

A0B17MTB – Matlab

# Part #1



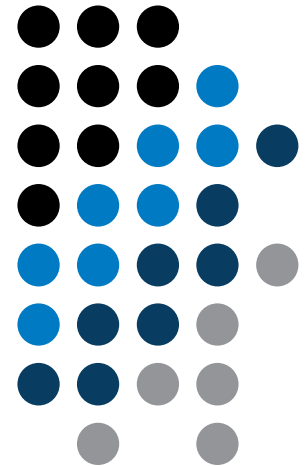
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# You will learn ...

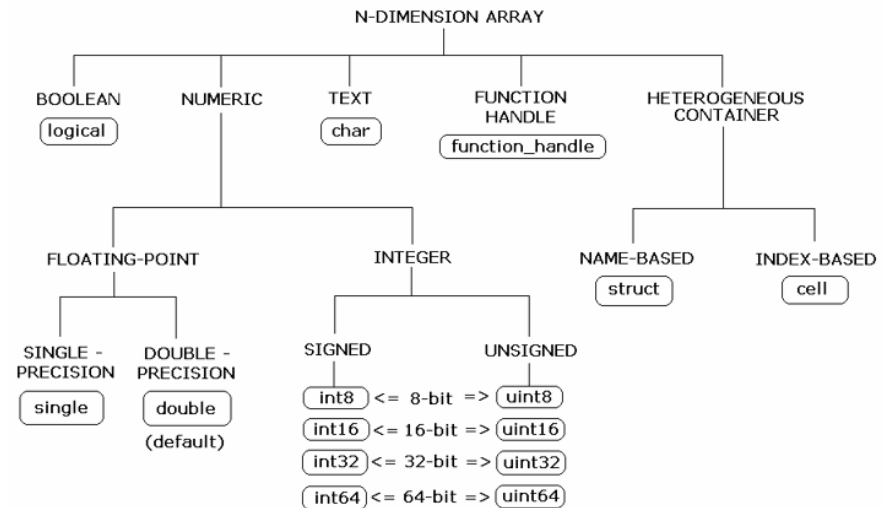
Scalars, vectors, matrices (class numeric)

Matrix operations

Command Window, Command History

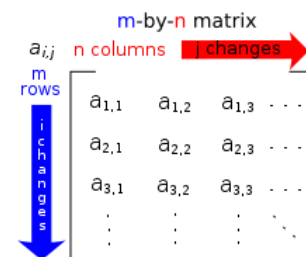
Saving and loading variables

Exercises



# Matrices v Matlabu

- matrix is a basic data structure in Matlab
- there are following types depending on size :
  - $1 \times 1$  (scalar)
  - $M \times 1$ , or  $1 \times N$  (vector)
  - $M \times N$  (matrix)
  - array (multidimensional matrices)  $M \times N \times P \times Q \times R \times \dots$
- can be complex
- Can contain text as well (beware the length)



# Matrix creation

- Following techniques are available:
  - element-by-element entering (suitable for small matrices only)
  - colon notation „:“ to define elements of a series
  - generation by built-in functions
  - generation of matrices in m-files
  - import and export from/to external files (.mat, .txt, .xls)

# Matrix construction element-by-element

- test following commands to construct matrices by element enumeration
  - suitable for small matrices only

```
>> a1 = -1
>> a2 = [-1] % brackets are redundant
```

$$a_1 = a_2 = -1$$

$$\mathbf{v}_1 = (-1 \ 0 \ 1)$$

$$\mathbf{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

```
>> v1 = [-1 0 1]
>> v2 = [-1;0;1]
```

```
>> M1 = [-1 0 1; -2 0 2]
>> M2 = [-1 -2; 0 0; 1 2]
>> M3 = [[-1 -2]; [0 0]] % inner brackets are redundant
```

$$\mathbf{M}_1 = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \end{pmatrix}, \quad \mathbf{M}_2 = \begin{pmatrix} -1 & -2 \\ 0 & 0 \\ 1 & 2 \end{pmatrix}, \quad \mathbf{M}_3 = \begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix}$$

# Matrix construction element-by-element

90 s ↑

- Construct following matrices:
  - matrix values are defined inside square brackets [ ]
  - semicolon „;“ separates individual rows of a matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

# Matrix construction

120 s ↑

- semicolon placed at the end of a command suppresses display of the output in Command Window

```
>> a = 1
>> b = 5;
```

- when more than one command on the same line, coma is used to separate each command

```
>> a = 1, b = 5
>> a = 1; b = 5;
```

- note: it is possible to copy and paste code including ">>"

```
>> c = [1 0 0]
>> d = [0; 0; 1]
```

$$\mathbf{c} = (1 \ 0 \ 0) \quad \mathbf{d} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- "row" vs. "column" vektor

# Workspace browser

- new variables
- deleting / modification of existing variables
- saving / loading
- size, elements of variables
  - Other information can be added
- fast data plotting option
- all operations can be carried out using Matlab functions that we learn later, e.g.
  - min, max, size, length

Name	Value	Size	Min	Max
A	[-1, 1; 1, -2]	2x2	-2	1
B	[1, 2, 3; 4, 5, 6; 7, 8, 9]	3x3	1	9
a	1	1x1	1	1
b	5	1x1	5	5
c	[1, 0, 0]	1x3	0	1
d	[0; 0; 1]	3x1	0	1

	1	2	3	4	5	6	7
1	-1	1					
2	1	-2					
3							
4							
5							
6							
7							

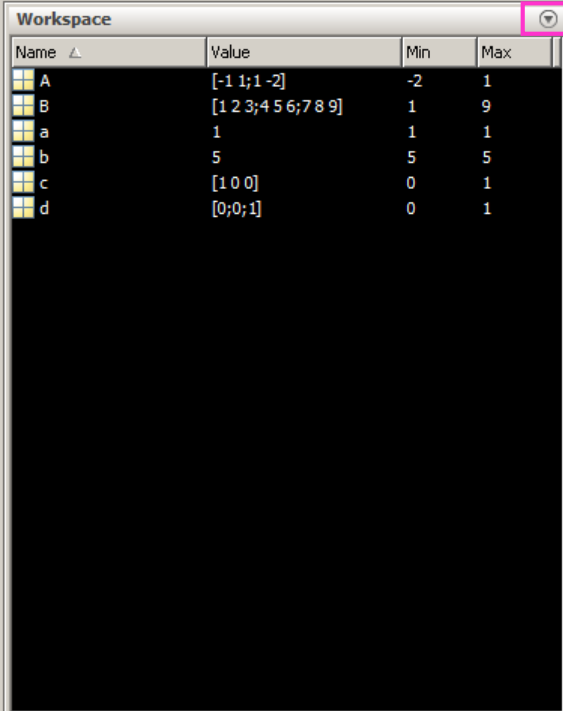


# Workspace browser

- Workspace now contains variables A, B, a, b, c, d (from previous slides)
  - all variables in the base workspace are displayed
- variable `ans` contains the last result
  - can be used for calculation
  - overwritten by each command input!

```
>> 2*2, ans^2
```

- Try to edit variables A, a
  - by a Matlab command directly
  - by change of value in `Workspace` browser
- Try to delete variables B, c



Name	Value	Min	Max
A	[-1 1;-2]	-2	1
B	[1 2 3;4 5 6;7 8 9]	1	9
a	1	1	1
b	5	5	5
c	[1 0 0]	0	1
d	[0;0;1]	0	1

# Basic math operators

- of several types:
  - arithmetic
    - matrix
    - vector
  - relational
  - logical
  - and other (to be mentioned later...)
  
- other operations using Matlab functions
  - complex conjugate,
  - sum, determinant, square root
  - and hundreds of other functions ...

+	addition
-	subtraction
*	multiplication
^	power
'	transpose
\	left matrix division
/	right matrix division
.	dot notation

# Operator Precedence in Matlab

- According to the following table
  - see Matlab → Language Fundamentals → Operators and Elementary Operations → Arithmetic

1	parentheses	()					
2	transpose, power	'	.'	^	.'	^	
3	unary plus, unary minus, logical negation	+	-	~			
4	multiplication, division	*	.*	/	\	./	.\
5	addition, subtraction	+	-				
6	colon operator	:					
7	relational operators	<	>	<=	>=	==	~=
8	logical AND (element-wise)	&					
9	logical OR (element-wise)						
10	logical AND (short-circuit)	&&					
11	logical OR (short-circuit)						

# Basic math operators

200 s



- type in following commands
  - zero can be omitted with a decimal number beginning with zero (not recommended!)

```
>> a3 = -2/4
>> a4 = -0.5
>> a5 = -.5
```

- what is the difference between a3, a4, a5?
- beware the precedence of operators (we see in the next slides):

```
>> 3*5*6
>> a1 = 15
>> a2 = 10;
>> a2/a3
>> a2/a3*a4
>> a2/(a3*a4)
```

- explain the difference between  $a2/a3*a4$  and  $a2/(a3*a4)$
- verify the rules of operator precedence from the previous slide

# Lengthy commands in Matlab

120 s ↑

- it is suitable to structure command blocks for clarity:
  - Next line: SHIFT+ENTER

```
>> A = [1 1 1]; B = [2 2 2]; % SHIFT+ENTER  
C = [2 3 2];
```

- three dots notation
  - for continuation of the same command on the next line
  - compare results:

```
>> A = [1 1 ...  
2 3]
```

```
>> A = [1 1  
2 3]
```

# Basic math functions

- Math functions in Matlab are generally divided in three groups:
  - scalar
    - function operates over individual elements of a matrix
    - e.g.: `sin`, `sqrt`, `log`, `factorial`
  - vector
    - Function operates over individual rows/columns of a matrix
    - e.g.: `sum`, `max`
  - maticové
    - function operates over whole matrix
    - e.g.: `det`, `trace`

# Basic math functions #1

600 s ↑

- using Matlab help, calculate following expression:  $a \sin^2(\alpha) + a \cos^2(\alpha) - a$ 
  - use following values

- verify following logarithmic identity:  $\log_{10}(a) + \log_{10}(b) - \log_{10}(ab) = 0$

- find sum of all elements in individual rows of the following matrix

$$\mathbf{T} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 6 & 7 & 8 & 9 \\ 0.2 & 0.3 & 0.4 & 0.5 \end{pmatrix}$$

# Basic math functions #2

600 s ↑

- assume following vectors  $\mathbf{u}$ ,  $\mathbf{v}$  :

$$\mathbf{u} = (1 \ 2 \ 3), \quad \mathbf{v} = (3 \ 2 \ 1)$$

- calculate

$$\mathbf{u}\mathbf{v}^T, \quad \mathbf{v}\mathbf{u}^T,$$

$$\mathbf{v}^T\mathbf{u}, \quad \mathbf{u}^T\mathbf{v},$$

$$\mathbf{u} \cdot \mathbf{v}, \quad \mathbf{u} \times \mathbf{v},$$

A

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

- following functions are needed:

- transpose of a matrix
- dot / scalar product
- cross product

- what is the result of the above mentioned operations?

10,

10,

$$\begin{pmatrix} 3 & 6 & 9 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix},$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 6 & 4 & 2 \\ 9 & 6 & 3 \end{pmatrix},$$

10,

$$(-4 \ 8 \ -4).$$

wikipedia.org



# Basic math functions #3

420 s ↑

- use following code and round the resulting number to:

```
>> r = 1 + 10*rand(1)
```

- (a) nearest integer
  - (b) nearest integer greater than  $r$
  - (c) nearest integer lower than  $r$
  - (d) zero
  - (e) zero with precision of 2 decimal digits
- find remainder after  $r$  is divided by  $0.1$ 
    - *modulus vs. remainder after division*

note: one of the functions is called *round*

# Matrix division in Matlab

- matrix operation
- two cases are distinguished: left division („\“) and right division („/“)
- $A$  is invertible (regular),  $b$  je row (column) vector

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

```
>> x = A \ b
```

solution to linear  
system of equations

$$\mathbf{xA} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{bA}^{-1}$$

```
>> x = b / A
```

# Basic math functions #4

500 s ↑

- find the sum of diagonal elements (trace of a matrix) of the matrix  $\mathbf{T}$  with elements coming from normal distribution with mean equal to 10 and standard deviation equal to 4

```
>> T = 10 + 4*randn(7, 7);
```

- find determinant of matrix  $\mathbf{U}$

$$\mathbf{U} = \begin{pmatrix} 1 & 2 & \frac{17}{81} \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{pmatrix}$$

- solve the linear system of equations

$$x_1 + 2x_2 + 3x_3 = 6$$

$$4x_1 + 5x_2 + 6x_3 = 15$$

$$7x_1 + 8x_2 + x_3 = 16$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

# Matlab commands

- Matlab is **cAsE sEnSiTiVe**

- almost entirely, with certain exceptions (properties of graphic objects, ...)
- pay attention to typos and variable names (see later)
  - new versions of Matlab offer certain options

```
>> AA = [1 1 1]
>> Aa
```

- beware of different syntax in Mathematica

- following syntax is incorrect both in Matlab and Mathematica:

```
>> Sin(pi/2) % function names start with lower case
>> cos[pi/3] % function input is in parentheses ()
```

# Predefined values in Matlab

- Matlab contains several predefined values
  - `eps` – precision of single/double numbers
    - `eps` determines the shortest distance between two single/double numbers
  - `ans` – most recent answer
  - `NaN` – *not a number* (every expression containing `NaN` results is `NaN`)
    - `NaN` can be used advantageously in some cases
  - `Inf` – *infinite number* (variable `Inf` can be used in calculation :))
    - pay attention to `Inf` propagation throughout your code (use allowed operations only)
  - `i`, `j` – complex unit
    - they are all basically functions (without input parameters)
- check results of the following expressions:
 

```

>> t1 = 10/0      % t1 = Inf
>> t2 = 0/0      % t2 = NaN
>> t3 = t1*5     % t3 = Inf
>> t4 = t1 + t2  % t4 = NaN
      
```
- `pi`, `intmin`, `intmax`, `realmin`, `realmax`, ... (functions)

# Workspace – output deletion #1

- to clean (erase) the command window:

```
>> home % cursor (>>) is shifted to the top-left position  
>> clc % Command Window is erased
```

- try and compare

```
Command Window  
>> a2 = 10  
a3 = -2/4  
a2/a3  
  
a2 =  
  
    10  
  
a3 =  
  
   -0.5000  
  
ans =  
  
   -20  
fx >> |
```

```
>> home
```

```
>> clc
```

```
Command Window  
fx >>
```

# Workspace – output deletion #2

- to clean one (or more) variable, use `clear`

```
>> clear           % whole Workspace is deleted
>> clear XX       % variable XX is deleted
>> clear XX YY    % variables XX and YY are deleted
>> clear z*       % everything starting with 'z' is deleted
```

- `clear` has a number other options (graphics, I/O)
- **try** to delete selected variables in workspace

# Workspace – output deletion #3

---

- to delete all variables except for one (or several):

```
>> clearvars -except a3 % clears everything except a3
```

- further information in doc `clear`, doc `clearvars`



# Command History window

- Command History window stores all commands from the command line
- command history accessible through ( $\uparrow$  or  $\downarrow$ )
- it is possible to filter out past commands by

- e.g. `>> A = [` +  $\uparrow$

- It is possible to copy-and-paste entire command history

- SHIFT / CTRL / CTRL+A  $\rightarrow$  CTRL+C

```

Command History
U = [1 2 17/81; 0 2 0; ...
0 -2 -1];
det(U)
clear,clc
T = 10 + 4*randn(10,10);
trace(T)
U = [1 2 17/81; 0 2 0; ...
0 -2 -1];
det(U)
A = [1 2 3; 4 5
b = [6 15 16]';
x = inv(A)*b;
x = A \ b;
t1 = 10/0 % t
t2 = 0/0 % t
t3 = t1*5 % t
t4 = t1 + t2 % t
home % vrátí pro
clc % vymaže Co
  
```

- later on, we will work with scripts and functions to store all the commands/code

# Variables storing and loading

- existing variables in Matlab Workspace can be stored on disk

```
>> save % stores all variables in matlab.mat in current folder  
>> save task1 % stores all variables in task1.mat  
>> save task1 a b c % stores variables „a“, „b“ and „c“ in task1.mat
```

- CTRL+S in Command Window / Command History
- loading variables is analogical

```
>> load % loads matlab.mat in current folder  
>> load task1 % loads all variables from task1.mat  
>> load task1 a b c % loads variables „a“, „b“ and „c“ from task1.mat
```

- alternatively, drag & drop the file from Current Folder in Command Window

# Storing history and variables

180 s ↑

- save today's Command History
  - use \*.txt file
- store all variables from Workspace in Data.mat

```
>> save Data
>> save('Data')
```

- try to store selected variables only

```
>> save Data_selected A T
>> save('Data_selected', 'A', 'T')
```

- clear Workspace and load above mentioned files

```
>> load Data_selected
>> load('Data_selected')
```

- both storing and loading can be carried out using mouse!!

# .mat file structure

- .mat files of the 7.3 version have the HDF5 format
  - HDF = Hierarchical Data Format
  - enable to store variables exceeding 2GB (64-bit system)
  - scientific format for data storing
- advantages of accessing HDF directly for certain applications:
  - speed
  - it is possible to define structure of the file and the stored data
  - Matlab *High-Level* functions and HDF *Low-Level* functions are available
- for more detailed information see:
  - MATLAB → Data and File Management → Data Import and Export → Scientific Data

# Variable names #1

- max. 63 characters starting with a letter (`>> namelengthmax`)
  - underscore is allowed in the variable name „\_“ (not at the beginning!)
  - characters not allowed are colon „:“, hyphen „-“ and others
- lowercase letters in the names of scalars and variables (`a = 17.59;`)
- matrix names usually start with a capital letter (`A = [ ... ]`)
  - clear huge matrices after they are used (`clear ..., memory'`)
- iteration variables, variables used in `for` cycles usually named `m`, `n`, `k` etc.
  - it is advisable to avoid `i`, `j` (complex unit)
- chose the names to correspond to the purpose of the variable
- avoid, if possible, standalone letter 'l' (to be confused with 1) and predefined variables in Matlab environment

# Variable names #2

---

- exceeding the maximum variable's name length :

```
>> a0123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789 = 10
Warning: 'a0123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789'
exceeds the MATLAB maximum name length of 63 characters and will be truncated to
'a0123456789012345678901234567890123456789012345678901'.

a01234567890123456789012345678901234567890123456789012345678901 =

10
```

# Variable names #3

- samples of valid variable names

```
a, A, b, c, x1, x2, M_12, test1, matrix_A, fx, fX
```

- samples of invalid variable names

```
lvar      % starts with a number (not possible in Matlabu)
matrix A  % contains space
coef.a    % possible only if coef is of type 'struct'
Test-1    % algebraic expression: ans = Test - 1
f(y)      % makes sense when using symbolic expressions
```

- samples of valid numbers in Matlab
  - pay attention to miss inserted spaces after exponent and imaginary unit

```
3, -66, +0.0015, .015, 1.6025e-10, 3i, 3.17e5i, -3.51j
```

# Discussed functions

---

<code>sin, cos</code>	trigonometric functions	
<code>sqrt</code>	square root	
<code>max</code>	largest element of column of a matrix; largest element of a vector	●
<code>sum</code>	sum of elements of column of a matrix; sum of elements of a vector	●
<code>log, log10</code>	natural logarithm, logarithm with base 10	
<code>factorial</code>	factorial	
<code>det, trace</code>	determinant of a (square) matrix, trace of a (square) matrix	
<code>transpose</code>	transpose	
<code>dot, cross</code>	scalar product, vector product	●
<code>inv</code>	invers of a matrix	
<code>round, ceil, floor, fix</code>	rounding	
<code>rem</code>	remainder after division	
<code>rand, randn</code>	random number generation	
<code>save, load</code>	storing, loading of variables	●
<code>clear, clearvars</code>	deleting variables and functions, deleting variables only	●
<code>home, clc</code>	command prompt shift, clears output	
<code>ans, eps</code>	returns last answer, numerical accuracy of Matlab	●

---



# Exercise #1

180 s ↑

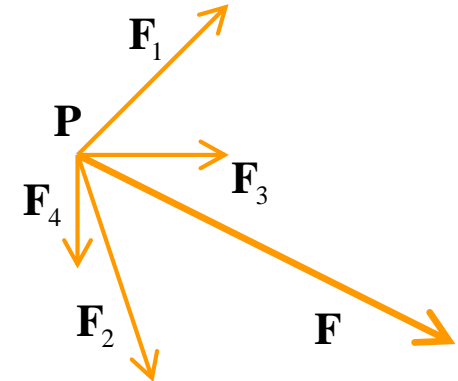
- forces were localized at point **P** :

$$\mathbf{F}_1 = (2 \ 2)$$

$$\mathbf{F}_3 = (2 \ 0)$$

$$\mathbf{F}_2 = (1 \ -3)$$

$$\mathbf{F}_4 = (0 \ -1.5)$$



- what is the direction of the resultant force **F**?

- normalize the resulting vector

$$\mathbf{n}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$

# Exercise #2

240 s ↑

- type-in following commands:

```
>> clear, clc;  
>> w1 = [1 2 3], w2 = [-2 -3 -4]',  
>> w3 = [-2; -3; -4],  
>> w4 = w2 - w3, w5 = w2 - w1
```

- compare differences
- the error of calculating w5 resides in what?

- try also

```
>> w1*3, w1 - 3,  
>> w1 + [5 5 5],  
>> w6 = 5*w1 - [3 5 6] - w2,
```

- calculate the norm (magnitude) of vector w1
- try more options

$$\hat{\mathbf{w}}_1 = \frac{\mathbf{w}_1}{|\mathbf{w}_1|}$$

- how to modify the calculation in the case of a complex vector?

# Exercise #3

180 s ↑

- calculate roots of the quadratic function  $-2x^2 - 5x = 3$ 
  - rearrange the terms of the function first

$$2x^2 + 5x + 3 = 0, \Rightarrow a = 2, b = 5, c = 3$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 24}}{4}$$

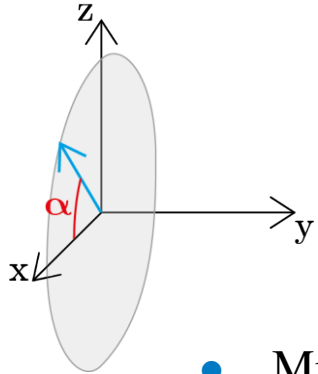
$$x_1 = -1, \quad x_2 = -\frac{3}{2}$$

- Matlab provides particular function for calculation of roots of a function, try to search it out

# Exercise #4

300 s ↑

- Consider matrices (prepare matrices for later use)
  - rotating by angle  $\alpha$  in  $x$ - $z$  plane



$$\mathbf{R} = \begin{pmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{pmatrix}$$

- Mirroring across plane
  - use Householder's transform

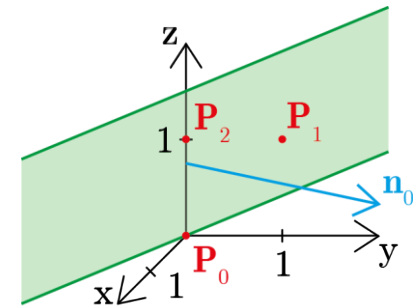
$$1x + 2y + 0z = 0$$

$$\mathbf{P} = \mathbf{I} - 2\mathbf{n}_0\mathbf{n}_0^T$$

$$\mathbf{n}_0 = \frac{\mathbf{v}_1 \times \mathbf{v}_2}{|\mathbf{v}_1 \times \mathbf{v}_2|} \quad \mathbf{P}_1 = [-2; 1; 0]$$

$$\mathbf{P}_2 = [0; 0; 1]$$

$$\mathbf{v}_k = (\mathbf{P}_k - \mathbf{0}) \begin{pmatrix} \mathbf{x}_0 \\ \mathbf{y}_0 \\ \mathbf{z}_0 \end{pmatrix}, \quad k \in \{1, 2\}$$



# Exercise #5

180 s ↑

- use rotation matrix  $\mathbf{R}$  to rotate vector  $\mathbf{k} = [1; 0; 0]$  by angle  $\alpha = \pi/2$

$$\mathbf{m} = \mathbf{R}\mathbf{k} = (0 \ 0 \ 1)^T$$

- use reflection matrix  $\mathbf{P}$  across plane:  $1x + 2y + 0z = 0$ 
  - to mirror vectors:

$$\mathbf{u}_1 = \mathbf{n}_0, \quad \mathbf{u}_2 = \left( \frac{5}{2} \ 0 \ 3 \right)^T$$

$$\mathbf{m}_1 = \mathbf{P}\mathbf{u}_1 = -\mathbf{n}_0, \quad \mathbf{m}_2 = \mathbf{P}\mathbf{u}_2 = \left( \frac{3}{2} \ -2 \ 3 \right)^T$$

- calculate the determinant of matrices  $\mathbf{R}$  and  $\mathbf{P}$ 
  - can you interpret the results?

$$\det \mathbf{R} = 1, \quad \det \mathbf{P} = -1$$

# Thank you!



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