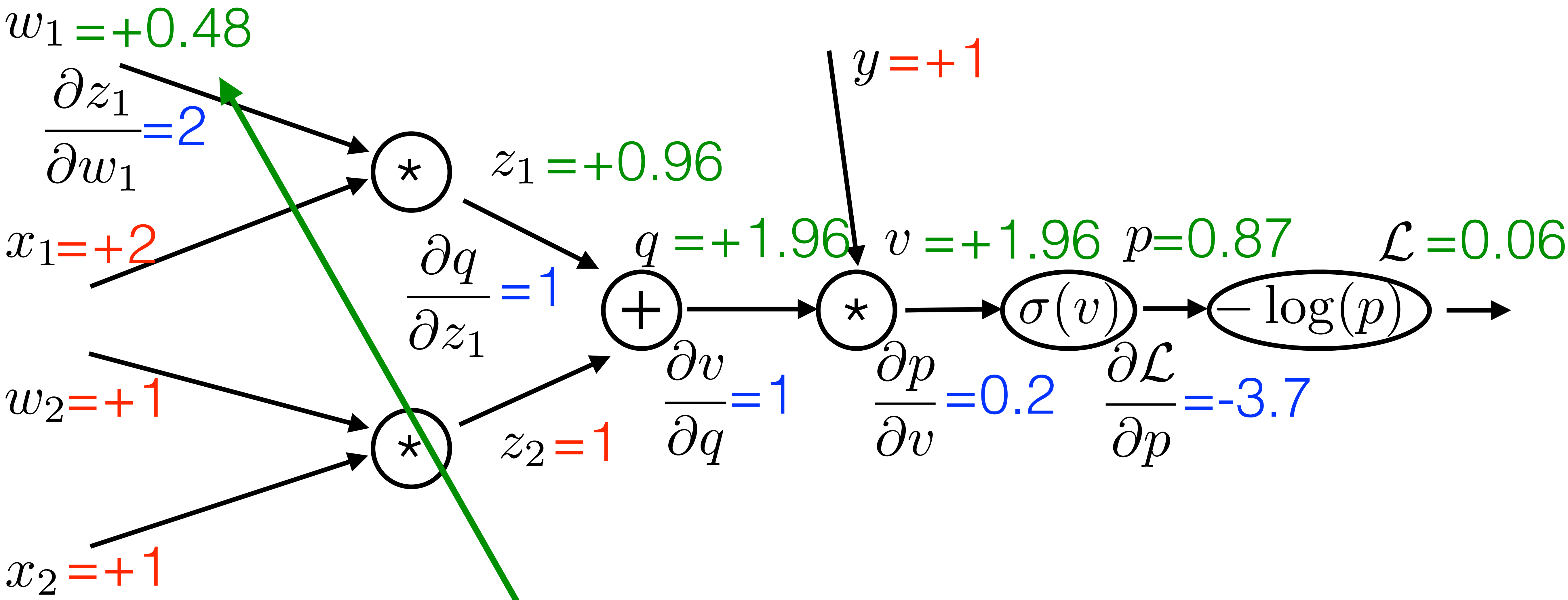

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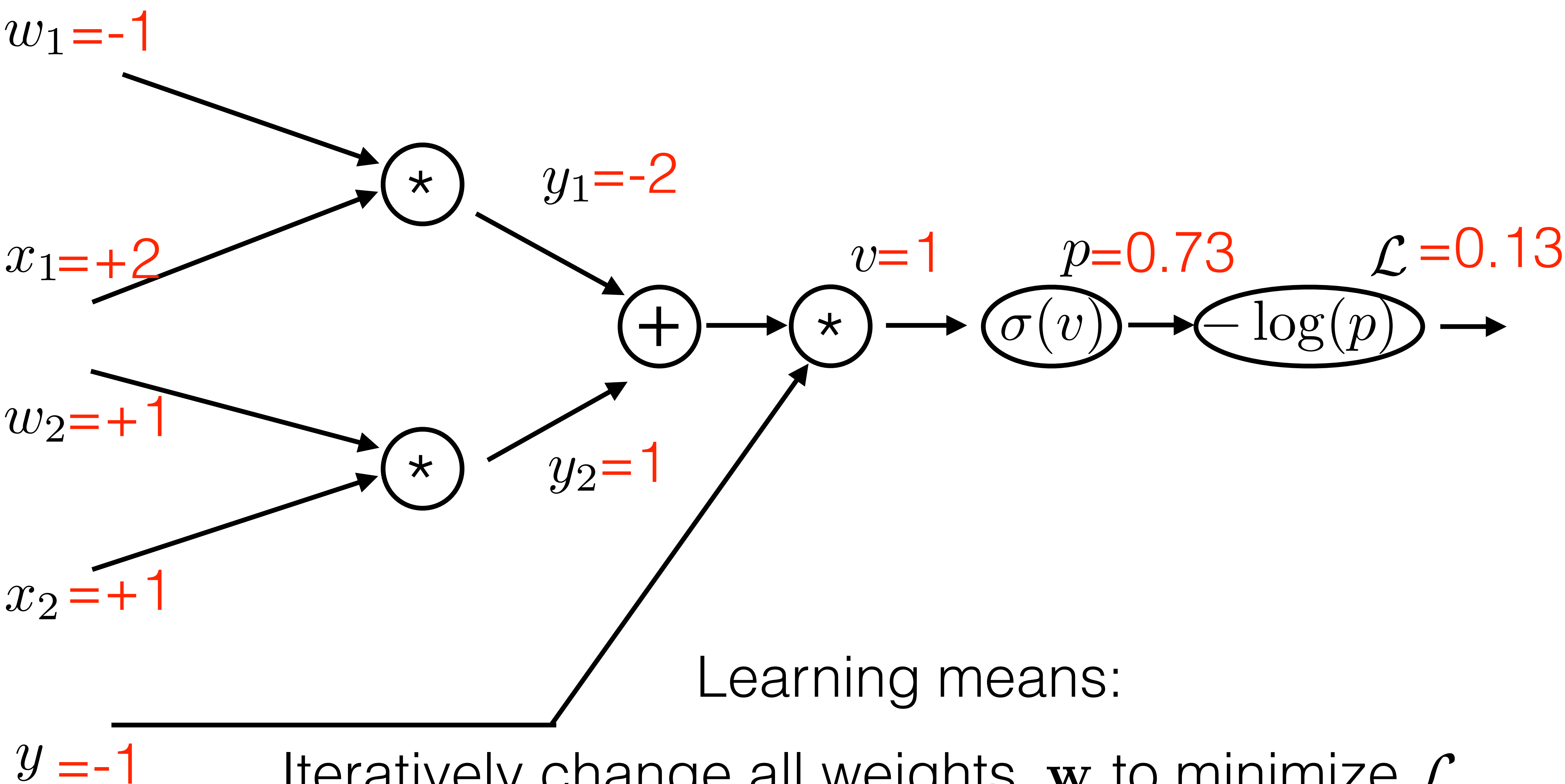
Example III: vector representation

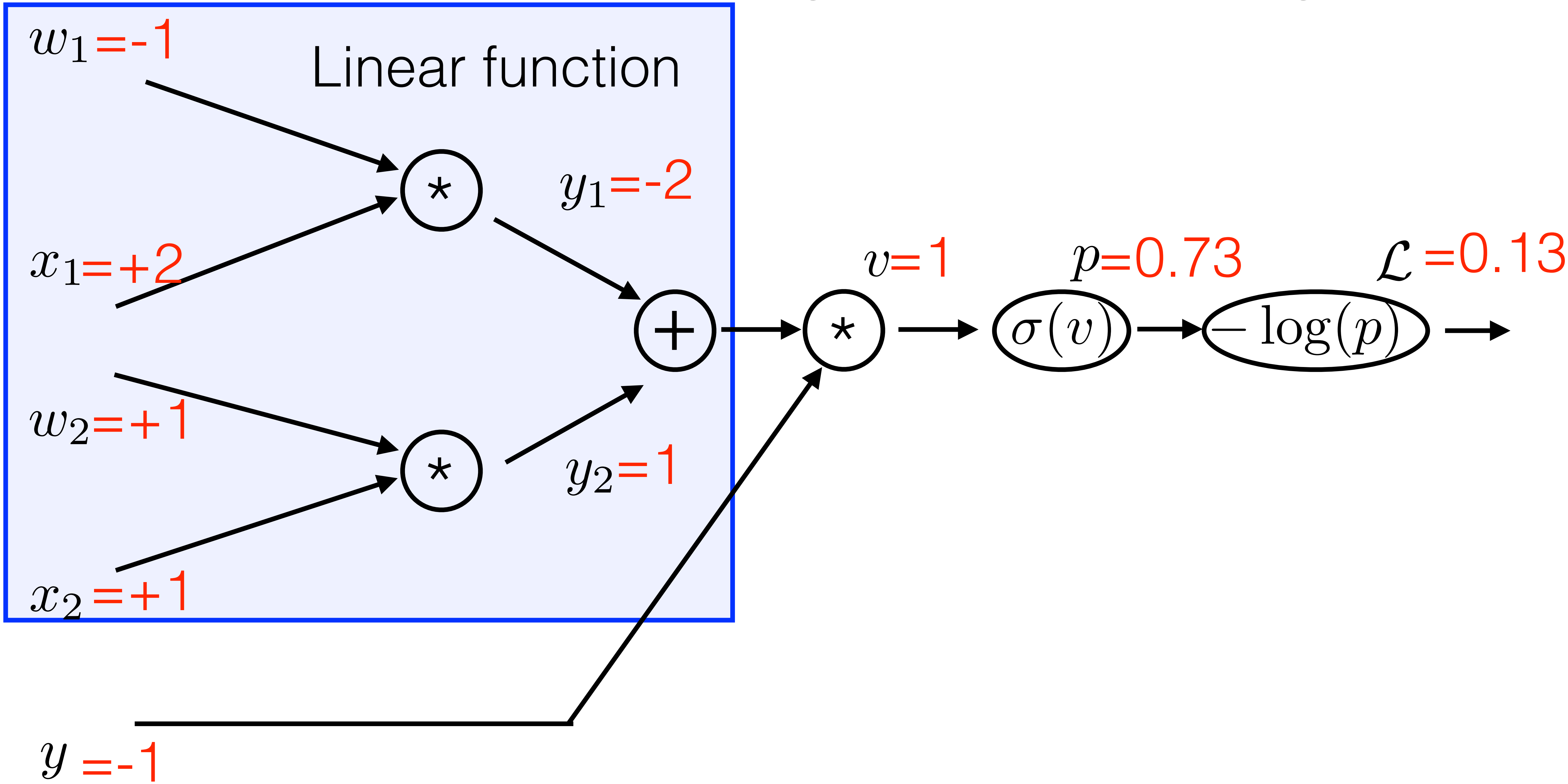


Learning means:

Iteratively change all weights $\mathbf{w}$ to minimize $\mathcal{L}$

$$\mathbf{w} = \mathbf{w} - \alpha \left[\frac{\partial \mathcal{L}(\mathbf{w})}{\partial \mathbf{w}}\right]^\top \quad \text{where} \quad \frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \left[\frac{\partial \mathcal{L}}{\partial w_1}, \frac{\partial \mathcal{L}}{\partial w_2}, \dots\right]$$

Computational graph of the learning



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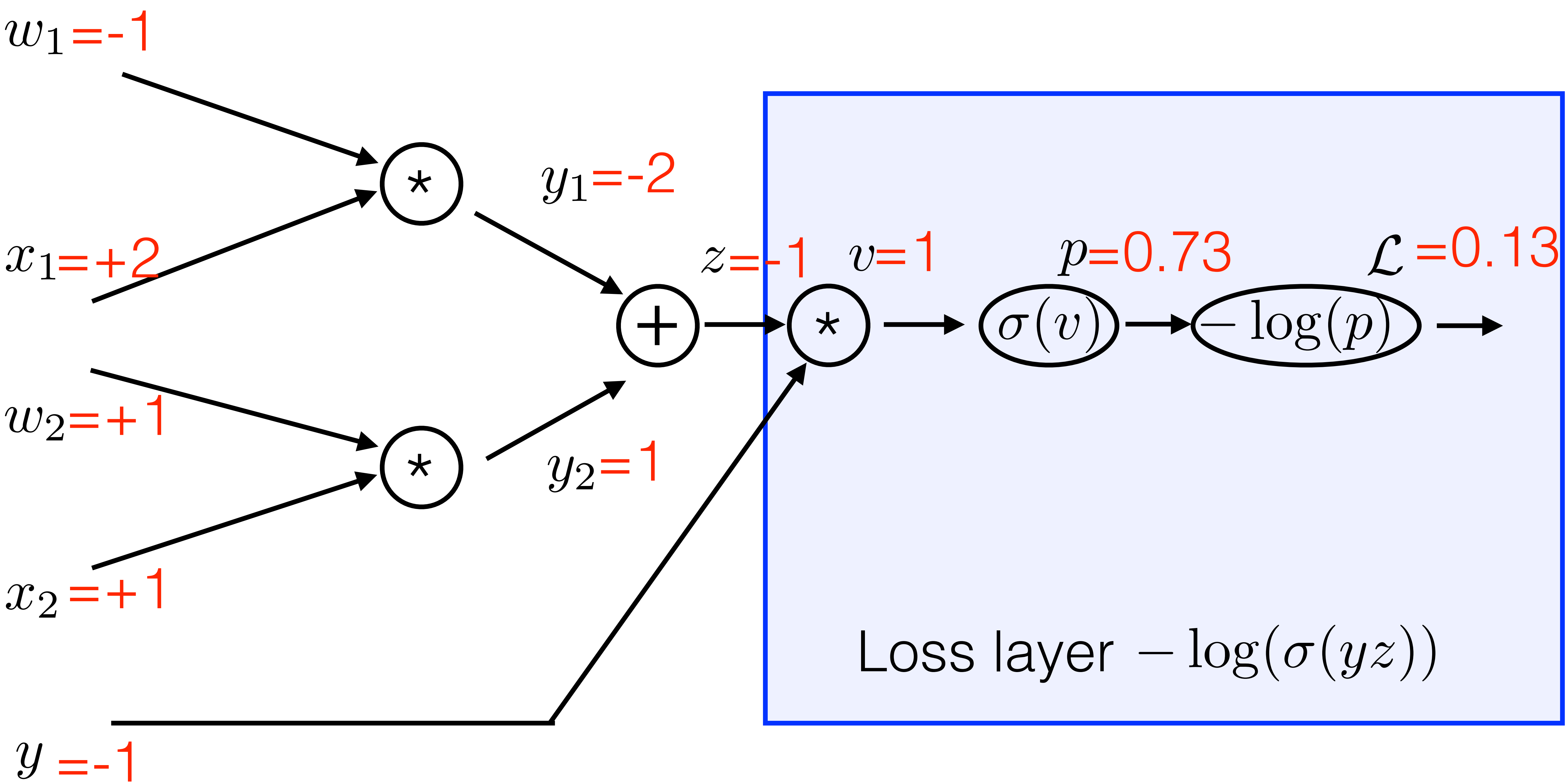
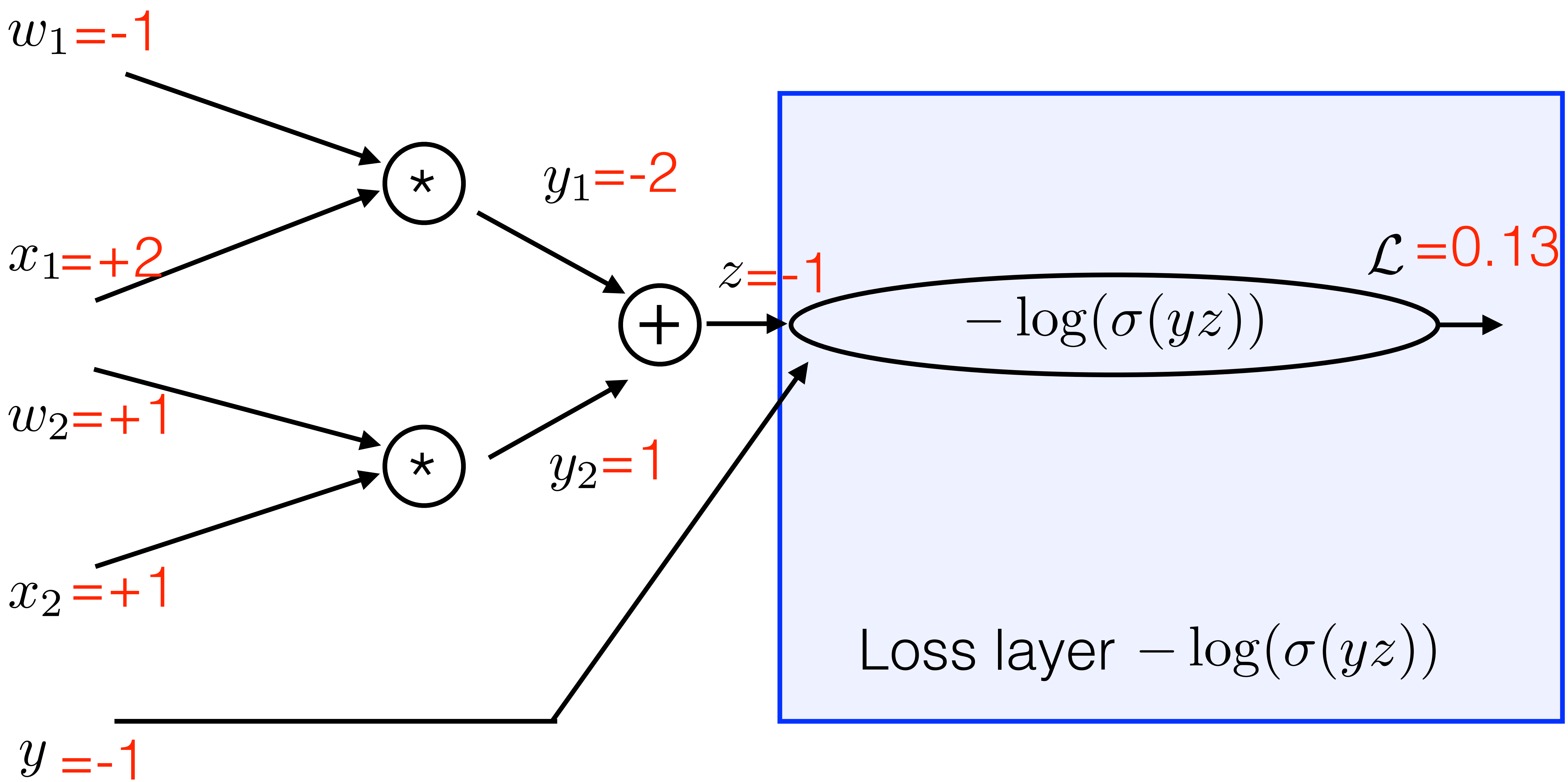


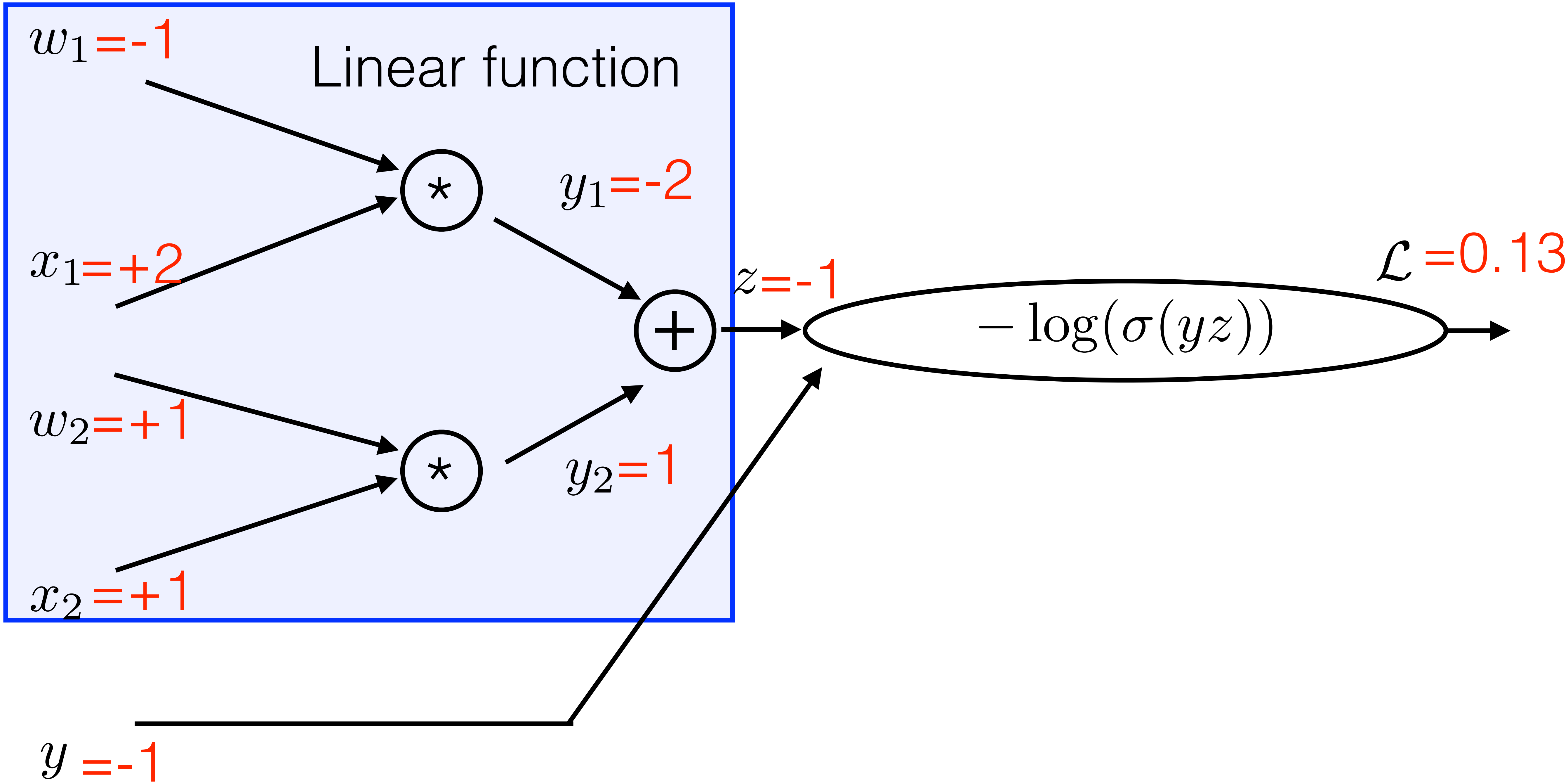
Backprop in vector representation



This is the logistic loss!

$$\mathcal{L}(y,z) = -\log(\sigma(yz)) = \log(1+\exp(-yz))$$

Backprop in vector representation



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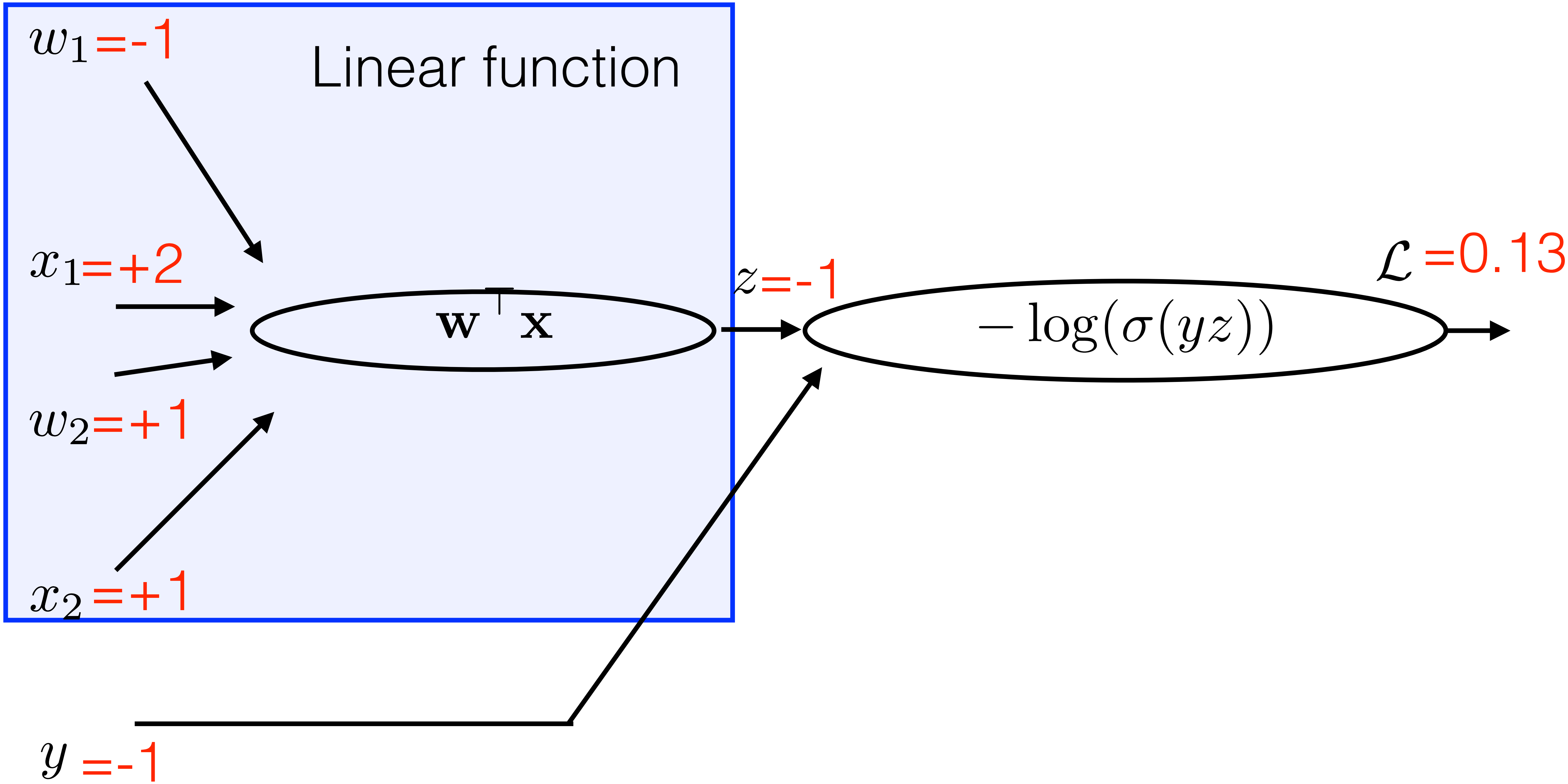

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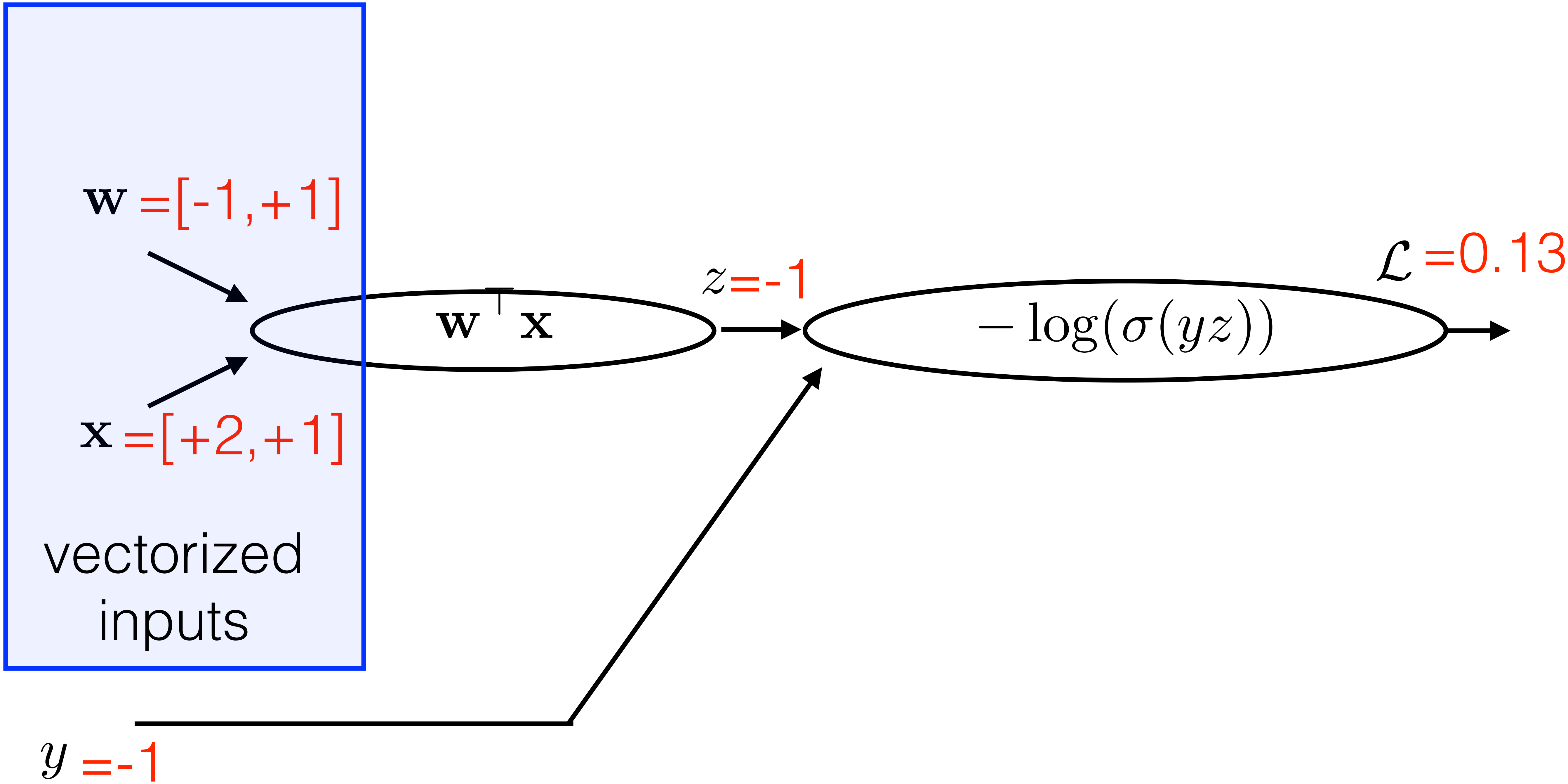

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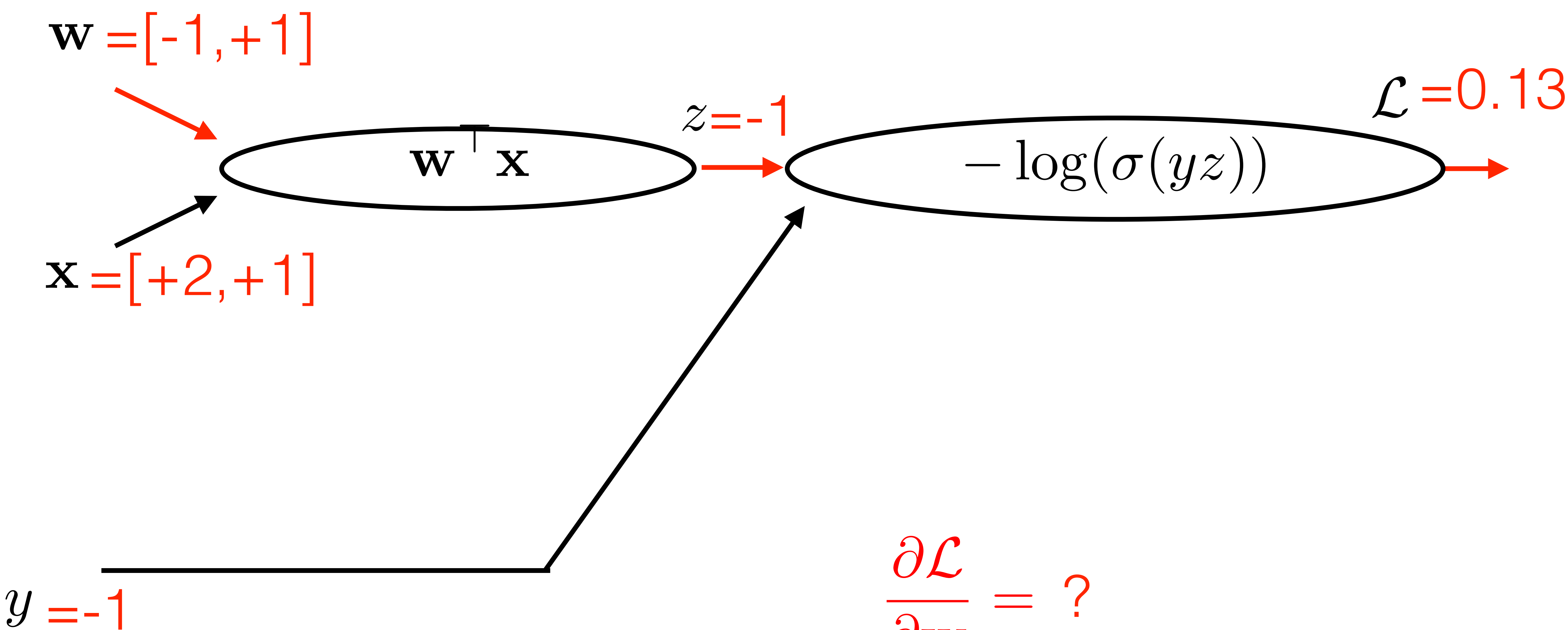
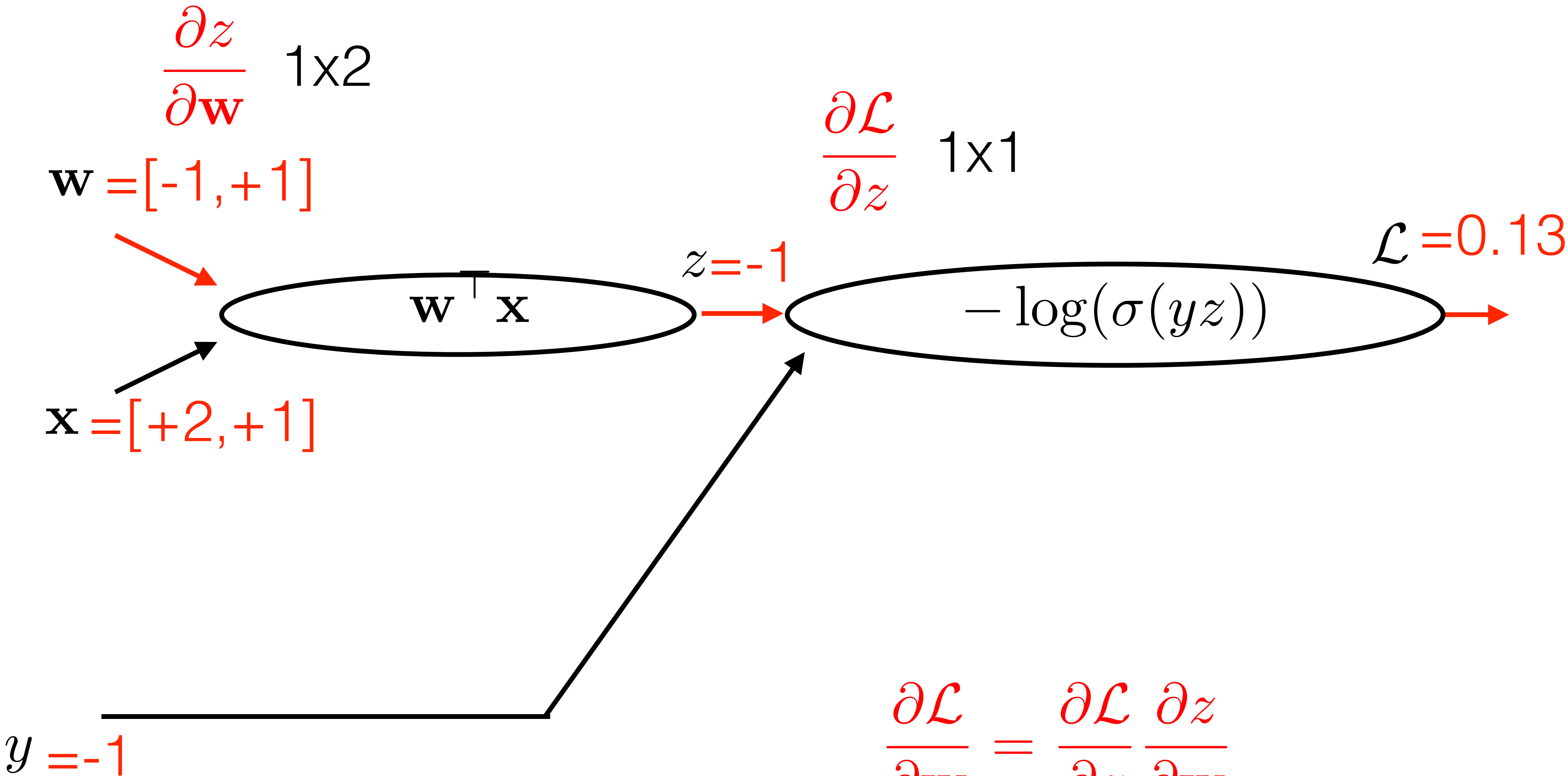


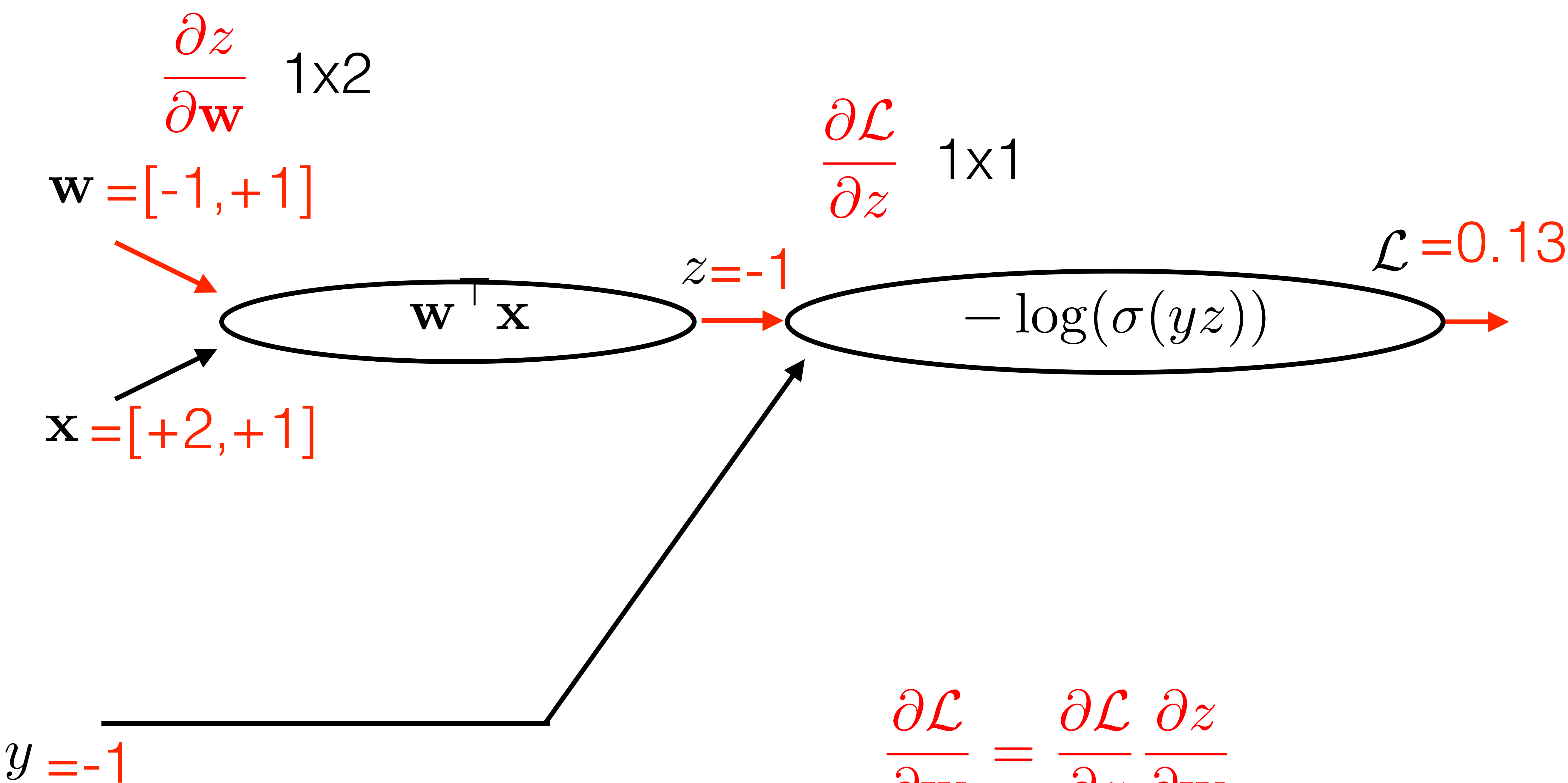
$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = ?$$

Backprop in vector representation



$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial z} \frac{\partial z}{\partial \mathbf{w}}$$

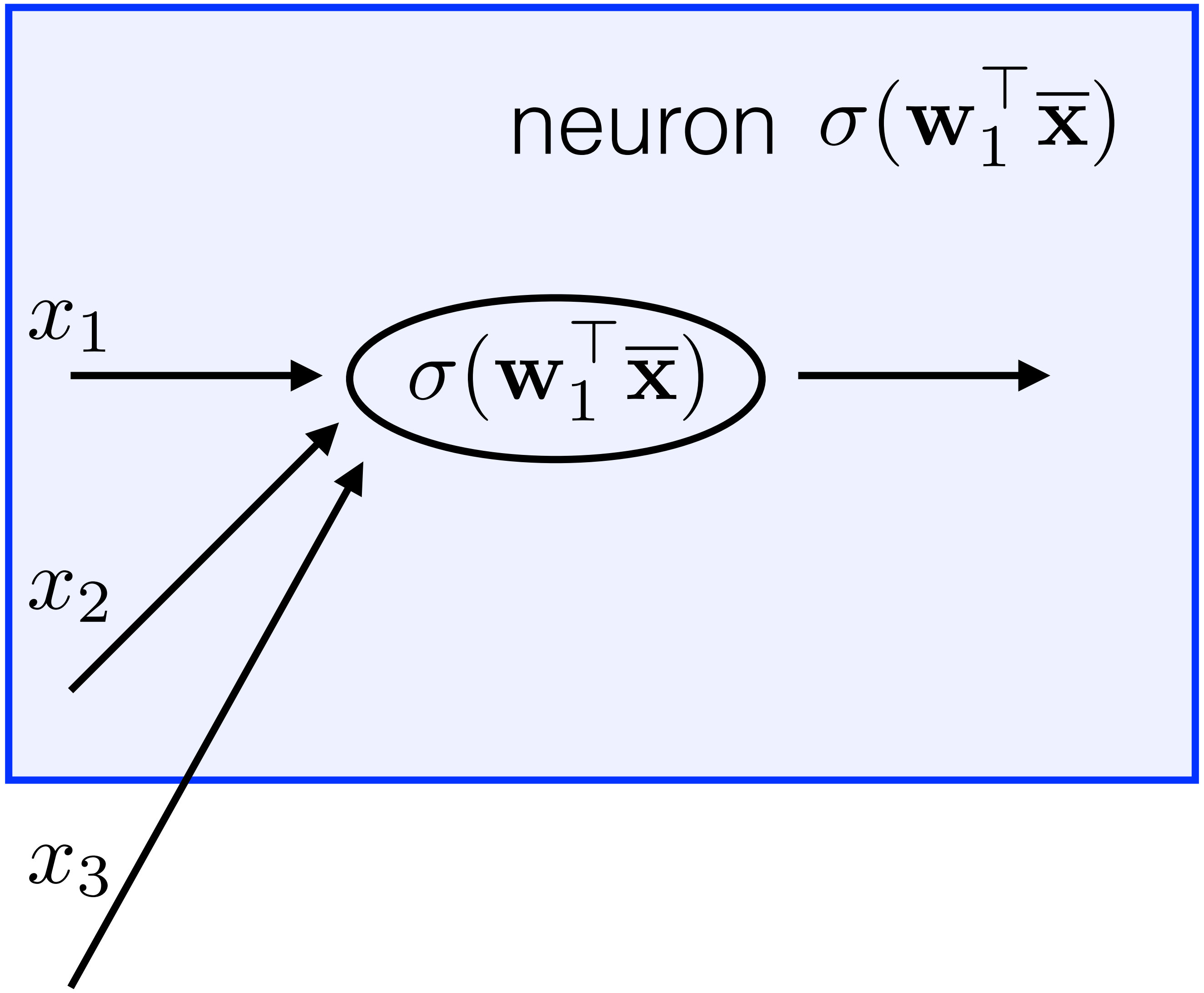
Backprop in vector representation



$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial z} \frac{\partial z}{\partial \mathbf{w}}$$

Learning from multiple training samples means summing up the gradient over all samples

Fully connected neural network



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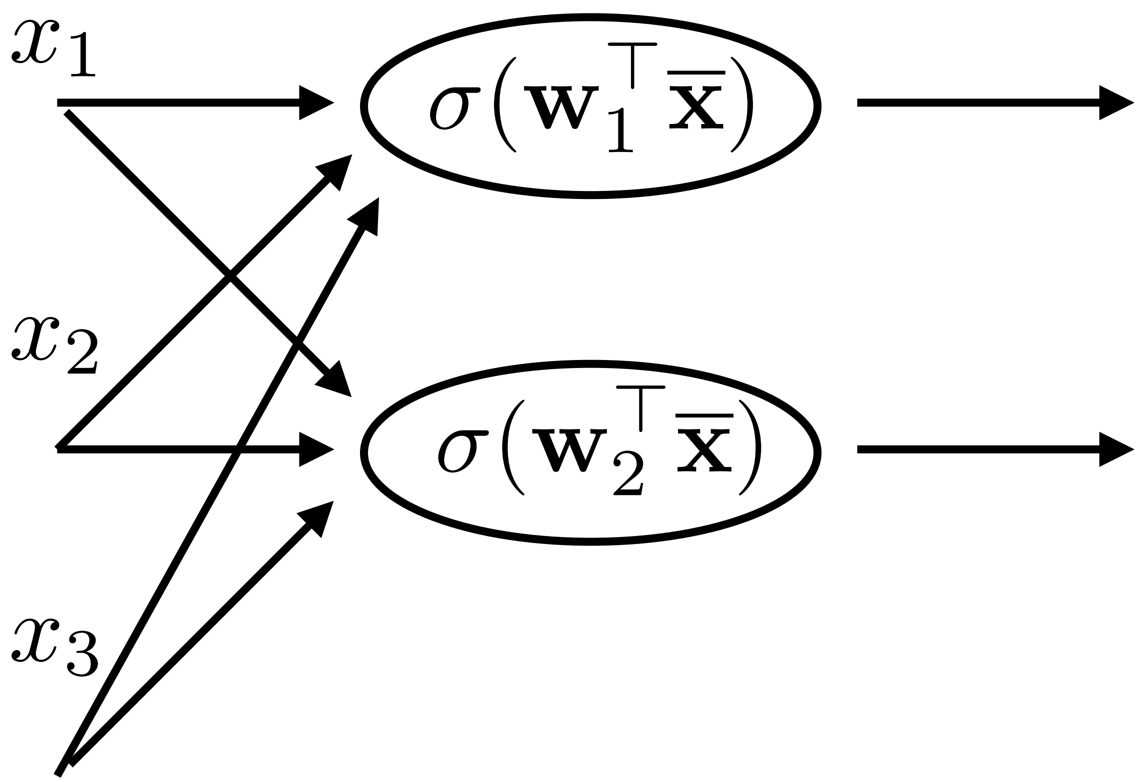

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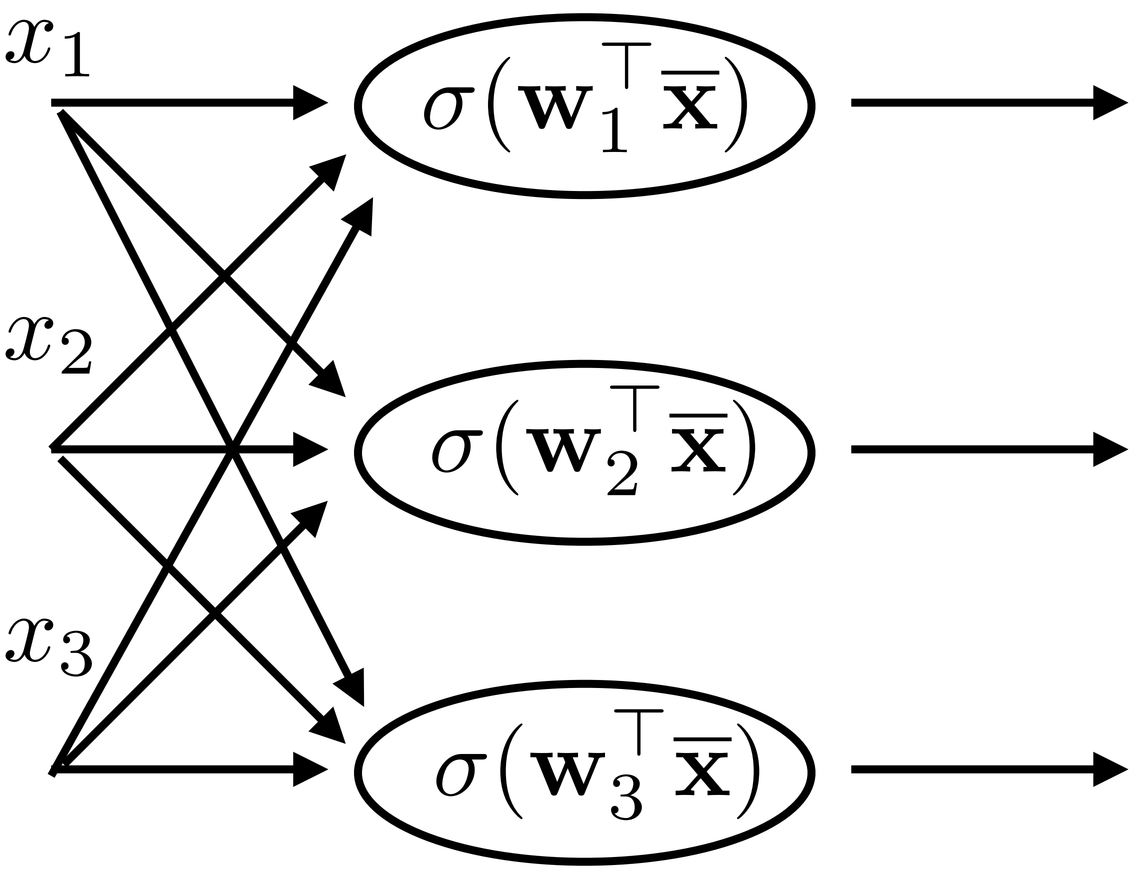

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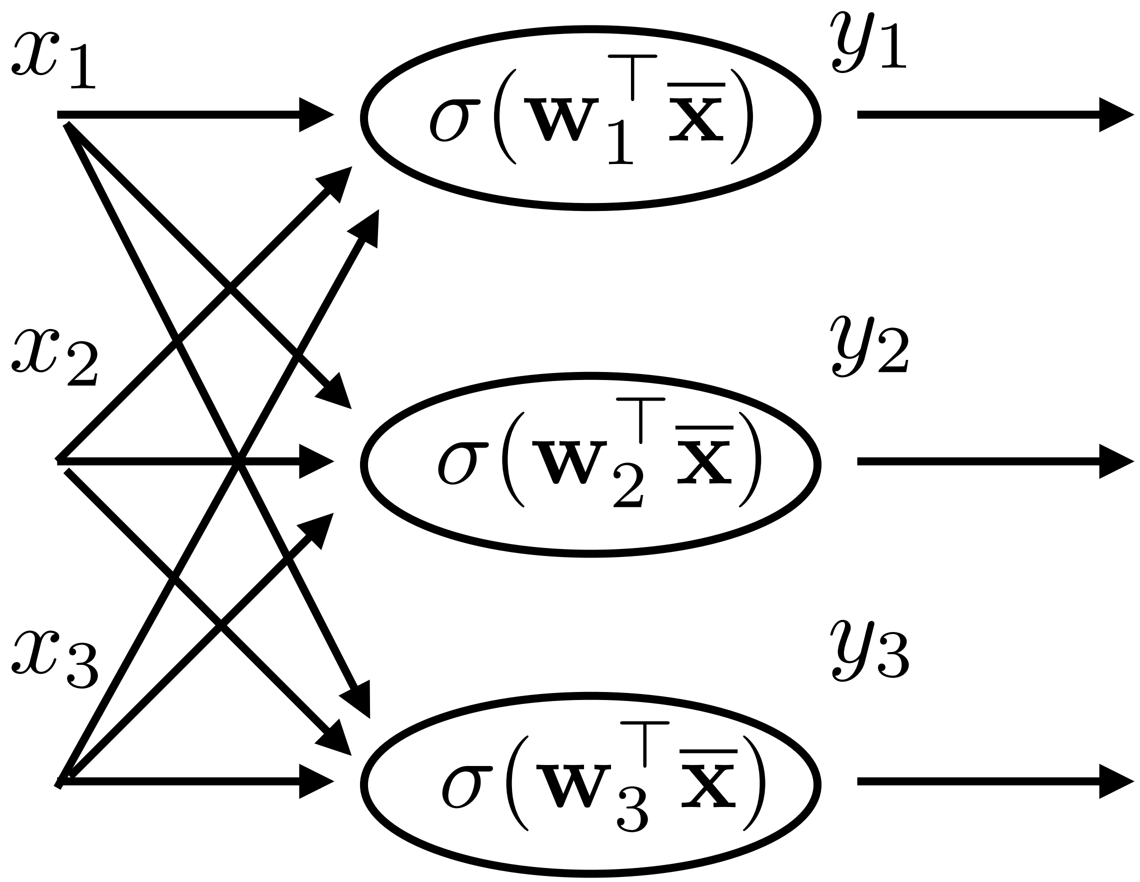

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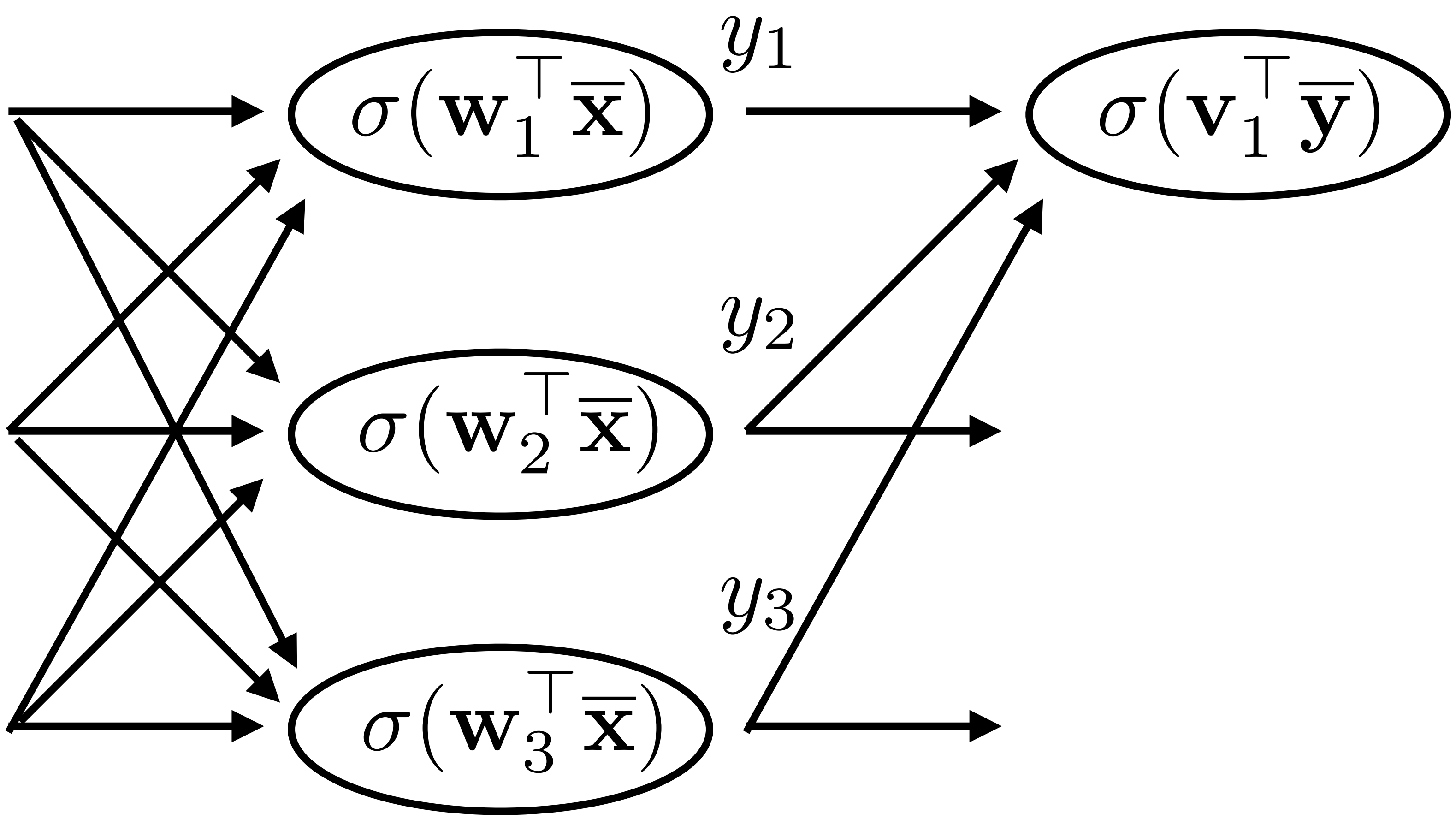

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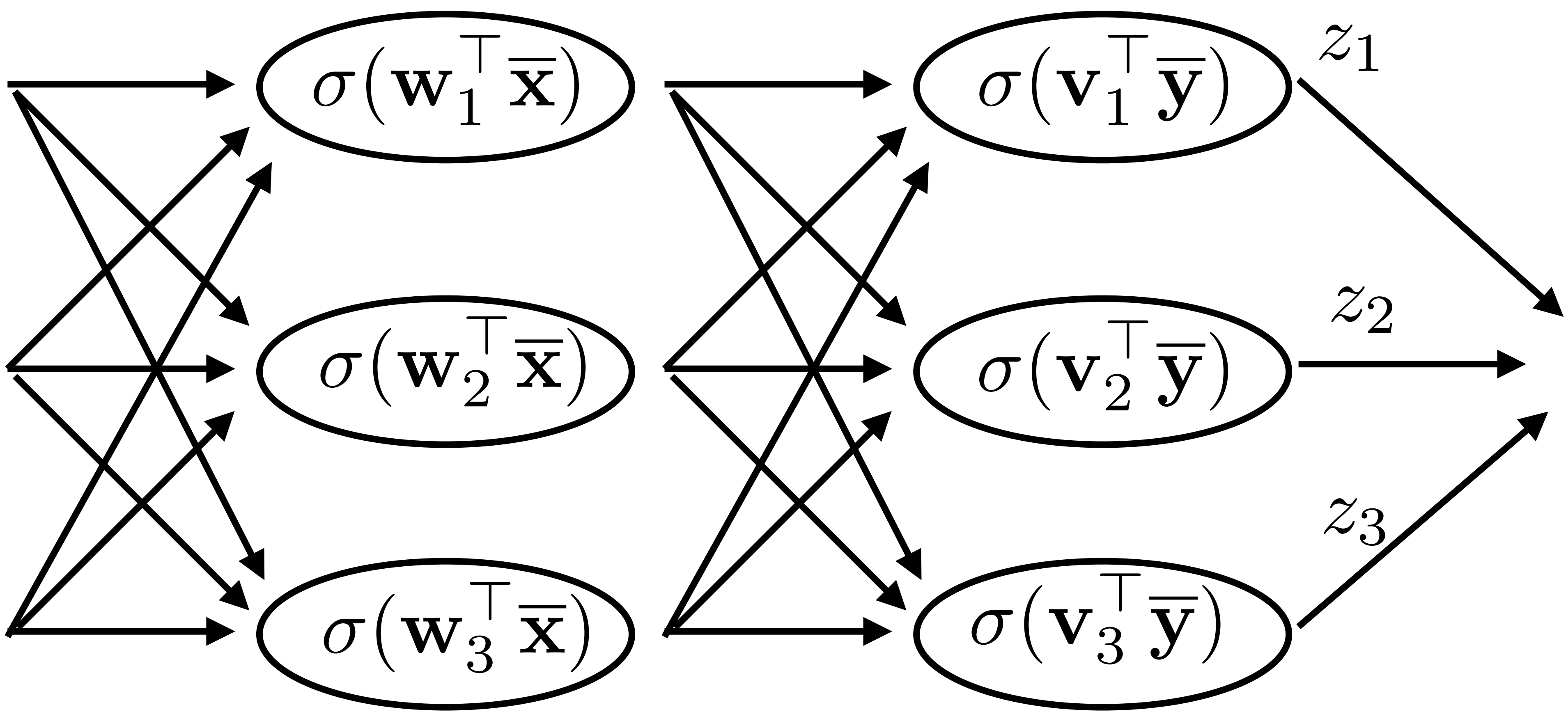

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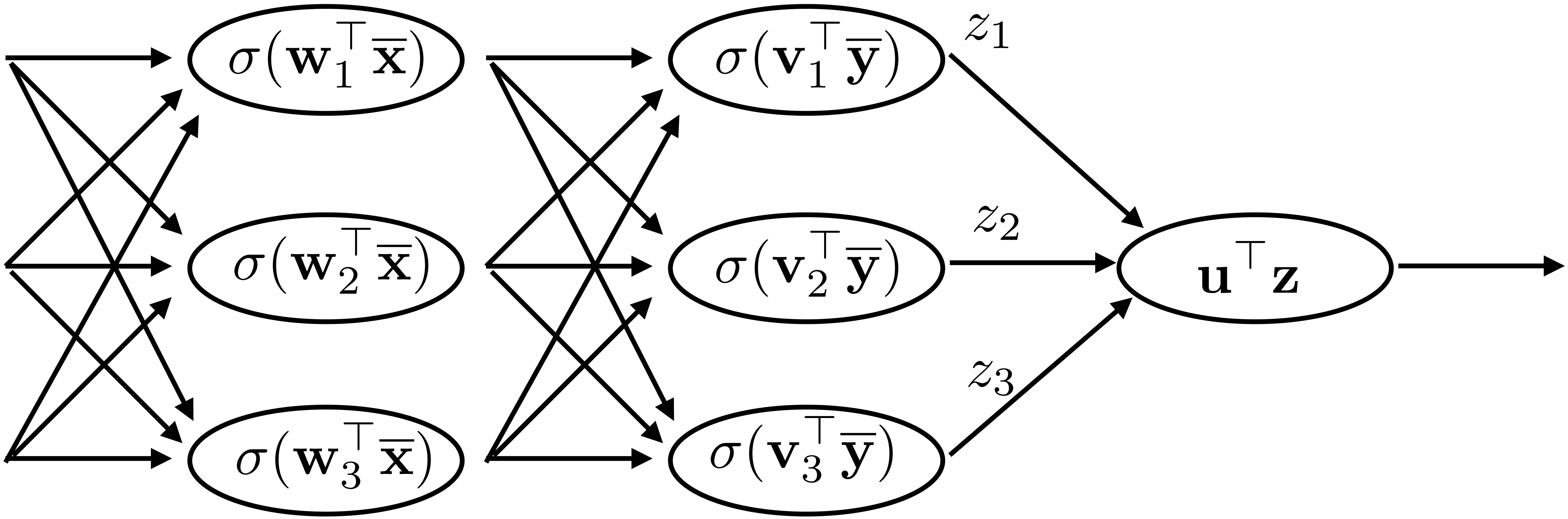

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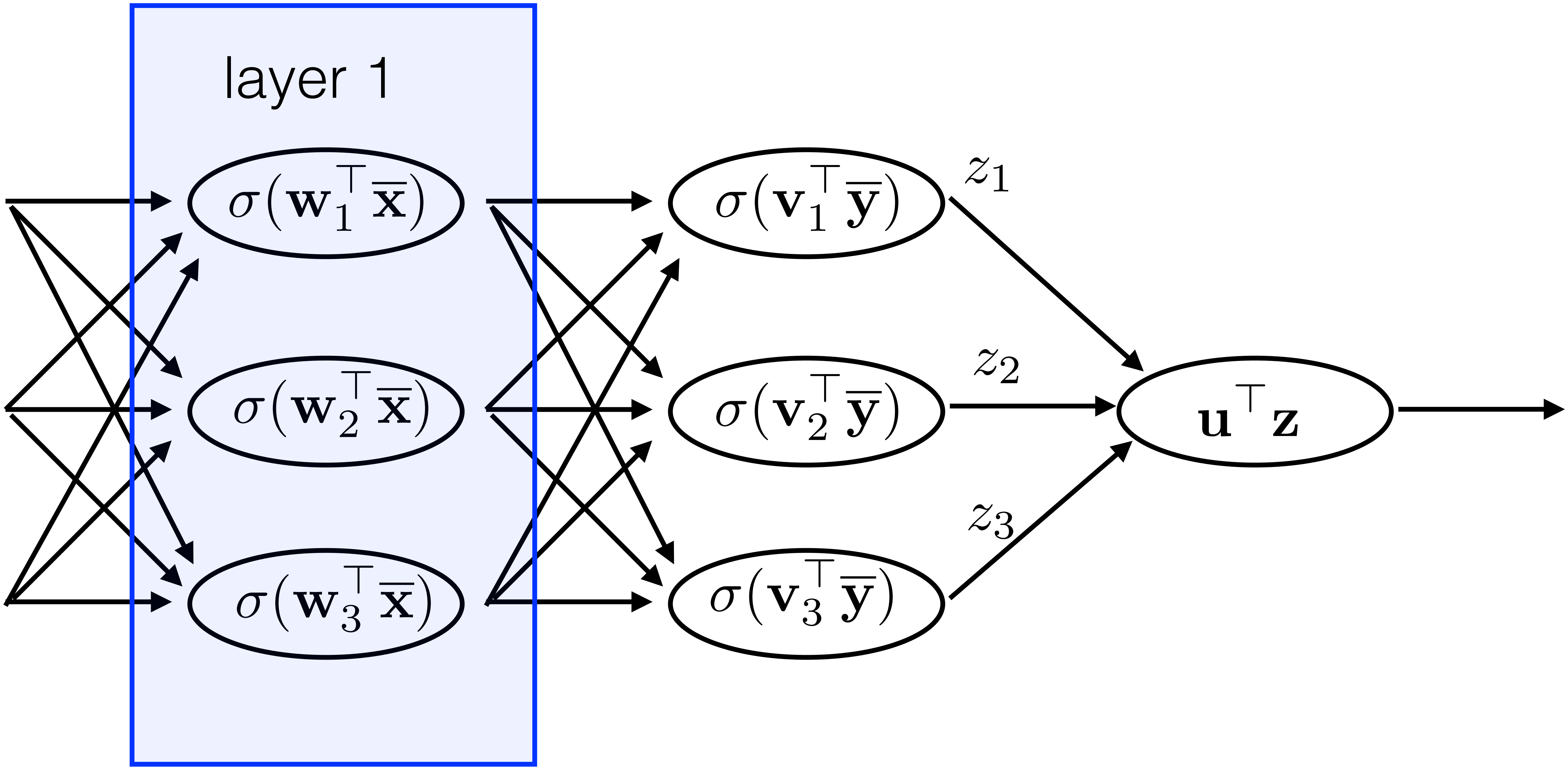

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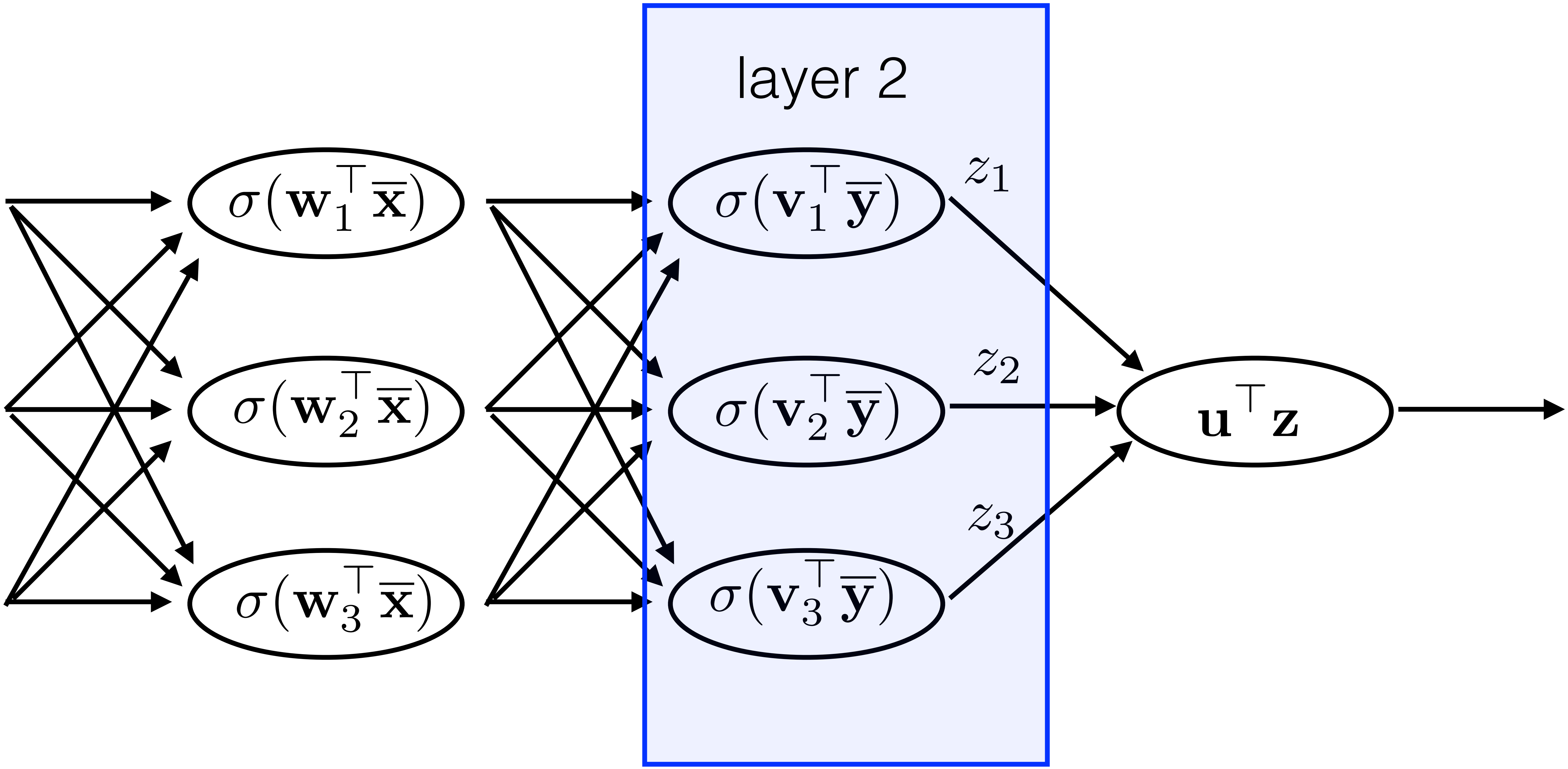

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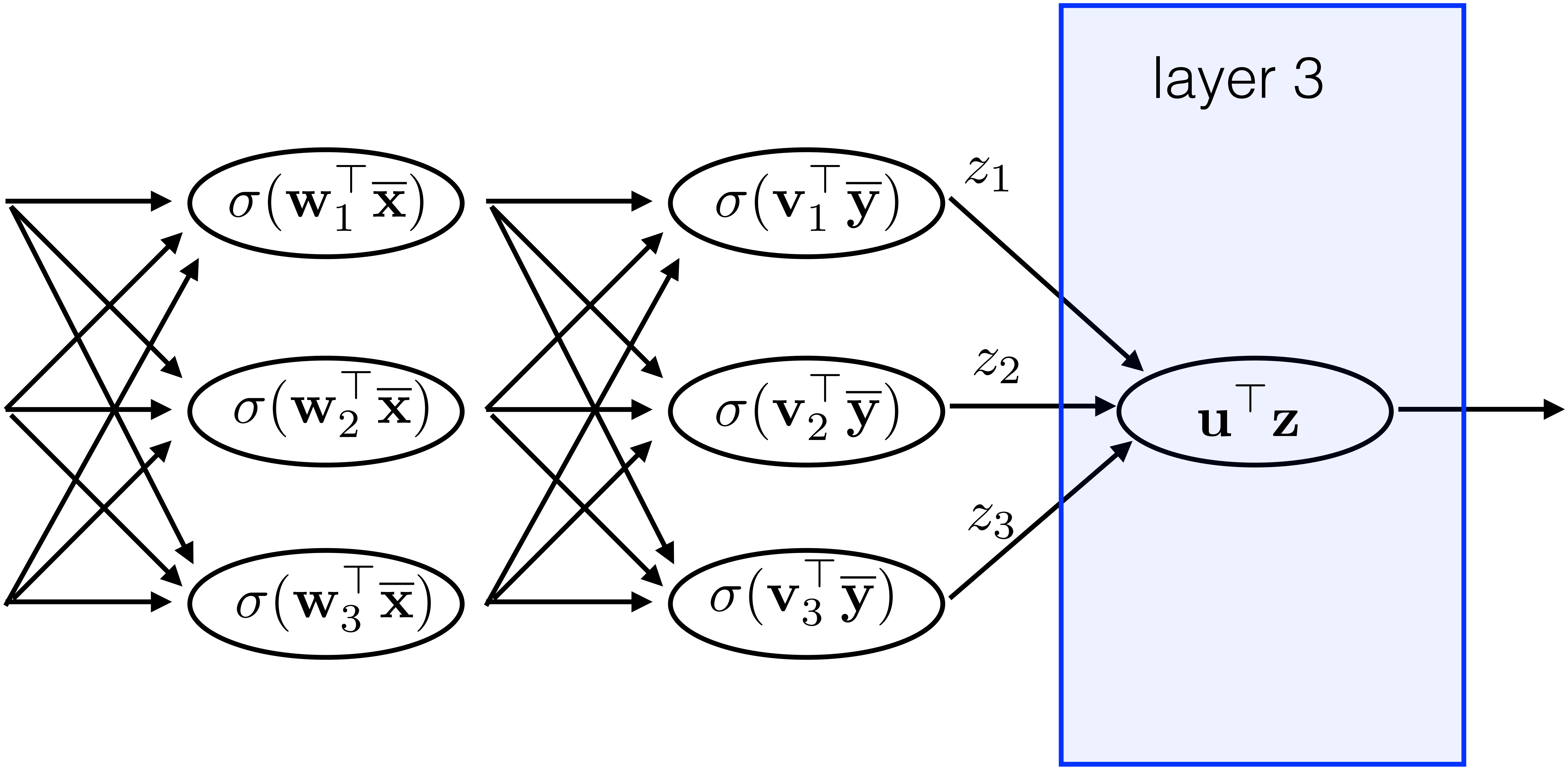

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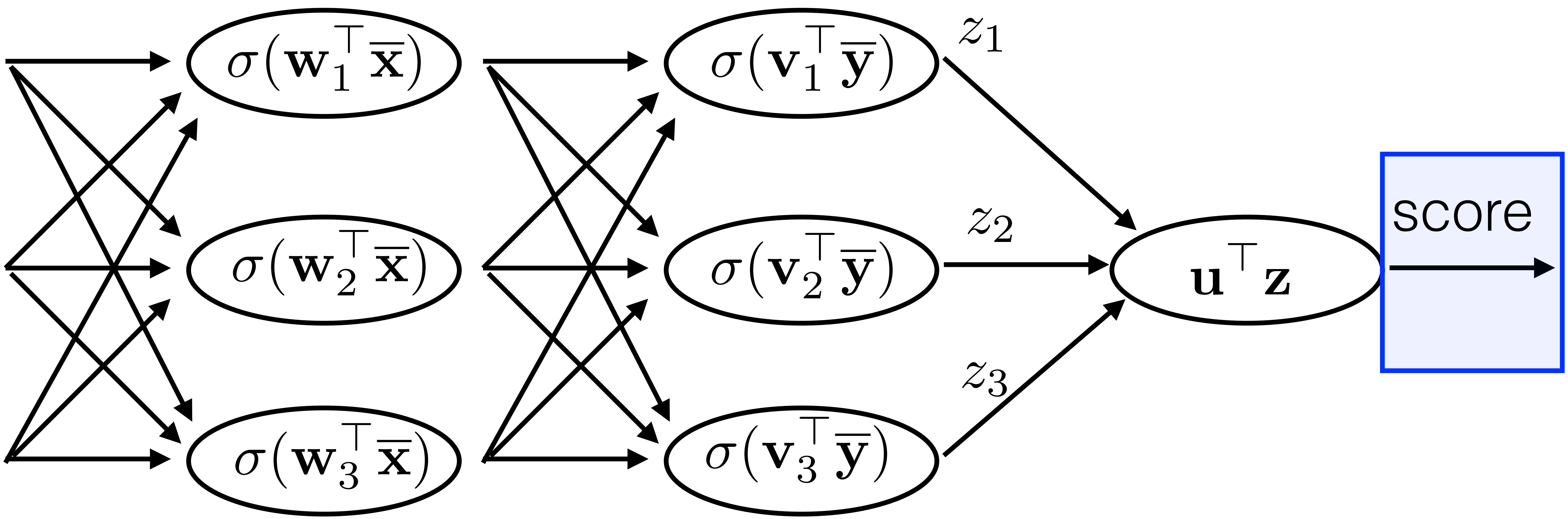

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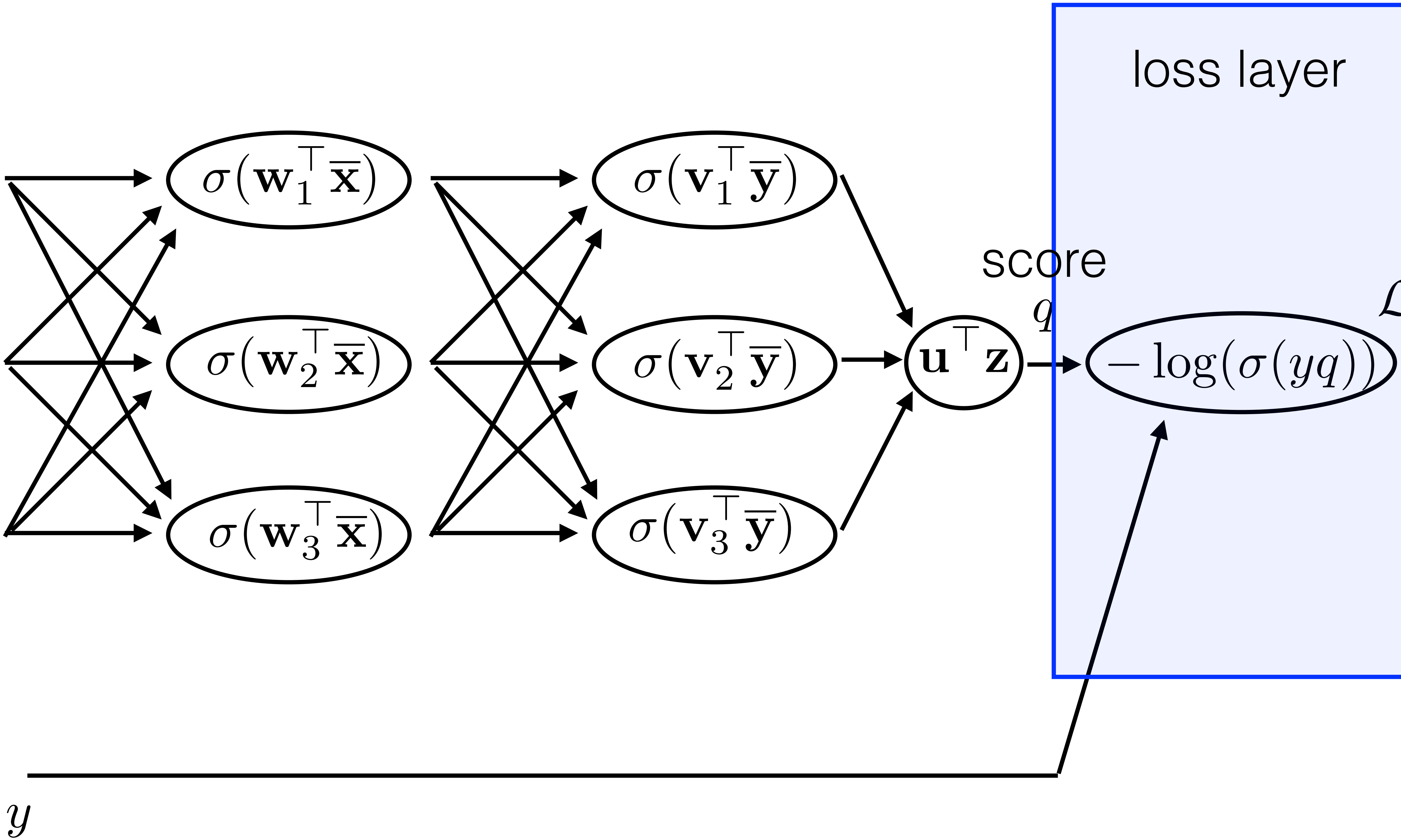

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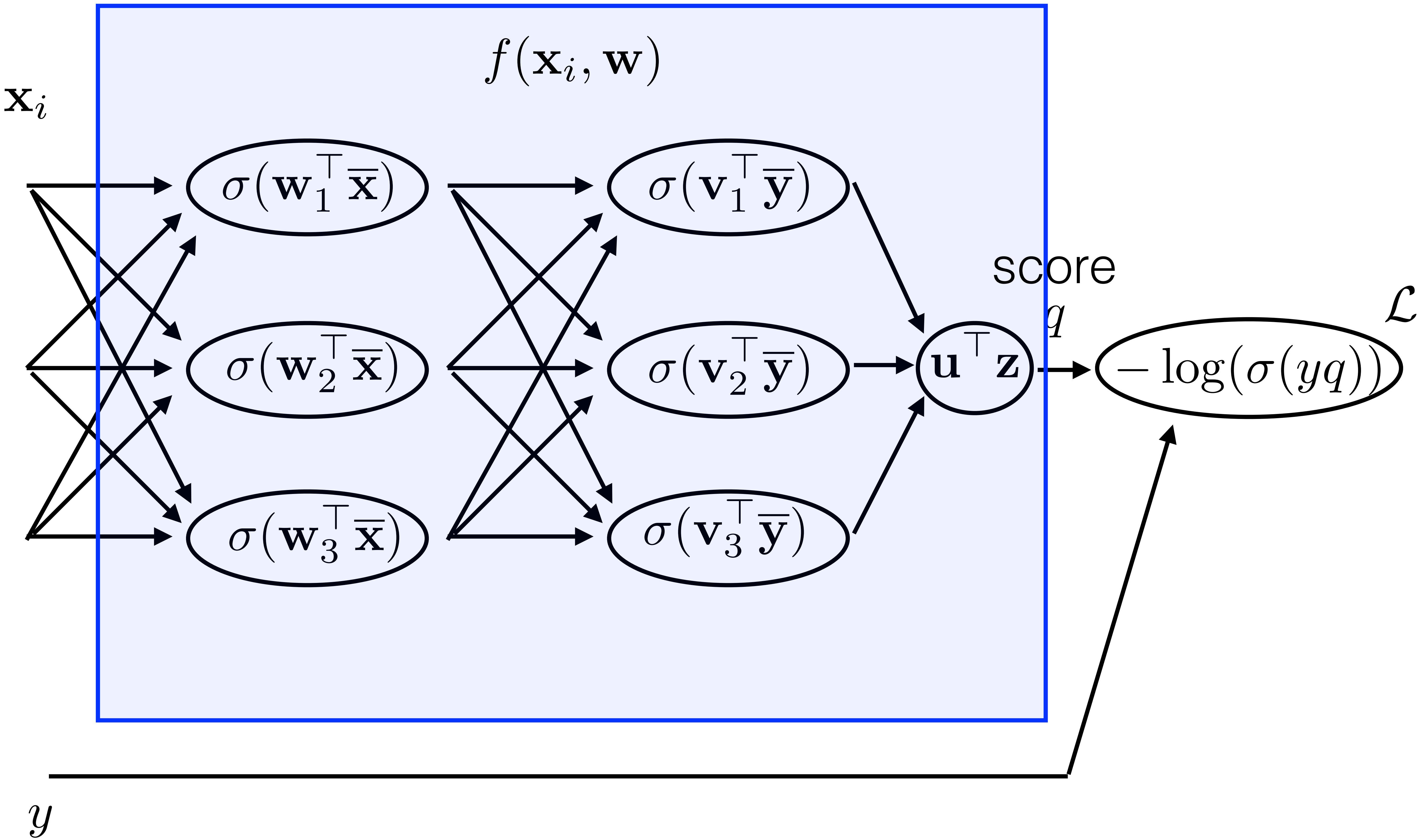

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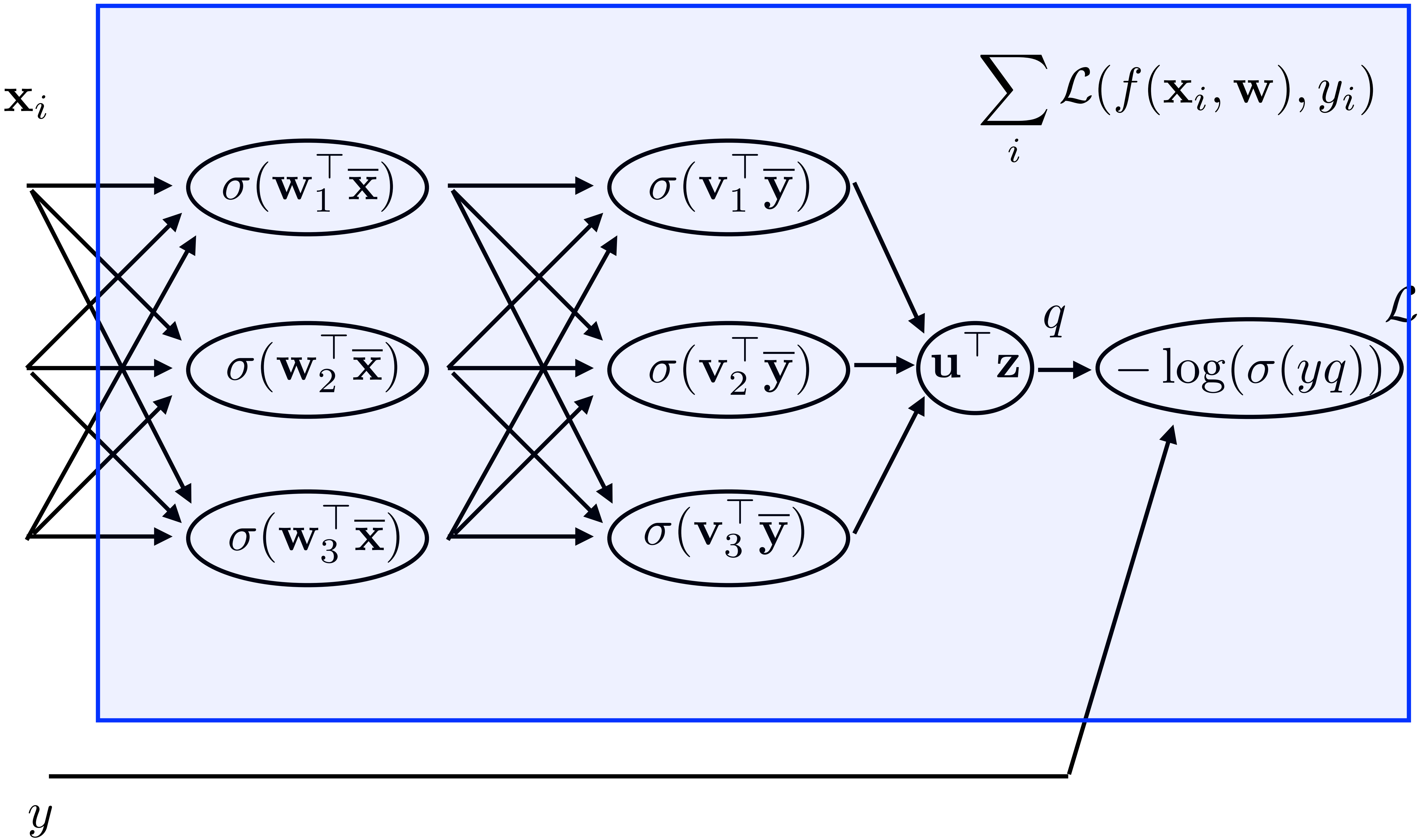


Learning of fully connected neural network

1. Estimate gradient

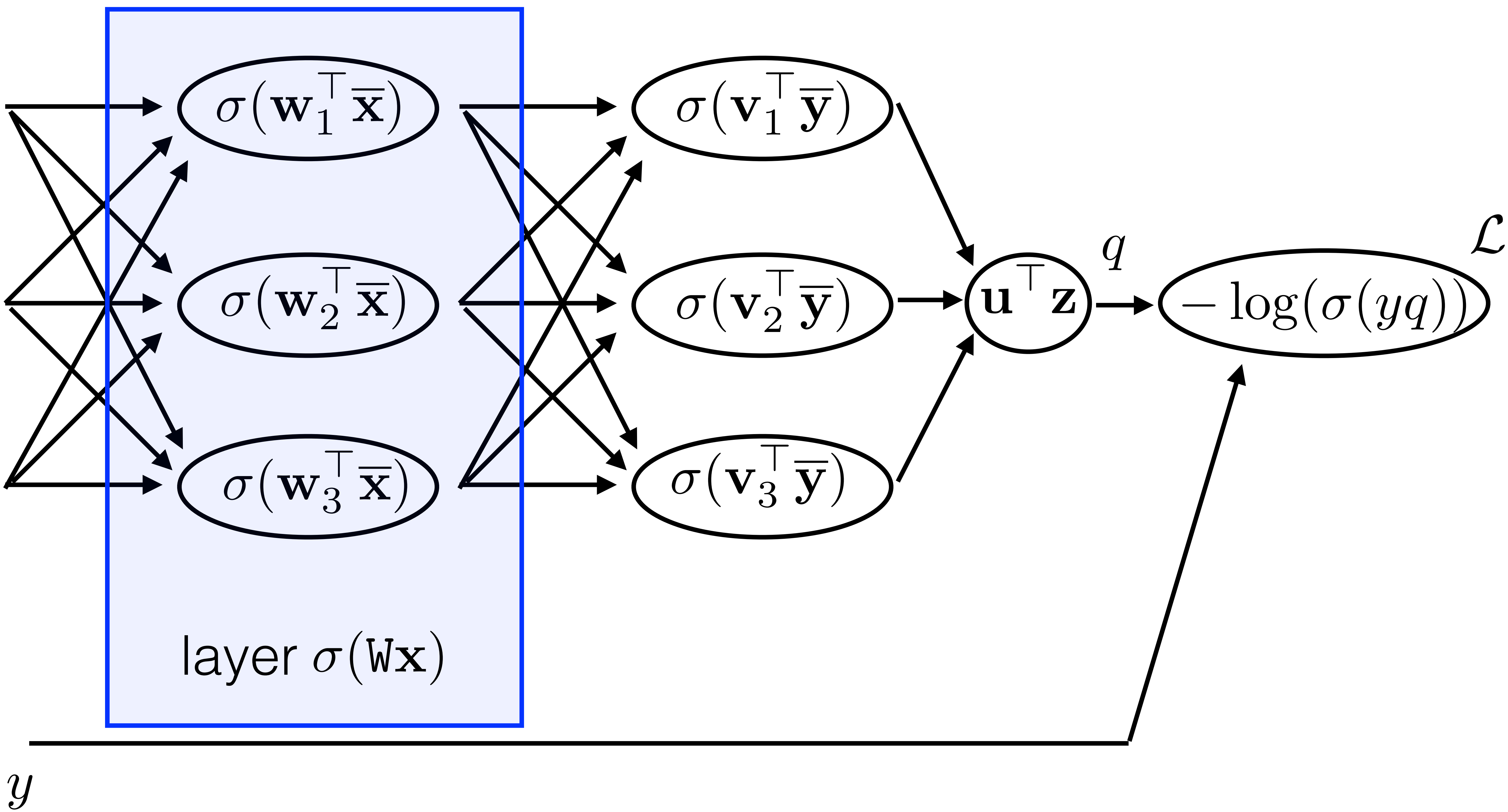
$$\sum_i \frac{\partial \mathcal{L}(f(\mathbf{x}_i, \mathbf{w}), y_i)}{\partial \mathbf{w}}$$

2. Update weights:

$$\mathbf{w} = \mathbf{w} - \alpha \sum_i \frac{\partial \mathcal{L}(f(\mathbf{x}_i, \mathbf{w}), y_i)}{\partial \mathbf{w}}$$

3. Optionally update learning rate α
4. Repeat until convergence

Learning of fully connected neural network



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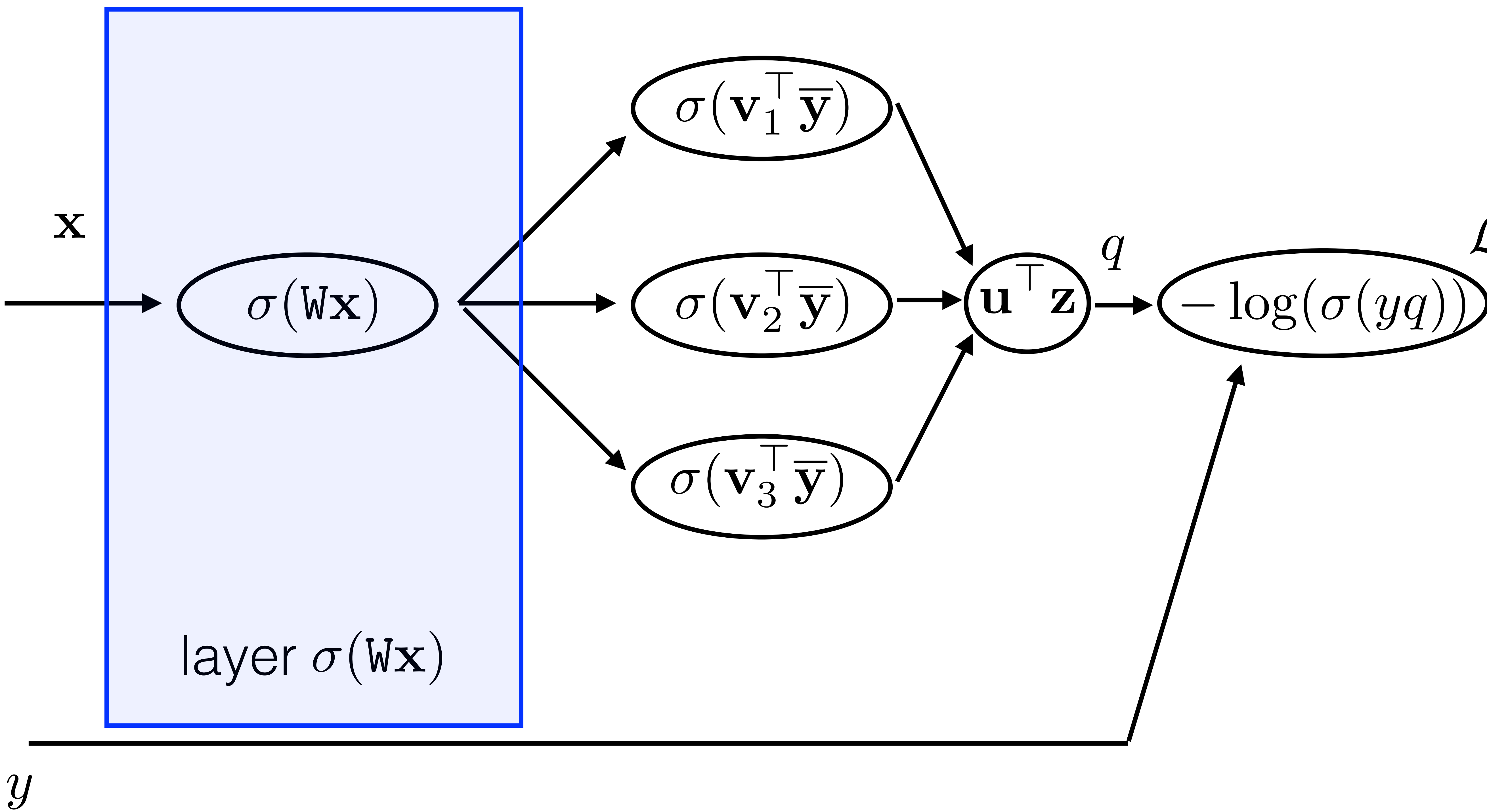

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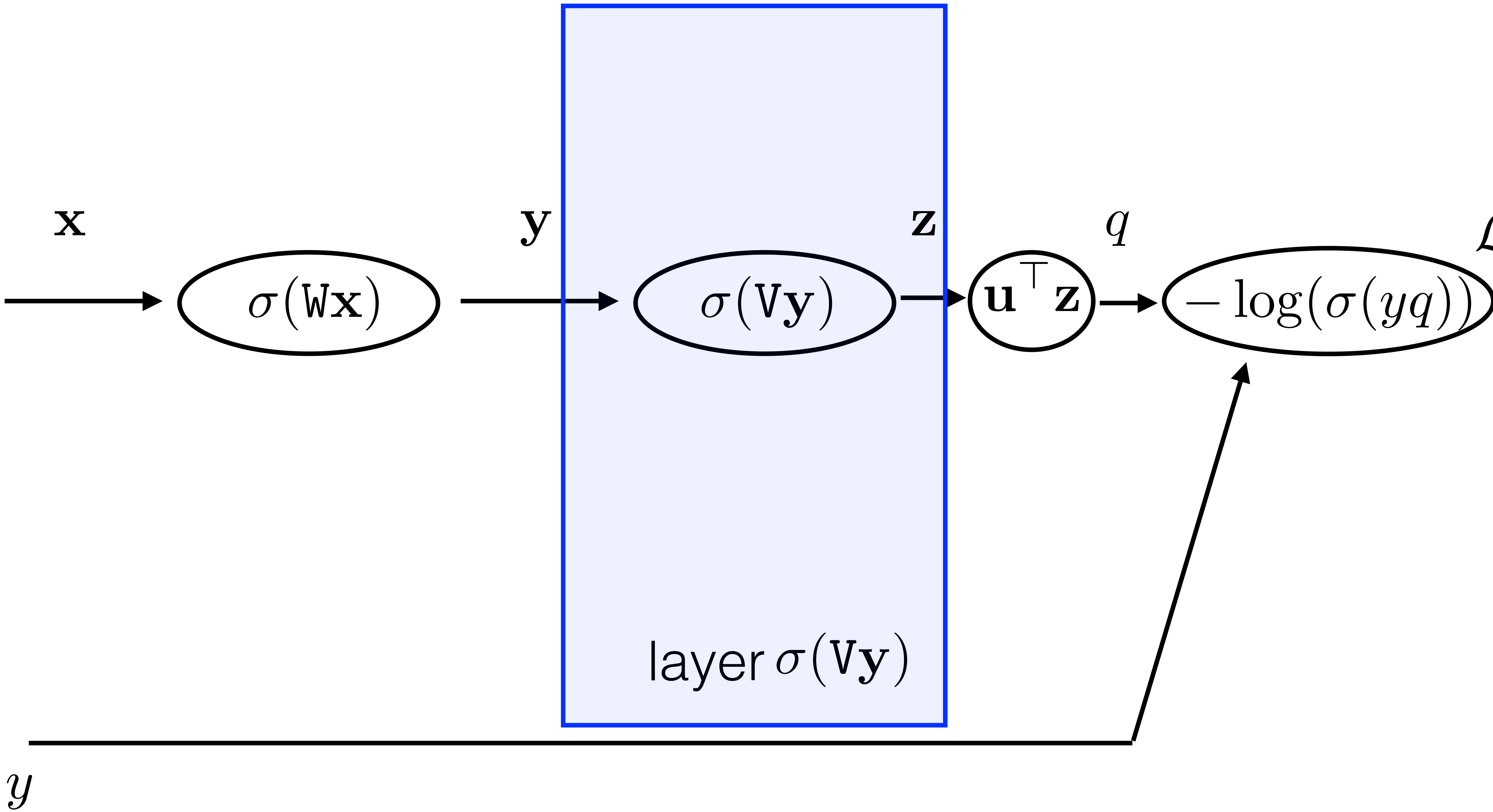
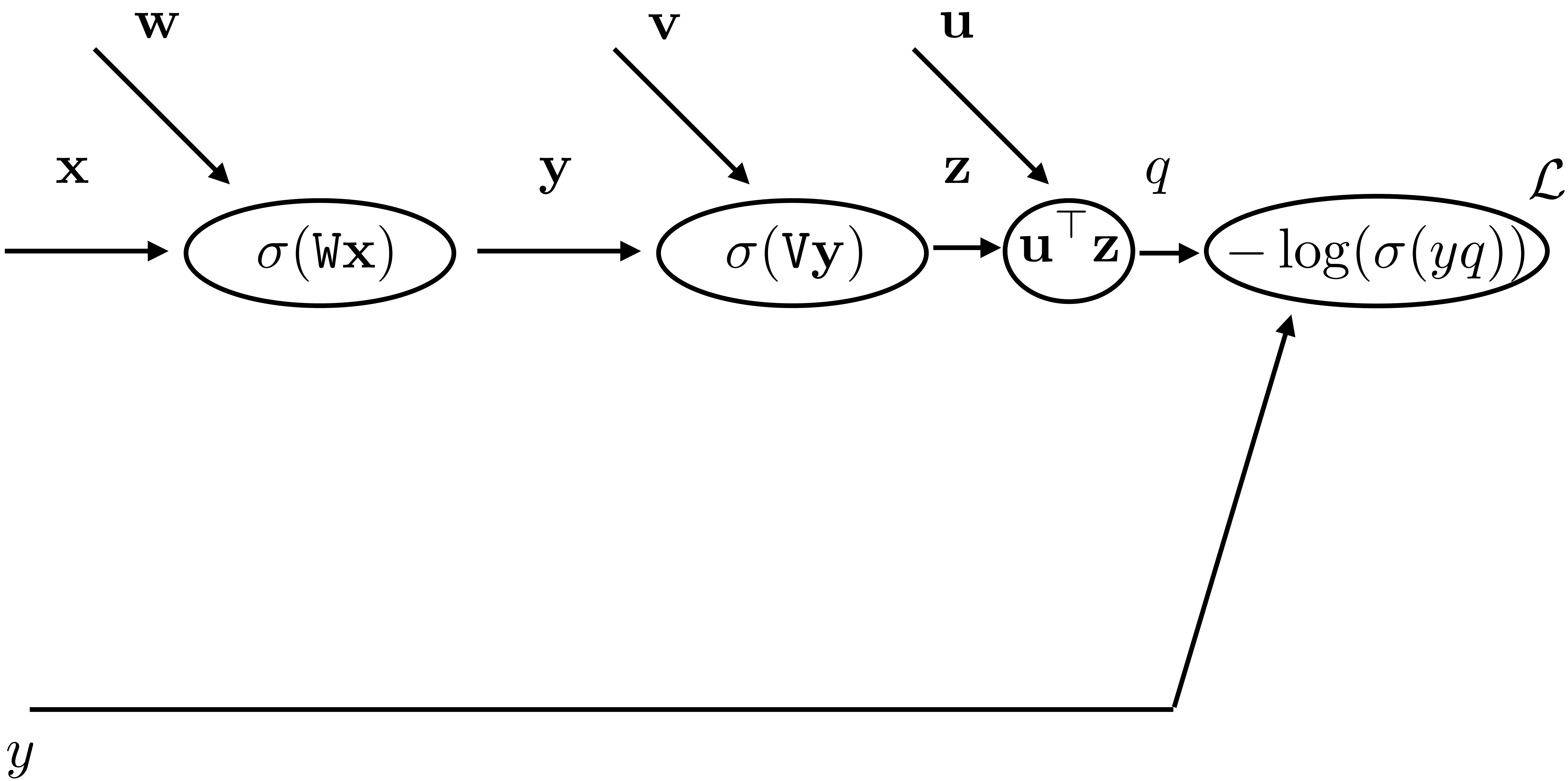


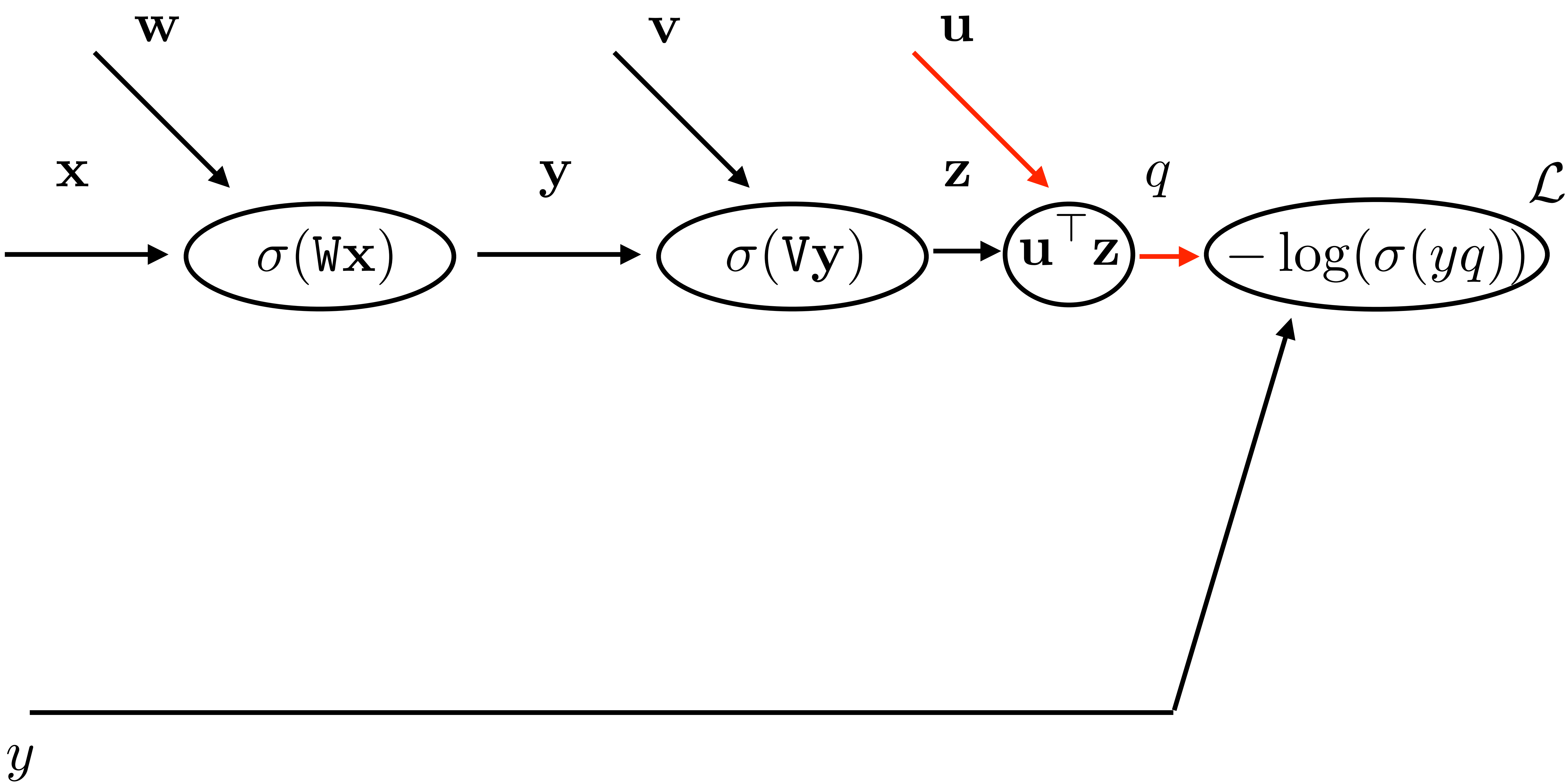
Learning of fully connected neural network

$\mathbf{w} = \mathrm{vec}(\mathtt{W}) \qquad \mathbf{v} = \mathrm{vec}(\mathtt{V})$



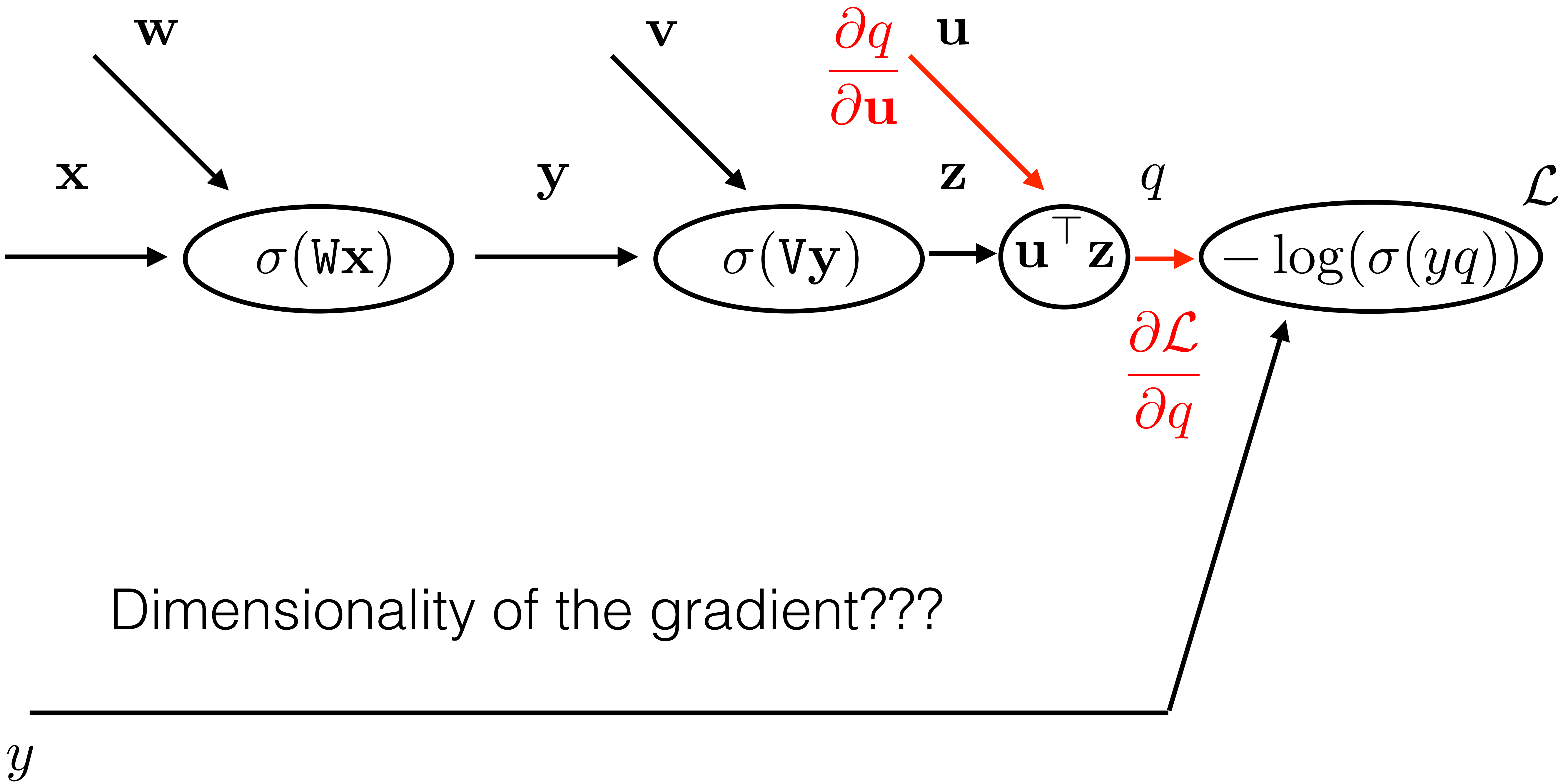
Learning of fully connected neural network

Derivative wrt $\mathbf{u}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = ?$



Learning of fully connected neural network

Derivative wrt $\mathbf{u}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = \frac{\partial \mathcal{L}}{\partial q} \frac{\partial q}{\partial \mathbf{u}}$

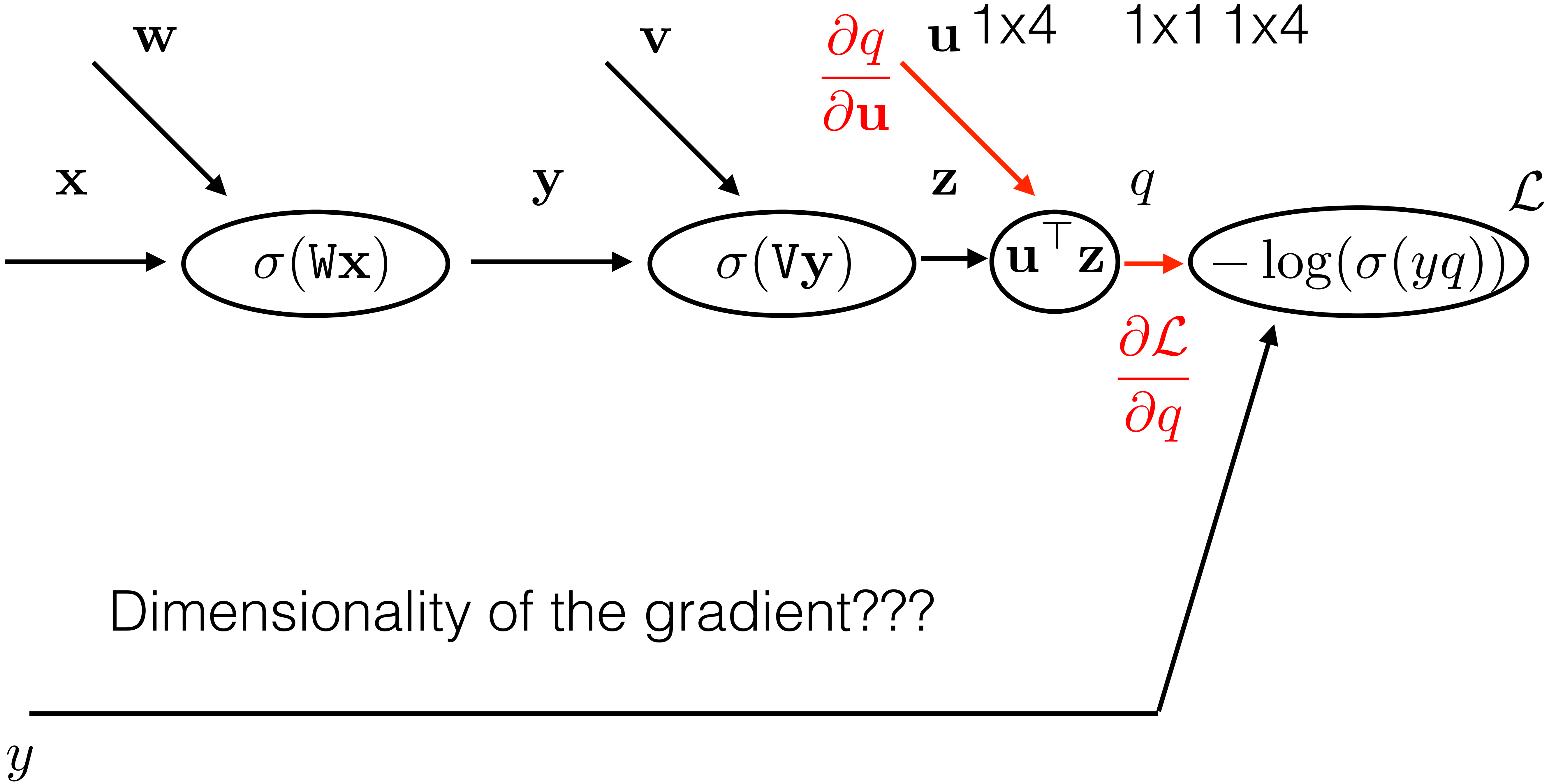


Dimensionality of the gradient???

Learning of fully connected neural network

Derivative wrt $\mathbf{u}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = \frac{\partial \mathcal{L}}{\partial q} \frac{\partial q}{\partial \mathbf{u}}$

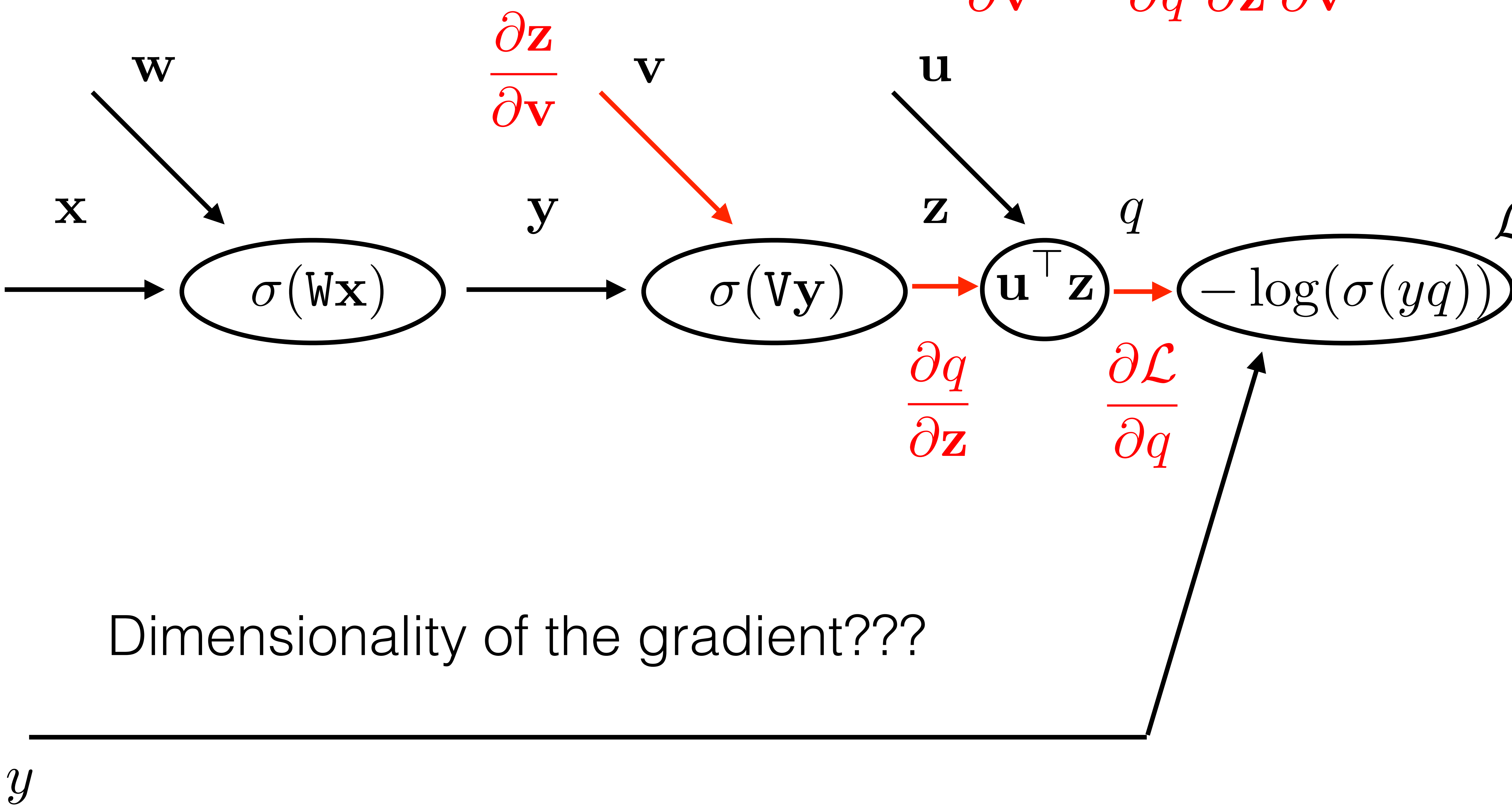
1x4 1x1 1x4



Dimensionality of the gradient???

Learning of fully connected neural network

Derivative wrt $\mathbf{v}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{v}}$

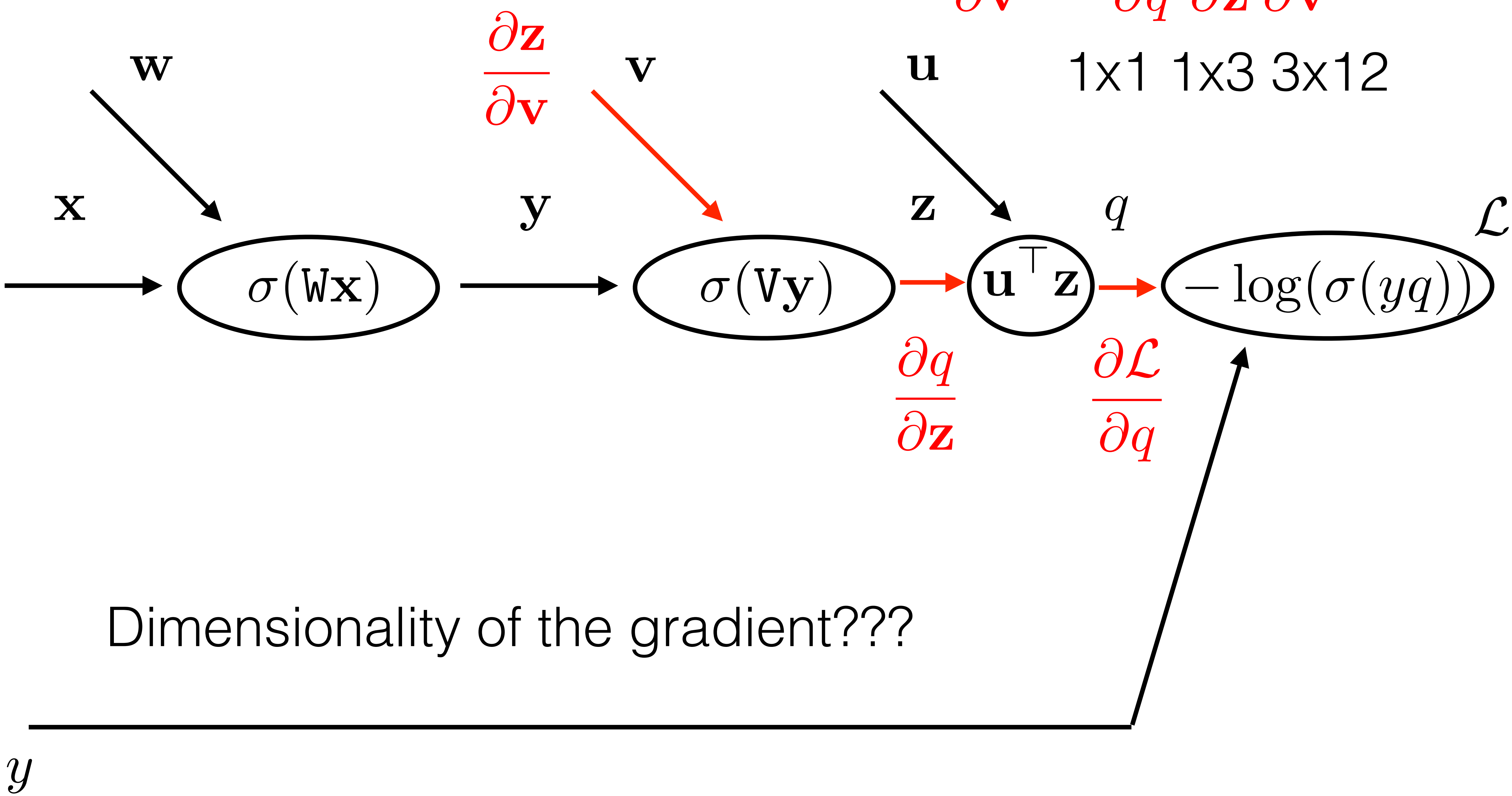


Dimensionality of the gradient???

Learning of fully connected neural network

Derivative wrt $\mathbf{v}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = \frac{\partial \mathcal{L}}{\partial q} \frac{\partial q}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial \mathbf{v}}$

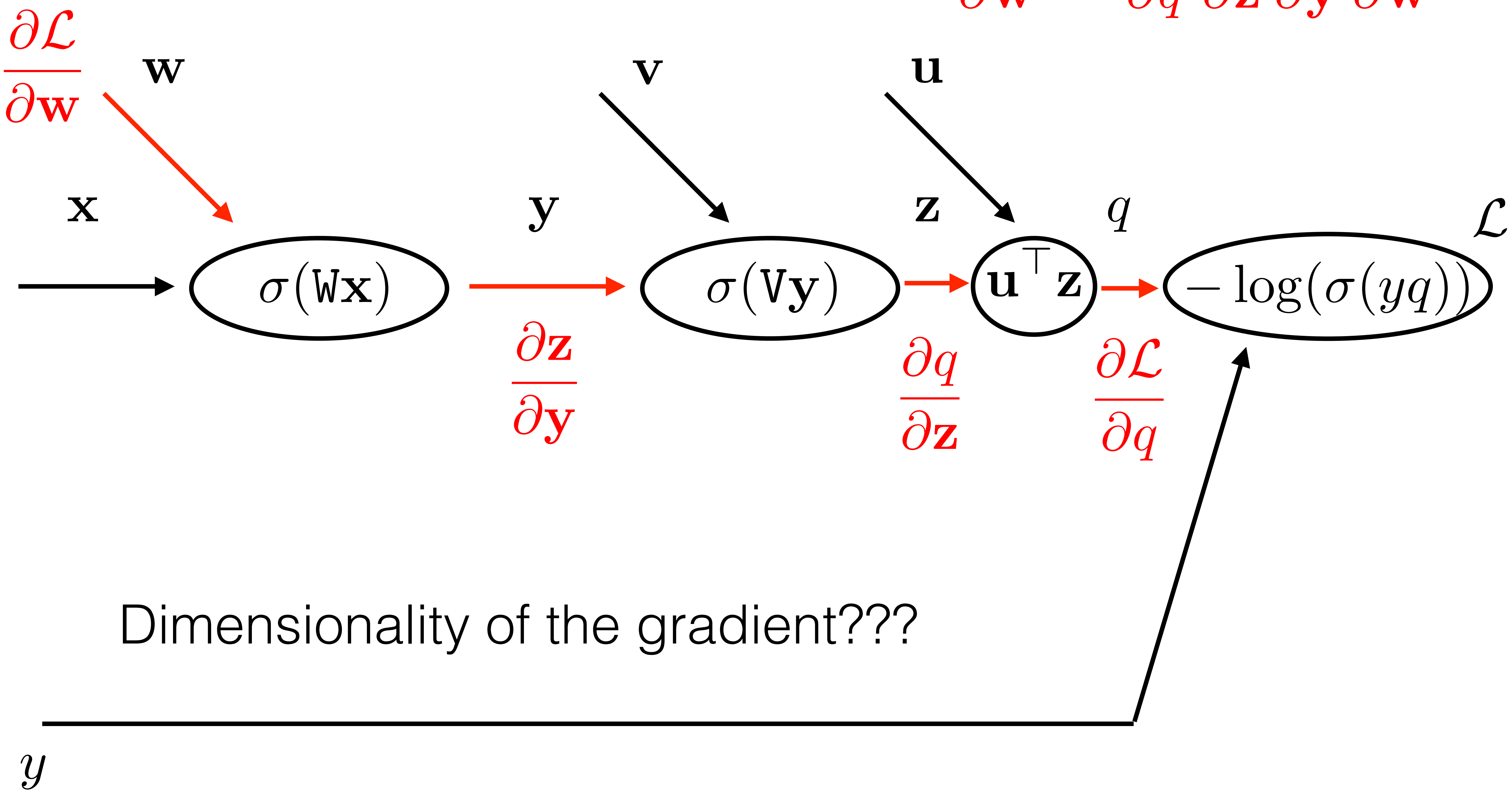
1x1 1x3 3x12



Dimensionality of the gradient???

Learning of fully connected neural network

Derivative wrt $\mathbf{w}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{w}}$

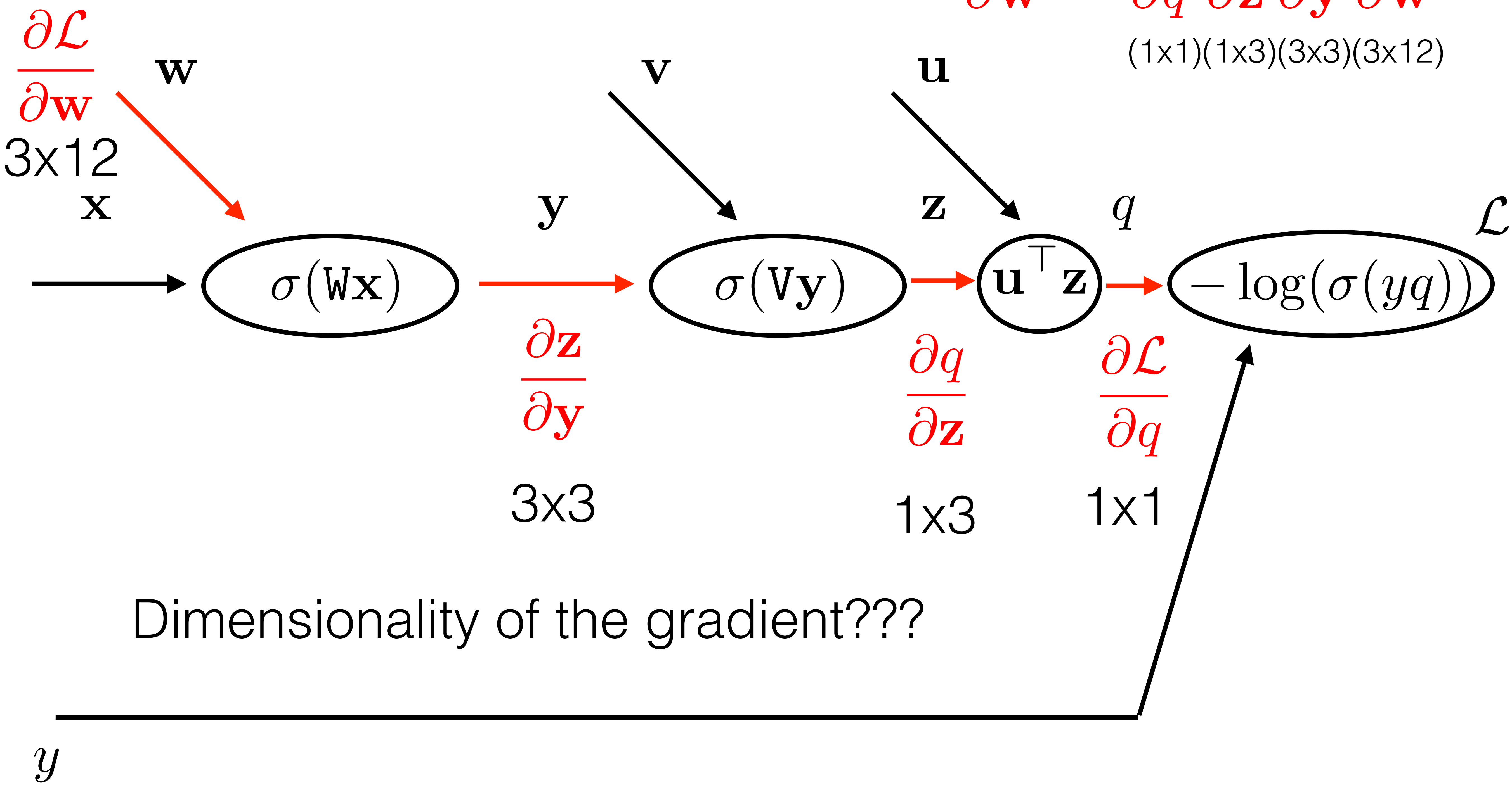


Dimensionality of the gradient???

Learning of fully connected neural network

Derivative wrt $\mathbf{w}$: $$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{w}}$$

(1x1)(1x3)(3x3)(3x12)



Dimensionality of the gradient???

Learning of fully connected neural network

1. Estimate all required local gradients
2. Update weights:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{u}} \qquad \mathbf{u} = \mathbf{u} - \alpha \left[\frac{\partial \mathcal{L}}{\partial \mathbf{u}}\right]^\top$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{v}} \qquad \mathbf{v} = \mathbf{v} - \alpha \left[\frac{\partial \mathcal{L}}{\partial \mathbf{v}}\right]^\top$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{w}} \qquad \mathbf{w} = \mathbf{w} - \alpha \left[\frac{\partial \mathcal{L}}{\partial \mathbf{w}}\right]^\top$$

3. Optionally update learning rate α
4. Repeat until convergence

Neural nets summary

- Neural net is a function created as concatenation of simplier functions (e.g. neurons or layers of neurons)
- Gradient optimization of the neural net is called backpropagation
- Neural net frameworks has many predefined layers
- **Spoiler alert:** It does not work (on images) at all - why?

$$\mathbf{w}^* = \arg\min_{\mathbf{w}} \left(\sum_i -\log(p(y_i|\mathbf{x}_i, \mathbf{w})) \right) + (-\log p(\mathbf{w}))$$

loss function prior/regulariser

- Class of function represented by a NN is too general.
- Naive regulariser helps a bit, but dimensionality/wildness is huge => curse-of-dimensionality, overfitting,…
- What is number of weights between two 1000-neuron layers?
- **Next lecture:** study animal cortex to find a stronger prior on the class of suitable functions.
- **Spoiler alert 2:**
  reduce very general class of functions ”neuron layer” to very specific sub-class of functions “convolution layer”