



## 1. Revision of Prerequisites. Hybrid Algorithms. Memetic Algorithms.

Petr Pošík

This lecture is based on the following works:  
'A taxonomy of hybrid metaheuristics'  
by E. G. Talbi, *Journal of Heuristics*, 2002,  
and  
'A tutorial for competent memetic algorithms'  
by N. Krasnogor and J. Smith, *IEEE Trans. on Evolutionary Computation*, 2005.



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Hybrid Optimization Algorithms

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# Revision



# Optimization, black-box, local-search, EAs

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- When should I use an EA? When should I use a hill-climber?
- Do you know other *metaheuristics*?





# Hybrid Optimization Algorithms



# Optimization Algorithms

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Single-state (single-solution) methods:

- local search (LS)
- greedy heuristic (GH)
- simulated annealing (SA)
- tabu search (TS)
- ...

Population-based methods:

- genetic and evolutionary algorithms (GA, EA, ES, EP, GP, ...)
- ant colonies (ACO)
- particle swarm optimization (PSO)
- scatter search (SS)
- ...

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- Hybridization and Classification of the approaches

- The 4 Main Groups of Hybrids

- Other Attributes of Hybrid Algorithms

- Hybridization Design Grammar

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# Hybridization and Classification of the approaches

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Hybrid algorithm:

- a combination of (at least some features of) at least 2 of the above methods

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Why?

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Talbi [Tal02] classifies hybrid approaches based on several independent design issues:

- Low-level vs. high-level hybrid:
  - In **low-level hybrid**, a particular function of one algorithm is replaced by another optimization algorithm.
  - In **high-level hybrid**, the different optimization algorithms are combined without changing their inner workings.
- Relay vs. teamwork hybrid:
  - In **relay hybridization**, the individual algorithms are applied one after another, each using the output of the previous algorithm as its input.
  - In **teamwork hybridization**, each algorithm performs the search independently of the others.

[Tal02] E. G. Talbi. A taxonomy of hybrid metaheuristics. *Journal of Heuristics*, 8(5):541–564, 2002.

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## The 4 Main Groups of Hybrids

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- LS builds init. population for EA. The results of EA are processed by TS.
- Iterated local search, ILS.
- Variable neighborhood search, VNS.
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- Island model of parallel GAs (one of the next lectures).

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The classification is not always unambiguous, it depends on personal view.

- Is GRASP a local search with its initialization phase replaced by a constructive heuristic (LRH), or
- is it a sequence of 2 self-contained algorithms (HRH)?



# Other Attributes of Hybrid Algorithms

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Again, following Talbi [Tal02]:

- Homogeneous vs. heterogeneous:
  - In a **homogeneous hybrid**, all the combined algorithms are of the same type. (They may have different parameters.)
  - In a **heterogeneous hybrid**, each algorithm may be of different type.

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  - In a **global hybrid**, all algorithms search the whole space.
  - In a **partial hybrid**, the problem is decomposed into subproblems and each algorithm is dedicated to the search in its own subspace. (See also the lecture on coevolution.)

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- General vs. specialist:
  - In a **general hybrid**, all the algorithms solve the same target optimization problem.
  - In a **specialist hybrid**, some of the algorithms may solve a different optimization problem, helping the others to solve the original one.

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**Example:** HTH(HRH(GH + LTH(GA(LS))))

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## Example: HTH(HRH(GH + LTH(GA(LS))))

- Island model containing HRH(GH + LTH(GA(LS))) in each node.
- In each node, initial population is generated using a greedy heuristic (GH)
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- ... with an embedded local search (LS) improving the results each generation.



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- HRH(GH + LS) is actually the GRASP algorithm, is it clearer now?



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<LTH> ::= LTH(<metaheuristic>(<metaheuristic>))
<HRH> ::= HRH(<metaheuristic> + <metaheuristic>)
<HTH> ::= HTH(<metaheuristic>)
           | HTH(<metaheuristic>, <metaheuristic>)
<flat> ::= (<nature>, <optimization>, <function>)
<nature> ::= homogeneous | heterogeneous
<optimization> ::= global | partial
<function> ::= general | specialist
<metaheuristic> ::= <hybrid-opt-algorithm>
                  | LS | TS | SA | GA | ES | GP | GH | ACO | SS | ...
```

## Example: HTH(HRH(GH + LTH(GA(LS))))

- Island model containing HRH(GH + LTH(GA(LS))) in each node.
- In each node, initial population is generated using a greedy heuristic (GH)
- ...and then a genetic algorithm (GA) was run
- ... with an embedded local search (LS) improving the results each generation.

## Example: HRH(HRH(GH + LS) + LTH(GA(HRH(GH + LS))))

- HRH(GH + LS) is actually the GRASP algorithm, is it clearer now?
- No? What about HRH(GRASP + LTH(GA(GRASP)))?





# Hybridization Design Grammar

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## Revision

### Hybrid Optimization Algorithms

- Optimization Algorithms
- Hybridization and Classification of the approaches
- The 4 Main Groups of Hybrids
- Other Attributes of Hybrid Algorithms
- [Hybridization Design Grammar](#)

### Memetic Algorithms

```
<hybrid-opt-algorithm> ::= <hierarchical><flat>
<hierarchical> ::= <LRH> | <LTH> | <HRH> | <HTH>
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## Example: HRH(HRH(GH + LS) + LTH(GA(HRH(GH + LS))))

- HRH(GH + LS) is actually the GRASP algorithm, is it clearer now?
- No? What about HRH(GRASP + LTH(GA(GRASP)))?
- GRASP creates the initial population for a memetic algorithm that uses GRASP to improve solutions of GA.



# Memetic Algorithms



# Memetic Algorithms

---

Memetic algorithm (MAs):

- hybrid of evolutionary algorithm with embedded local search, i.e.
- EA that includes one or more LS phases within its evolutionary cycle.
- MA is **LTH(EA(LS))** using Talbi's taxonomy.

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- *Local search* quickly identifies an optimum, but the optimum is often only local
- *EA* is more robust against getting stuck in local optima; it uses large population and searches several places of the search space in parallel. It is thus *slower*.

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When implemented well,

- MAs are faster than ordinary EAs, and
- MAs are more robust against getting stuck in local optima than local search.

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# Memetic Algorithms

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- MAs are more robust against getting stuck in local optima than local search.

Talbi's taxonomy ignores many MA design issues.



# Models of Evolution

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## Darwinian evolution

- the whole “plan” of an organism is inscribed in its genes
- the survival of the organism depends on its genetic constitution only
- used in ordinary EAs

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- characteristics acquired during the organism’s lifetime can be transferred to the offspring
- used in Lamarckian MAs:
  - an individual in EA is improved by a local search
  - the improved individual replaces the original individual

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## Baldwin effect

- the genetic information encodes the ability to learn
- organisms able to learn have higher chance of survival, which in turn allows the “right” genes to survive and breed
- used in Baldwinian MAs:
  - an individual in EA is improved by a local search
  - the original individual is preserved in the population, but its fitness is changed to the fitness of the improved individual

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# MA: Main Design Issues

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What is the best trade-off between local-search and the global search provided by evolution?

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- Where (in which phase of the evolutionary cycle) and when (in which evolutionary cycle) should the local search be applied?
- Which individuals should be improved? (The best ones? All? Random ones?)
- How long should each local search be allowed to run? (Make a complete run? Only perform one step of LS?)
- How can the genetic operators be best combined (modified) with local search to achieve the synergy?
- Should we use the Lamarckian or Baldwinian model?



# MAs Classification

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Krasnogor and Smith [KS05] introduce the concept of *schedulers*:

- *Fine-grained mutation scheduler*  $fS_M$  replaces the mutation operator and organizes the mutation and LS.
- *Fine-grained crossover scheduler*  $fS_R$  replaces the crossover operator and organizes the crossover and LS.
- *Coarse-grained scheduler*  $cS$  replaces selection and replacement, and organizes the interplay between selection, crossover and LS. It can supply the population statistics to LS.
- *Meta scheduler*  $mS$  provides a historical information to LS inside MA.

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A 4-bit number can then be assigned to each MA:

$$D = (b_4, b_3, b_2, b_1)$$

- $b_1 = 1 \iff$  MA contains  $fS_M$ .
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- $b_3 = 1 \iff$  MA contains  $cS$ .
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Higher value of  $D$  implies more complex algorithm, but not necessarily a better one!

[KS05] N. Krasnogor and J. Smith. A tutorial for competent memetic algorithms: model, taxonomy, and design issues. *Evolutionary Computation, IEEE Transactions on*, 9(5):474–488, 2005.



# Which LS should I choose?

---

It depends.

- Even within a single problem class, e.g. TSP, different LS operators are best for different instances of TSP. The choice is *instance-specific*.
- Even during one optimization run on one problem instance, different LS operators are best for different phases of evolution. The choice is *time-dependent*.

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Based on these observations, a sensible approach could be:

- do not decide a priori which LS should be used;
- incorporate several of them and consider methods to avoid spending time using the unproductive ones.
- This implies at least some way of adapting of operator probabilities, which in turn implies the presence of *cS* or *mS*.

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# How should I best integrate LS into EA?

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It depends. We can list some suggestions:

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# How should I best integrate LS into EA?

---

It depends. We can list some suggestions:

Lamarckian vs. Baldwinian:

- Lamarckian learning can happen before or after the application of other operators.
- There is a little point in applying Baldwinian search after parent selection, but before recombination or mutation.

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If Lamarckian search is continued to optimality:

- recombination and mutation will likely produce offspring worse than parents.
- We hope that the offspring will be in the basin of attraction of high-quality LO.
- To prevent selection discarding these new worse offspring, perform LS on them prior to selection.

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- We hope that the offspring will be in the basin of attraction of high-quality LO.
- To prevent selection discarding these new worse offspring, perform LS on them prior to selection.

Lamarckian approach is used most often, the LS is continued to optimality.

- This leads to the loss of diversity very quickly, MA gets stuck in LO, or stagnates on a plateau.
- A  $cS$  can monitor the population statistics and re-introduce diversity.

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# How should I set the global-local tradeoff?

---

The majority of MAs apply LS to every individual in every generation of the EA. But a  $cS$  can be used to

- choose individuals to be optimized by LS,
- set the intensity of LS,
- set the probability of applying LS.

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Example of an approach to select points for LS:

- Single-linkage clustering is used to prevent running LS too often in the same basin of attraction.
- Each basin is represented by a set of points used for LS.
- If a point is used in LS, it is assigned to the respective cluster.
- The number of clusters grows, the number of points in the cluster grows as well (if not limited somehow).
- A candidate for LS is not used, if it lies within a critical distance from an already clustered point.

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# Summary

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- Hybrid optimization algorithm is virtually any combination of 2 or more other algorithms.
  - Many existing algorithms can be described as hybrids.
  - Using the systematic taxonomy, many of non-existing hybrids can be easily created. (This does not mean they will be better than the existing ones.)
- MAs are a broad class of hybrid optimization algorithms combining
  - the robustness of evolutionary algorithms and
  - the speed of local search methods.
- Their design is
  - simple from a practitioner's point of view (very often they just work), but
  - becomes harder when we want to exploit the most of it (extensive testing and comparisons needed), and is
  - incredibly complex from the theoretical point of view when the exact interplay between particular algorithm features should be understood or modeled.

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