Finger Print Analysis and Matching

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18.10, 2011, Prague

Acknowledgments: Chris Miles,Tamer Uz, Andrzej Drygajlo

Handbook of Fingerprint Recognition, Chapter III Sections 1-6

laboratory Gerstner

Outline

- Introduction
- Morphology imaging processing
- Post - processing
- Singularity and Core Detection
- Matching

laboratory Gerstner

Morphology Image Processing

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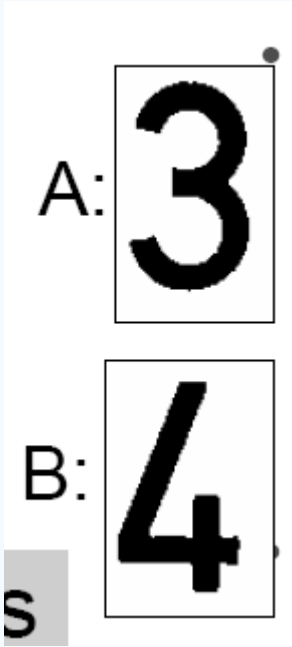
Acknowledgments: José Neira Parra

laboratory Gerstner

Morphology

- Morphology deals with form and structure
- Mathematical morphology is a tool for extracting image components useful in:
  - representation and description of region shape (e.g. boundaries)
  - pre- or post-processing (filtering, thinning, etc.)
- Based on set theory



Definitions: Let A and B be binary images, p and q two pixels with indices $[i,j]$ y $[k,l]$ respectively, and Ω the universal binary image.

Union:

$$A \cup B = \{p | p \in A \vee p \in B\}$$

Intersection:

$$A \cap B = \{p | p \in A \wedge p \in B\}$$

Complement:

$$\overline{A} = \{p | p \in \Omega \wedge p \notin A\}$$

Difference:

$$A - B = A \cap \overline{B}$$

Translation:

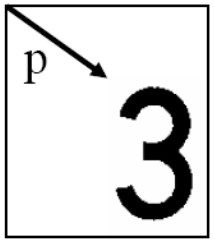
$$A_p = \{a + p | a \in A\}$$

- **Vectorial sum:**

$$p + q = [i + k, j + l]$$

- **Vectorial difference:**

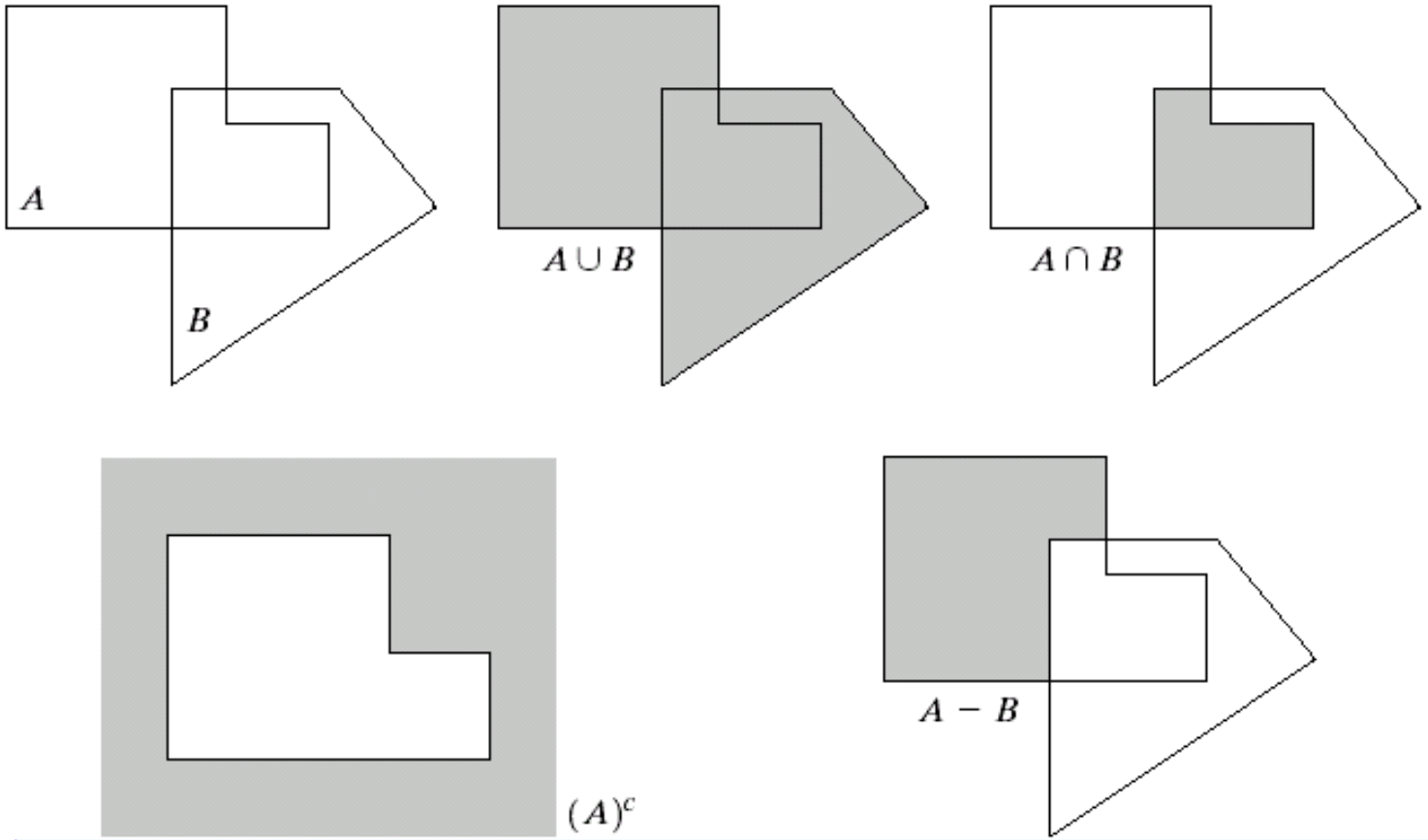
$$p - q = [i - k, j - l]$$



Reflex:

$$A' = \{-p | p \in A\}$$

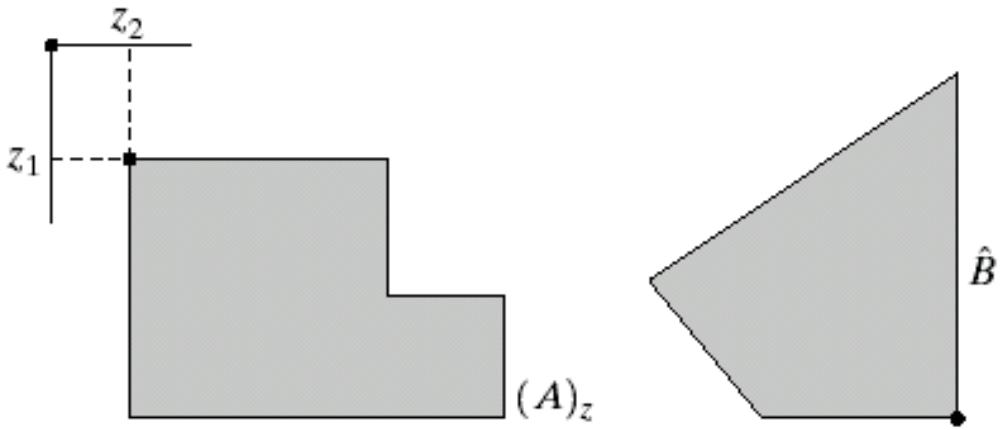
Morphological Image Processing



a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Morphological Image Processing



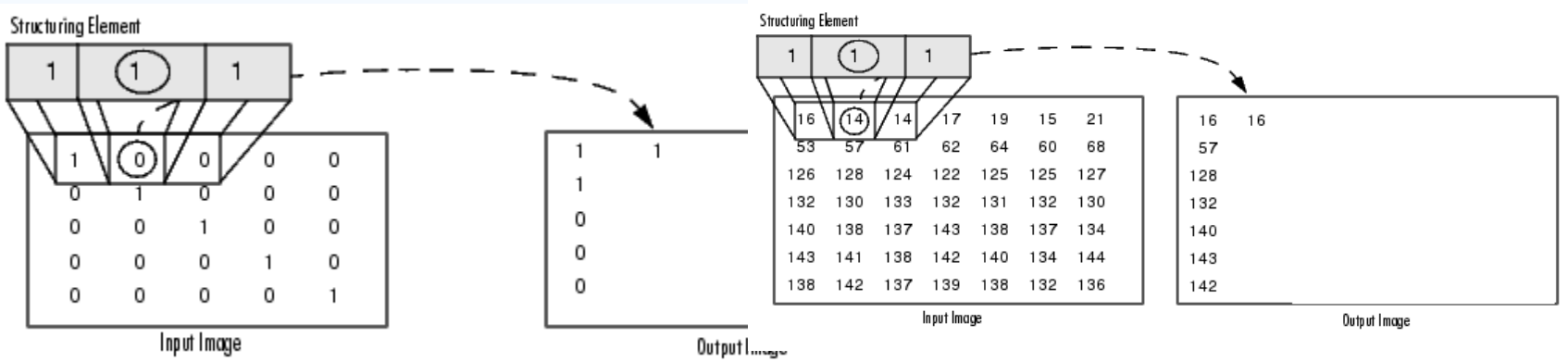
a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

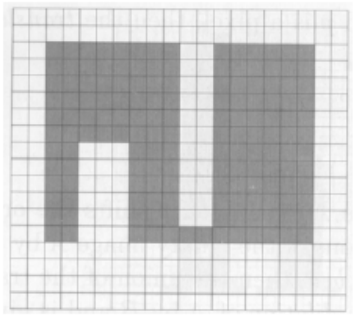
Dilation & Erosion

- The value of the output pixel is the *maximum* value of all the pixels in the input pixel's neighborhood.
- Dilation: $$A \oplus B = \{x \mid [(\hat{B})_x \cap A] \subseteq A\}$$
  - B is the structuring element in dilation.

- The value of the output pixel is the *minimum* value of all the pixels in the input pixel's neighborhood.
- Erosion: $$A \quad B = \{x \mid (B)_x \subseteq A\}$$

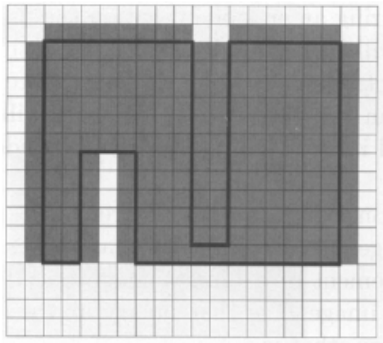


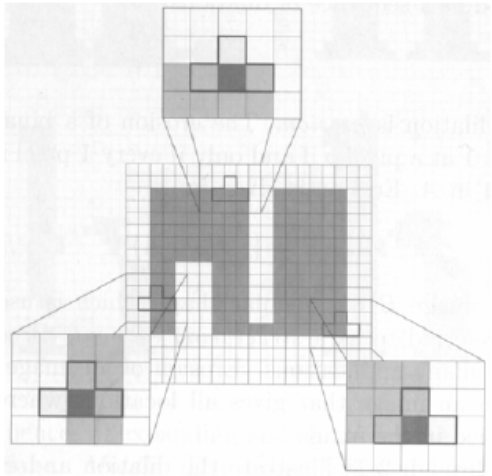
Dilation & Erosion

A  B 

- **Dilation:** union of translations of an image ***A*** by each pixel of image ***B***, called ***structural element***:

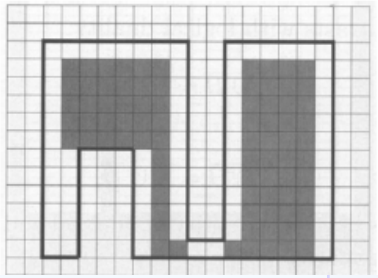
$$A \oplus B = \bigcup_{b_i \in B} A_{b_i}$$



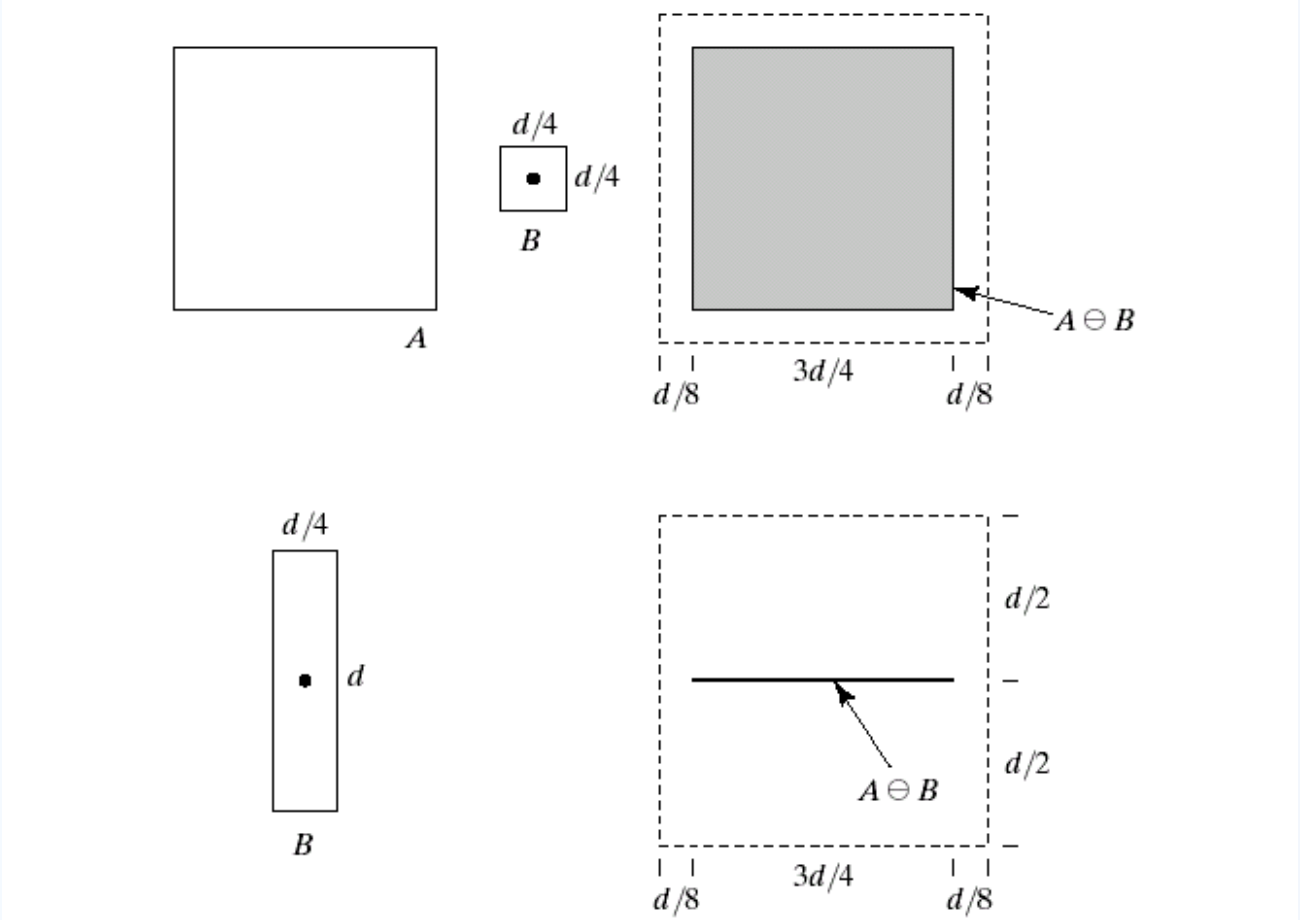


- **Erosion** inverse operation

$$A \ominus B = \{p | B_p \subseteq A\}$$



Morphological Image Processing



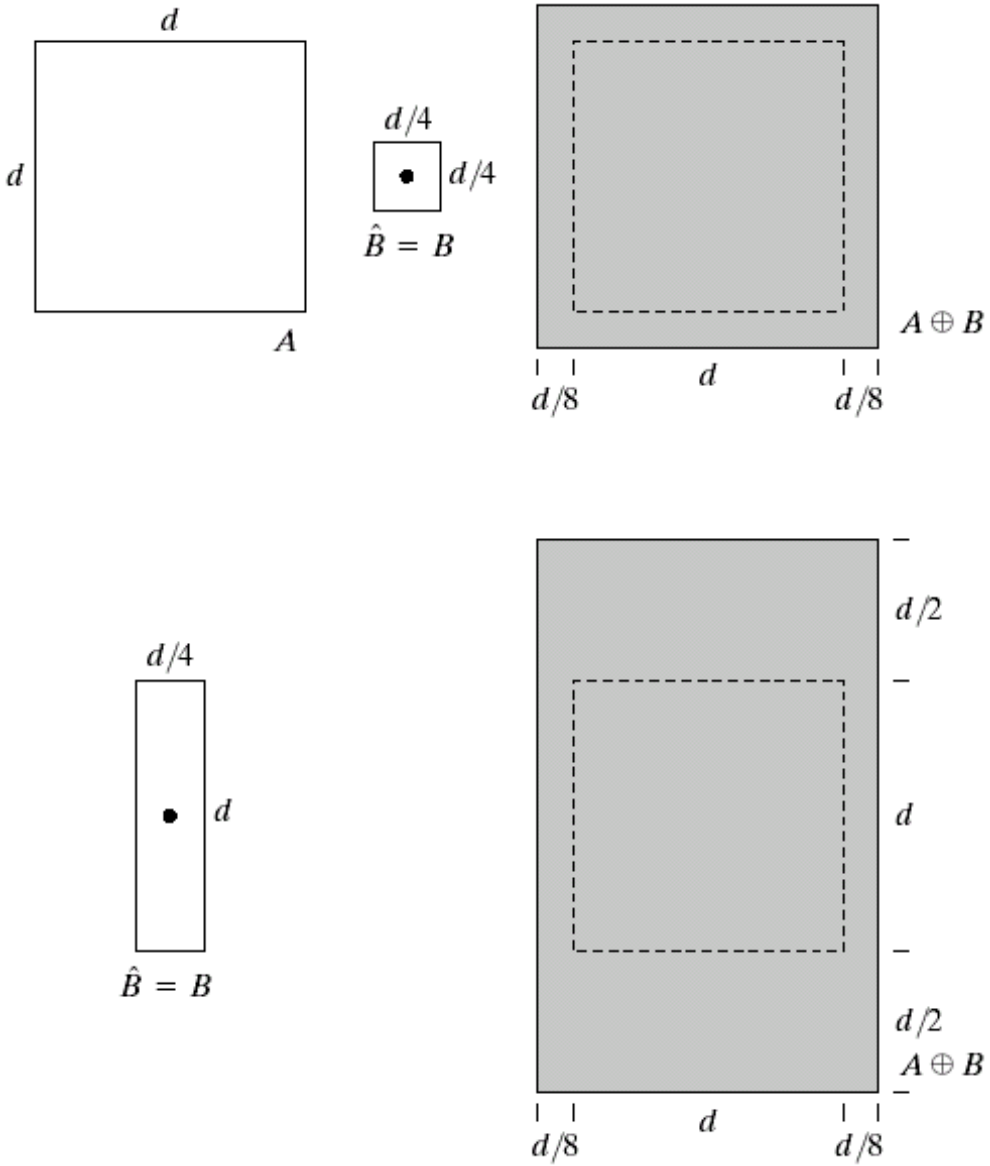
a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

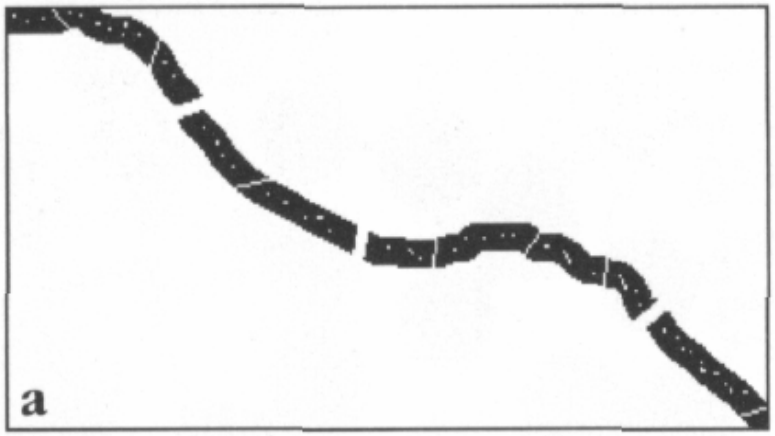
Morphological Image Processing

a b c
d e

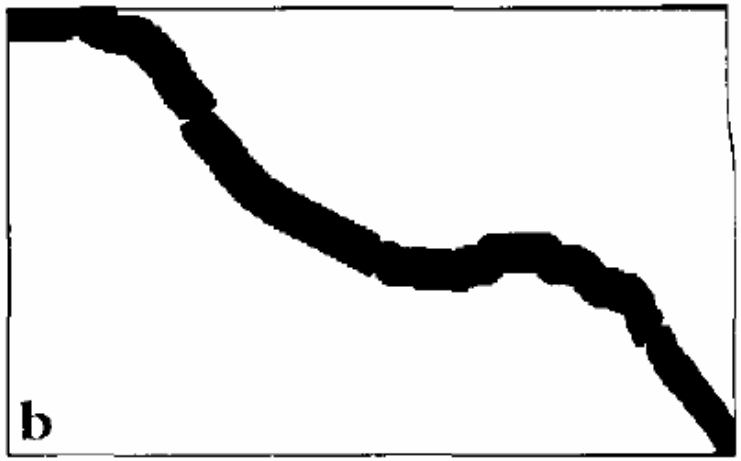
FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



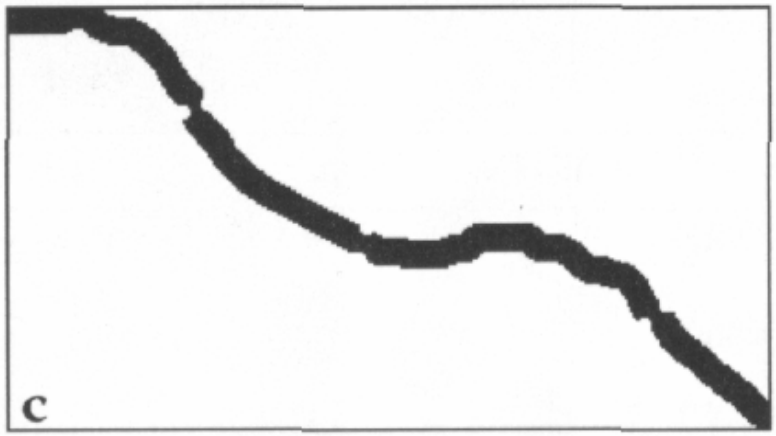
Object connection



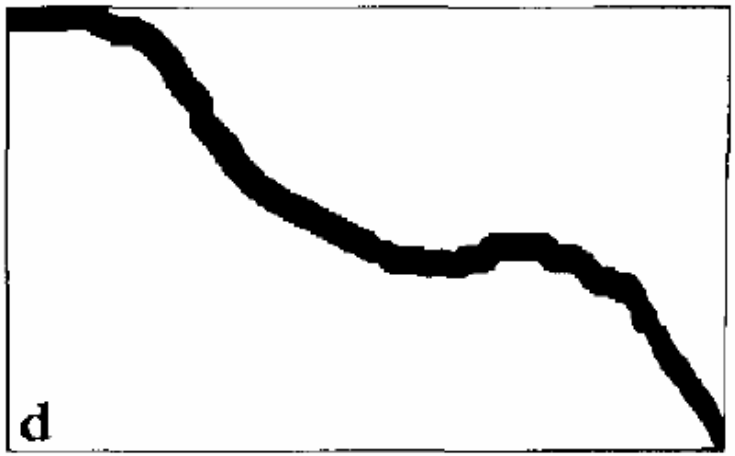
Original



Dilated twice



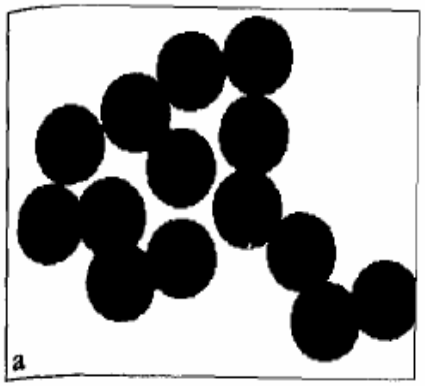
Eroded twice



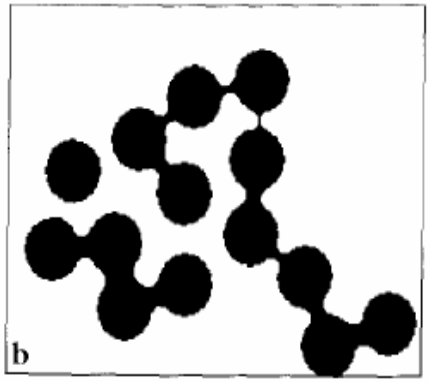
Dilated and eroded four times

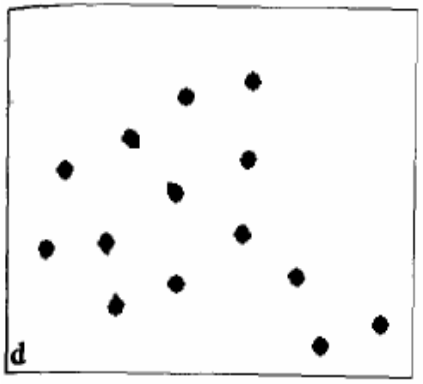
Object separation

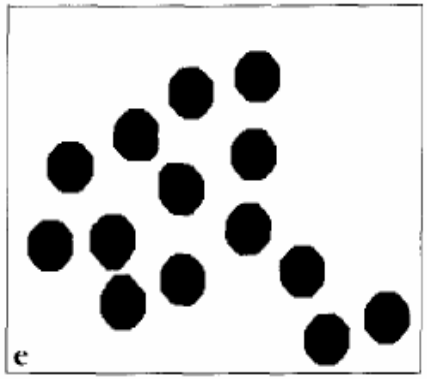
a. Original
b. Eroded twice
d. Eroded 7 times

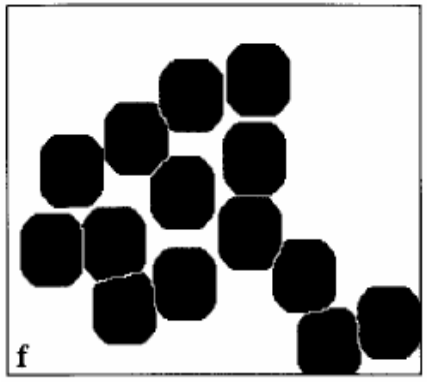


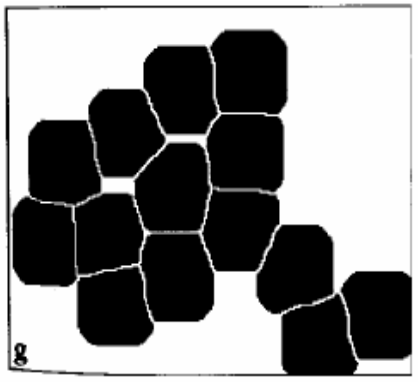
e. Dilated four times with XOR
f. Dilated seven times with XOR
g. Dilated nine times with XOR
h. AND with original image

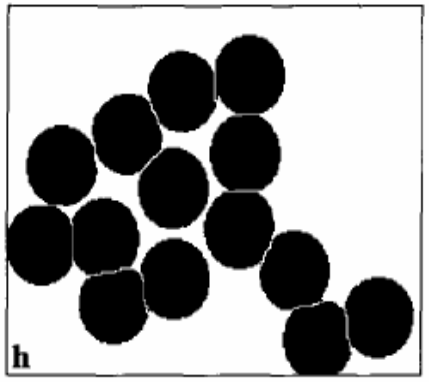




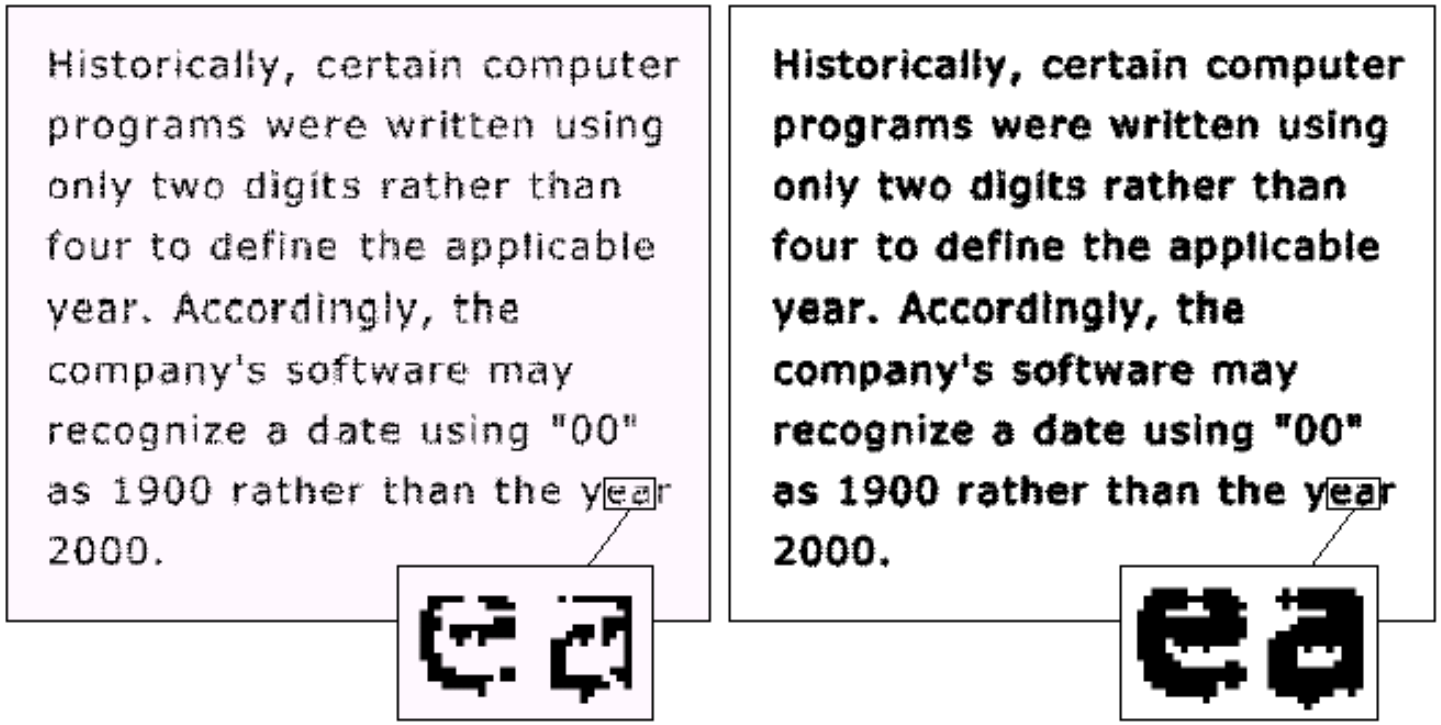








Morphological Image Processing

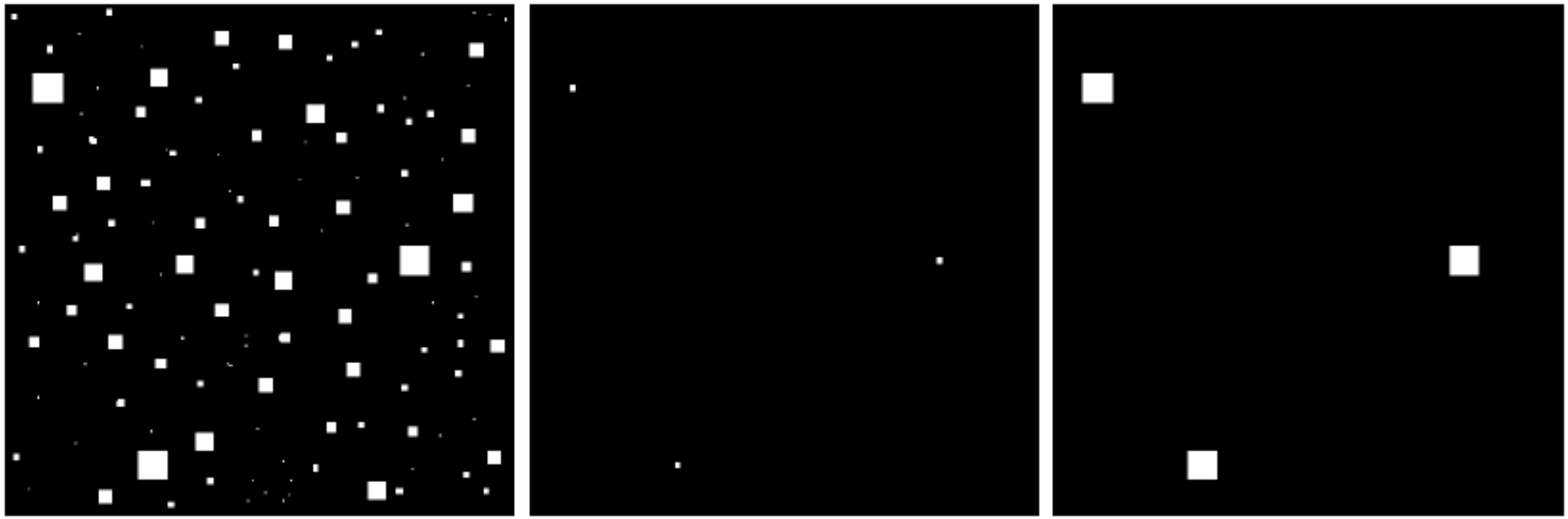


a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0

Morphological Image Processing

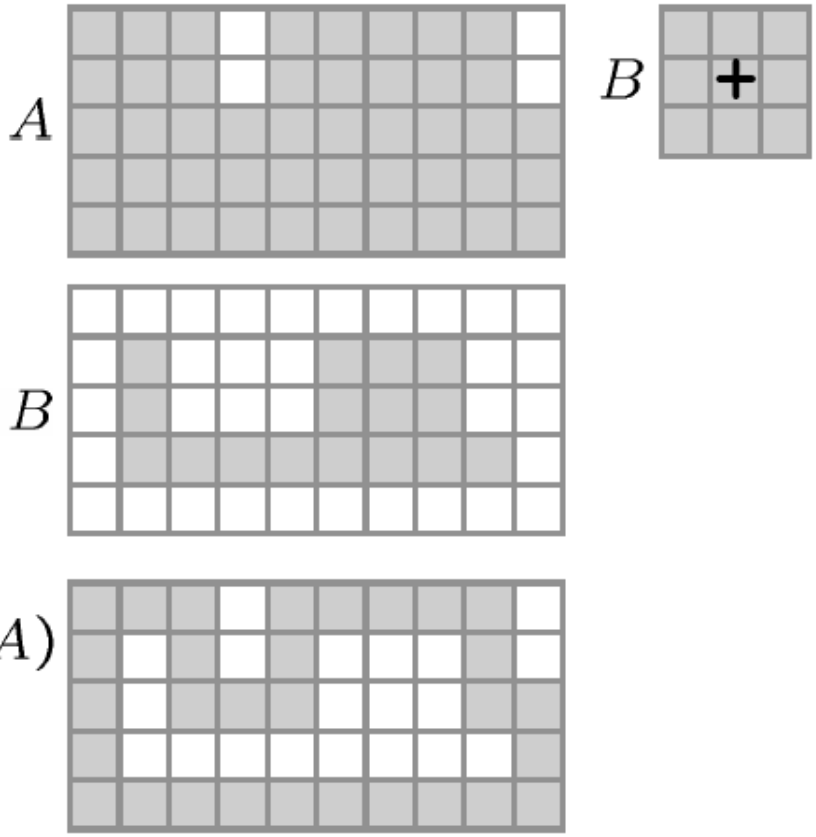


a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

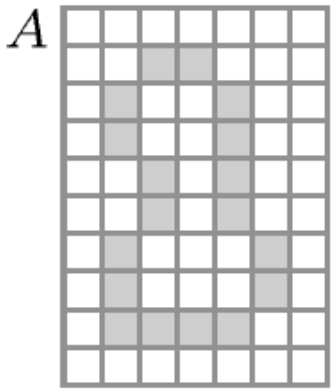
Contour extraction

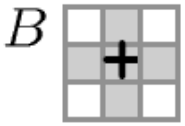
$$\beta(A) = A - (A \ominus B)$$

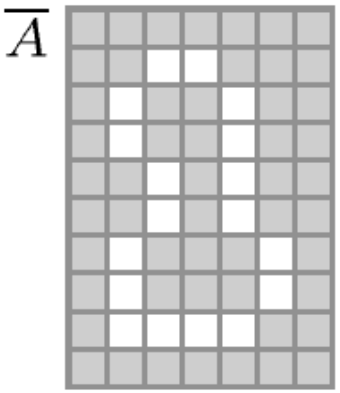


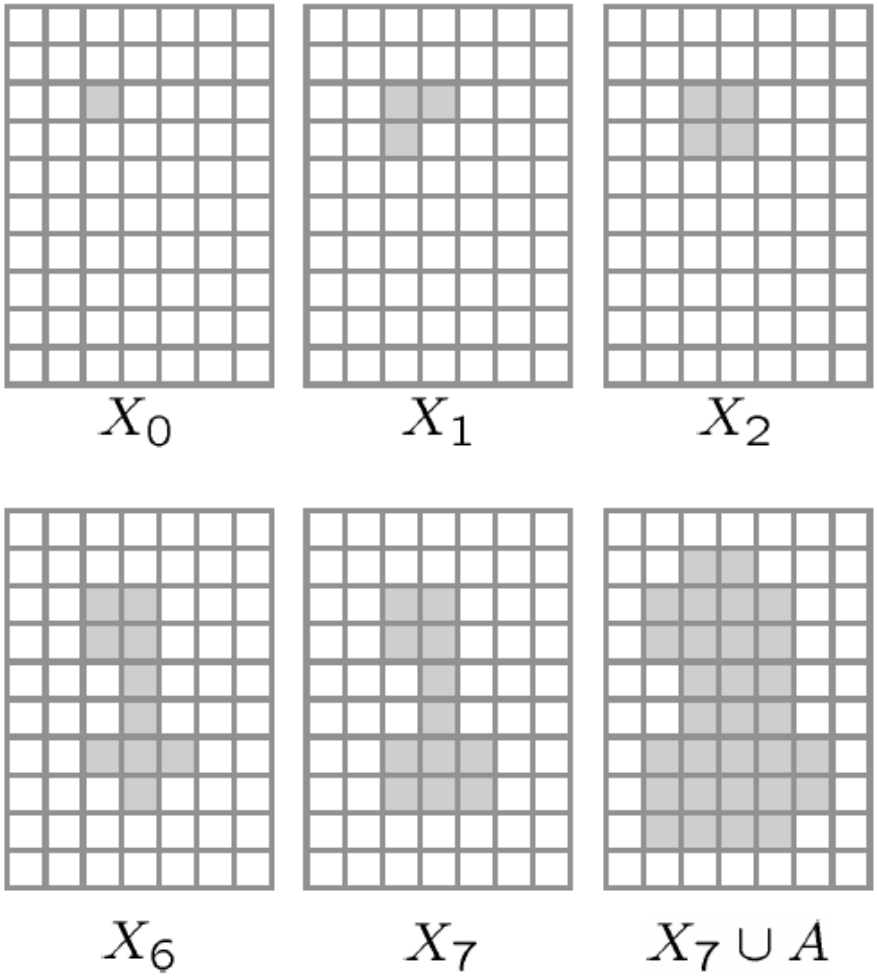
Region filling

$$X_0 = p$$
$$X_k = (X_{k-1} \oplus B) \cap \overline{A}$$









$$X_k = X_{k-1}$$

Opening & Closing

- In essence, dilation expands an image and erosion shrinks it.
- Opening:
  - generally smoothes the contour of an image, breaks isthmuses, eliminates protrusions.
- Closing:
  - smoothes sections of contours, but it generally fuses breaks, holes, gaps, etc.

– Opening of A by structuring element B:

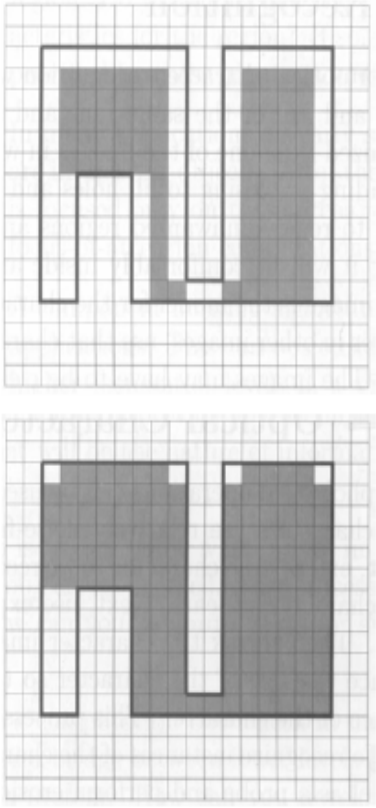
$$A \circ B = (A \ominus B) \oplus B$$

- Closing:

$$A \bullet B = (A \oplus B) \ominus B$$

- **Opening:** erosion + dilation with the same element

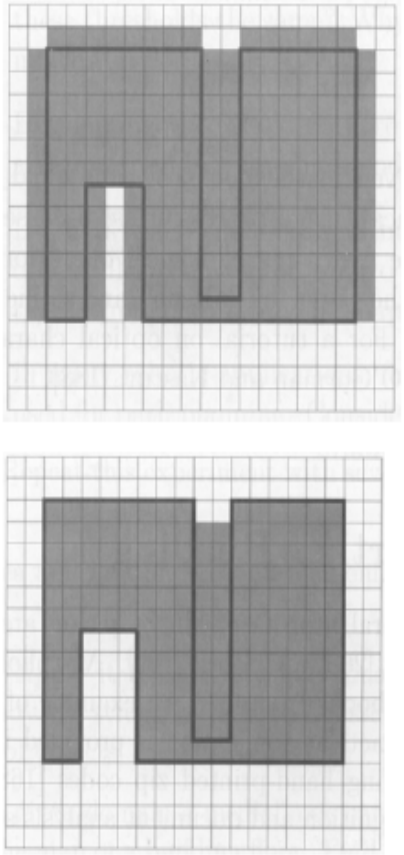
$$A \circ K = (A \ominus K) \oplus K$$



- Eliminates all regions too small to contain the structural element

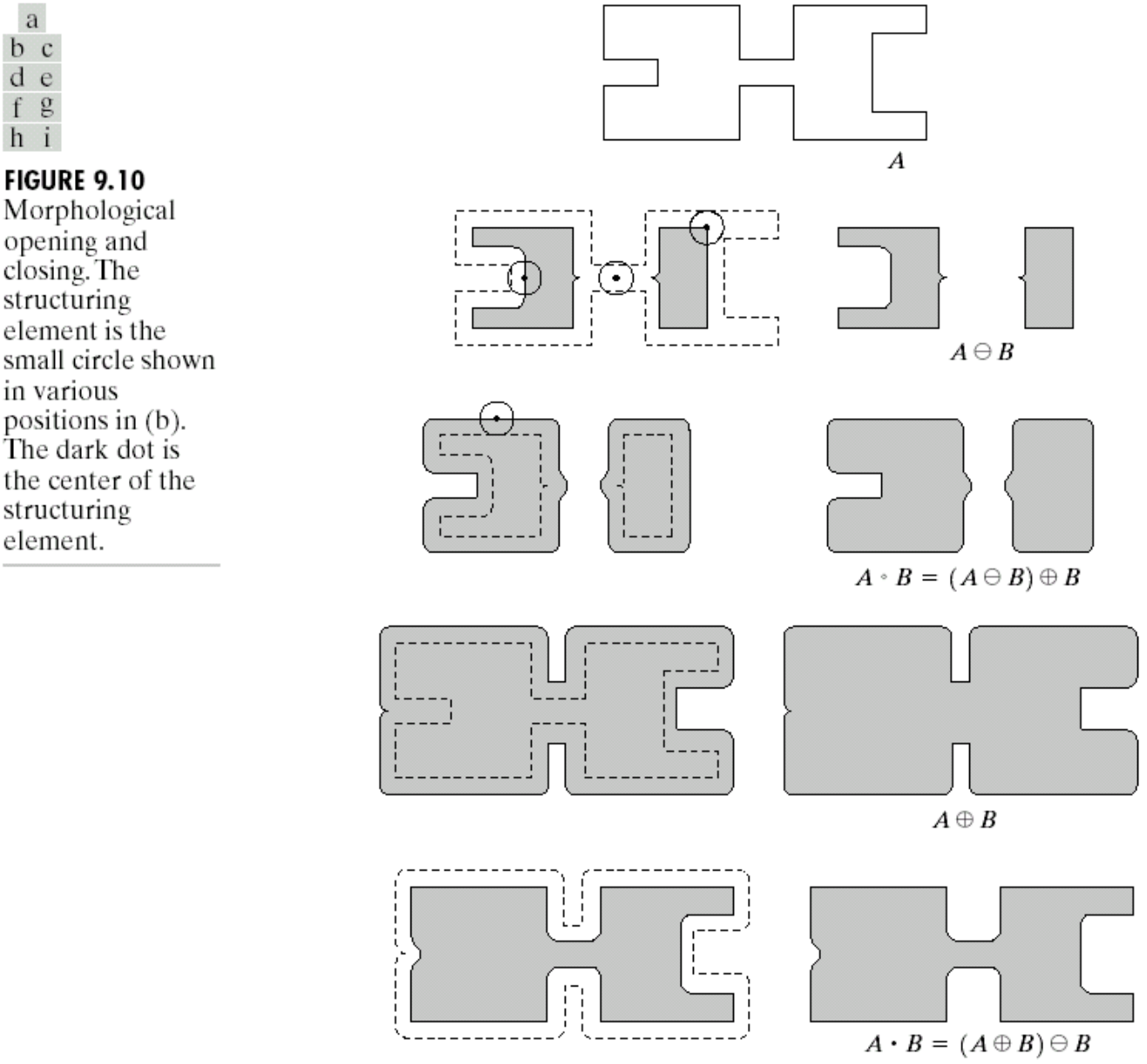
- **Closing:** dilation + erosion with the same element

$$A \bullet K = (A \oplus K) \ominus K$$



- Fills all holes and cavities smaller than the structural element

Morphological Image Processing



a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

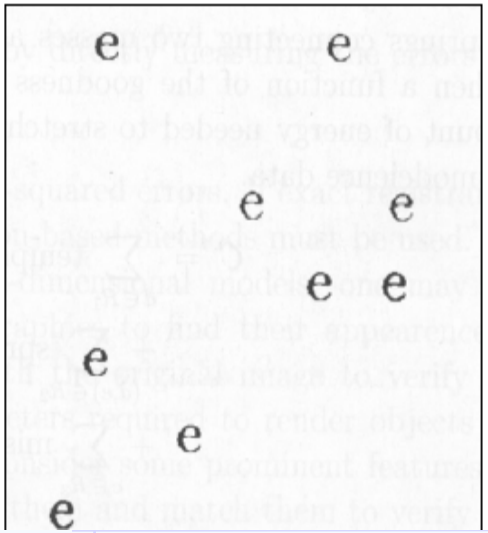
- **Template matching**

A

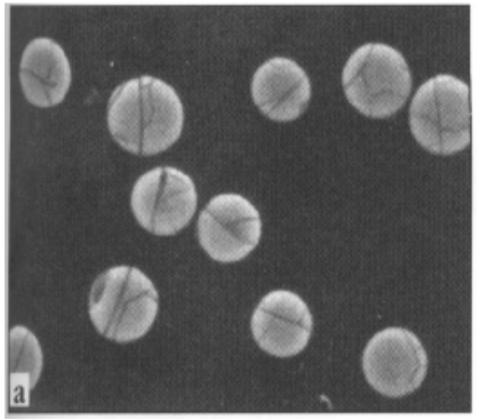
objects in the *real*
bject models. This t
cognition effortlessl
ask for implementa
r we will discuss d
echniques that hav
We will discuss difl

B

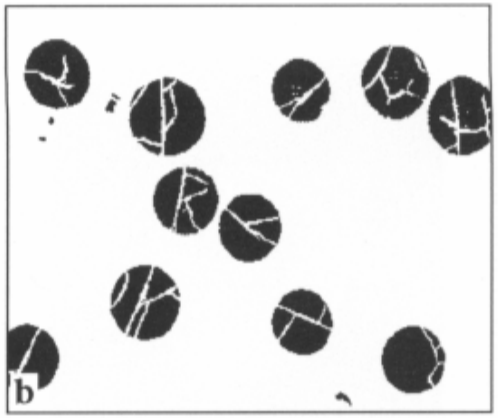
$A \circ B$



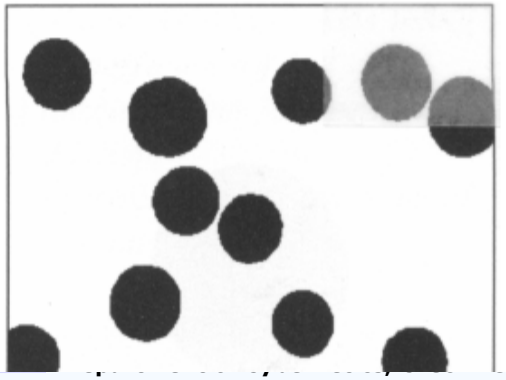
- **Reconstruction**



Original

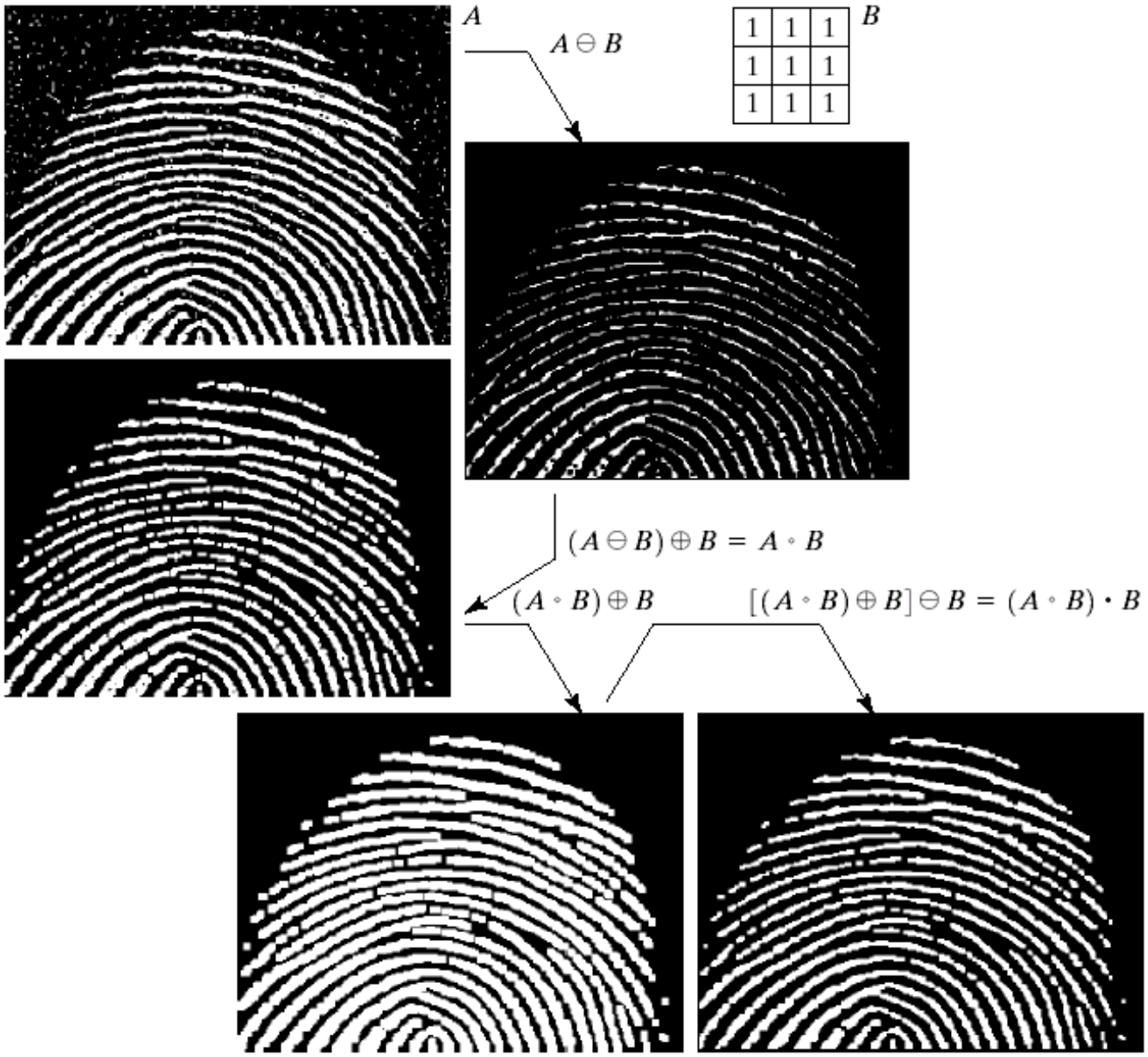


Thresholded



Closing

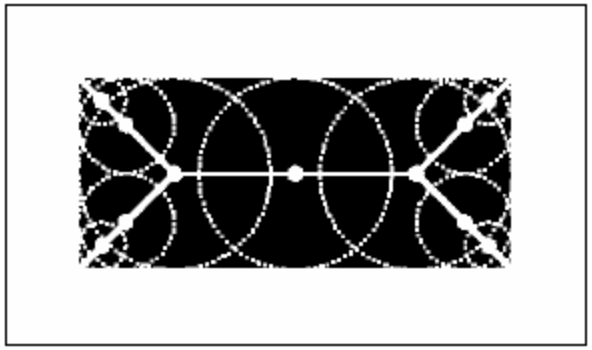
Morphological Image Processing

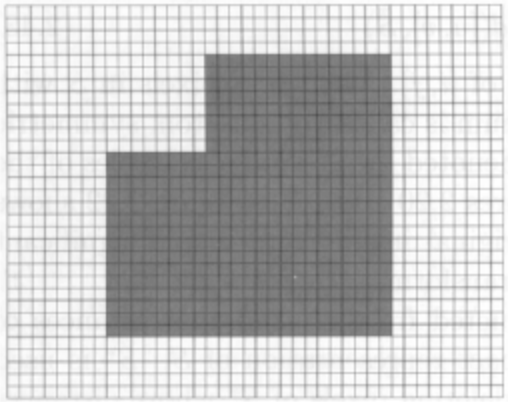


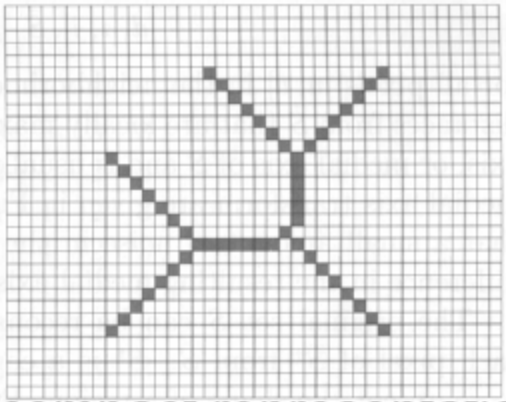
a b
d c
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

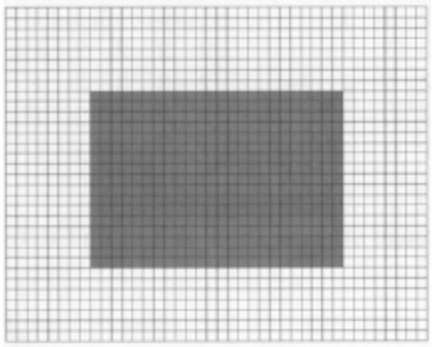
- **Skeletons, axis of symmetry S*:** geometric place of the centers of all at least bi-tangent circles.

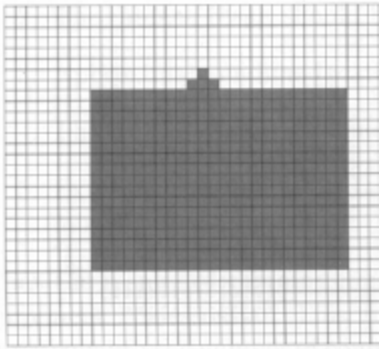


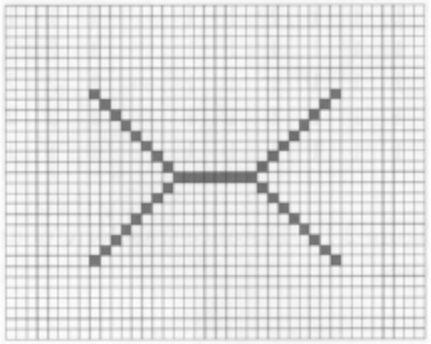


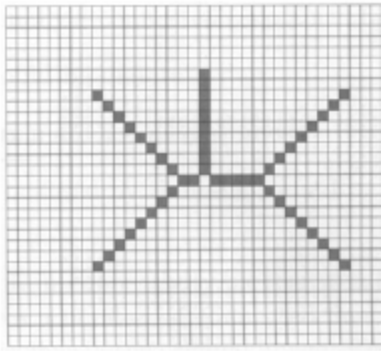


- Highly sensitive to noise.





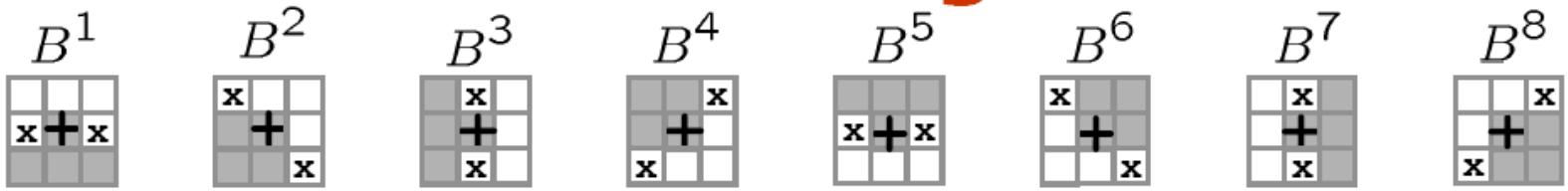




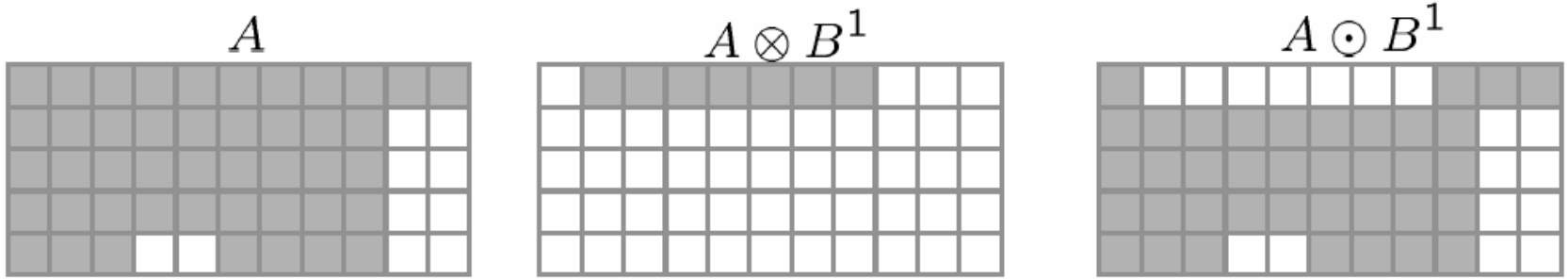
- S* is a compact representation of S; it represents the *shape* of the region.

The frontier is also a compact representation of shape.

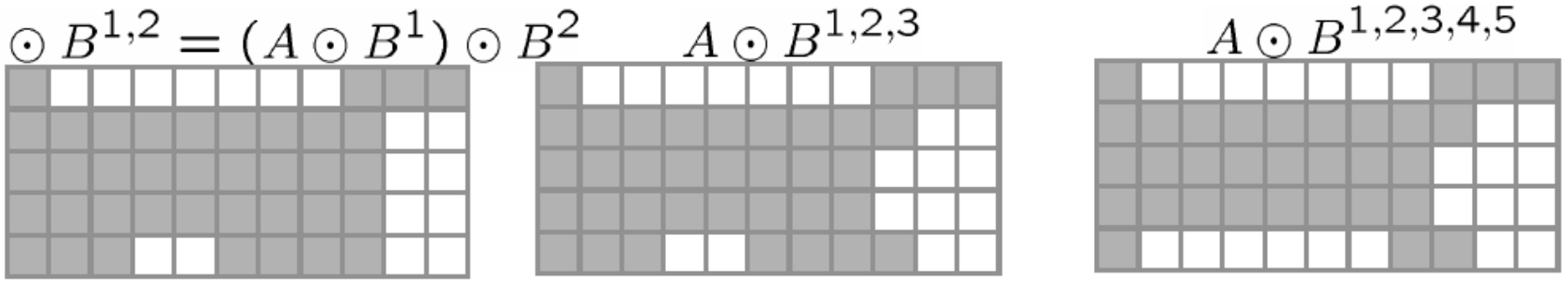
Thinning



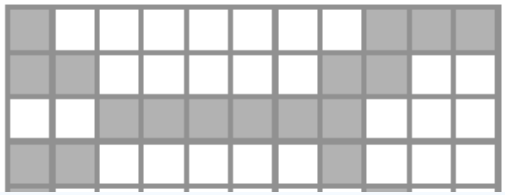
$$A \odot B^i = A - (A \otimes B^i)$$



$A \odot B^{1,2} = (A \odot B^1) \odot B^2$

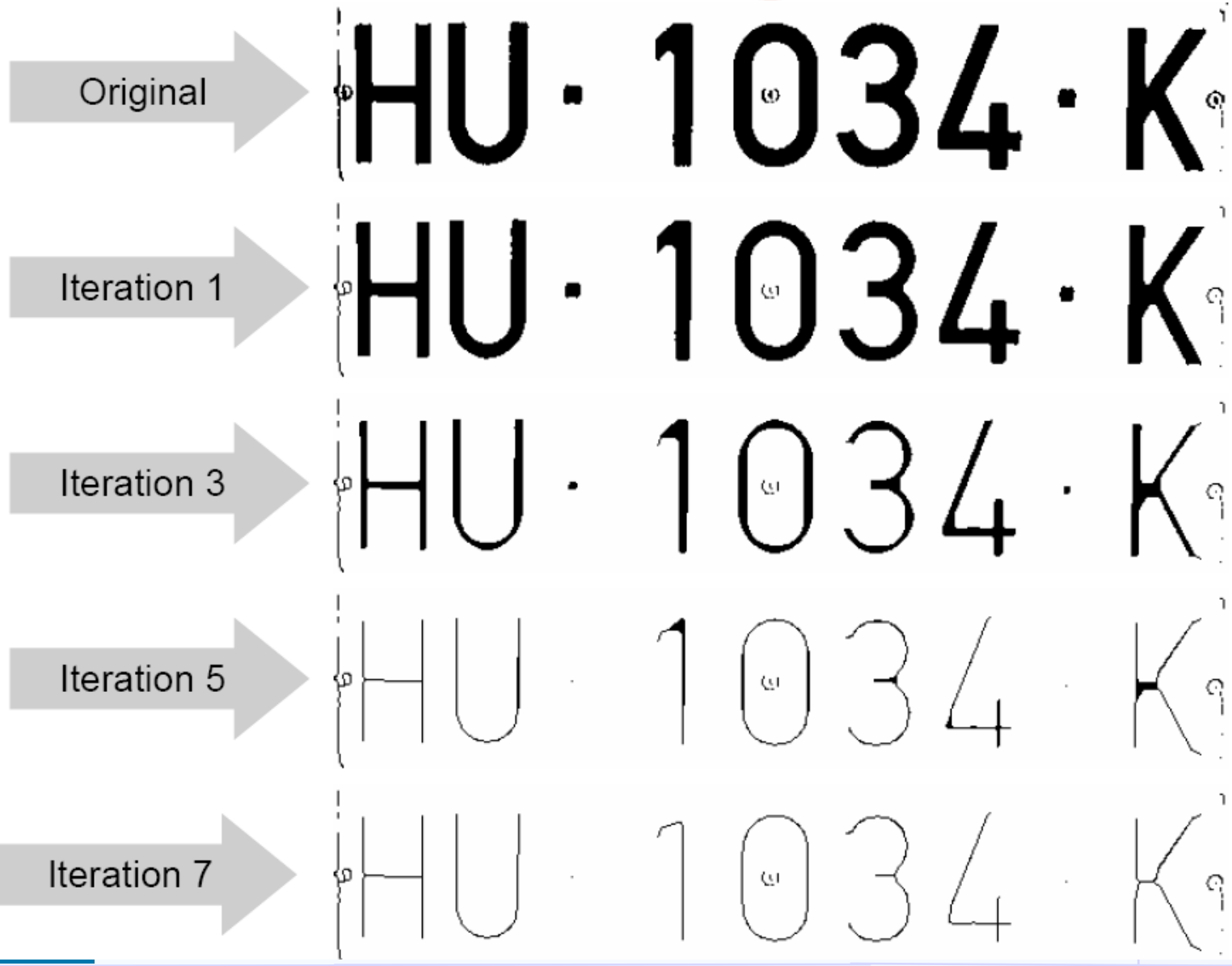


$$X_0 = A$$
$$X_k = X_{k-1} \odot B^{\{1,\cdots,n\}}$$



Final result

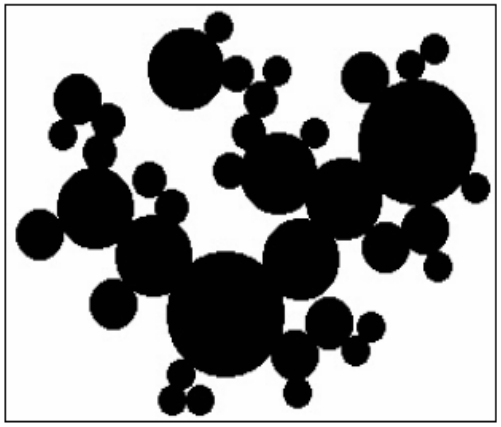
Thinning

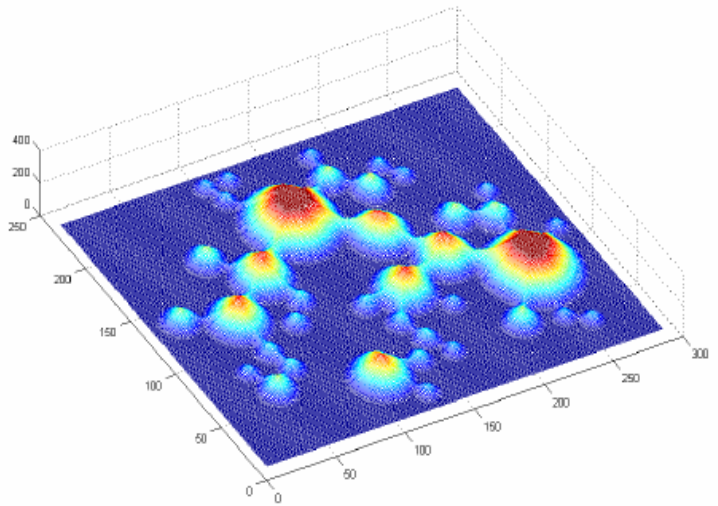


6/6. Euclidean distance maps (EDM)

- Image representing the smallest distance of e/pixel to the background.

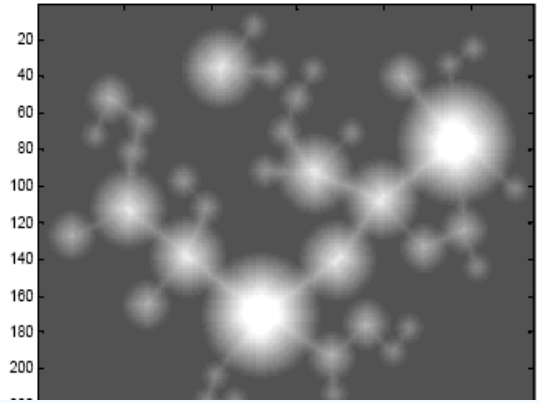
A





$D(3d)$

D



There are several possible definitions for distance

Distance measurements

- Fundamental properties:

$$\forall p, q, r:$$
$$1.\ d(p,q) \geq 0,$$
$$d(p,q) = 0 \Leftrightarrow p = q$$
$$2.\ d(p,q) = d(q,p)$$
$$3.\ d(p,r) \leq d(p,q) + d(q,r)$$

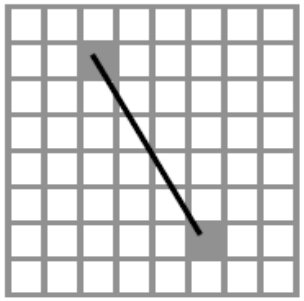
$$d([i_1, j_1], [i_2, j_2]) =$$

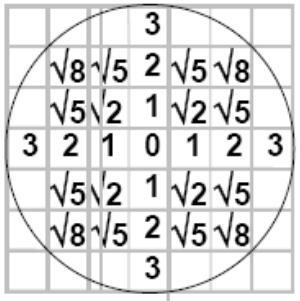
1. Euclidean:
$$\sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2}$$
2. Manhattan:
$$|i_1 - i_2| + |j_1 - j_2|$$
3. Chess:
$$\max(|i_1 - i_2|, |j_1 - j_2|)$$

Euclidean is closest to the real case; Mosr costly to compute

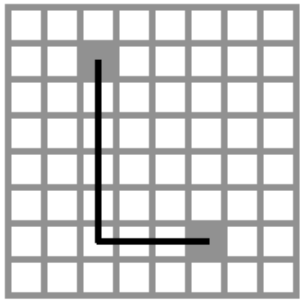
Discs: pixels at distance <= k.

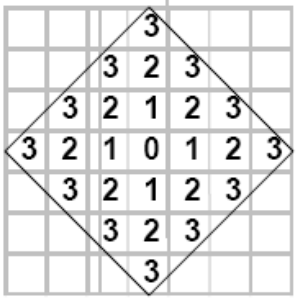
1.



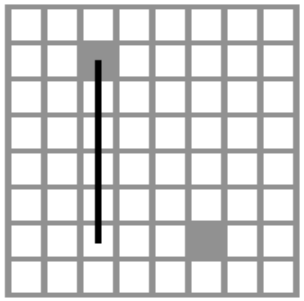


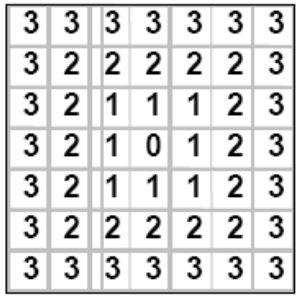
2.





3.





Gerstner

Obtaining distance maps

$$
\begin{aligned}
f^0[i,j] &= B[i,j] \\
f^m[i,j] &= f^0[i,j] \\
&+ \min\left(f^{m-1}[u,v]\right)
\end{aligned}
$$

$$\forall [u,v] : d([u,v],[i,j]) = 1$$

1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

1	1	1	1	1	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	1	1	1	1	1

- **Iteration 0:** original image.
- **Iteration 1:** All pixels not adyacent to background change to 2.
- **Next iterations:** pixels farther from background change.
- No pixels changes when the distances to all have been computed.

1	1	1	1	1	1
1	2	2	2	2	1
1	2	3	3	2	1
1	2	3	3	2	1
1	2	2	2	2	1
1	1	1	1	1	1

Post-processing

- Reduces the width of the ridges to one pixel
- *Skeletons* , spikes
- Filling holes, removing small breaks, eliminating bridges between ridges etc.
- ***cleanskeleton:*** *removespur, linkbreak, removebridge*

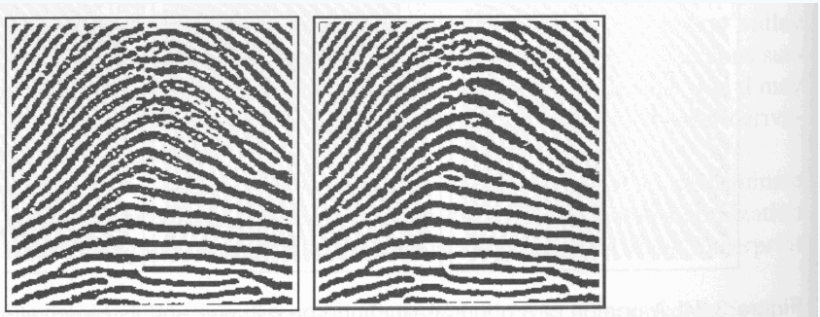


Figure 3.35. The result of eliminating small holes from both the ridges and valleys of the binary image; input image is shown on the left and the output is shown on the right. The filtering is performed by computing the connected components of the image and by removing the components whose area (number of pixels) is smaller than a given threshold.

laboratory Gerstner

Thinning

- enhance2ridgevalley.m
- imOutput = bwmorph(imcomplement (imReconstruct),'thin', 'Inf'); %thins the reconstructed image

laboratory Gerstner

Singularity and Core Detection

- Poincare Method

$$P_{G,C}(i,j) = \begin{cases} 0° & \text{if } [i,j] \text{ does not belong to any singular region} \\ 360° & \text{if } [i,j] \text{ belongs to a whorl type singular region} \\ 180° & \text{if } [i,j] \text{ belongs to a loop type singular region} \\ -180° & \text{if } [i,j] \text{ belongs to a delta type singular region.} \end{cases}$$

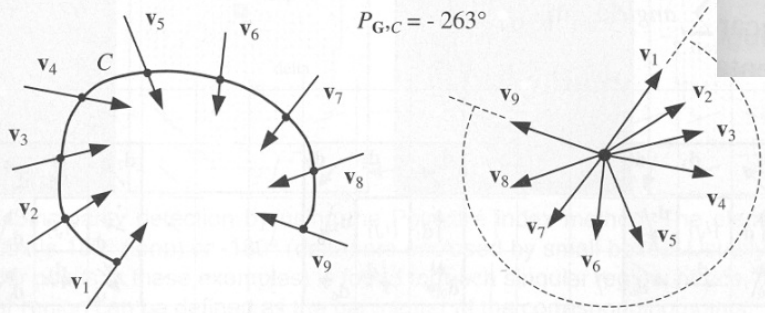
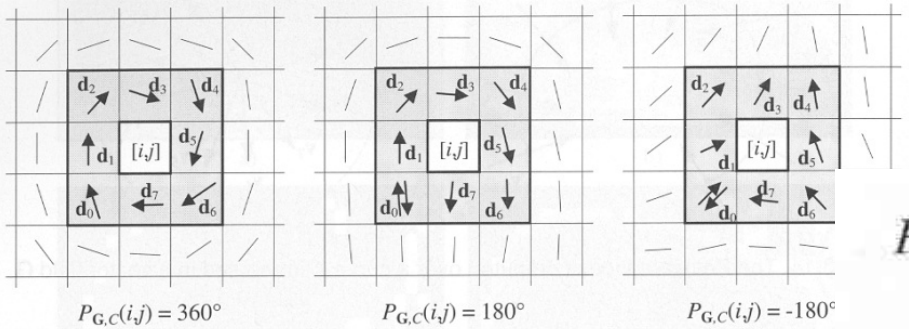


Figure 3.14. The Poincaré index computed over a curve *C* immersed in a vector field **G**.



$$P_{G,C}(i,j) = \sum_{k=0..7} angle(\mathbf{d}_k, \mathbf{d}_{(k+1) \bmod 8}).$$

Figure 3.15. Example of computation of the Poincaré index in the 8-neighborhood of points belonging (from the left to the right) to a whorl, loop, and delta singularity, respectively. Note that for the loop and delta examples (center and right), the direction of $\mathbf{d}_0$ is first chosen upward (to compute the angle between $\mathbf{d}_0$ and $\mathbf{d}_1$) and then successively downward (when computing the angle between $\mathbf{d}_7$ and $\mathbf{d}_0$).

Singularity and Core Detection

–Poincare Method

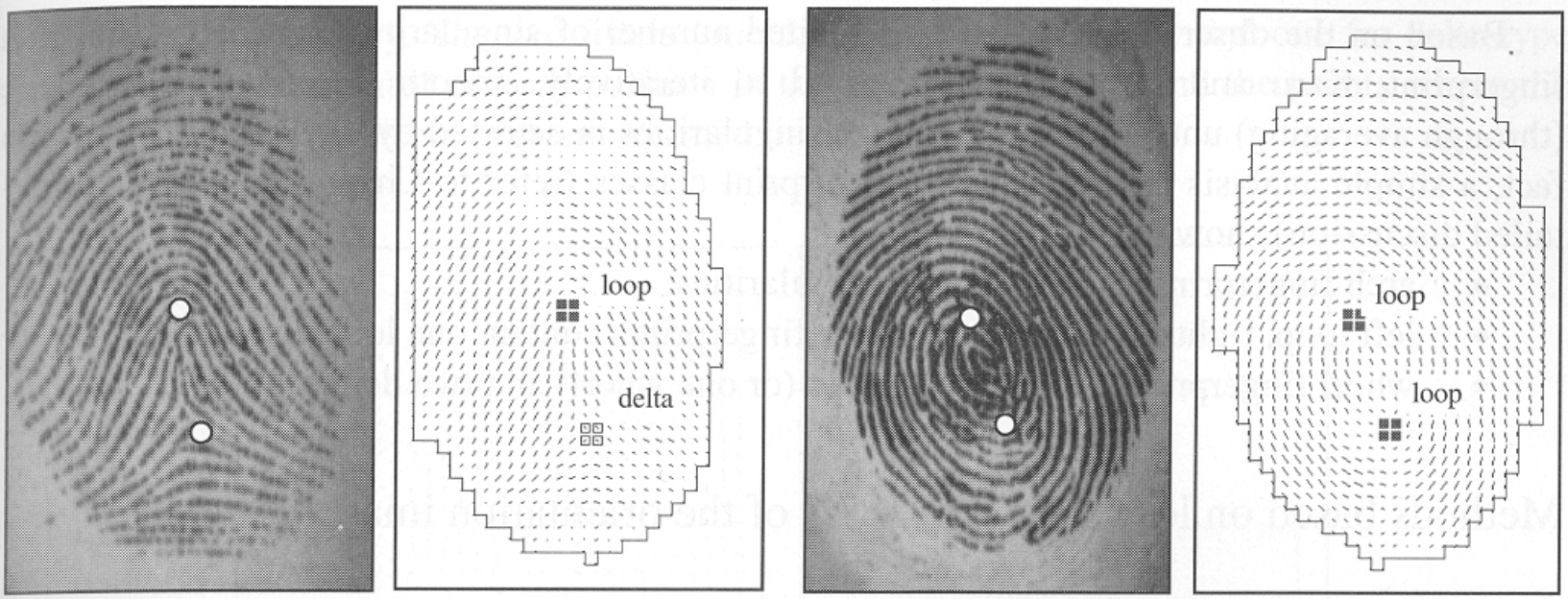


Figure 3.16. Singularity detection by using the Poincaré index method. The elements whose Poincaré index is 180° (loop) or -180° (delta) are enclosed by small boxes. Usually, more than one point (four points in these examples) is found for each singular region: hence, the center of each singular region can be defined as the barycenter of the corresponding points.

Singularity and Core Detection

– Poincare Method

If we know the type of the fingerprint beforehand, false singularities can be eliminated by iteratively smoothing the image with the help of the following observation:

- Arch fingerprints do not contain singularities
- Left loop, right loop and tented arch fingerprints contain one loop and one delta
- Whorl fingerprints contain two loops and two deltas

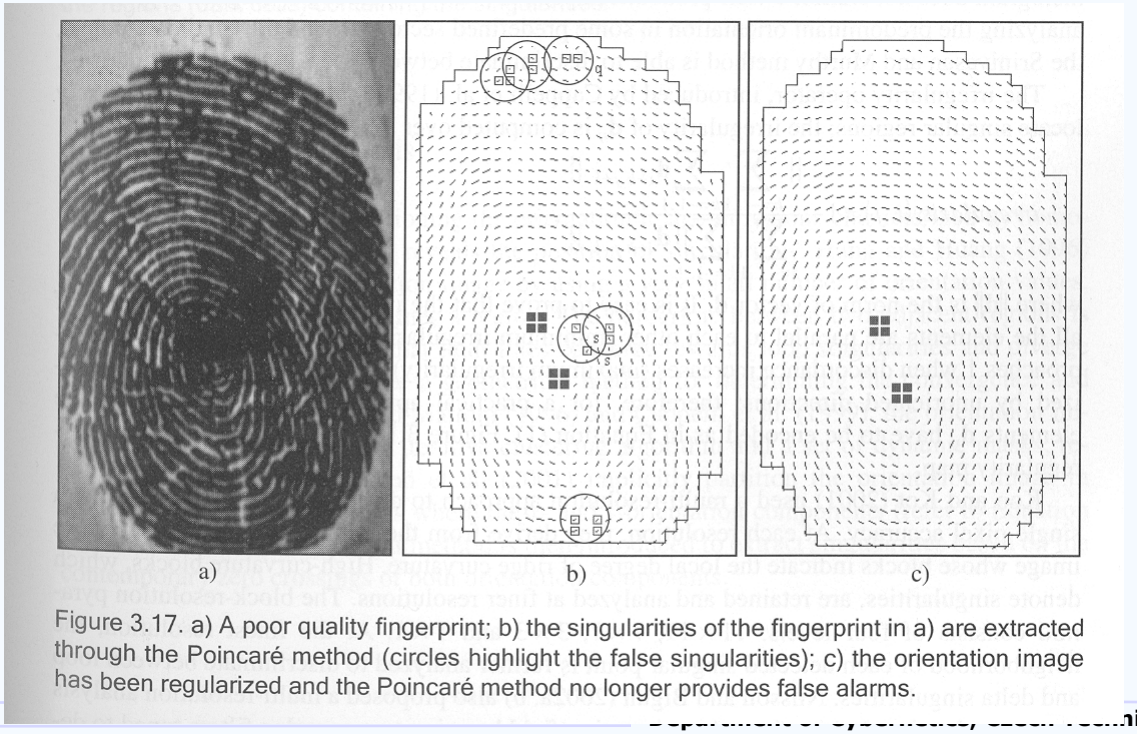


Figure 3.17. a) A poor quality fingerprint; b) the singularities of the fingerprint in a) are extracted through the Poincaré method (circles highlight the false singularities); c) the orientation image has been regularized and the Poincaré method no longer provides false alarms.

Singularity and Core Detection

- Methods based on local features
  - Orientation histograms at local level (3x3)
  - Irregularity
  - **Fp1 = findsingularitypoint(Fp1);**
  - $d_{ij} = [r_{ij}.cos2\theta_{ij}, r_{ij}sin2\ \theta_{ij}]$

$$irregularity(i,j) = 1 - \frac{\| \sum_{h=-1..1} \sum_{k=-1..1} \mathbf{d}_{i+h\ j+k} \|}{\sum_{h=-1..1} \sum_{k=-1..1} \| \mathbf{d}_{i+h\ j+k} \|},$$

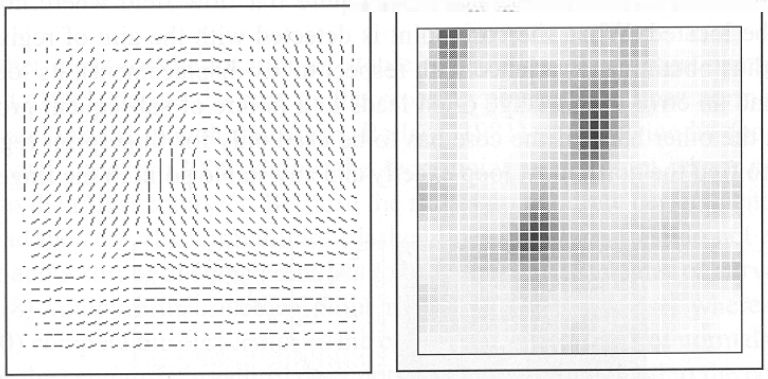


Figure 3.18. An orientation image and the corresponding irregularity map that clearly identifies the regions (dark cells) containing the singularities.

Singularity and Core Detection

– Partitioning based methods

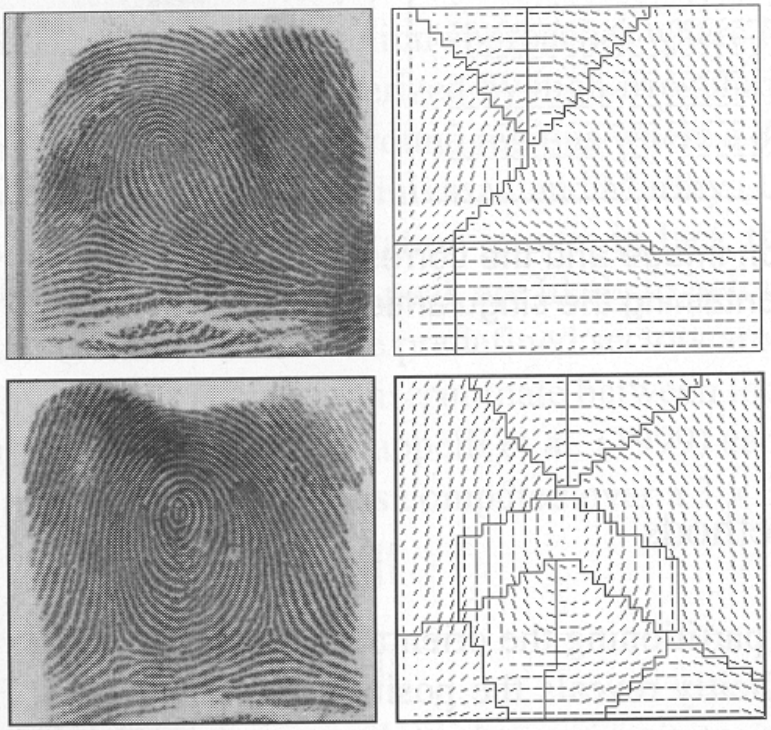
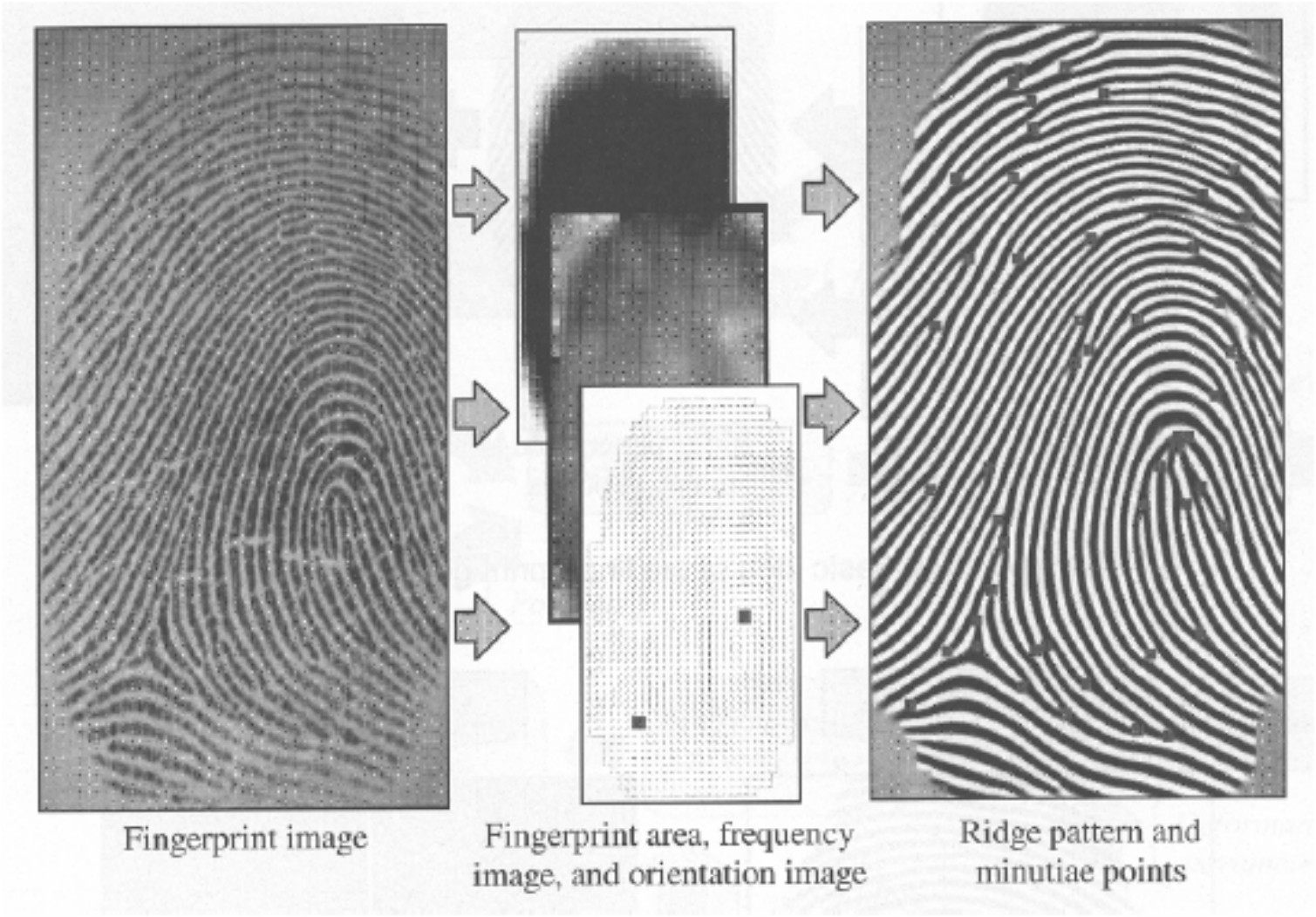


Figure 3.19. Orientation image partitioning with the MASK approach (Cappelli et al., 1999). The intersections between region border lines denote fingerprint singularities. ©IEEE.

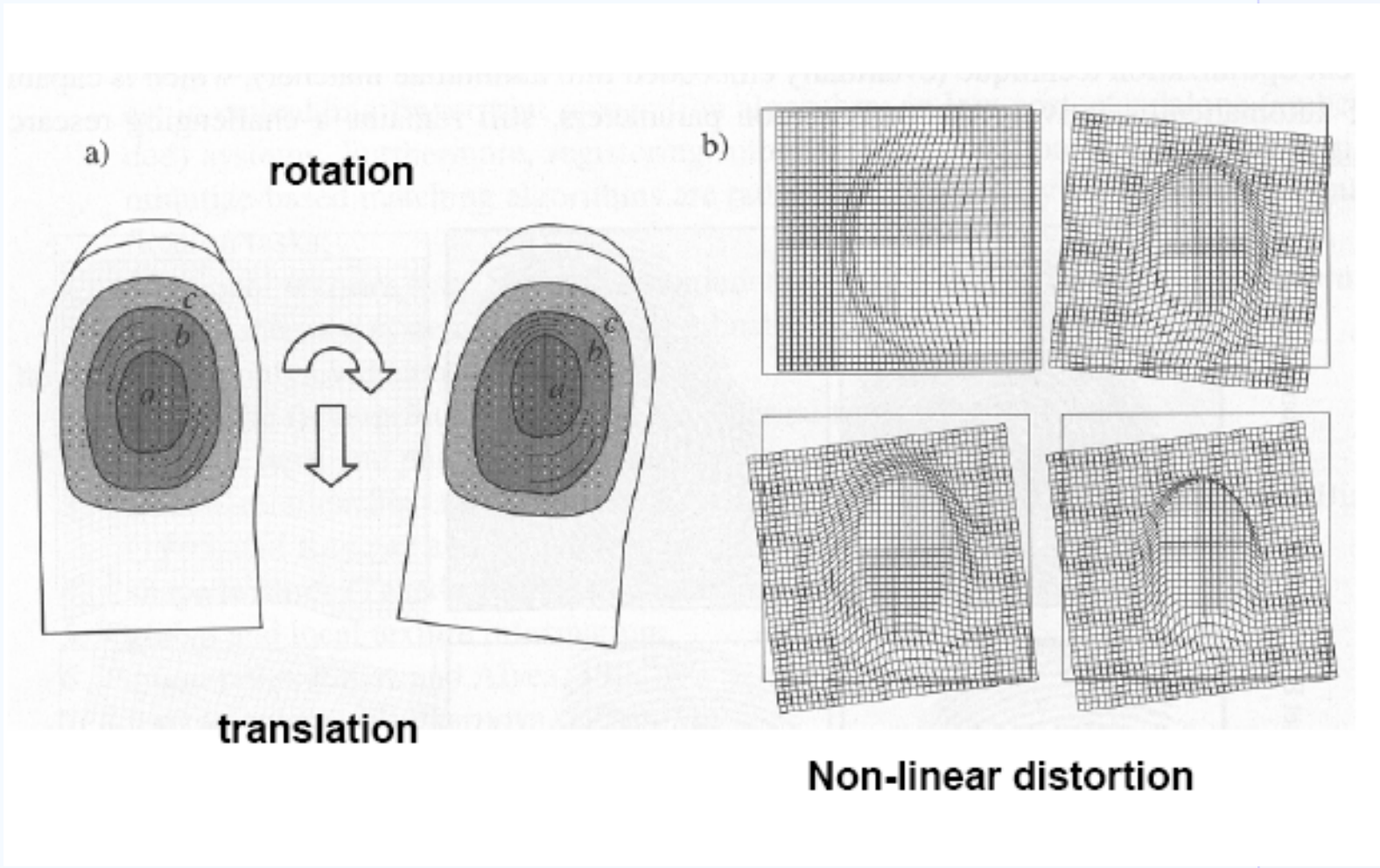
Feature Extraction



Feature Extraction Errors

- The feature extraction algorithms are imperfect and often introduce measurement errors
- Errors may be made during any of the feature extraction stages, e.g., estimation of orientation and frequency images, detection of the number, type, and position of the singularities and minutiae, segmentation of the fingerprint area from bacground, etc.
- Aggressive enhancement algorithms may introduce inconsistent biases that perturb the location and orientation of the reported minutiae from their gray-scale counterparts
- In low-quality fingerprint images, the minutiae extraction process may introduce a large number of spurious minutiae and may not be able to detect all the true minutiae

Non-linear distortion

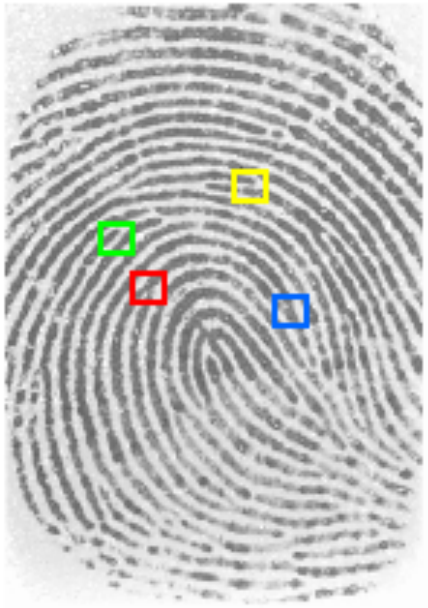


Intra-variability

- Matching fingerprint images is an extremely difficult problem, mainly due to the large variability in different impressions of the same finger (intra-variability). The main factors are:
  - Displacement (global translation of the fingerprint area)
  - Rotation
  - Partial overlap
  - Non-linear distortion:
    - *the act of sensing maps the three-dimensional shape of a finger onto the two-dimensional surface of the sensor*
    - *skin elasticity*
  - Pressure and skin condition
  - Noise: introduced by the fingerprint sensing system
  - Feature extraction errors

Fingerprint matching

Reference fingerprint



Test fingerprint

- Comes the test- and the reference fingerprint from the same finger?
  the two fingerprints can be *translated* and *rotated* relative to each other.
- Minutiae based matching: by *direct use of the local structure* of the fingerprints extract common minutiae points.

MATCHING: Approaches

- Correlation-based matching
  - Superimpose images – compare pixels
- Minutiae-based matching
  - Classical Technique – Most popular
  - Compare extracted minutiae
- Ridge Feature-based matching
  - Compare the structures of the ridges
  - Everything else

laboratory Gerstner

Fingerprint Matching

- Compare two given fingerprints *T,I*
  - Return degree of similarity *(0->1)*
  - Binary Yes/No
- *T* -> template, acquired during enrollment
- *I* -> Input
- Either input images, or feature vectors (minutiae) extracted from them

- Pressure and Skin condition
  - Pressure, dryness, disease, sweat, dirt, grease, humidity
- Noise
  - Dirt on the sensor
- Feature Extraction Errors

- Many algorithms match high quality images
- Challenge is in low-quality and partial matches
- **20% of the problems (low quality) at FVC2000 caused 80% of the false non-matches**
- Many were correctly identified at FVC2002 though

Correlation-based Techniques

- T and I are images
- Sum of squared Differences
  - $SSD(T,I) = ||T-I||^2 = (T-I)^T(T-I) = ||T||^2 + ||I||^2 - 2T^TI$
  - Difference between pixels
- $||T||^2 + ||I||^2$ are constant under transformation
- Try to maximize correlation – Minimizes difference
  - $CC(T,I) = T^TI$
  - Can't be used because of displacement / rotation

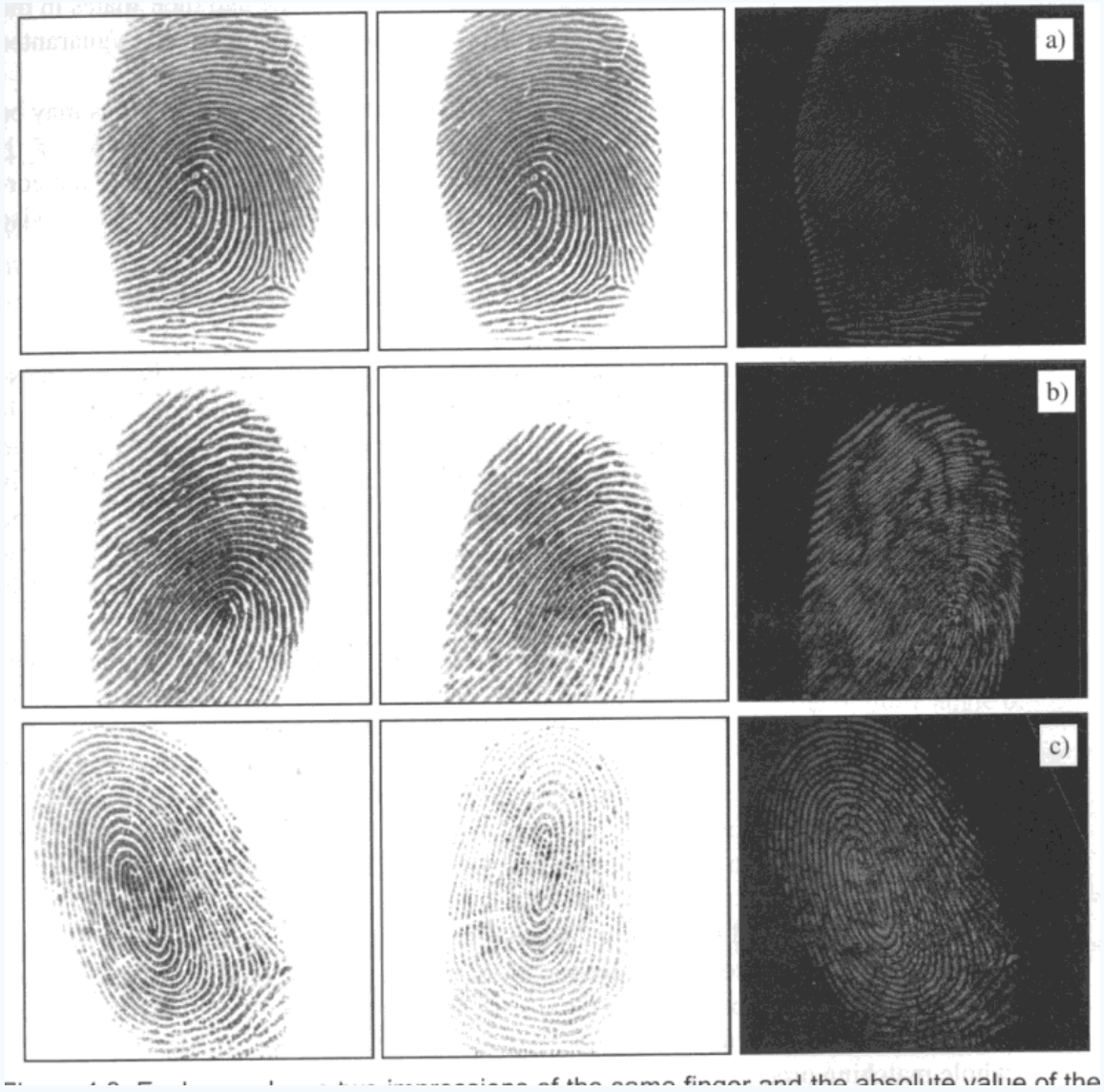
Maximizing Correlation

- $I^{(\Delta x,\Delta y,\theta)}$
  - Transformation of I
  - Rotation around the origin by θ
  - Translation by x,y
- $S(T,I) = \max CC(T,I^{(\Delta x,\Delta y,\theta)})$
  - Try them all – take max

$$S(\mathbf{T},\mathbf{I}) = \max_{\Delta x,\Delta y,\theta} CC\left(\mathbf{T},\mathbf{I}^{(\Delta x,\Delta y,\theta)}\right)$$

laboratory Gerstner

Correlation Results



Minutiae-based Methods

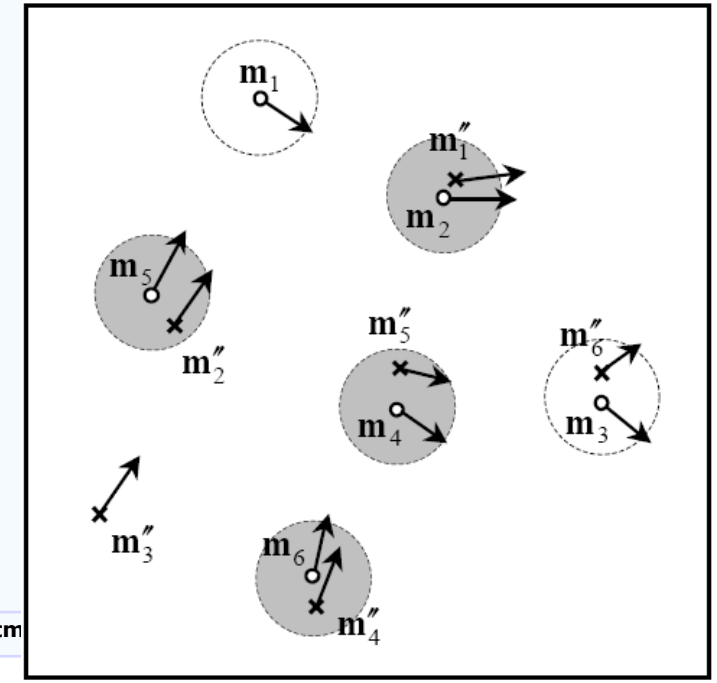
$$\mathbf{m} = \{x, y, \theta\}$$

- Classical Technique
- T, I are feature vectors of minutiae
- Minutiae = (x,y,θ)
- Two minutiae match if
  - Euclidean distance < r_0
  - Difference between angles < θ_0
  - Tolerance Boxes
    - r_0
    - θ_0

$$\mathbf{T} = \{\mathbf{m}_1, \mathbf{m}_2, \ldots \mathbf{m}_m\}, \quad \mathbf{m}_i = \{x_i, y_i, \theta_i\}, \quad i = 1 \ldots m$$
$$\mathbf{I} = \{\mathbf{m}'_1, \mathbf{m}'_2, \ldots \mathbf{m}'_n\}, \quad \mathbf{m}'_j = \{x'_j, y'_j, \theta'_j\}, \quad j = 1 \ldots n,$$

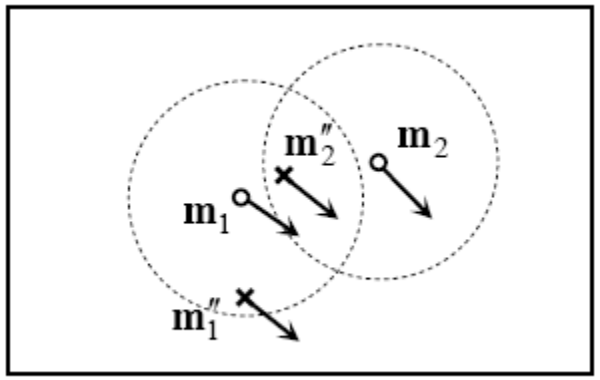
$$sd(\mathbf{m}'_j, \mathbf{m}_i) = \sqrt{(x'_j - x_i)^2 + (y'_j - y_i)^2} \leq r_0, \text{ and}$$

$$dd(\mathbf{m}'_j, \mathbf{m}_i) = min\left(\left|\theta'_j - \theta_i\right|, 360° - \left|\theta'_j - \theta_i\right|\right) \leq \theta_0 .$$



Formulation

- M''_j = map(m'_j)
  - Map applies a geometrical transformation
- mm(m''_j, m_i) returns 1 if they match
- Matching can be formulated as



$$\underset{\Delta x,\Delta y,\theta,P}{\text{maximize}} \sum_{i=1}^{m} mm\left(map_{\Delta x,\Delta y,\theta}\left(\mathbf{m}'_{P(i)}\right),\mathbf{m}_i\right)$$

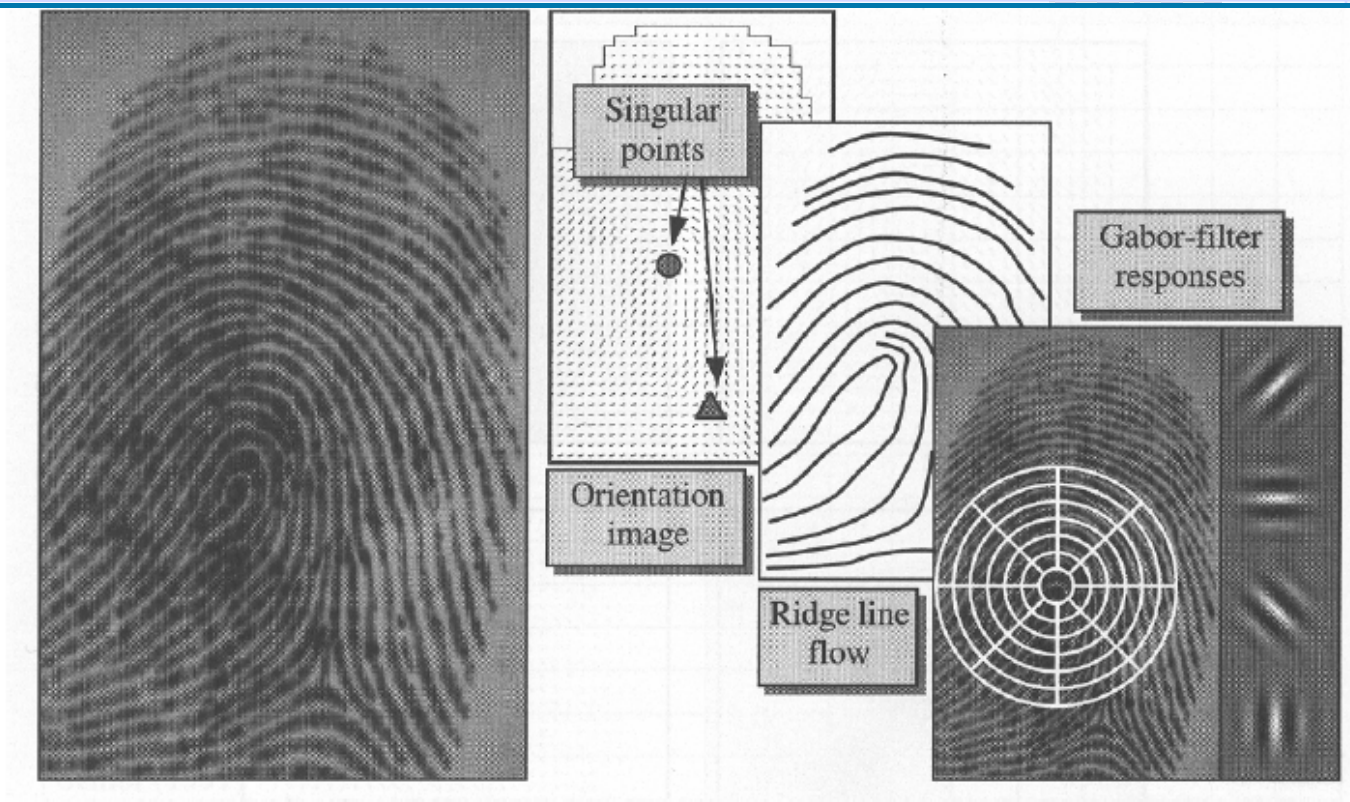
- **P is an unknown function which pairs the minutiae**
  - Which minutiae in I corresponds to which in T: ***allign2***

$$map_{\Delta x,\Delta y,\theta}\left(\mathbf{m}'_j = \{x'_j, y'_j, \theta'_j\}\right) = \mathbf{m}''_j = \{x''_j, y''_j, \theta'_j + \theta\}, \text{ where}$$

$$\begin{bmatrix} x''_j \\ y''_j \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x'_j \\ y'_j \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}.$$

$$mm\left(\mathbf{m}''_j,\mathbf{m}_i\right) = \begin{cases} 1 & sd\left(\mathbf{m}''_j,\mathbf{m}_i\right) \le r_0 \text{ and } dd\left(\mathbf{m}''_j,\mathbf{m}_i\right) \le \theta_0 \\ 0 & \text{otherwise.} \end{cases}$$

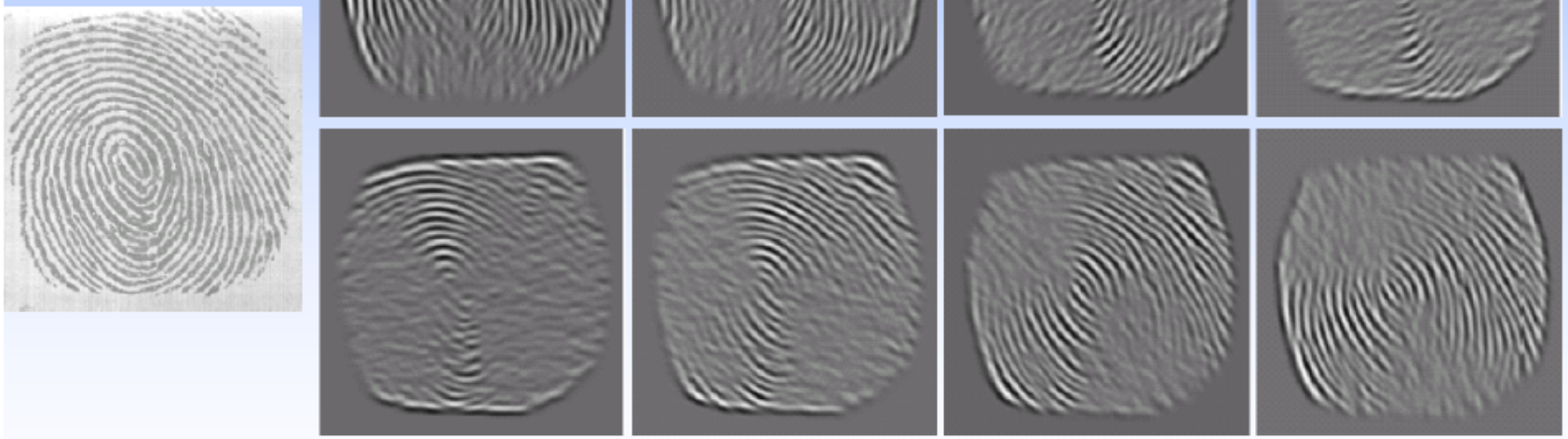
Ridge feature based matching



Most frequently used features for fingerprint matching:
-Orientation image
-Singular points (loop and delta)
-Ridge line flow
-Gabor filter responses

Gabor filters

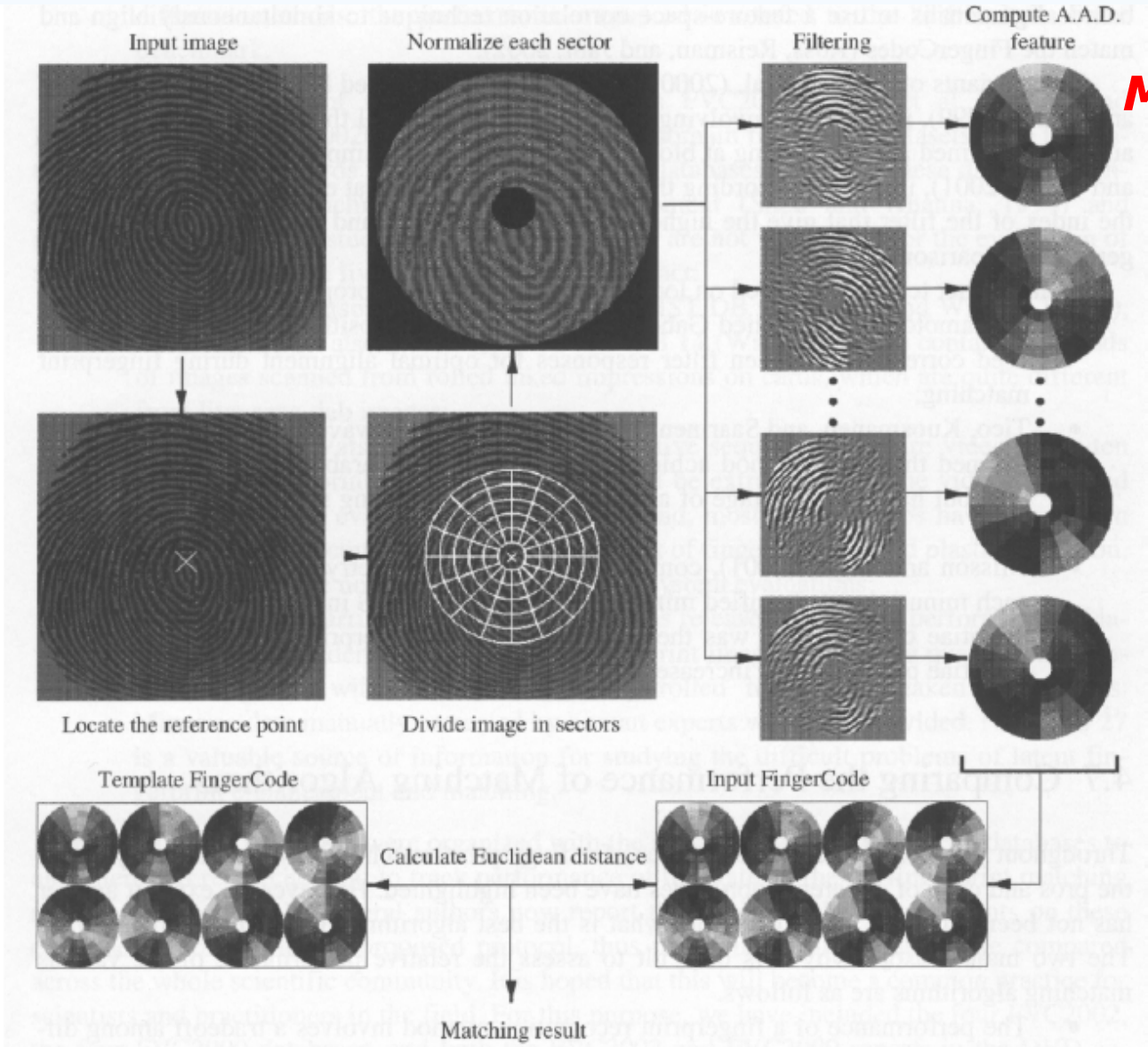
- An input fingerprint image is filtered using 8 Gabor filters all having the same frequency but different orientations (0°, 22.5°, 45°, 67.5°, 90°, 112.5°, 135°, 157.5°)



Local texture analysis

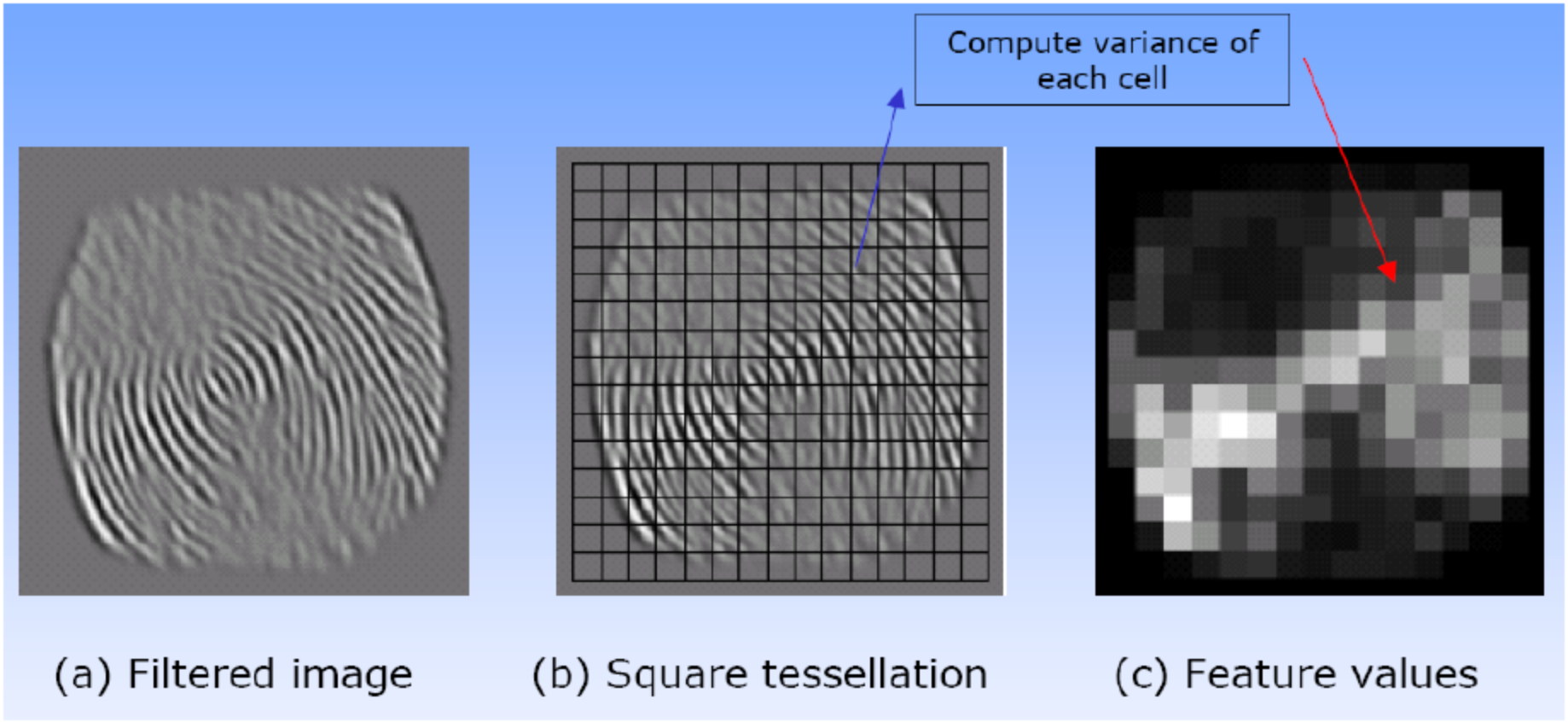
- The fingerprint area of interest is tesselated with respect to the core point
- The local texture information in each sector is decomposed into separate channels by using a Gabor filterbank
- Feature vector:
  - 80 cells (5 bands and 16 sectors)
  - Filterbank of 8 Gabor filters (8 orientations, 1 scale =1/10 for 500 dpi fingerprint images)
  - Each fingerprint is represented by a 80x8 = 640 fixed-size feature vector, called the FingerCode
- Computation of average absolute deviation (AAD)

Finger code approach



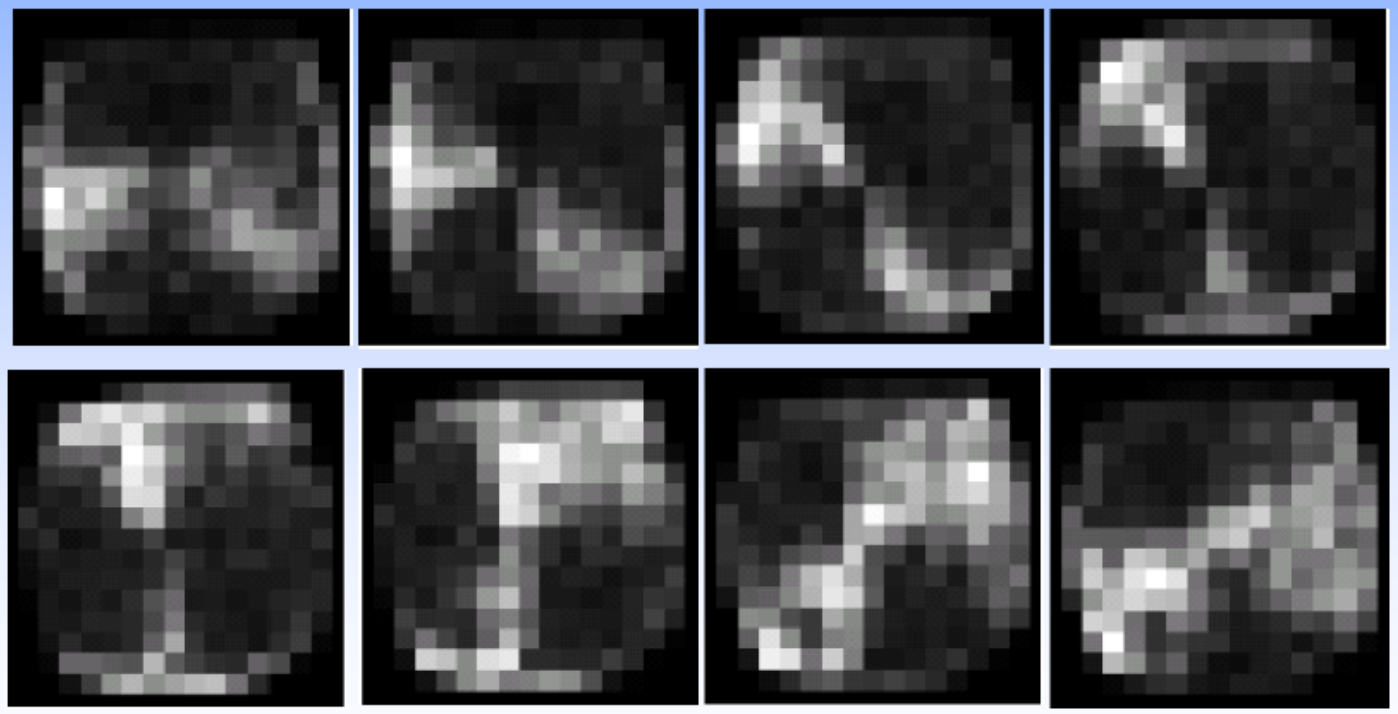
MatchGaborFeat

Texture based representation



Ridge feature map

- The filtered images are examined using a square tessellation and the variance of pixel intensities in every cell is used as a feature value

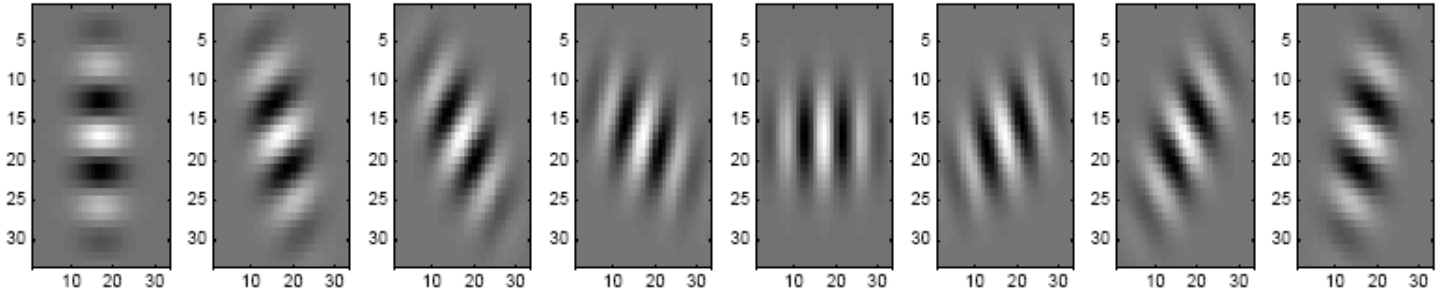


The ridge feature map is a fixed-length feature vector

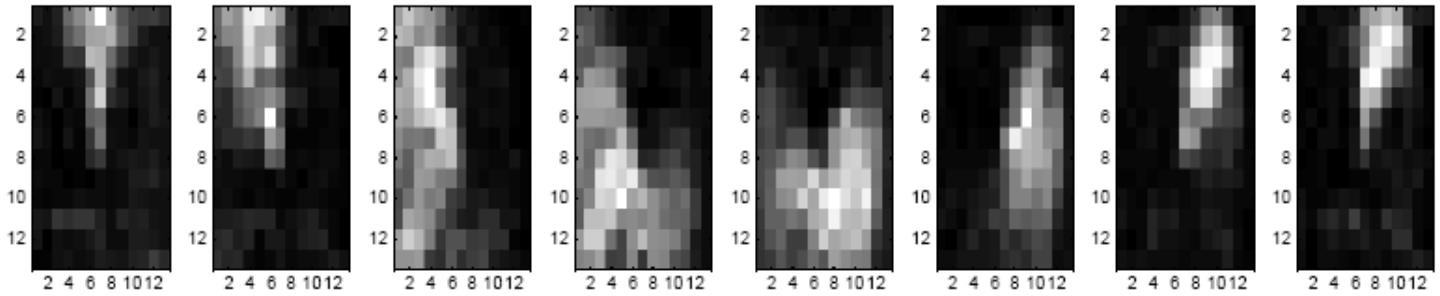
Ross, et al, "A Hybrid Fingerprint Matcher", Pattern Recognition, Vol. 36, July 2003

Ridge feature based matching

Responses of directional Gabor filters



Fingerprint 1



Fingerprint 2 (same finger)