F-Race [Birattari02] – procedure that empirically evaluates a set of candidate configurations by discarding bad ones as soon as statistically sufficient evidence is gathered against them.

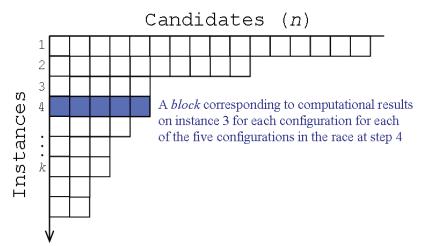
- The process starts from a given finite pool of candidate configurations.
- If sufficient evidence is gathered that some candidate is inferior to at least another one, such a candidate is dropped from the pool and the procedure is iterated over the remaining ones.

The methodology can be applied to repetitive problems – problems where many similar instances appear over time.

- k is the current step of the race process and $n = |\Theta_{k-1}|$ configurations are still in the race.
- \underline{i} is a random sequence of training instances; \underline{i}_k is drawn from I according to P_I , independently for each k.
- $\underline{c}^k(\theta, \underline{i})$ is an array of k terms; $c(\theta, \underline{i}_l)$ is the cost of the best solution found by configuration θ on instance \underline{i}_l .

For a given θ , the array \underline{c}^k of length k can be obtained from \underline{c}^{k-1} by appending the cost concerning the k-th instance in \underline{i} .

 A block is n-variate random variable (<u>c</u>^k(θ₁, <u>i</u>_l), <u>c</u>^k(θ₂, <u>i</u>_l),..., <u>c</u>^k(θ_n, <u>i</u>_l)) that corresponds to the computational results on instance <u>i</u>_l for each configuration in the race at step k.



• Null hypothesis – all possible rankings of the candidates within each block are equally likely.

The optimization problem is tackled by generating a sequence $\Theta_0 = \Theta \supseteq \Theta_1 \supseteq \Theta_2 \supseteq \ldots$

The step from a set Θ_{k-1} to Θ_k is realized as follows

- 1. At step k, a new instance \underline{i}_k is considered; each candidate $\theta \in \Theta_{k-1}$ still in the race is executed on \underline{i}_k and each observed cost $c(\theta, \underline{i}_k)$ is appended to its $\underline{c}^{k-1}(\theta, \underline{i})$.
- An aggregate comparison of the arrays <u>c</u>^k(θ, <u>i</u>) for all θ ∈ Θ_{k-1} is carried out by a statistical test non-parametric Friedman 2-way analysis of variance by ranks.
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3. If the null hypothesis is rejected, pairwise comparisons between the best candidate and each other one are carried out by means of the t-test. All candidates that result significantly worse than the best one are discarded.

Otherwise, all candidates in Θ_{k-1} pass to Θ_k .

F-Race: Algorithm

```
\Theta^* = \Theta_0, ni = 0
repeat
     randomly choose instance i from set I; run all configurations on \Theta^* on i
     ni = ni + 1
     if (ni \geq ni_{min})
           perform rank-based Friedman test on results for configurations in \Theta^*
           on all instances in I evaluated so far
           if (test indicates significant performance differences)
                \theta^* = \text{best configuration in } \Theta^* according to the statistical population parameter
                 over instances evaluated so far
                      for all \theta \in \Theta^* \setminus \{\theta^*\} do
                            perform pairwise t-test on \theta and \theta^*
                            if (test indicates significant performance differences)
                                  eliminate \theta from \Theta^*
                            end if
                      end for
           end if
     end if
until (termination condition)
return \Theta^*
```

Good technique, but:

- not suited for applications with large configuration spaces;
- thus, mainly used for configuration problems with few parameters and rather small configuration spaces.