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Largest Empty Circle Problem

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Problem

- “Given set **P** of **n** points in the plane, find a largest circle that contains no points of the set **P** and whose center is internal to the convex hull of those points.”
- $\mathbf{r} = \max(\min((x_i - x_0)^2 + (y_i - y_0)^2))$
  - $\mathbf{r} \in \mathrm{Hull}(\mathbf{P})\ i$

Problem

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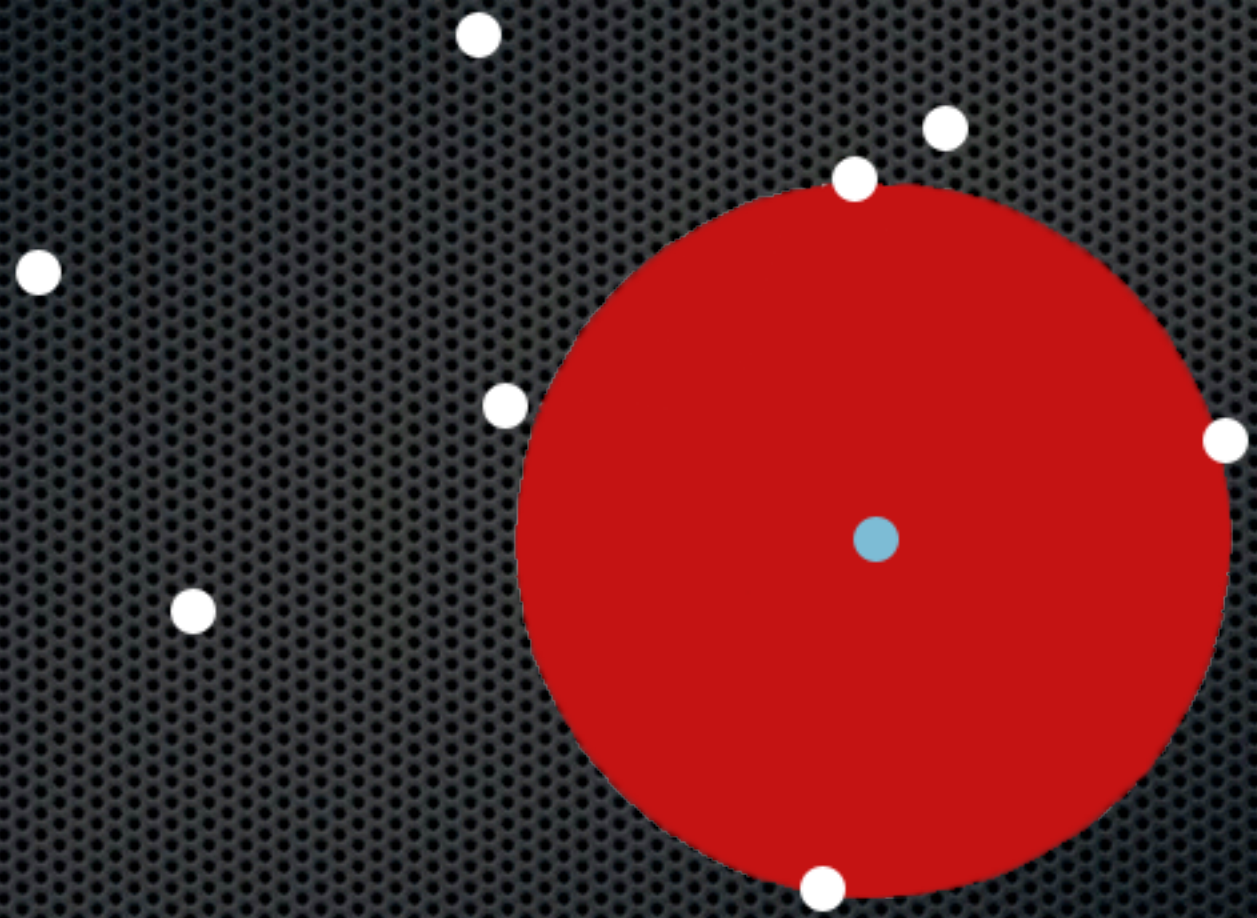
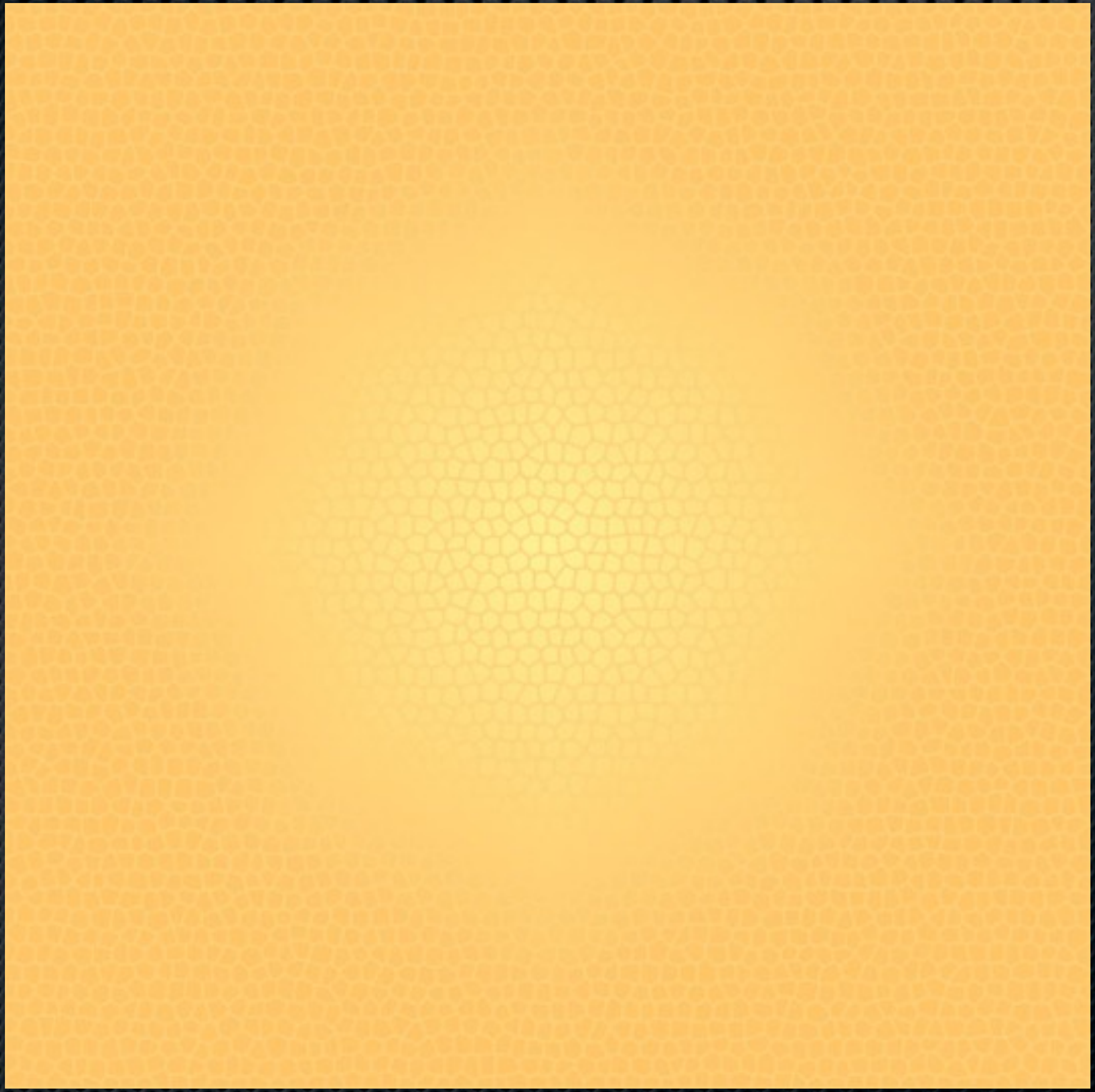


What is this good for?

Credits: http://weheartit.com/lyolya_lu

My idea



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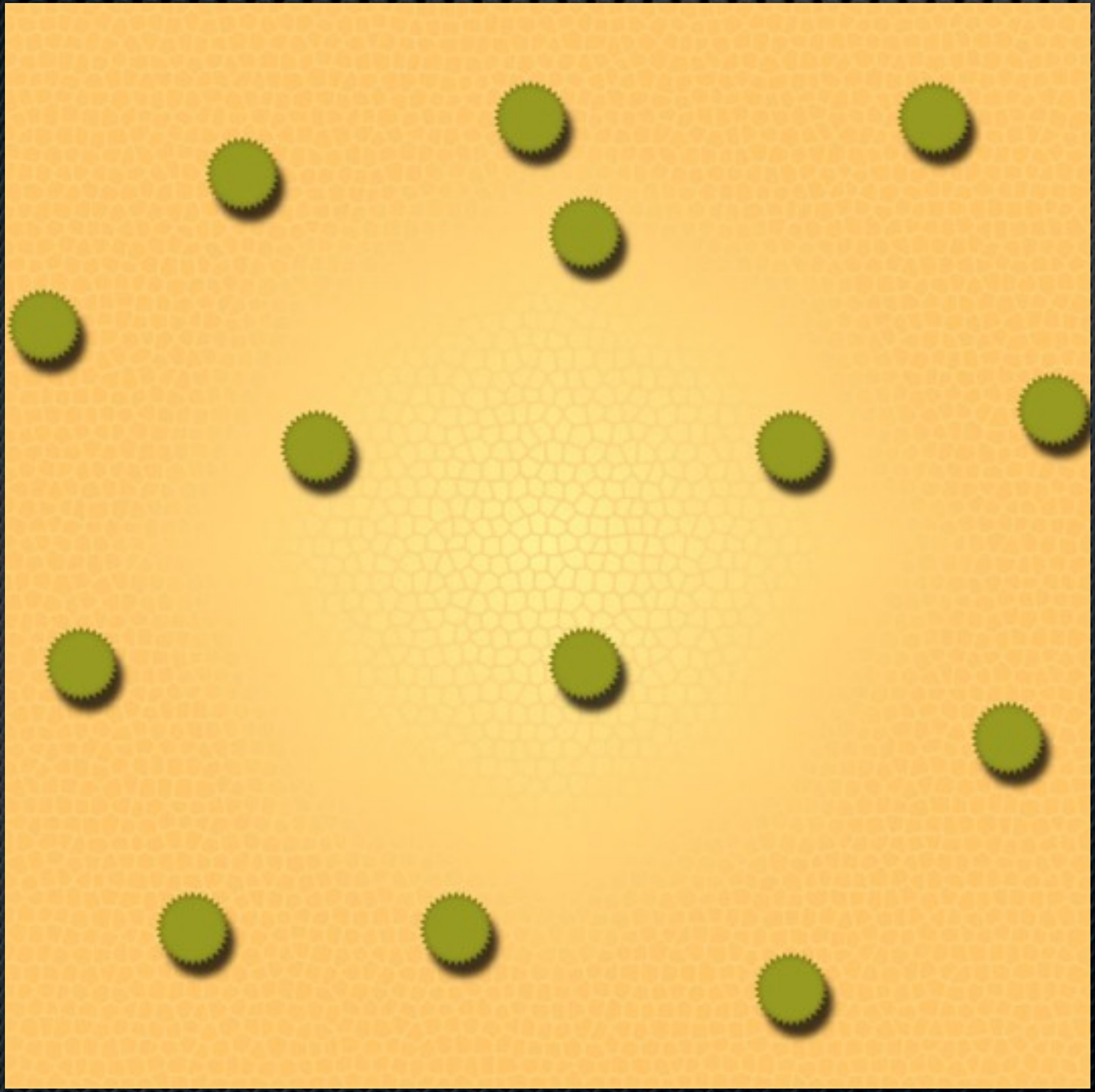

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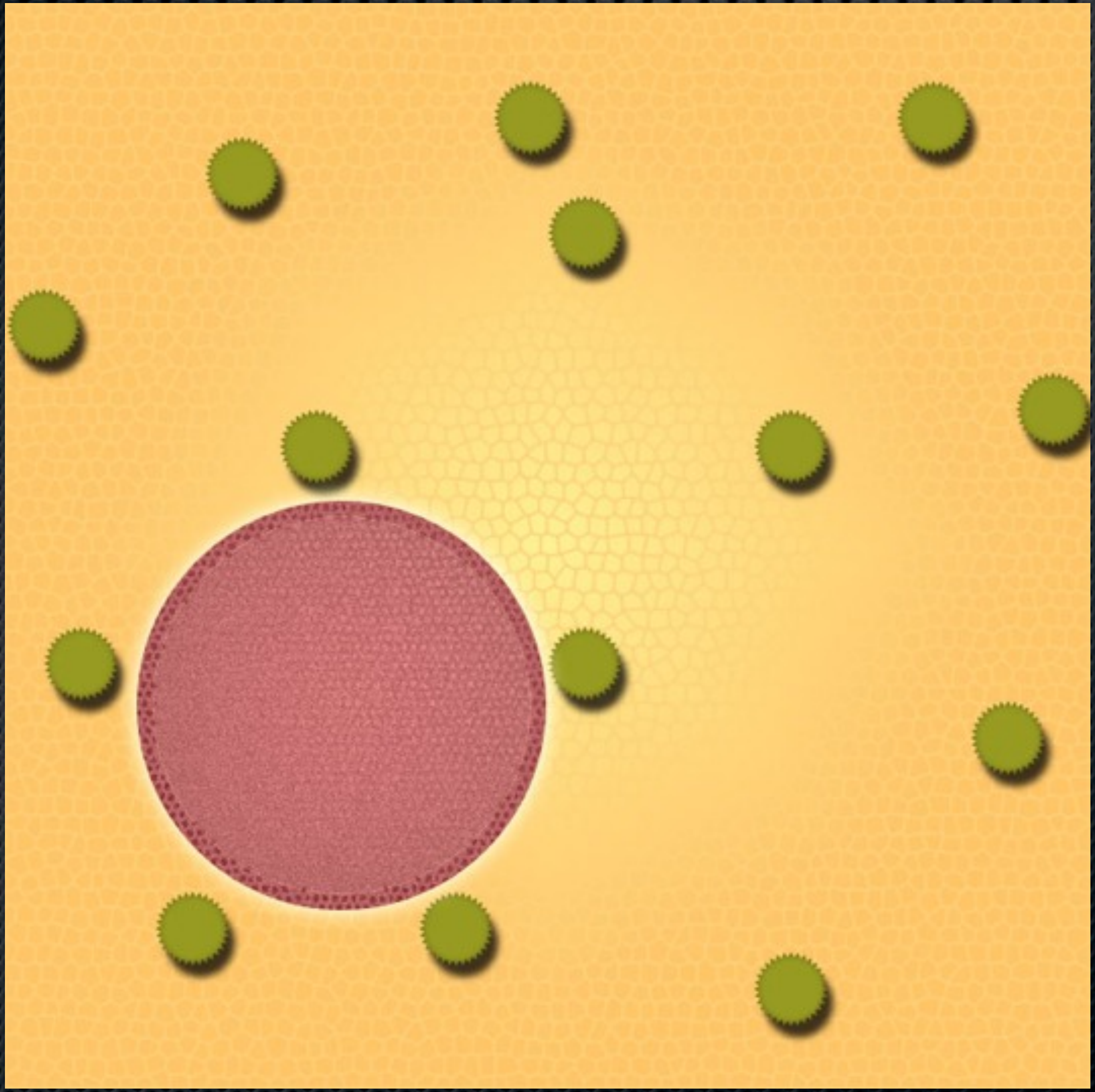
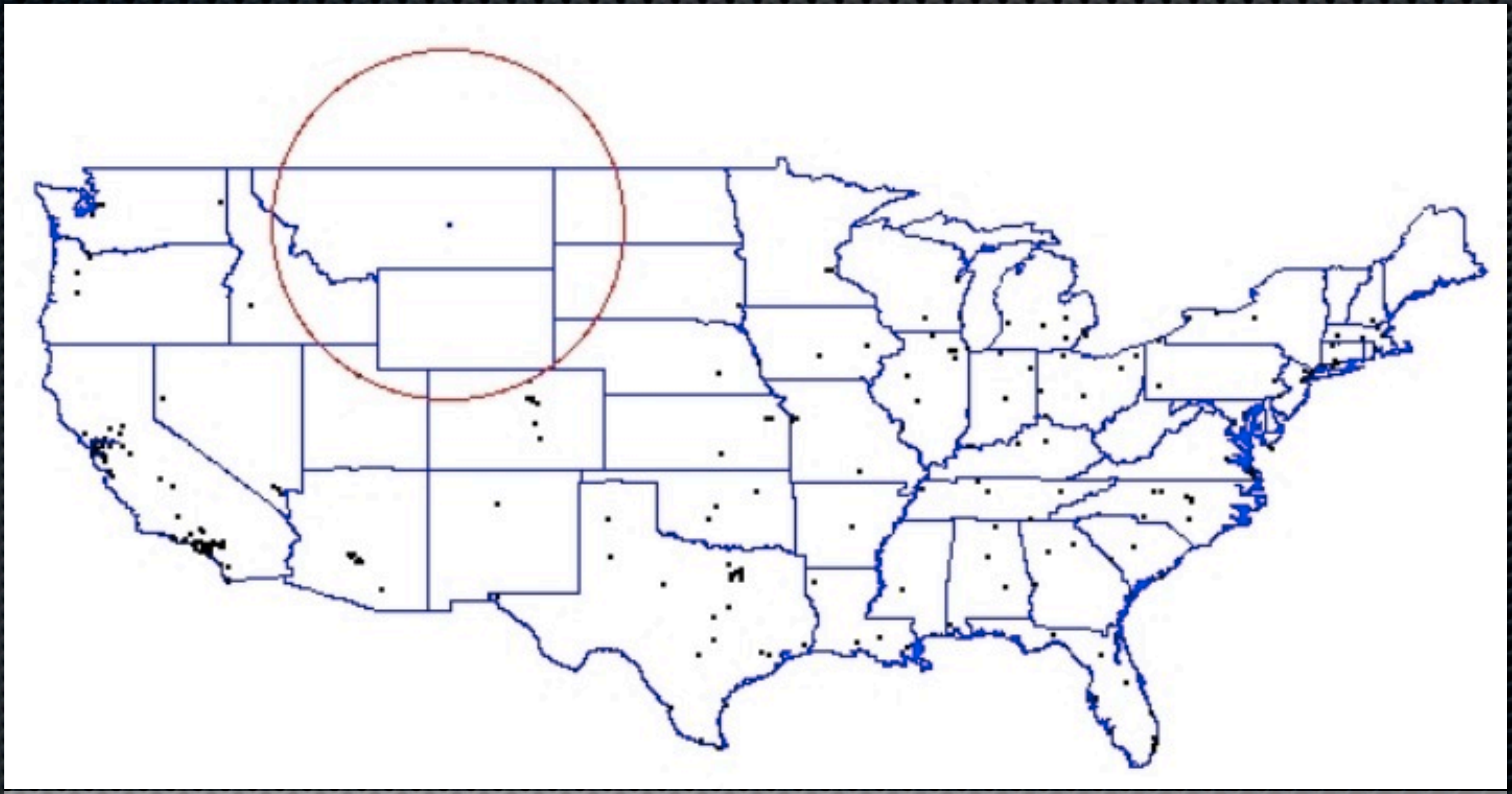


Toxic Waste Dump problem

- “What to do with toxic waste?”
- cities with more than 100 000 citizens
- 251 cities in USA

Credits: http://cz.123rf.com/

Toxic Waste Dump problem



Credits: Megan Schuster

Locations for new stores

- Points are current stores
- Best location = center of largest circle

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Credits: www.mcdonalds.com

Facts

- Largest Empty Circle = LEC
- LEC center is in the convex hull of set **P**
  - else center in infinity

Facts

- LEC is centered in Voronoi vertex
- or in the intersection between Voronoi vertex and convex hull

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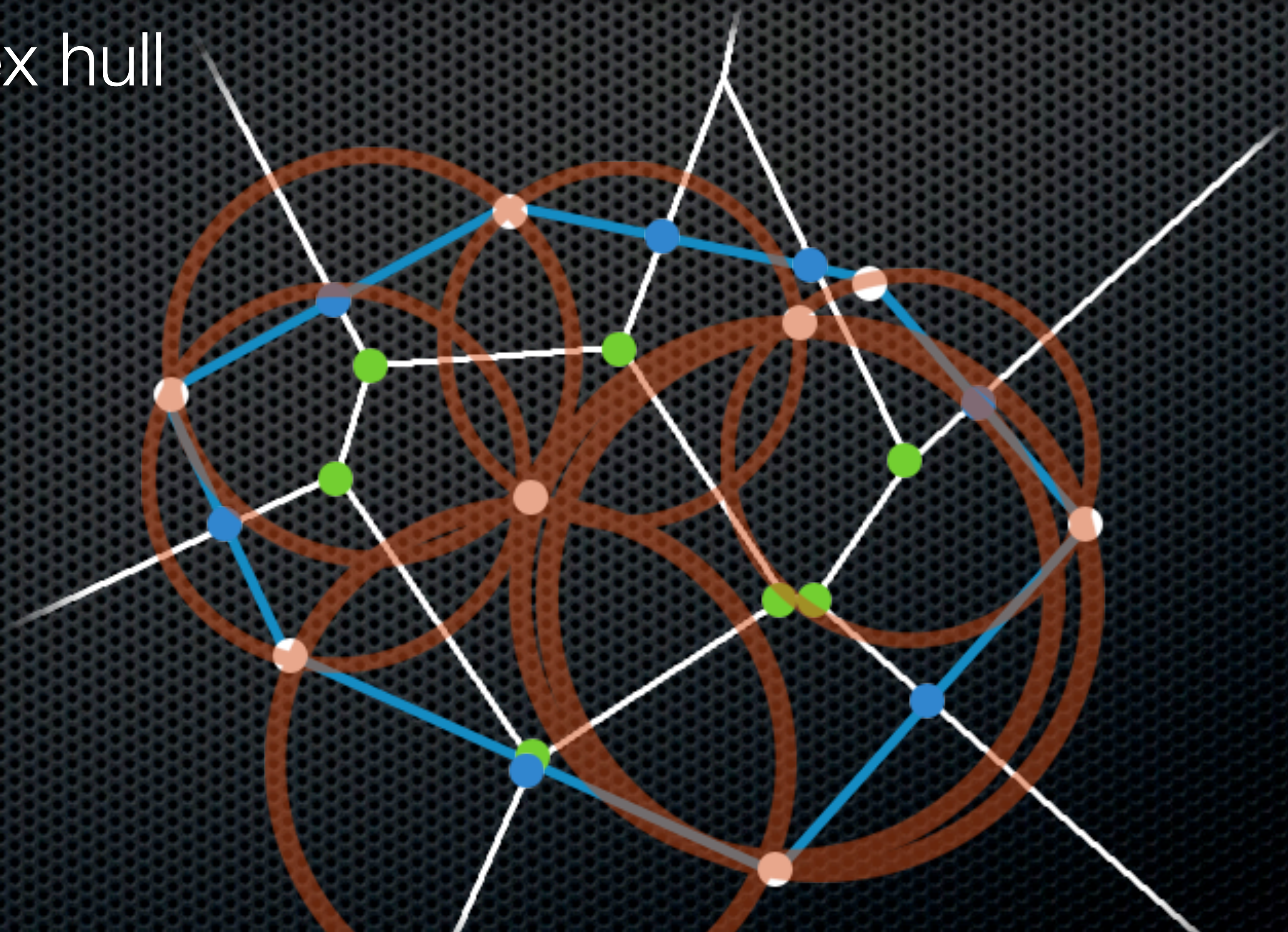

Facts

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- or in the intersection between Voronoi vertex and convex hull

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- Voronoi diagram is dual to Delaunay triangulation
  - every Voronoi vertex == Delaunay triangle
  - neighbors triangles are in Voronoi connected by edge

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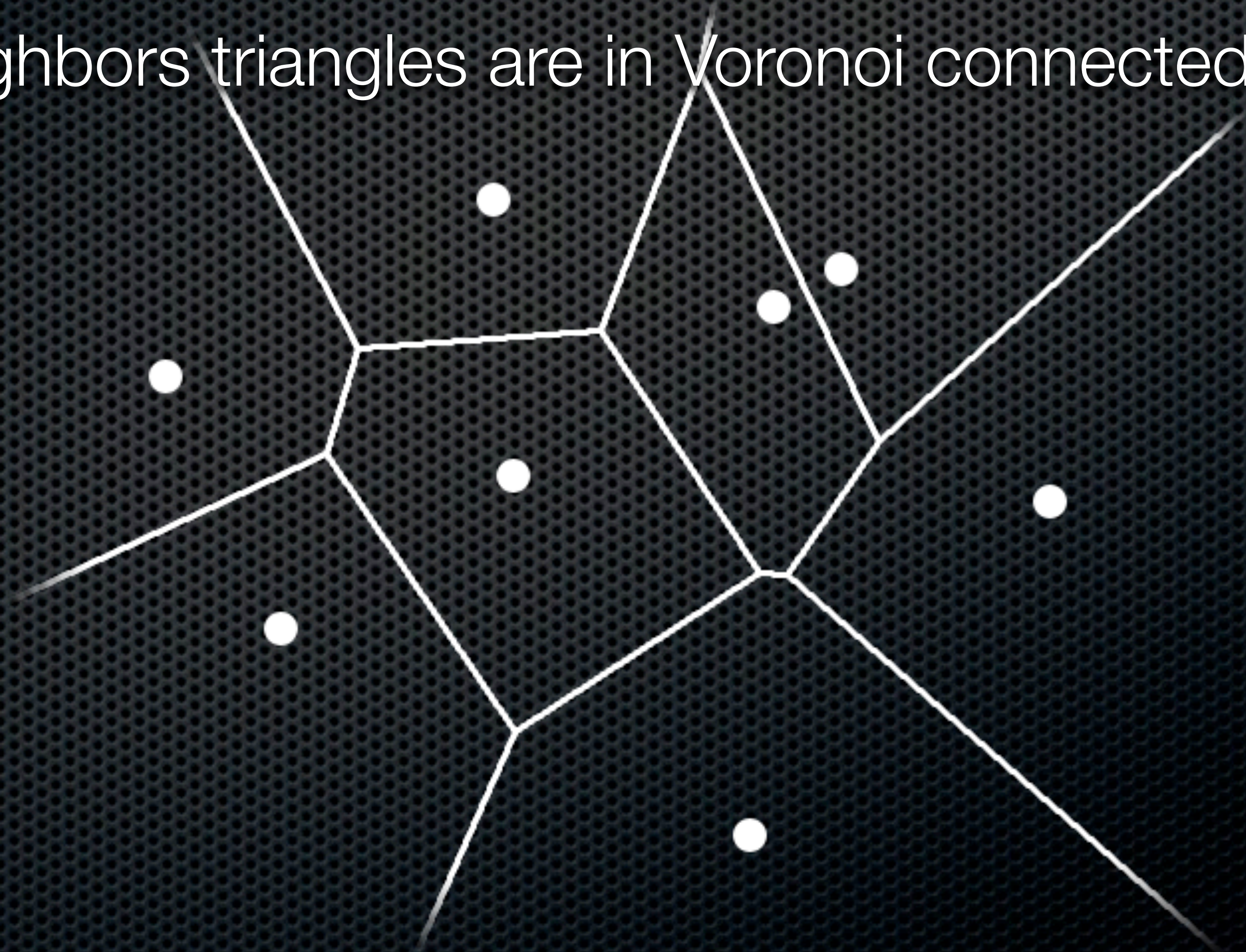

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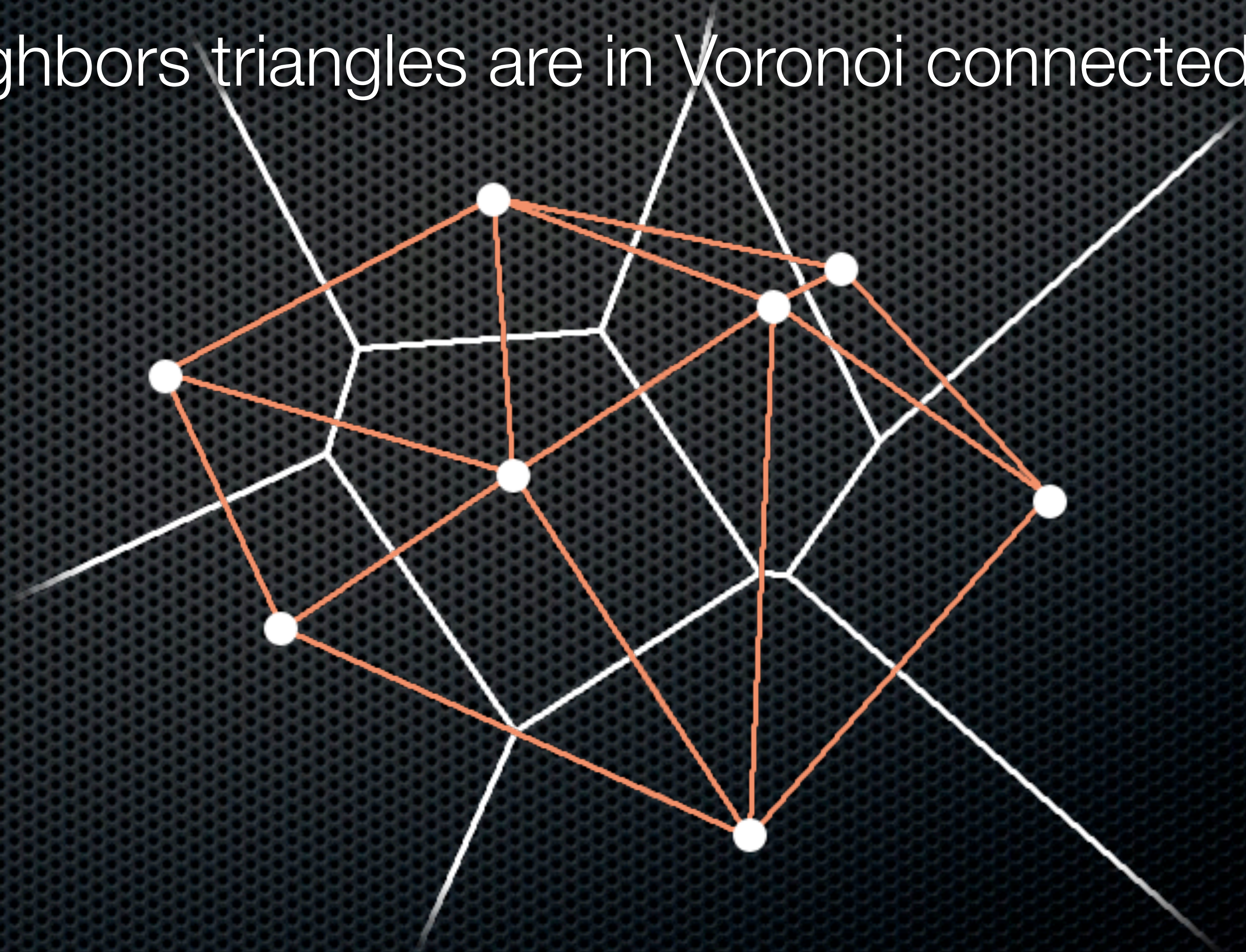


Voronoi Diagram

- Georgy Voronoi (1908)
- proximity information
- complexity: **O(n log n)**

Delaunay triangulation

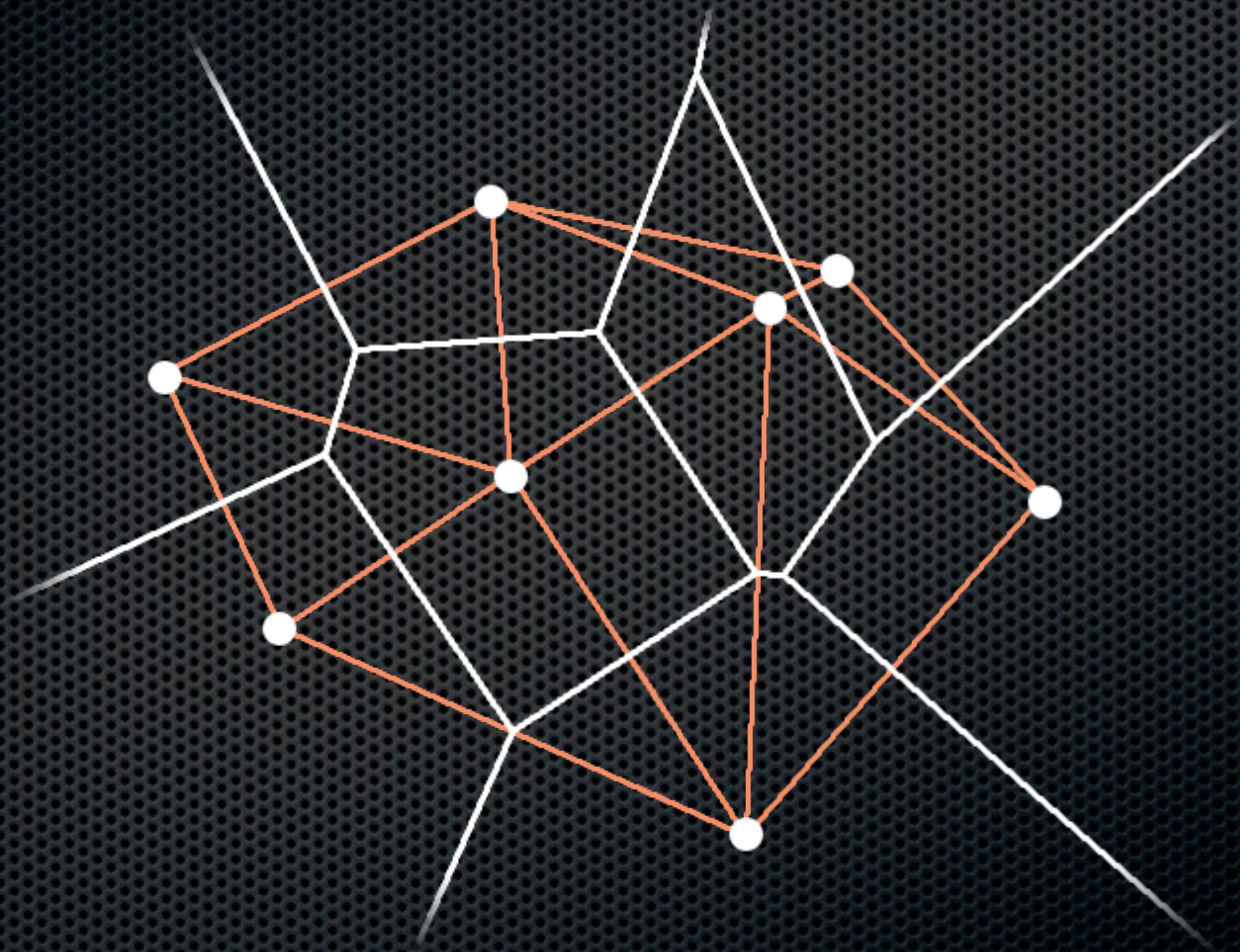
- Boris Delaunay
- 1934
- no skinny triangles
- no point in set **P** is in circumcircle of any triangle

Credits: http://feeldesign.com

Delaunay triangulation

Delaunay triangulation

Delaunay triangulation



Delaunay triangulation

Algorithm

- Delaunay triangulation
- Voronoi diagram
- Convex hull
- Check interior nodes
- Find intersection between Voronoi edges and convex hull
- Find the LEC

Delaunay Triangulation

- incremental algorithm
- maximalize minimal angle
- represented by acyclic graph

Delaunay Triangulation algorithm:

- start with enclosing triangle
- adding points one by one
- locate triangle **T** which contain point **r** (the new one)
- draw edges between **r** and points in **T**

Delaunay Triangulation example:

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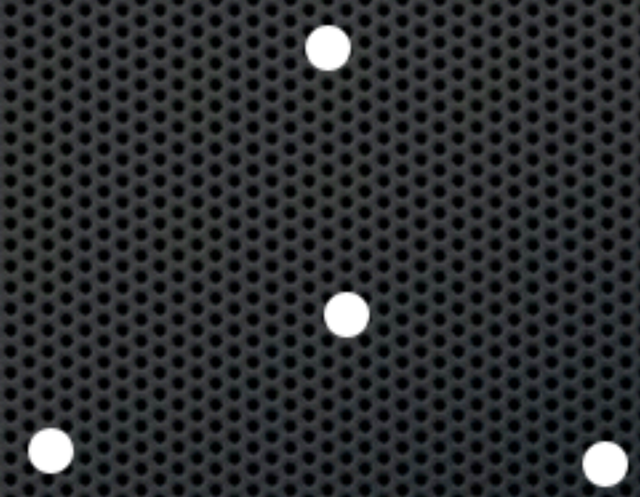

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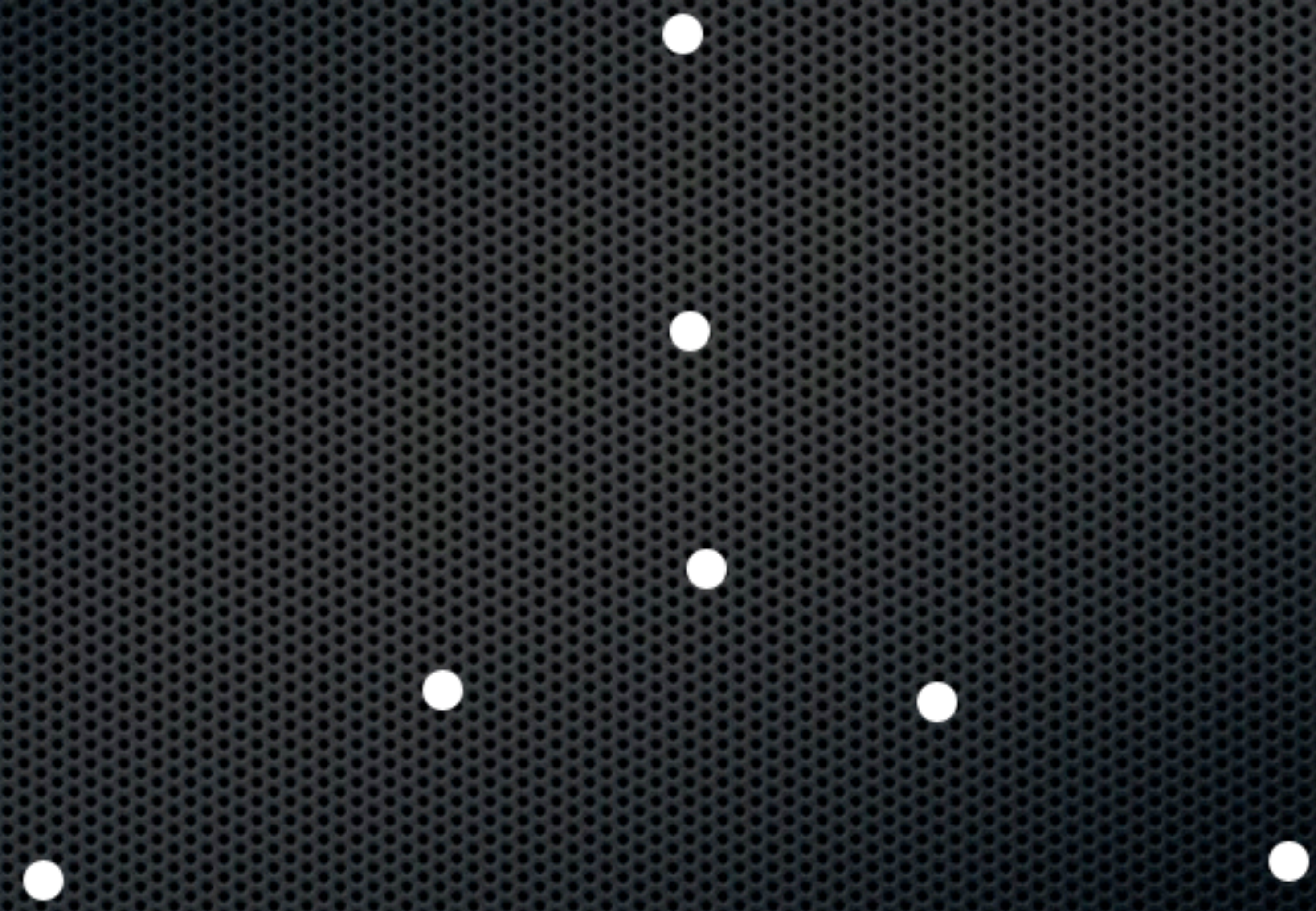

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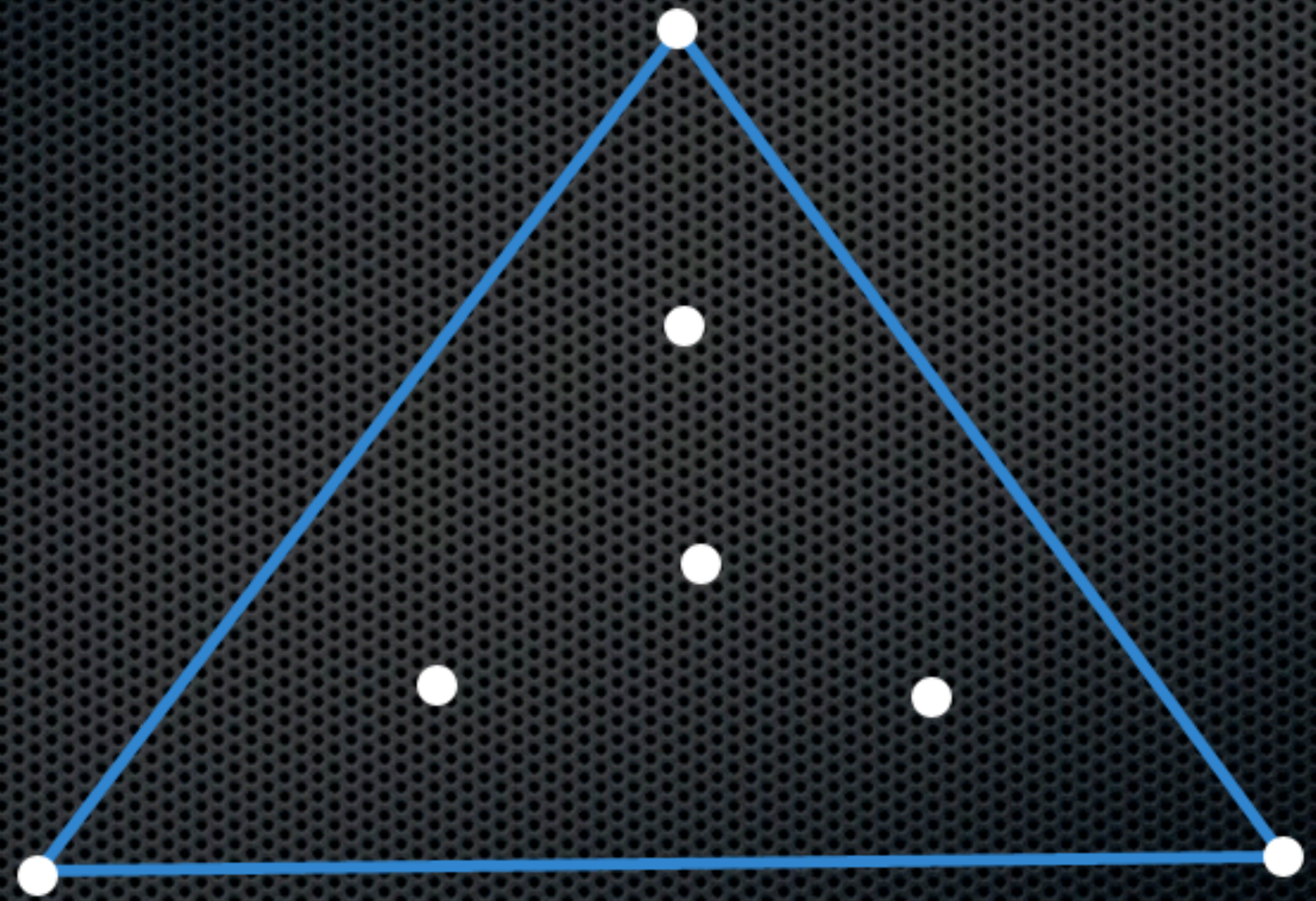

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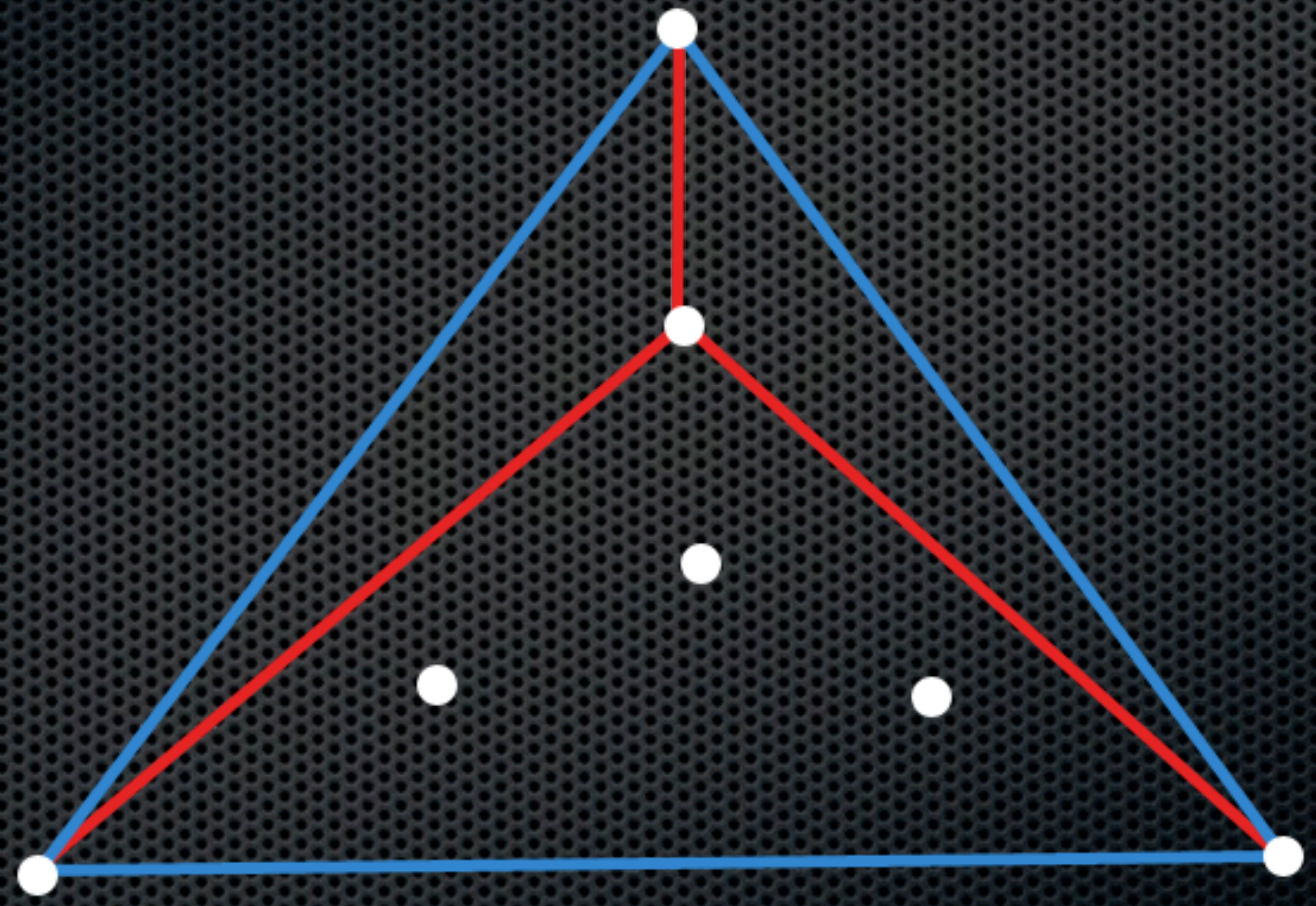

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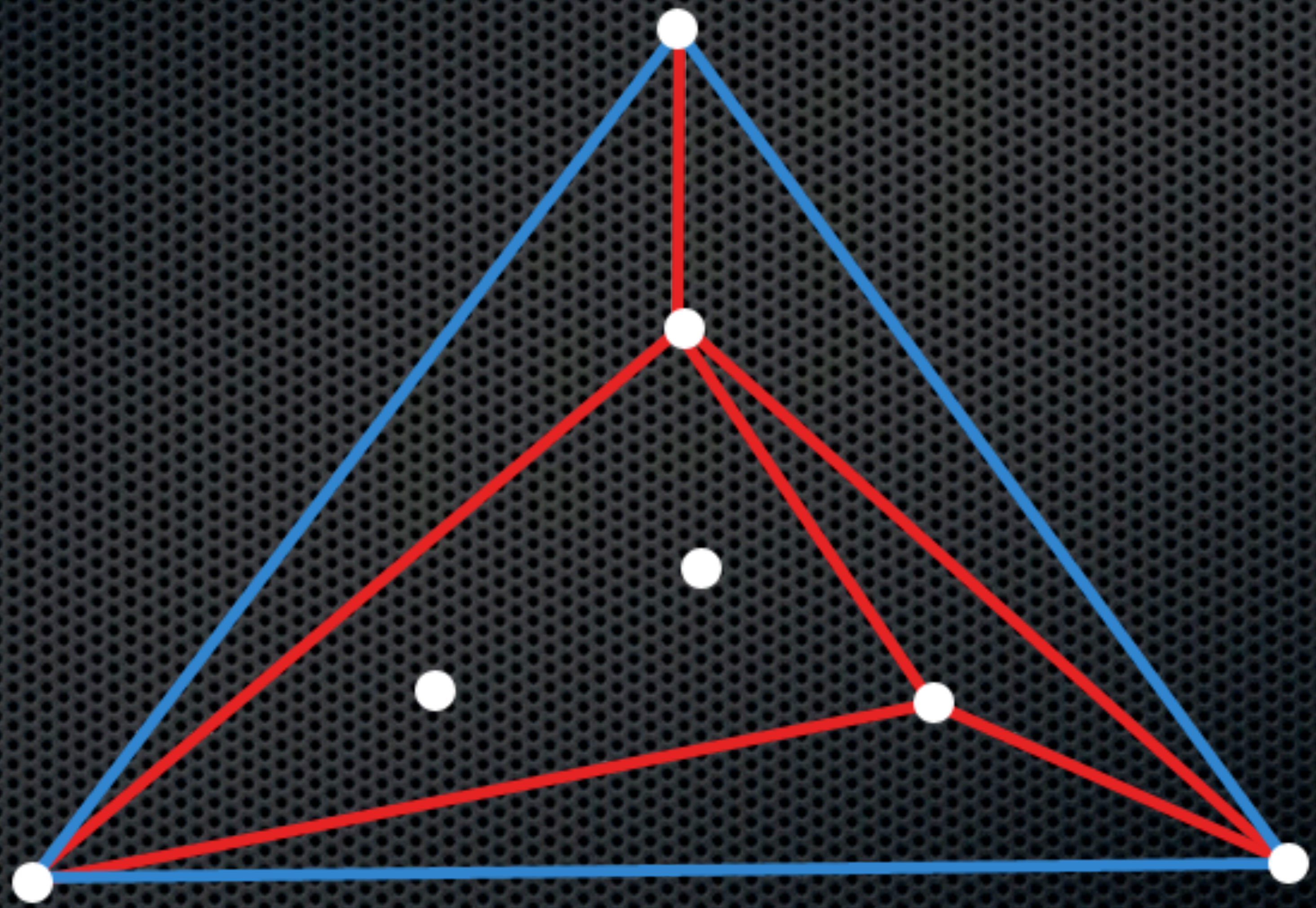

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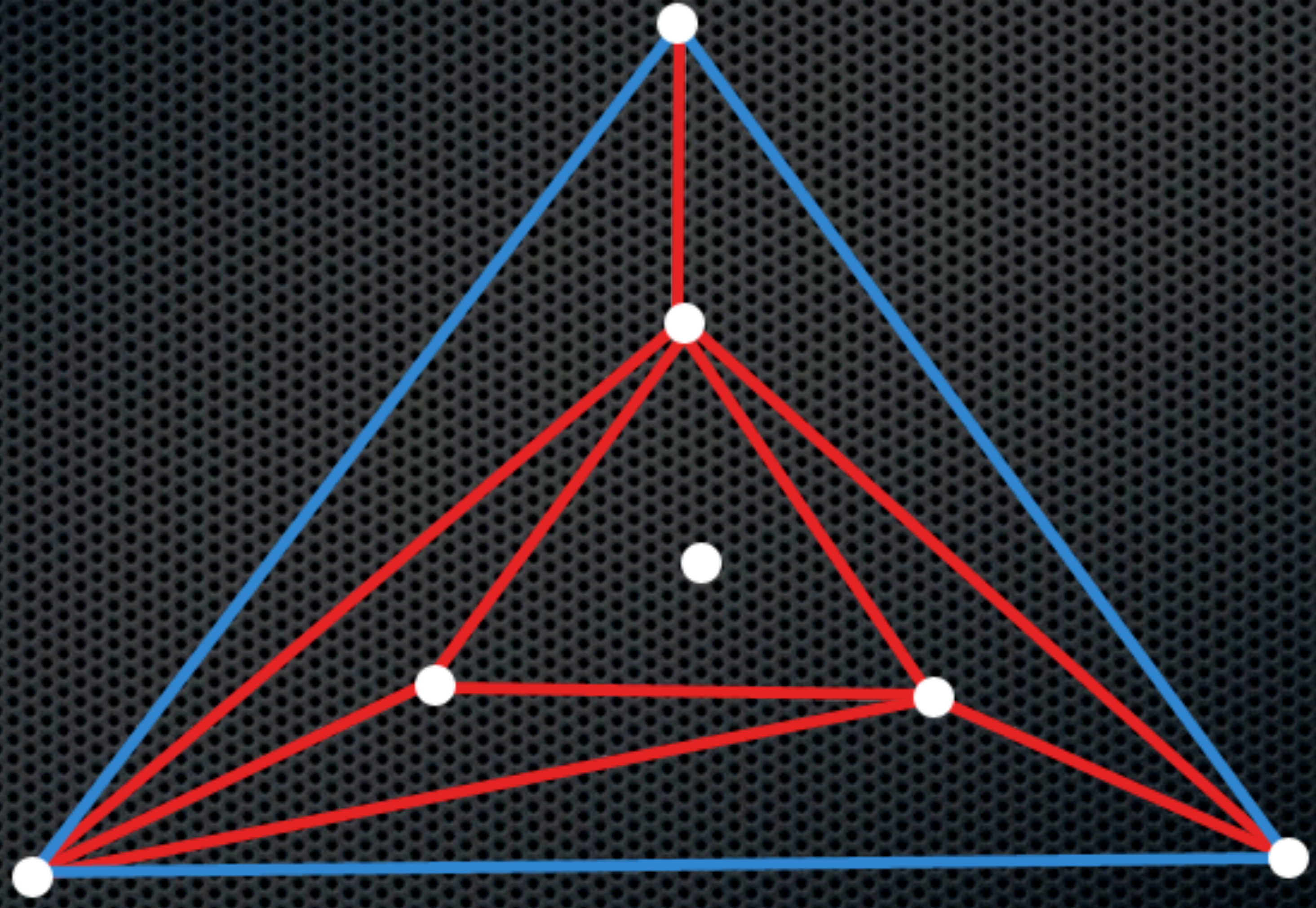

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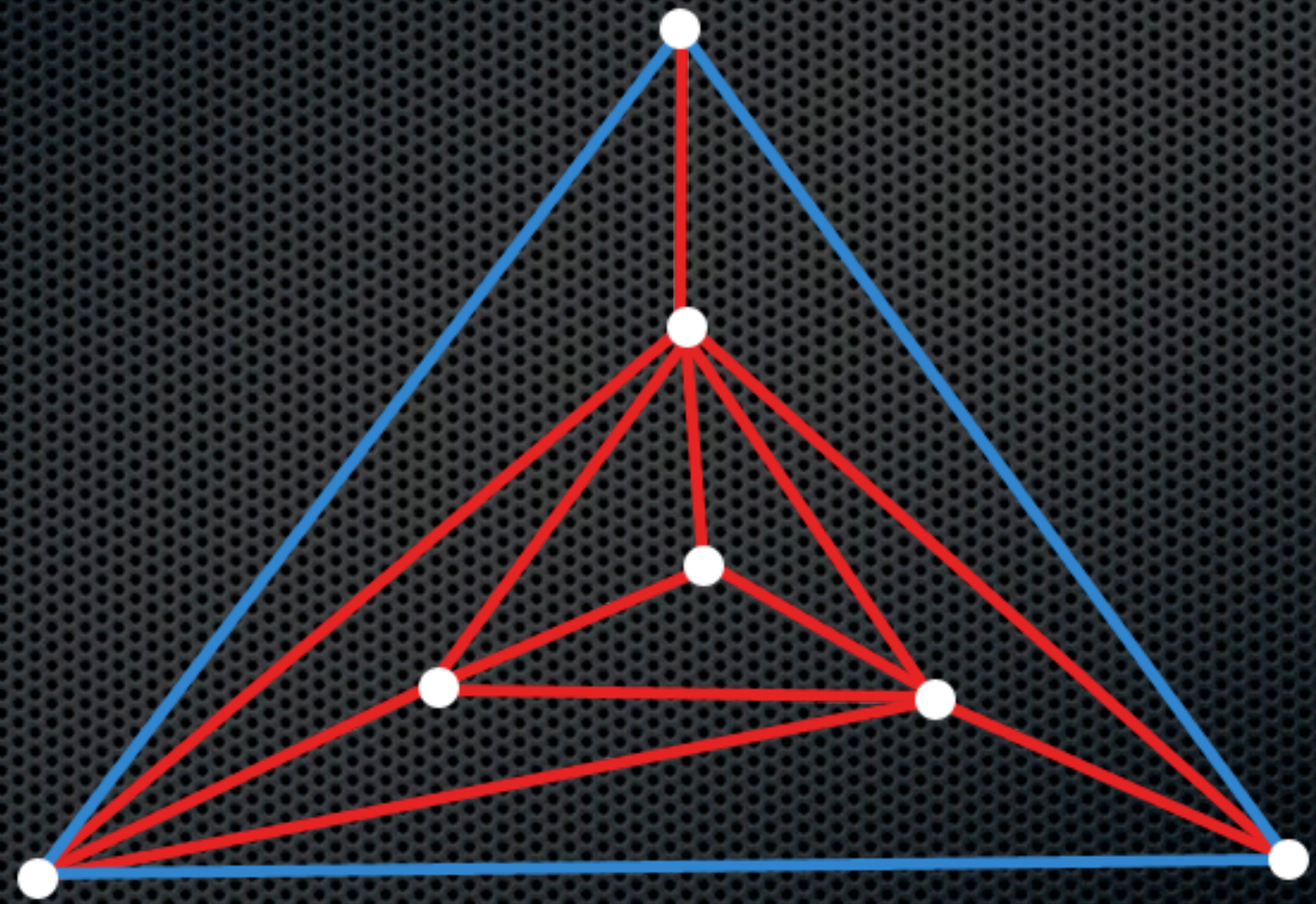

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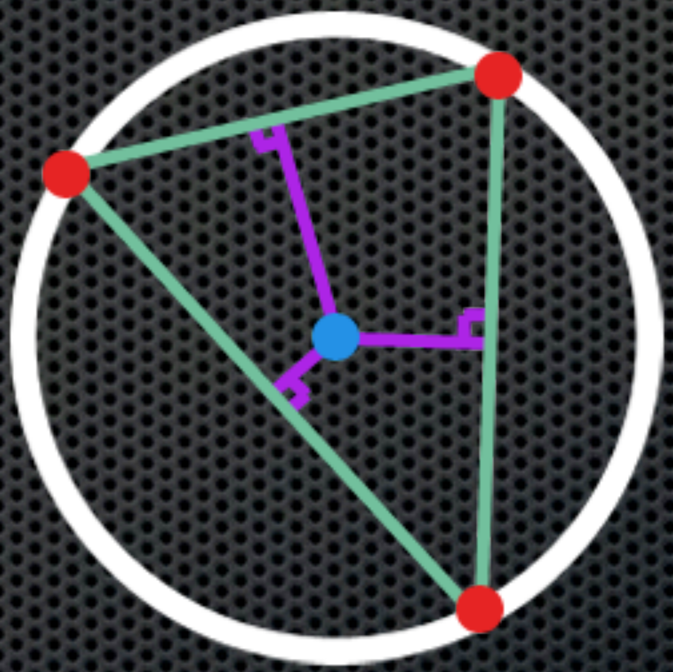

Credits: Megan Schuster

Delaunay Triangulation -> Voronoi diagram

- iterate over all triangles **O(n)**
- compute Voronoi vertex (centers of triangles) **O(1)**
- connect Voronoi vertex (3 neighbor triangles) **O(1)**

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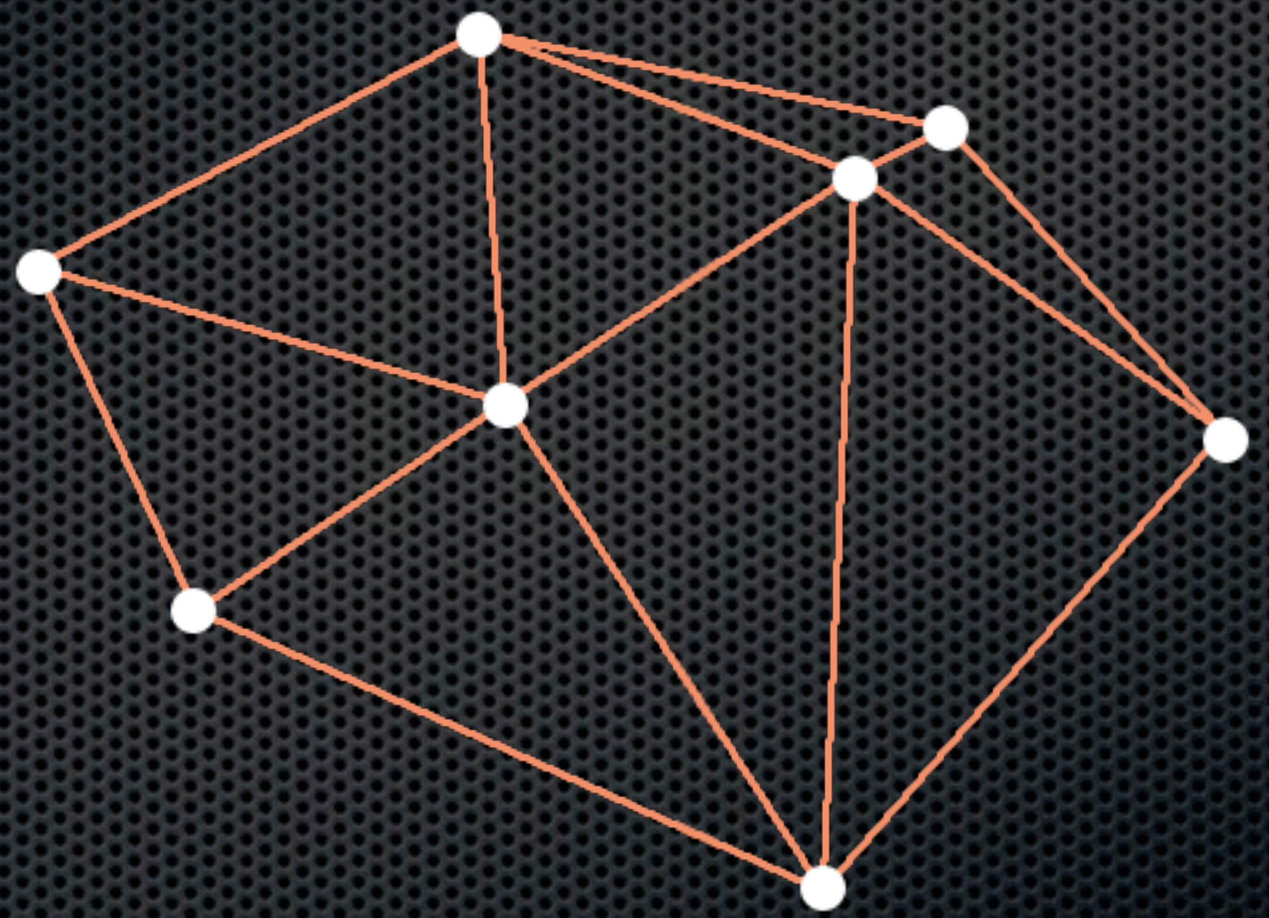

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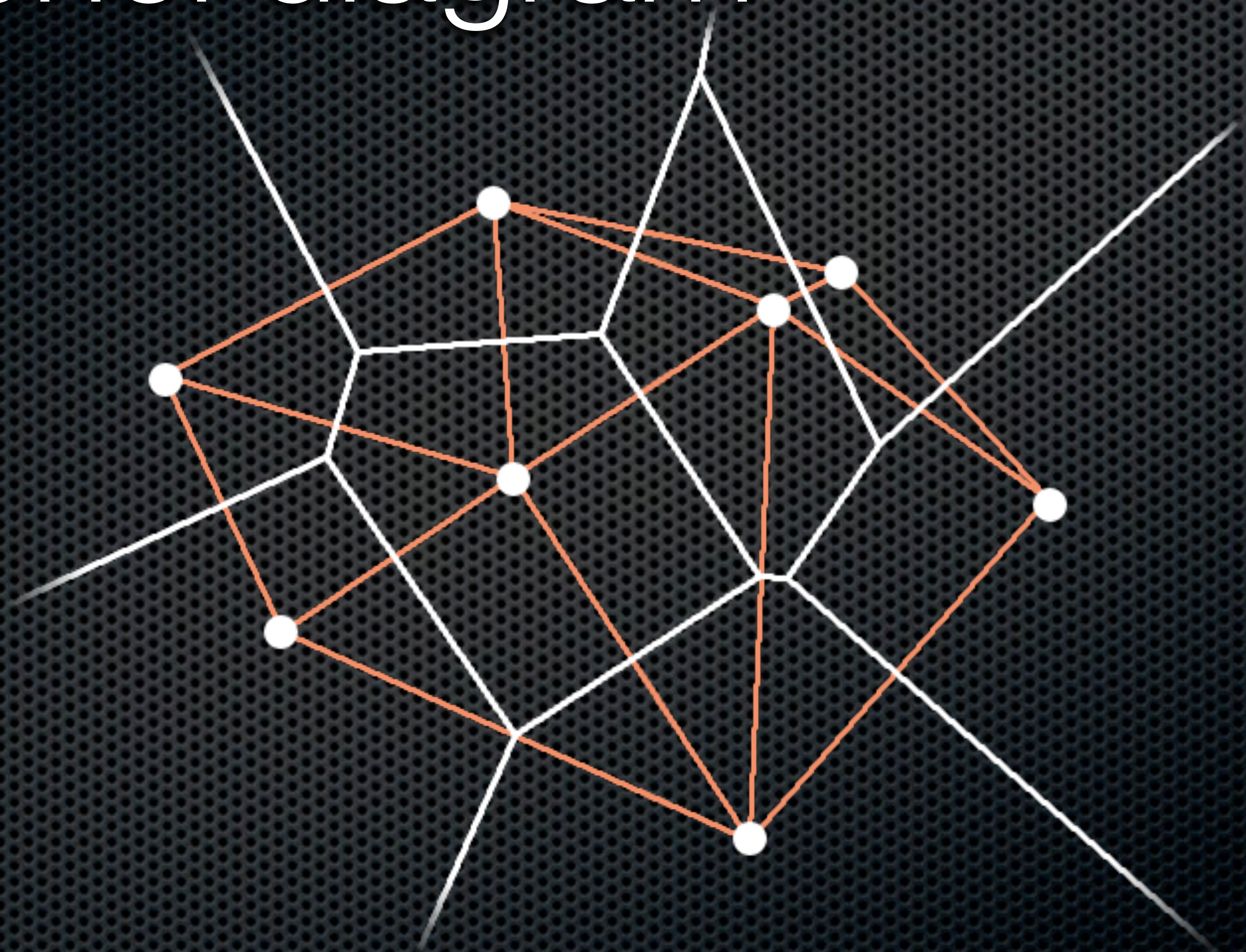

Convex Hull

- Jarvis March
- complexity **O(nh)**

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- complexity **O(nh)**

Interior Nodes

- **for all** Voronoi vertices
  - **for** convex hull points //CCW
    - **if**(Voronoi verices is right) **return false** //exterior
- **return true** //interior

- complexity: **O(nh)**

Interior Nodes

Interior Nodes

Finding Convex hull and Voronoi edge intersection

- **for all** Voronoi edges (two points)
  - **if**((first_point==interior)&&(second_point==exterior))
    - find_intersection()

- complexity: **O(nh)**

Finding LEC

- draw circles and find the biggest
- **O(n)**

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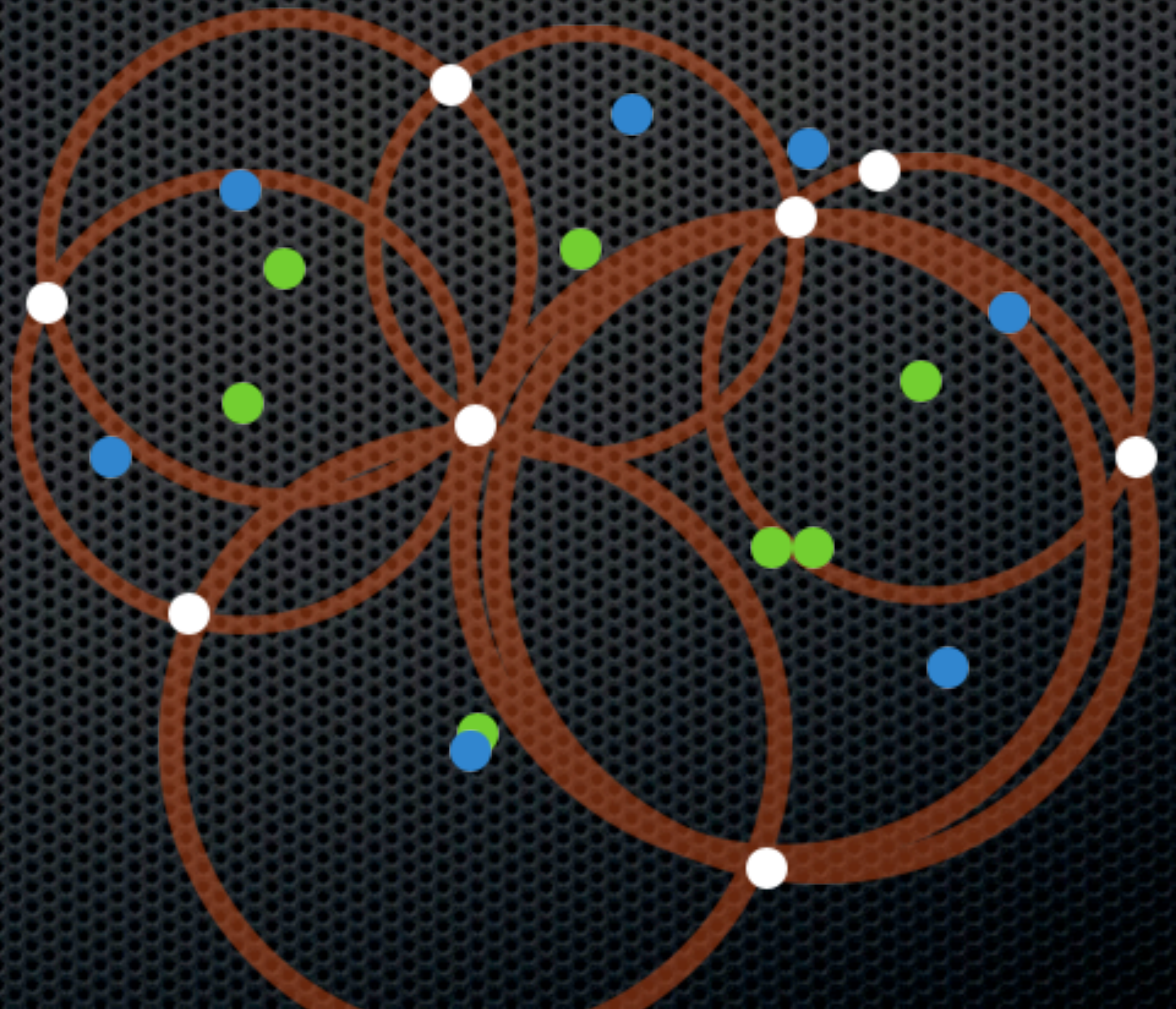

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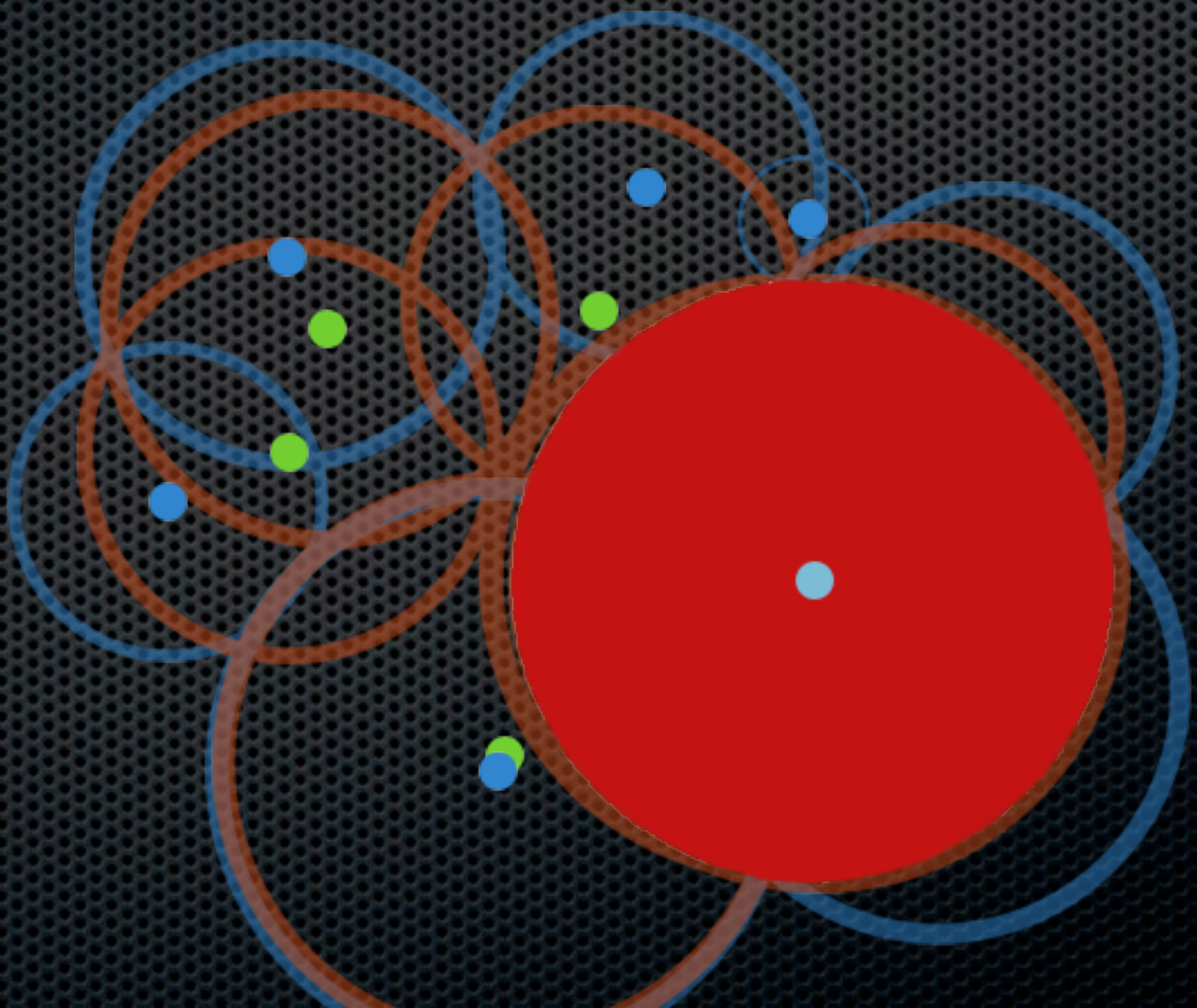


Brief history about LEC

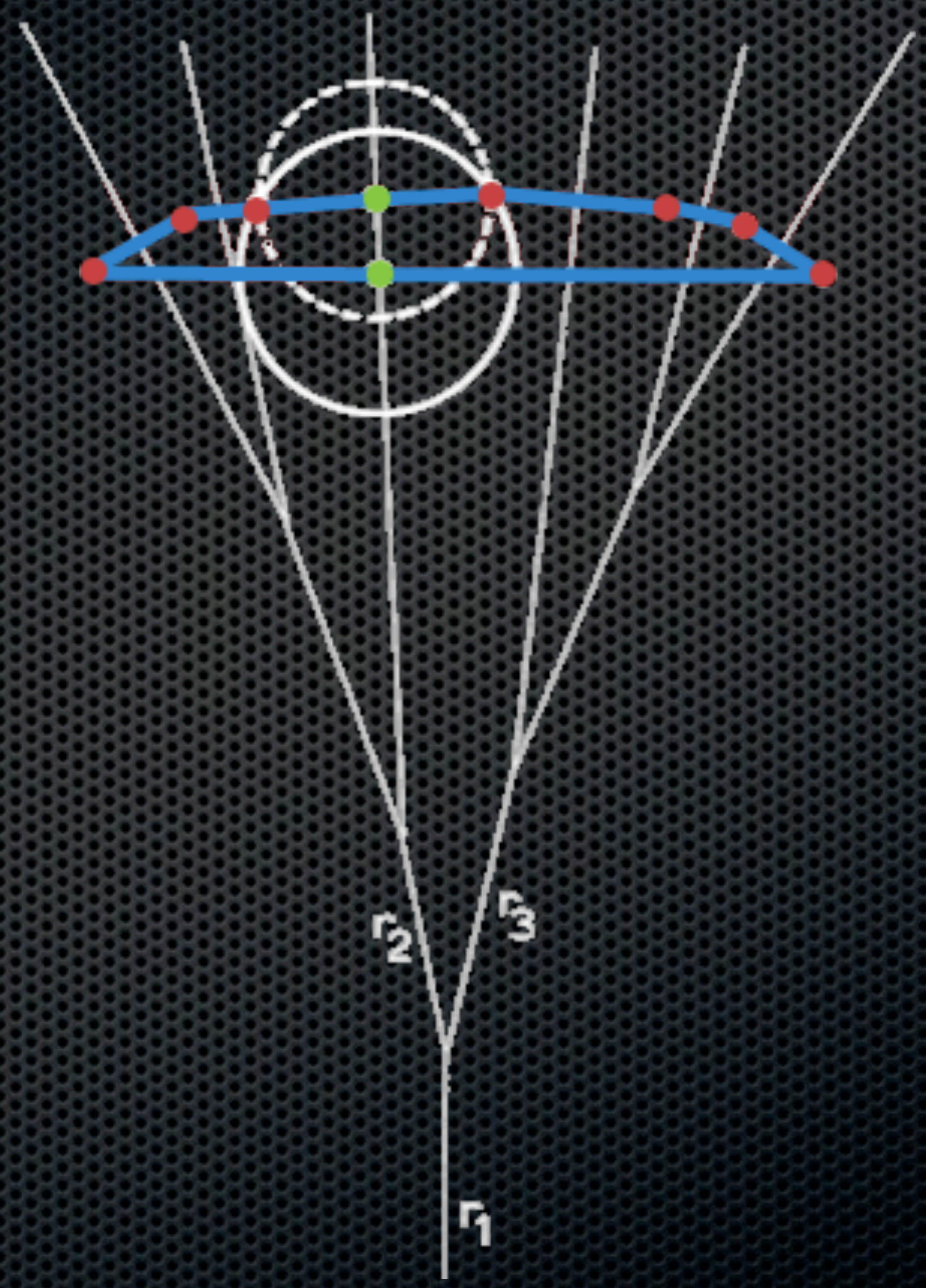
Dasarathy and White

- first attempt (1975)
- worst case complexity: **O (n^3)**
- imperfect analysis of the problem

Michael Shamos

- PhD thesis (1978)
- complexity: **O(n)**
- wrong assumption:
  - every convex hull edge is intersect at most by two Voronoi edges

Example



Credits: Toussaint

Godfried Toussaint

- present correct algorithm (1983)
- complexity: **O(n log h)**

Algorithm

- compute LEC for each interior Voronoi vertex **O(n log h)**
  - **O(n)** Voronoi vertices
  - **O(log h)** check if is interior (h = edge in convex hull)
  - **O(1)** computing LEC
- compute all intersection between Voronoi edges and convex hull **O(n log h)**
  - **O(n)** Voronoi edges
  - **O(log h)** Chazelle (1980) - intersection between line segment and convex n-gon

Preparata & Shamos

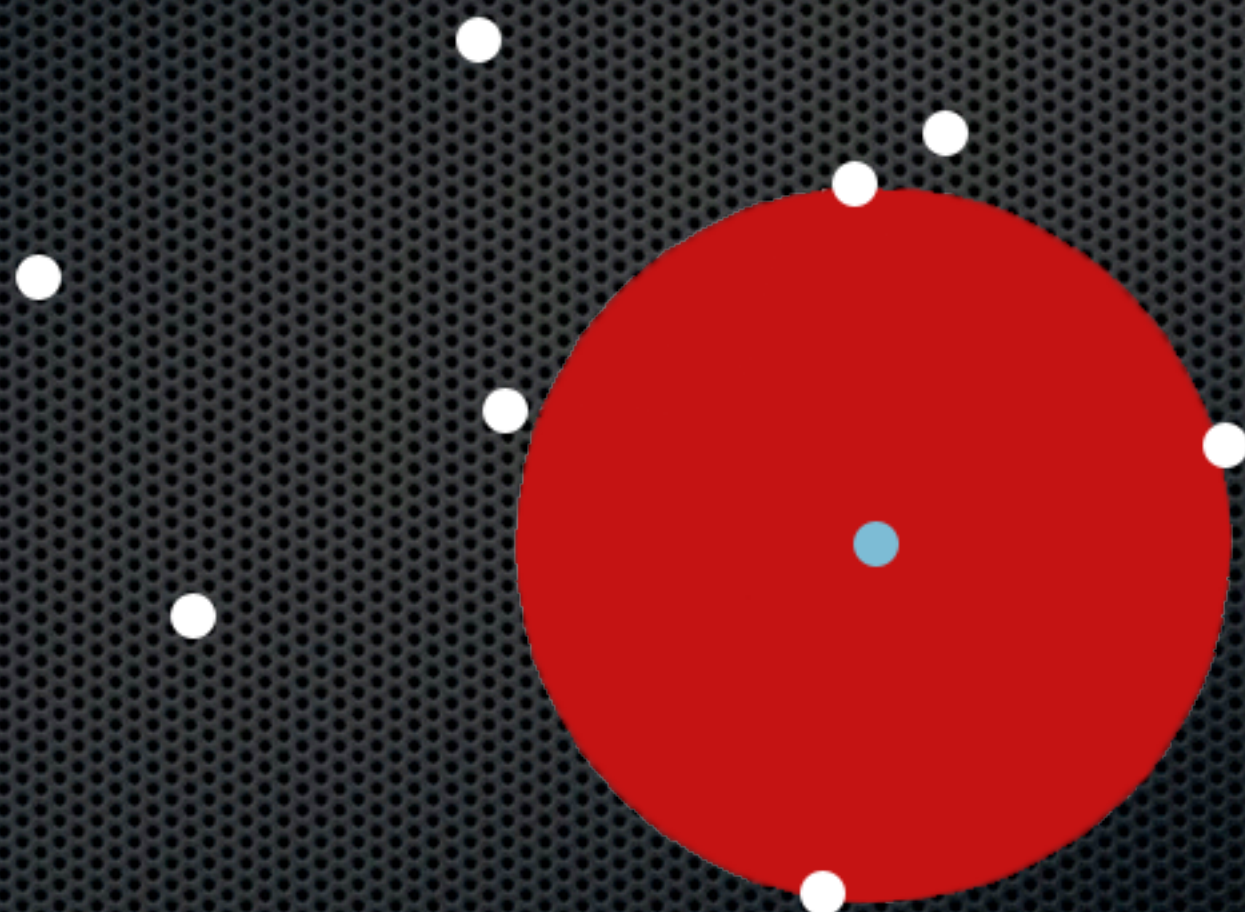
 

- slightly improvement (1985)
- marching method **O(n)**
  - finding intersection between Voronoi edge and convex hull
- all this algorithms need **O(n log n)** for Voronoi diagram

Reference

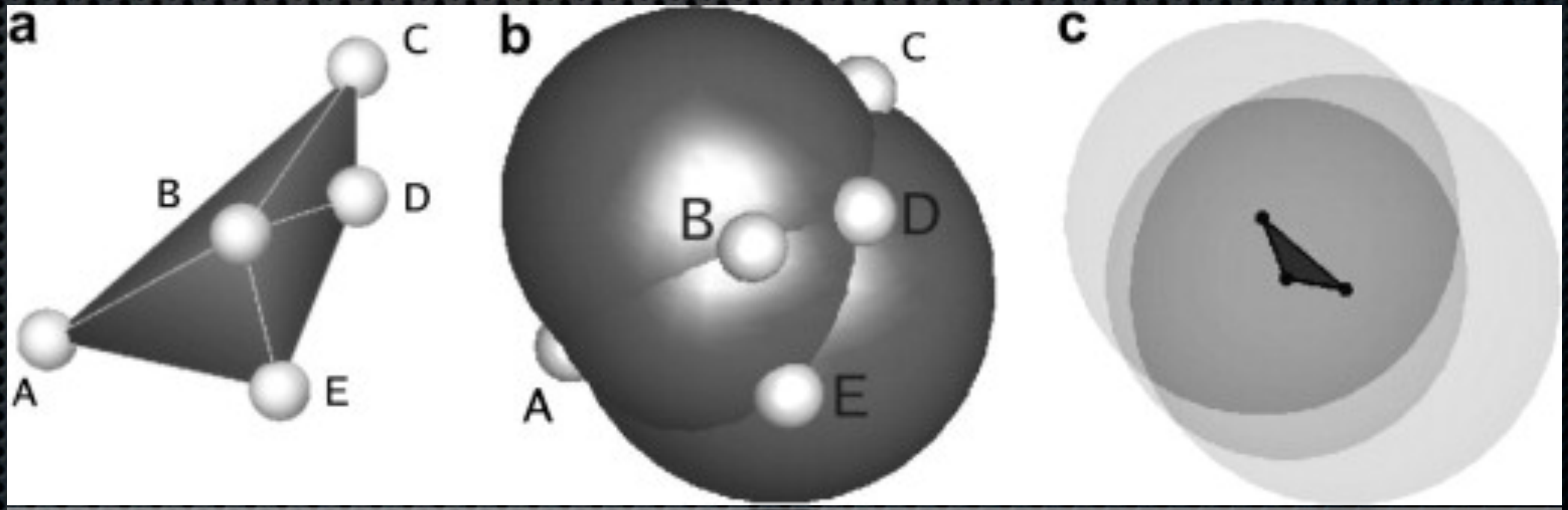
- Preparata: *Computational Geometry: An Introduction*
- Schuster: *The Largest Empty Circle Problem*
- Toussaint: *Computing Largest Empty Circles with Locations Constrains*
- Shamos: *Computional Geometry*
- M. de Berg: *Computational Geometry: Algorithms and Applications*
- Dasarathy, White: *A Maxmin Location Problem*

Questions & Comments



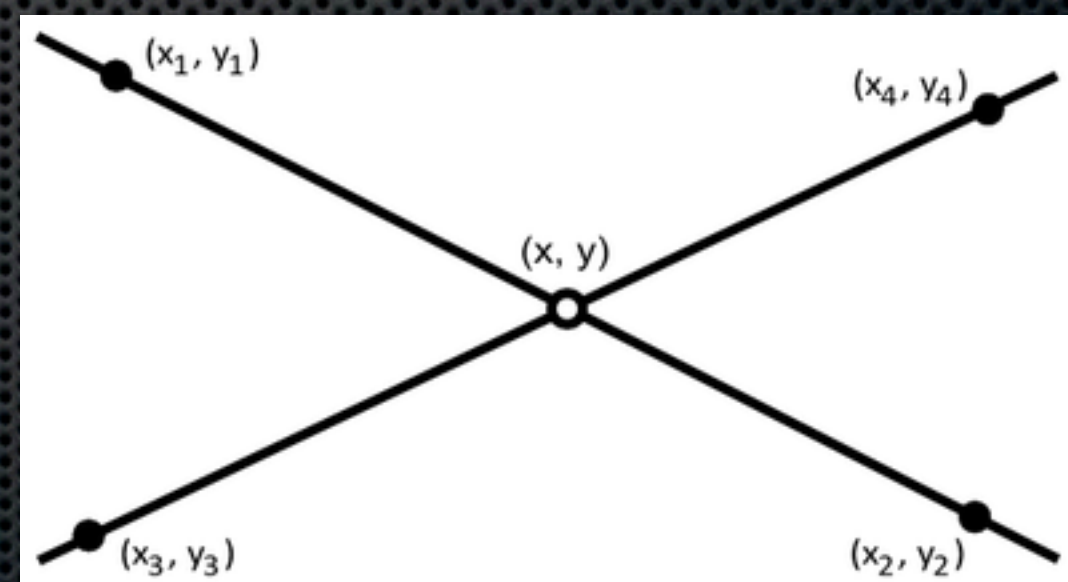
Is it possible in 3D?

- **YES!**
- Largest Empty Sphere problem
- it is possible in d-dimension



Two lines intersection

$$(P_x, P_y) = \left(\frac{(x_1y_2 - y_1x_2)(x_3 - x_4) - (x_1 - x_2)(x_3y_4 - y_3x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)}, \right.$$
$$\left. \frac{(x_1y_2 - y_1x_2)(y_3 - y_4) - (y_1 - y_2)(x_3y_4 - y_3x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)} \right)$$



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