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# Cheat sheet

AE4M33RZN

## 1 Bayesian networks

### Evaluation functions

- Bayesian Information Criterion (BIC)

$$BIC(G : D) = -\frac{K}{2} \log_2 M + \log_2 L(G : D) = -\frac{K}{2} \log_2 M - M \sum_{i=1}^n H(P_i | \text{rodice}(P_i)^G)$$

$$K = \sum_{i=1}^n q_i (r_i - 1)$$

$q_i$  ...the number of unique instantiations of  $P_i$  parents,  $n$  ...the number of variables,

$r_i$  ...the number of distinct  $P_i$  values,  $M$  ...the number of observations,

$$H(P_i | \text{rodice}(P_i)^G) = - \sum_{j=1}^{q_i} \sum_{k=1}^{r_i} \frac{N_{ij}}{M} \frac{N_{ijk}}{N_{ij}} \log_2 \frac{N_{ijk}}{N_{ij}} = - \sum_{j=1}^{q_i} \sum_{k=1}^{r_i} \frac{N_{ijk}}{M} \log_2 \frac{N_{ijk}}{N_{ij}}$$

$N_{ij}$  ...the number of samples, where  $\text{parents}(P_i)$  take the  $j$ -th instantiation of values,

$N_{ijk}$  ...the number of samples, where  $P_i$  takes the  $k$ -th value and  $\text{parents}(P_i)$  the  $j$ -th instantiation of values.

- Bayesian score

$$\ln \Pr(D|G) = \ln \prod_{i=1}^n g(P_i, \text{rodice}(P_i)^G)$$

$$g(P_i, \text{rodice}(P_i)) = \prod_{j=1}^{q_i} \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \prod_{k=1}^{r_i} N_{ijk}!$$

### Dynamic Bayesian Networks (DBNs)

- the recursive definition of filtering task

$$\Pr(X_{t+1} | e_{1:t+1}) = \alpha \Pr(e_{t+1} | X_{t+1}) \Pr(X_{t+1} | e_{1:t}) = \alpha \Pr(e_{t+1} | X_{t+1}) \sum_{x_t} \Pr(X_{t+1} | x_t) \Pr(x_t | e_{1:t})$$

$X_t$  -- the set of unobservable state variables at time  $t$ ,

$E_t$  -- the set of observable evidence variables at time  $t$ .

## 2 Description Logics

$\rightarrow_{\sqcap}$  rule

if  $(C_1 \sqcap C_2) \in L_G(a)$  and  $\{C_1, C_2\} \notin L_G(a)$  for some  $a \in V_G$ .

then  $S' = S \cup \{G'\} \setminus \{G\}$ , where  $G' = (V_G, E_G, L_{G'})$ , and  $L_{G'}(a) = L_G(a) \cup \{C_1, C_2\}$  and otherwise is the same as  $L_G$ .

$\rightarrow_{\sqcup}$  rule

if  $(C_1 \sqcup C_2) \in L_G(a)$  and  $\{C_1, C_2\} \cap L_G(a) = \emptyset$  for some  $a \in V_G$ .

then  $S' = S \cup \{G_1, G_2\} \setminus \{G\}$ , where  $G_{(1|2)} = (V_G, E_G, L_{G_{(1|2)}})$  and  $L_{G_{(1|2)}}(a) = L_G(a) \cup \{C_{(1|2)}\}$  and otherwise is the same as  $L_G$ .

$\rightarrow_{\exists}$  rule

if  $(\exists R \cdot C) \in L_G(a)$  and there exists no  $b \in V_G$  such that  $R \in L_G(a, b)$  and at the same time  $C \in L_G(b)$ .

then  $S' = S \cup \{G'\} \setminus \{G\}$ , where  $G' = (V_G \cup \{b\}, E_G \cup \{(a, b)\}, L_{G'})$  and  $L_{G'}(b) = \{C\}$ ,  $L_{G'}(a, b) = \{R\}$  and otherwise is the same as  $L_G$ .

$\rightarrow_{\forall}$  rule

if  $(\forall R \cdot C) \in L_G(a)$  and there exists no  $b \in V_G$  such that  $R \in L_G(a, b)$  and at the same time  $C \notin L_G(b)$ .

then  $S' = S \cup \{G'\} \setminus \{G\}$ , where  $G' = (V_G, E_G, L_{G'})$ , and  $L_{G'}(b) = L_G(b) \cup \{D\}$  and otherwise is the same as  $L_G$ .

$\rightarrow_{\perp}$  rule

if  $\top_C \notin L_G(a)$  for some  $a \in V_G$ .

then  $S' = S \cup \{G'\} \setminus \{G\}$ , where  $G' = (V_G, E_G, L_{G'})$ , and  $L_{G'}(a) = L_G(a) \cup \{\top_C\}$  and otherwise is the same as  $L_G$ .

## 3 Basics of fuzzy logic

Inverse membership	$\mu_A^{-1}(M) = \{x \in X \mid A(x) \in M\}$
Height	$\text{Height}(A) = \sup \{\alpha \mid x \in \Delta, A(x) = \alpha\}$
Support	$\text{Supp}(A) = \{x \in X \mid A(x) > 0\}$
Core	$\text{Core}(A) = \{x \in X \mid A(x) = 1\}$
Vertical to horizontal	$R_A(\alpha) = \{x \in X \mid A(x) \geq \alpha\}$
Horizontal to vertical	$\mu_A(x) = \max\{\alpha \in [0, 1] \mid x \in R_A(\alpha)\}$
Fuzzy inclusion	$A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ for all $x \in \Delta$
Cutworthiness	$P(A_1, \dots, A_n) \Rightarrow P(R_{A_1}(\alpha), \dots, R_{A_n}(\alpha))$
Cut-consistency	$P(A_1, \dots, A_n) \Leftrightarrow P(R_{A_1}(\alpha), \dots, R_{A_n}(\alpha))$

### 3.1 Negation

Fuzzy negation	if $\alpha \leq \beta$ then $\neg_{\circ} \beta \leq \neg_{\circ} \alpha$ (N1)
	$\neg_{\circ} \neg_{\circ} \alpha = \alpha$ (N2)
Standard negation	$\neg_S \alpha = 1 - \alpha$
Cosine negation	$\neg_{\cos} \alpha = (\cos(\pi\alpha) + 1)/2$
Sugeno negation	$\neg_{S\lambda} \alpha = \frac{1-\alpha}{1+\lambda\alpha}$
Yager negation	$\neg_{Y\lambda} \alpha = (1 - \alpha^\lambda)^{1/\lambda}$
Non-involutive negation	$\neg_{\circ} \neg_{\circ} 0 = 1$ and $\neg_{\circ} \neg_{\circ} 1 = 0$ (No)
	if $\alpha \leq \beta$ then $\neg_{\circ} \beta \leq \neg_{\circ} \alpha$ (N1)
Gödel negation	$\neg_G \alpha = \begin{cases} 1 & \alpha = 0 \\ 0 & \text{otherwise} \end{cases}$

### 3.2 Implication

General fuzzy implication	$(x \overset{\circ}{\Rightarrow} y) = (x \Rightarrow y)$ on $x, y \in \{0, 1\}$
Residue implication	$\alpha \overset{R}{\Rightarrow} \beta = \sup\{\gamma \mid \alpha \wedge \gamma \leq \beta\}$
R-impl. properties	$\alpha \overset{R}{\Rightarrow} \beta = 1$ iff $\alpha \leq \beta$ (I1)
	$1 \overset{R}{\Rightarrow} \beta = \beta$ (I2)
	$\alpha \overset{R}{\Rightarrow} \beta$ is not increasing in $\alpha$
	and not decreasing in $\beta$ (I3)
Standard implication	$\alpha \overset{R}{S} \beta = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ \beta & \text{otherwise} \end{cases}$
Lukasiewicz implication	$\alpha \overset{R}{L} \beta = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ 1 - \alpha + \beta & \text{otherwise} \end{cases}$
Algebraic implication	$\alpha \overset{R}{A} \beta = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ \frac{\beta}{\alpha} & \text{otherwise} \end{cases}$
S-implication	$\alpha \overset{S}{\Rightarrow} \beta = \neg_S \alpha \overset{\vee}{\circ} \beta$
Kleene-Dienes implication	$\alpha \overset{S}{\Rightarrow} \beta = \max(1 - \alpha, \beta)$
Generalized fuzzy inclusion	$A \overset{\circ}{\subseteq} B = \inf\{A(x) \overset{\circ}{\Rightarrow} B(x) \mid x \in \Delta\}$

### 3.3 Conjunction

Definition

$$\alpha \wedge \beta = \beta \wedge \alpha \text{ (T1)}$$

$$\alpha \wedge (\beta \wedge \gamma) = (\alpha \wedge \beta) \wedge \gamma \text{ (T2)}$$

$$\text{if } \beta \leq \gamma \text{ then } (\alpha \wedge \beta) \leq (\alpha \wedge \gamma) \text{ (T3)}$$

$$(\alpha \wedge 1) = \alpha \text{ (T4)}$$

Standard c.  $\alpha \wedge_S \beta = \min(\alpha, \beta)$

Łukasiewicz c.  $\alpha \wedge_L \beta = \max(\alpha + \beta - 1, 0)$

Algebraic product  $\alpha \wedge_A \beta = \alpha \cdot \beta$

Weak conjunction  $\alpha \wedge_W \beta = \begin{cases} \alpha & \text{if } \beta = 1 \\ \beta & \text{if } \alpha = 1 \\ 0 & \text{otherwise} \end{cases}$

### 3.5 Relations

Composition  $R \circ S(x, z) = \sup_{y \in Y} R(x, y) \wedge S(y, z)$

Comp. properties  $R \circ (S \circ T) = (R \circ S) \circ T \text{ (C3)}$

$$R \circ E = R, E \circ R = R \text{ (C1)}$$

$$(R \overset{S}{\cup}) \circ T = (R \circ T) \overset{S}{\cup} (S \circ T) \text{ (C4)}$$

$$(R \circ S)^{-1} = S^{-1} \circ R^{-1} \text{ (C2)}$$

$$R \circ (S \overset{S}{\cup} T) = (R \circ S) \overset{S}{\cup} (R \circ T) \text{ (C5)}$$

First projection  $R^{(1)}(x) = \sup_{y \in Y} R(x, y)$

Second projection  $R^{(2)}(y) = \sup_{x \in X} R(x, y)$

Cylindrical extension  $A \times B(x, y) = A(x) \wedge_S B(y)$

Identity relation  $E = \{(x, x) \mid x \in \Delta\}$

Reflexivity  $E \subseteq R$

Symmetry  $R = R^{-1}$

◦-anti-symmetry  $R \cap R^{-1} \subseteq E$

◦-transitivity  $R \circ R \subseteq R$

◦-partial order **reflexive, ◦-transitive and ◦-anti-symmetric**

◦-equivalence **reflexive, ◦-transitive and ◦-symmetric**

### 3.4 Disjunction

Definition

$$\alpha \vee \beta = \beta \vee \alpha \text{ (S1)}$$

$$\alpha \vee (\beta \vee \gamma) = (\alpha \vee \beta) \vee \gamma \text{ (S2)}$$

$$\text{if } \beta \leq \gamma \text{ then } (\alpha \vee \beta) \leq (\alpha \vee \gamma) \text{ (S3)}$$

$$(\alpha \vee 0) = \alpha \text{ (S4)}$$

Standard d.  $\alpha \overset{S}{\vee} \beta = \max(\alpha, \beta)$

Łukasiewicz d.  $\alpha \overset{L}{\vee} \beta = \min(\alpha + \beta, 1)$

Algebraic sum  $\alpha \overset{A}{\vee} \beta = \alpha + \beta - \alpha \cdot \beta$

Weak disjunction  $\alpha \overset{W}{\vee} \beta = \begin{cases} \alpha & \text{if } \beta = 0 \\ \beta & \text{if } \alpha = 0 \\ 1 & \text{otherwise} \end{cases}$

## 4 Fuzzy description logic

non-atomic c. interpretation

$$\perp \quad 0$$

$$\top \quad 1$$

$$A \quad A^{\mathcal{F}}(x)$$

$$\neg C \quad \overline{C^{\mathcal{F}}(x)}$$

$$C \sqcap D \quad C^{\mathcal{F}}(x) \wedge D^{\mathcal{F}}(x)$$

$$C \sqcup D \quad C^{\mathcal{F}}(x) \vee D^{\mathcal{F}}(x)$$

$$C \overset{\circ}{\supset} D \quad C^{\mathcal{F}}(x) \overset{\circ}{\supset} D^{\mathcal{F}}(x)$$

$$\exists R \cdot C \quad \sup_y R^{\mathcal{F}}(x, y) \wedge C^{\mathcal{F}}(y)$$

$$\forall R \cdot C \quad \inf_y R^{\mathcal{F}}(x, y) \overset{\circ}{\supset} C^{\mathcal{F}}(y)$$

$$(n C) \quad n \cdot C(x)$$

$$\text{mod}(C) \quad \text{mod}(C^{\mathcal{F}}(x))$$

$$w_1 C_1 + \dots + w_k C_k \quad w_1 C_1^{\mathcal{F}}(x) + \dots + w_k C_k^{\mathcal{F}}(x)$$

axiom satisfied if

$$\langle i : C \mid \alpha \rangle \quad C^{\mathcal{F}}(i^{\mathcal{F}}) \geq \alpha$$

$$\langle (i, j) : R \mid \alpha \rangle \quad R^{\mathcal{F}}(i^{\mathcal{F}}, j^{\mathcal{F}}) \geq \alpha$$

$$\langle C \sqsubseteq D \mid \alpha \rangle \quad C \overset{\circ}{\supset} D \geq \alpha$$

$$\langle R_1 \sqsubseteq R_2 \rangle \quad R_1^{\mathcal{F}} \subseteq R_2^{\mathcal{F}}$$

$$\langle \text{transitive } R \rangle \quad R \text{ is } \circ\text{-transitive}$$

$$\langle R_1 = R_2^{-1} \rangle \quad R_1^{\mathcal{F}} = (R_2^{\mathcal{F}})^{-1}$$



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