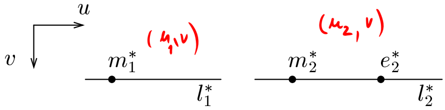
▶Rectification Homographies

Cameras $(\mathbf{P}_1, \mathbf{P}_2)$ are rectified by a homography pair $(\mathbf{H}_1, \mathbf{H}_2)$:

$$\mathbf{P}_i^* = \mathbf{H}_i \mathbf{P}_i = \mathbf{H}_i \mathbf{K}_i \mathbf{R}_i \begin{bmatrix} \mathbf{I} & -\mathbf{C}_i \end{bmatrix}, \quad i = 1, 2$$

rectified entities: $\mathbf{F}^*$, $\underline{\mathbf{l}}_2^*$, $\underline{\mathbf{l}}_1^*$, etc:



corresponding epipolar lines must be:

1. parallel to image rows $\Rightarrow$ epipoles become $e_1^* = e_2^* = (1, 0, 0)$
2. equivalent $l_2^* = l_1^* \;\Rightarrow\; \underline{\mathbf{l}}_2^* \simeq \underline{\mathbf{l}}_1^* \simeq \underline{\mathbf{e}}_1^* \times \underline{\mathbf{m}}_1 = [\underline{\mathbf{e}}_1^*]_\times \underline{\mathbf{m}}_1 = \mathbf{F}^* \underline{\mathbf{m}}_1$

both conditions together give the rectified fundamental matrix

$$\mathbf{F}^* \simeq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$u_1 = u_2$

A two-step rectification procedure

1. Find some pair of <u>primitive rectification homographies</u> $\hat{\mathbf{H}}_1$, $\hat{\mathbf{H}}_2$
2. Upgrade them to a pair of <u>optimal rectification homographies</u> from the class preserving $\mathbf{F}^*$.

▶Geometric Interpretation of Linear Rectification

What pair of physical cameras is compatible with $\mathbf{F}^*$?

- we know that $\mathbf{F} = (\mathbf{Q}_1 \mathbf{Q}_2^{-1})^\top [\underline{\mathbf{e}}_1]_\times$ Slide 77
- we choose $\mathbf{Q}_1^* = \mathbf{K}_1^*$, $\mathbf{Q}_2^* = \mathbf{K}_2^* \mathbf{R}^*$; then

$$(\mathbf{Q}_1^* \mathbf{Q}_2^{*-1})^\top [\underline{\mathbf{e}}_1^*]_\times = (\mathbf{K}_1^* \mathbf{R}^{*\top} \mathbf{K}_2^{*-1})^\top \mathbf{F}^*$$

- we look for $\mathbf{R}^*$, $\mathbf{K}_1^*$, $\mathbf{K}_2^*$ compatible with

$$(\mathbf{K}_1^* \mathbf{R}^{*\top} \mathbf{K}_2^{*-1})^\top \mathbf{F}^* = \lambda \mathbf{F}^*, \qquad \mathbf{R}^* \mathbf{R}^{*\top} = \mathbf{I}, \qquad \mathbf{K}_1^*, \mathbf{K}_2^* \text{ upper triangular}$$

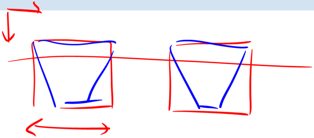
- we also want $\mathbf{b}^*$ from $\underline{\mathbf{e}}_1^* \simeq \mathbf{P}_1^* \underline{\mathbf{C}}_2^* = \mathbf{K}_1^* \mathbf{b}^*$ $\mathbf{b}^*$ in cam. 1 frame
- result:

$$\mathbf{R}^* = \mathbf{I}, \quad \mathbf{b}^* = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{K}_1^* = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K}_2^* = \begin{bmatrix} k_{21} & k_{22} & k_{23} \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (29)$$

- rectified cameras are in canonical position with respect to each other

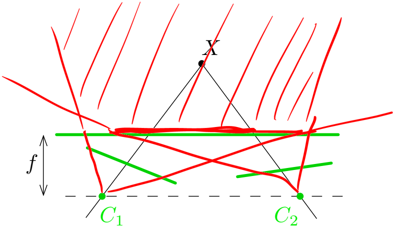
  not rotated, canonical baseline
- rectified calibration matrices can differ in the first row only
- when $\mathbf{K}_1^* = \mathbf{K}_2^*$ then the rectified pair is called the standard stereo pair and the homographies standard rectification homographies

▶cont'd



- rectification is a homography (per image)
  - ⇒ rectified camera centers are equal to the original ones
- standard rectified cameras are in canonical orientation
  - ⇒ rectified image projection planes are coplanar
- standard rectification guarantees equal rectified calibration matrices
  - ⇒ rectified image projection planes are equal

standard rectification homographies reproject onto a common image plane parallel to the base-line



Corollary

- the standard rectified stereo pair has vanishing disparity for 3D points at infinity
  - but known $\mathbf{F}$ alone does not give any constraints to obtain standard rectification homographies
  - for that we need either of these:
    1. projection matrices, or
    2. calibrated cameras, or
    3. a few points at infinity calibrating k_{1i}, k_{2i}, $i = 1, 2, 3$ in (29)

▶Primitive Rectification

Goal: Given fundamental matrix $\mathbf{F}$, derive some simple rectification homographies $\mathbf{H}_1$, $\mathbf{H}_2$

1. Let the SVD of $\mathbf{F}$ be $\mathbf{U}\mathbf{D}\mathbf{V}^\top = \mathbf{F}$, where $\mathbf{D} = \mathrm{diag}(1,\, d^2,\, 0)$, $\quad 1 \geq d^2 > 0$
2. decompose $\mathbf{D} = \mathbf{A}^\top \mathbf{F}^* \mathbf{B}$, where ($\mathbf{F}^*$ is given → Slide 151)

$\underline{m}_2^\top F \underline{m}_1 = 0$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & d & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -d & 0 \end{bmatrix}$$

$m_2'^\top \underbrace{\hat{H}_2^\top F^* \hat{H}_1}\, m_1' = 0$

$F^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

3. then

given F find $\hat{H}_1, \hat{H}_2$

$$\mathbf{F} = \mathbf{U}\mathbf{D}\mathbf{V}^\top = \underbrace{\mathbf{U}\mathbf{A}^\top}_{\hat{\mathbf{H}}_2^\top} \overbrace{\mathbf{F}^*}^{D} \underbrace{\mathbf{B}\mathbf{V}^\top}_{\hat{\mathbf{H}}_1}$$

and the primitive rectification homographies are

$$\hat{\mathbf{H}}_2 = \mathbf{A}\mathbf{U}^\top, \qquad \hat{\mathbf{H}}_1 = \mathbf{B}\mathbf{V}^\top$$

⊛ P1; 1pt: derive some $\mathbf{A}$, $\mathbf{B}$ from the admissible class

- rectification homographies do exist
- there are other primitive rectification homographies, these suggested are just simple to obtain

▶Primitive Rectification Suffices for Calibrated Cameras

Obs: calibrated cameras: $d = 1 \Rightarrow \hat{\mathbf{H}}_1,\ \hat{\mathbf{H}}_2$ are orthogonal

1. determine primitive rectification homographies $(\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2)$ from the essential matrix
2. choose a suitable common calibration matrix $\mathbf{K}$, e.g.

$$\mathbf{K} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \quad f = \frac{1}{2}(f^1 + f^2), \quad u_0 = \frac{1}{2}(u_0^1 + u_0^2), \quad \text{etc.}$$

3. the final rectification homographies are

$$\mathbf{H}_1 = \mathbf{K}\hat{\mathbf{H}}_1, \quad \mathbf{H}_2 = \mathbf{K}\hat{\mathbf{H}}_2$$

- we got a standard camera pair and non-negative disparity

$$\mathbf{P}_i^+ \stackrel{\text{def}}{=} \widetilde{\mathbf{K}_i^{-1}\mathbf{P}_i} = \mathbf{R}_i \begin{bmatrix} \mathbf{I} & -\mathbf{C}_i \end{bmatrix}, \quad i = 1, 2$$

note we started from $\mathbf{E}$, not $\mathbf{F}$

$$\mathbf{H}_1\mathbf{P}_1^+ = \not{\mathbf{K}}\hat{\mathbf{H}}_1\mathbf{P}_1^+ = \mathbf{K}\,\underbrace{\mathbf{B}\mathbf{V}^\top\mathbf{R}_1}_{\mathbf{R}^*} \begin{bmatrix} \mathbf{I} & -\mathbf{C}_1 \end{bmatrix} = \mathbf{K}\mathbf{R}^* \begin{bmatrix} \mathbf{I} & -\mathbf{C}_1 \end{bmatrix}$$

$$\mathbf{H}_2\mathbf{P}_2^+ = \not{\mathbf{K}}\hat{\mathbf{H}}_2\mathbf{P}_2^+ = \mathbf{K}\,\underbrace{\mathbf{A}\mathbf{U}^\top\mathbf{R}_2}_{\mathbf{R}^*} \begin{bmatrix} \mathbf{I} & -\mathbf{C}_2 \end{bmatrix} = \mathbf{K}\mathbf{R}^* \begin{bmatrix} \mathbf{I} & -\mathbf{C}_2 \end{bmatrix}$$

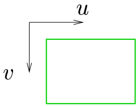
- one can prove that $\mathbf{B}\mathbf{V}^\top\mathbf{R}_1 = \mathbf{A}\mathbf{U}^\top\mathbf{R}_2$ with the help of (11)
- points at infinity project to $\mathbf{K}\mathbf{R}^*$ in both images $\Rightarrow$ they have zero disparity

Slide 159

▶The Degrees of Freedom in Epipolar Rectification

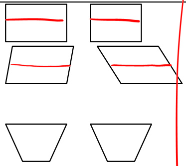
Proposition 1 Homographies $\mathbf{A}_1$ and $\mathbf{A}_2$ are rectification-preserving if the images stay rectified, i.e. if $\mathbf{A}_2^{-\top}\,\mathbf{F}^*\,\mathbf{A}_1^{-1} \simeq \mathbf{F}^*$, which gives

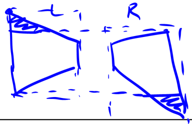
$$\mathbf{A}_1 = \begin{bmatrix} l_1 & l_2 & l_3 \\ 0 & s_v & t_v \\ 0 & q & 1 \end{bmatrix}, \qquad \mathbf{A}_2 = \begin{bmatrix} r_1 & r_2 & r_3 \\ 0 & s_v & t_v \\ 0 & q & 1 \end{bmatrix},$$



where $s \neq 0,\ u_0,\ l_1,\ l_2 \neq 0,\ l_3,\ r_1,\ r_2 \neq 0,\ r_3,\ q$ are 9 free parameters.

general	transformation	canonical	type
$l_1,\ r_1$	horizontal scales	$l_1 = r_1$	algebraic
$l_2,\ r_2$	horizontal skews	$l_2 = r_2$	algebraic
$l_3,\ r_3$	horizontal shifts	$l_3 = r_3$	algebraic
q	common special projective		geometric
s_v	common vertical scale		geometric
t_v	common vertical shift		algebraic
9 DoF		$9 - 3 = 6$ DoF	





- q is rotation about the baseline
- s_v changes the focal length

proof: find a rotation $\mathbf{G}$ that brings $\mathbf{K}$ to upper triangular form via RQ decomposition: $\mathbf{A}_1\mathbf{K}_1^* = \hat{\mathbf{K}}_1\mathbf{G}$ and $\mathbf{A}_2\mathbf{K}_2^* = \hat{\mathbf{K}}_2\mathbf{G}$

The Rectification Group

Corollary for Proposition 1 Let $\bar{\mathbf{H}}_1$ and $\bar{\mathbf{H}}_2$ be (primitive or other) rectification homographies. Then $\mathbf{H}_1 = \mathbf{A}_1 \bar{\mathbf{H}}_1, \quad \mathbf{H}_2 = \mathbf{A}_2 \bar{\mathbf{H}}_2$ are also rectification homographies.

Proposition 2 Pairs of rectification-preserving homographies $(\mathbf{A}_1, \mathbf{A}_2)$ form a group with group operation $(\mathbf{A}'_1, \mathbf{A}'_2) \circ (\mathbf{A}_1, \mathbf{A}_2) = (\mathbf{A}'_1 \mathbf{A}_1, \mathbf{A}'_2 \mathbf{A}_2)$.

Proof:

- closure by Proposition 1
- associativity by matrix multiplication
- identity belongs to the set
- inverse element belongs to the set by $\mathbf{A}_2^\top \mathbf{F}^* \mathbf{A}_1 \simeq \mathbf{F}^* \Leftrightarrow \mathbf{F}^* \simeq \mathbf{A}_2^{-\top} \mathbf{F}^* \mathbf{A}_1^{-1}$

Optimal choice for the free parameters

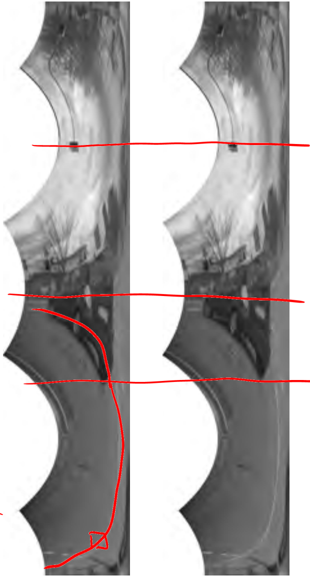
- by minimization of residual image distortion, eg. [Gluckman & Nayar 2001]

$$\mathbf{A}_1^* = \arg\min_{\mathbf{A}_1} \iint_\Omega \left(\det J(\mathbf{A}_1 \hat{\mathbf{H}}_1 \underline{\mathbf{x}}) - 1\right)^2 d\mathbf{x}$$

image domain

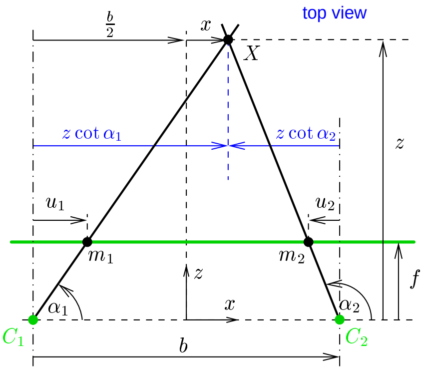
- by minimization of image information loss [Matoušek, ICIG 2004]
- non-linear rectification suitable for forward motion [Pollefeys et al. 1999], [Geyer & Daniilidis 2003]

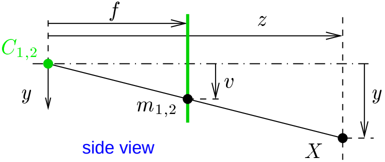
forward egomotion



rectified images, Pollefeys' method

▶Binocular Disparity in Standard Stereo Pair





- Assumptions: single image line, standard camera pair

$$b = z \cot \alpha_1 - z \cot \alpha_2$$

$$u_1 = f \cot \alpha_1 \qquad\qquad u_2 = f \cot \alpha_2$$

$$b = \frac{b}{2} + x - z \cot \alpha_2$$

$X = (x, z)$ from **disparity** $d = u_1 - u_2$:

$$z = \frac{b\,f}{d}, \quad x = \frac{b}{d}\,\frac{u_1 + u_2}{2}, \quad y = \frac{b\,v}{d}$$

$z' = bf/2$

f, d, u, v in pixels, b, x, y, z in meters

Observations

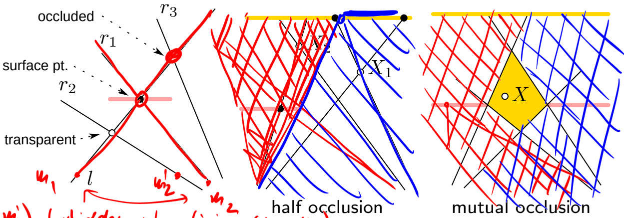
- constant disparity surface is a frontoparallel plane
- distant points have small disparity
- relative error in z is large for small disparity

$$\frac{1}{z}\,\frac{dz}{dd} = -\frac{1}{d}$$

- increasing baseline increases disparity and reduces the error

▶Understanding Basic Occlusion Types

assumption: opaque objects



(m_1, m_2') forbidden when fixing (m_1, m_2)

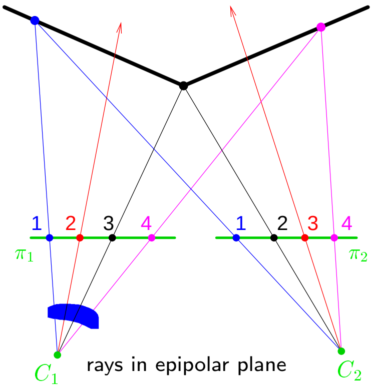
- surface point at the intersection of rays l and r_1 occludes a world point at the intersection (l, r_3) and implies the world point (l, r_2) is transparent, therefore

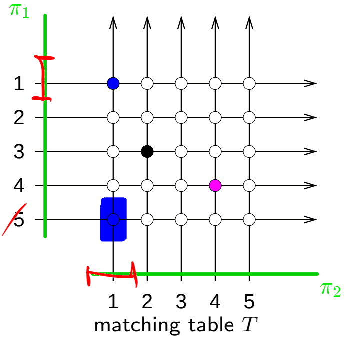
(l, r_3) and (l, r_2) are excluded by (l, r_1)

- in half-occlusion, every world point such as X_1 or X_2 is excluded by a binocularly visible surface point
  ⇒ decisions on correspondences are not independent
- in mutual occlusion this is no longer the case: any X in the yellow zone is not excluded
  ⇒ decisions in the zone are independent on the rest

▶Matching Table

Based on the observation on mutual exclusion we expect each pixel to match at most once.





matching table

- rows and columns represent optical rays
- nodes: possible correspondence pairs
- full nodes: correspondences
- numerical values associated with nodes: descriptor similarities

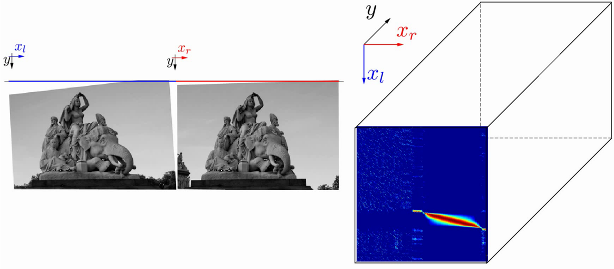
see next

Image Point Descriptors And Their Similarity

Descriptors: Tag image points by their (viewpoint-invariant) physical properties:

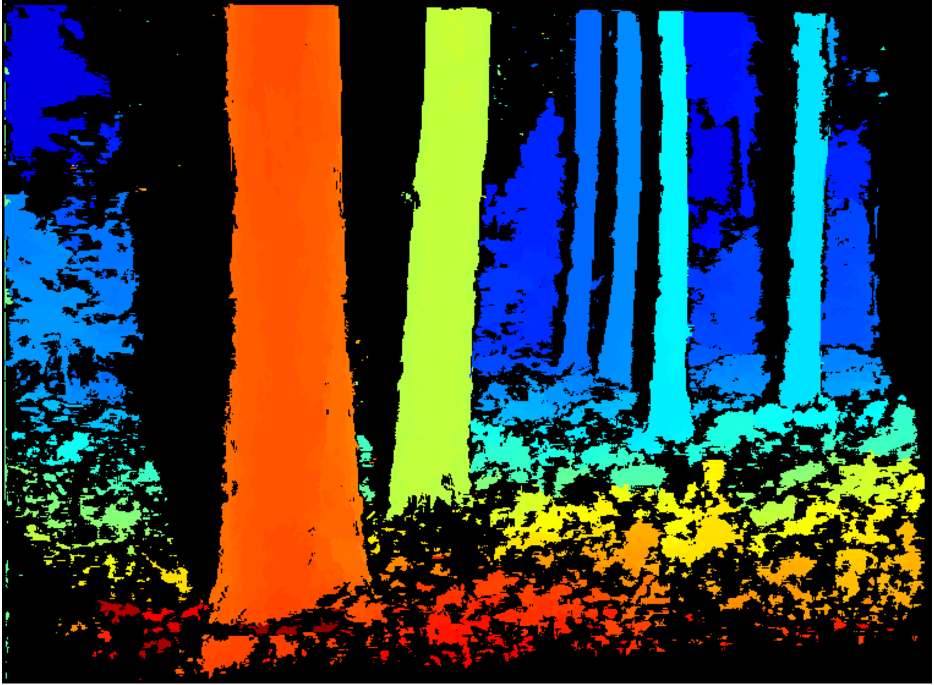
- texture window [Moravec 77]
- reflectance profile under a moving illuminant
- photometric ratios [Wolff & Angelopoulou 93-94]
- dual photometric stereo [Ikeuchi 87]
- polarization signature
- ...

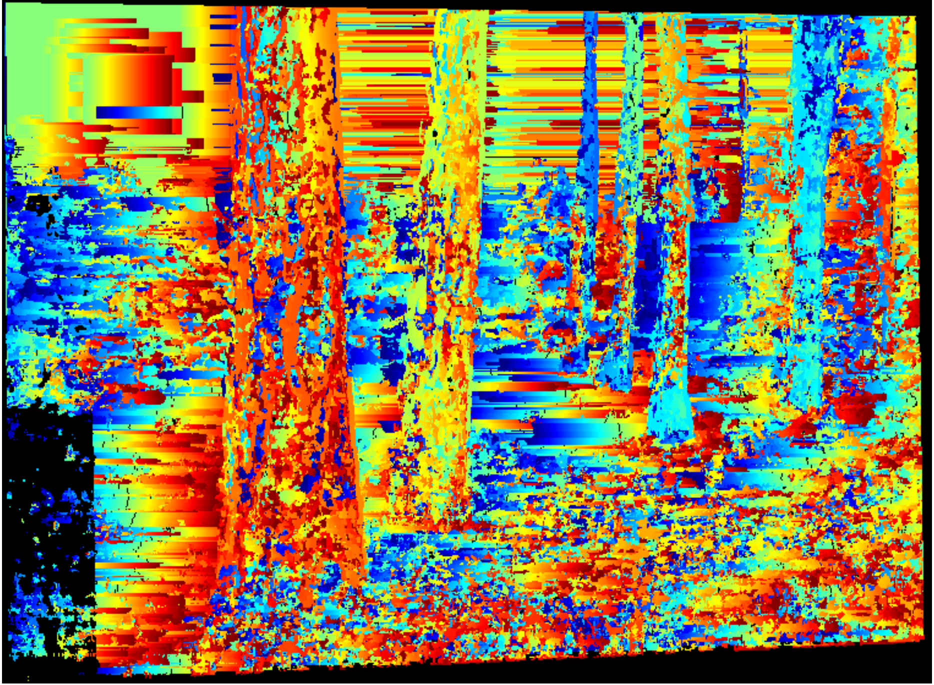
- similar points are more likely to match
- we will compute image similarity for all 'match candidates' and get the matching table

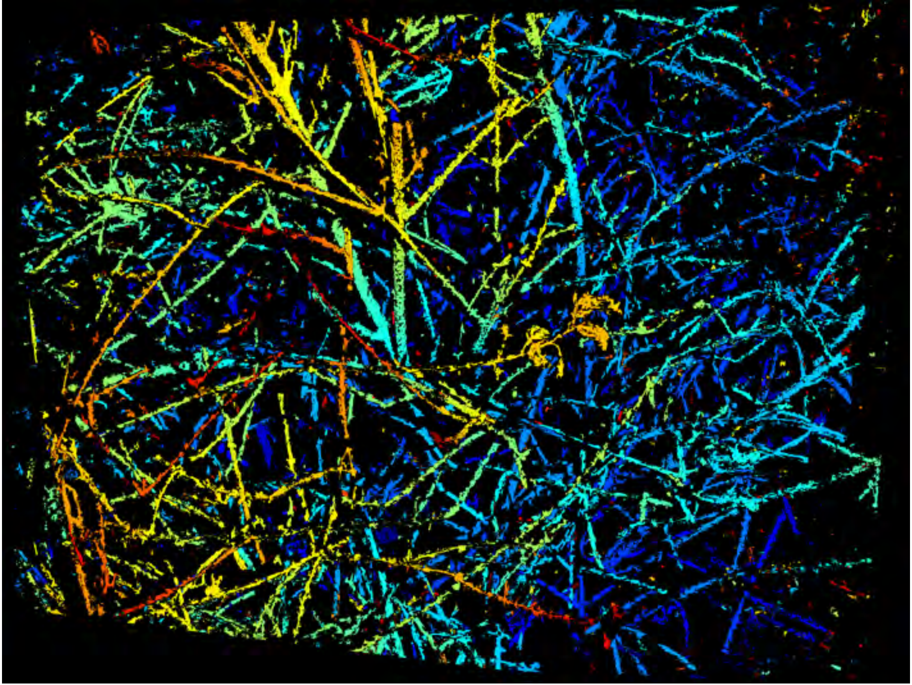


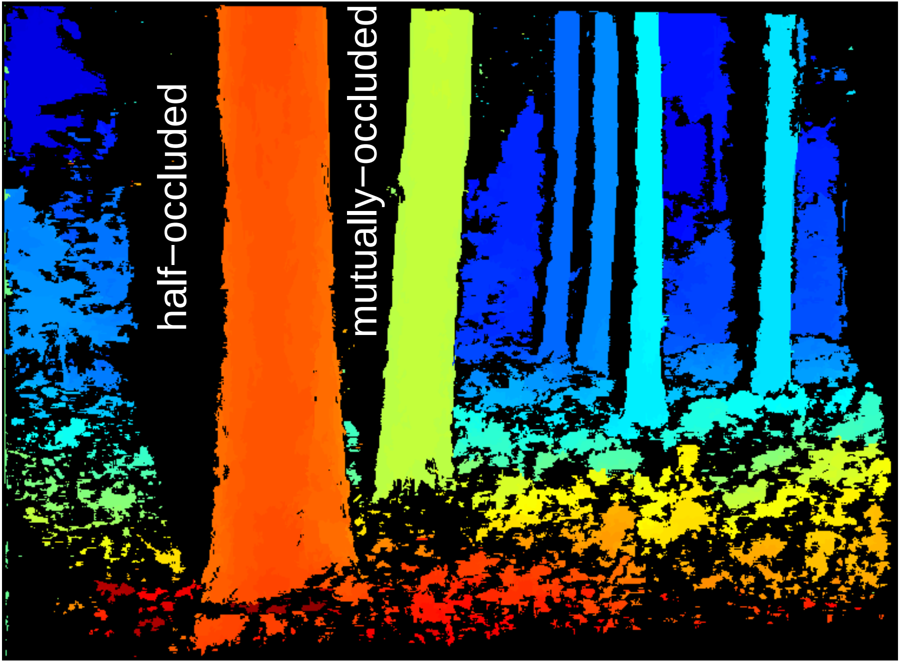
video

Thank You








half–occluded
mutually–occluded