GVG'2017-Test-α-EN

Jméno: **Body:**

1. Complete vectors $\vec{b}_2$ and $\vec{b}_3$ to form a basis in $\mathbb{R}^3$: $\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\vec{b}_2 = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$ $\vec{b}_3 = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$

2. Use LD, resp. LI, to mark linearly dependent, resp. linearly independent, sets of vectors in ( )

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \right\} \quad \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

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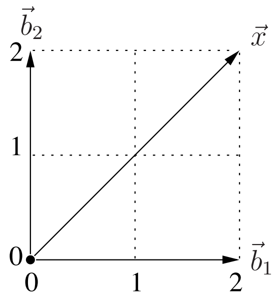
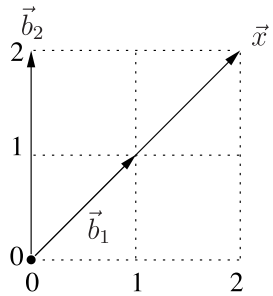
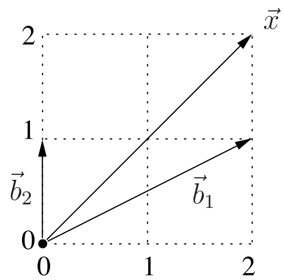
3. Write the coordinates of vector $\vec{x}$ in ordered basis $\beta = (\vec{b}_1, \vec{b}_2)$:



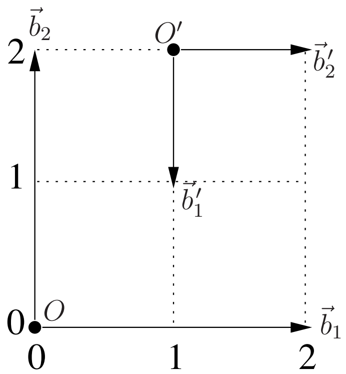




$\vec{x}_\beta = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$ $\vec{x}_\beta = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$ $\vec{x}_\beta = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$

4. Vector $\vec{x}$ has coordinates $(1, -1)$ in ordered basis $\vec{b}_1$, $\vec{b}_2$. What are its coordinates in basis $\vec{b}'_1 = 2\,\vec{b}_1$, $\vec{b}'_2 = \vec{b}_1 - \vec{b}_2$?

5. The following figure shows two coordinate systems (O, β) and (O', β'), with bases $\beta = (\vec{b}_1, \vec{b}_2)$ and $\beta' = (\vec{b}'_1, \vec{b}'_2)$



   (a) Write down cordinates of vectors of basis β in basis β'.

   (b) Write down cordinates of vectors of basis β' in basis β.

   (c) Write down the general formula for transforming coordinates of $\vec{x}_\beta$ representing a general point X in (O, β) into coordinates of $\vec{x}'_{\beta'}$ representing X in (O', β') and fill in the concrete numerical values for the situation in the figure.

6. Write down the basis of the one-dimensional subspace of $\mathbb{R}^3$, which results as the intersetion of two two-dimensional subspaces of $\mathbb{R}^3$, which are determined by their bases

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

7. Change one element of the following matrix so it becomes a rank one matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 4 & 0 \end{bmatrix}$$

8. Find all solutions to the systems

$$\begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \ \\ \ \\ \ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \ \\ \ \\ \ \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

9. Find the eigenvalues and eigenvectors of matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

10. How many roots including multiples does the equation $x^6 + x^4 - x^2 - 1 = 0$ have in complex space? Find as many of its roots as possible.

K řešení použijte další papíry, podepište je a přiložte je.