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The running time of algorithm / program

1.
Which of the following two fragments is faster?

```
int n = 100;
int sum = 0;
for (i = 0; i < n; i++)
   for (j = 0; j < i; j++)
      sum += i+j;
```

```
int n = 75;
int sum = 0;
for (i = 0; i < n; i++)
   for (j = 0; j < n; j++)
      sum += i+j;
```

2.
Fill in the missing constant in the condition of the following code so that the procedure xyz () is called just 2100 times.

```
for (i=0; i < 70; i++) {
  j = 0;
  do {
    if (j > ___ ) xyz();
    j++;
  } while (j < 90);
}
```

3.
Fill in the missing constant in the condition of the following code so that the procedure xyz () is called just 2000 times.

```
i = 50;
do {
  for (j=0; j < 70; j++)
    if (j > ___ ) xyz();
  i++;
} while (i < 150);
```

4.
Fill in the missing expression in the condition of the following code so that the procedure uvw() is called just 49 times.

```
for (i = 0; i < 7; i++) {
  j = i;
  while (j < ___) {
    uvw();
    j++;
  }
}
```

5.
Fill in the missing constant in the condition of the following code so that the procedure `uvw` () is called just 85 times.

```
i = 0;
while (i < 10) {
  for (j = i; j < ___; i++)
    uvw();
```

```
  i++;
}
```

6.
The processing of the k-th row of a matrix of size nxn requires 2k operations. The total number of operations needed to handle the matrix processing is

a) $2n^2$
b) $(n^2)/2$
c) $n(n+1)/2$
d) $n(n-1)$
e) $n(n+1)$

7.
The task which time of a solution is $C \cdot n^2$, where n is the range of input data, is solved on the computer for n = 5000. A new computer is purchased that is about 2.5 times faster. How is it possible to extend the range of input data to be processed so that the task is solved on the new computer within the same time?
Solve for different conditions depending on the time scale of input data:
$C \cdot n^3$, $C \cdot n^{0.5}$, $C \cdot n \cdot \log_2(n)$ ...

8.
The machine performs 10^9 operations per second. 1 hour of processing time is available. Determine what can be the maximum value of n, which determines the size of input data, if the number of instructions required for the processing of data of n is: $n^{3/2}$, $n^{5/4}$, $n \cdot \log_2(n) \cdot \log_2(\log_2(n))$, $n^2 \cdot \log_2(n)$, ...and others.

9.
Method A needs $n^2 + 17$ operations to solve the task. Method B requires 2n + 80 operations. The integer n describes the range of input data. For what n is method A preferable to use?

Asymptotic complexity

10.
Let f(x) be a continuous increasing function and g(x) be a function so that $f(x) \in \Omega(g(x))$.Then it implies that

a) $f(x) \in O(g(x))$
b) $f(x) \in \Theta(g(x))$
c) $g(x) \in \Theta(f(x))$
d) $g(x) \in \Omega(f(x))$
e) $g(x) \in O(f(x))$

11.
Let f(x) be a continuous increasing function and g(x) be a function so $f(x) \in O(g(x))$. Then it implies that

a) $f(x) \in \Theta(g(x))$
b) $f(x) \in \Omega(g(x))$
c) $g(x) \in \Theta(f(x))$
d) $g(x) \in \Omega(f(x))$
e) $g(x) \in O(f(x))$

12.
If function f increases asymptoticly faster than function g (i.e.. f(x) $\notin$ O(g(x))), then the following relation holds

a) if both function are defined in a point x then f(x) > g(x)
b) difference f(x) - g(x) is always positive,
c) difference f(x) – c * g(x) is positive for every x > y, where y is a sufficiently large number and c is a positive number,
d) both function f and g are defined pro non-negative arguments only,
e) none of the previous.

13.
If function f increases asymptoticly equivalently as function g (i.e f(x) $\in$ Θ(g(x))), then just one of the following relation holds. Which one?

a) if both function are defined in a point x then f(x) = g(x)
b) neither ratio f(x)/g(x) nor ration g(x)/f(x) converge to zero with increasing x,
c) difference f(x) – c * g(x) is positive for every x > y, where y is a sufficiently large number and c is a positive number
d) both function f and g are defined pro non-negative arguments only,
e) none of the previous.

14.
Both continuous functions f(x) and g(x) are increasing on entire **R** and f(x) < g(x) for every $x \in$ **R**. It implies that

a) f(x) $\notin \Omega$(g(x))
b) f(x) $\notin$ O(g(x))
c) it is possible that f(x) $\in \Omega$(g(x))
d) g(x) $\notin \Theta$(f(x))
e) f(x) grows asympticly slower than g(x)

15.
Both continuous functions f(x) and g(x) are increasing on entire **R** and f(x) $\notin \Omega$ (g(x)), f(x) $\notin \Theta$(g(x)). Then

a) g(x) $\in$ O(f(x))
b) g(x) $\in \Theta$(f(x))
c) f(x) < g(x) for each x $\in$ **R**
d) f(x) $\leq$ g(x) for each x $\in$ **R**
e) there might exist $y \in$ **R** such that f(y) > g(y)

16.
An algorithm A processes successively all the elements in two-dimensional array of size $n \times n$. An action (unknown to us) of the complexity $\Theta(\log_2(n))$ is performed with each element.The overall asymptotic complexity of the algorithm A is

a) $\Theta(n \cdot \log_2(n))$
b) $\Theta(n^2)$
c) $\Theta(n^3)$
d) $\Theta(n^2+\log_2(n))$
e) $\Theta(n^2 \cdot \log_2(n))$

17.
Just one of the following proposition is not valid. Mark it.

a) $x \cdot \log_2(x) \in O(x^2 - x)$
b) $x \cdot \log_2(x) \in O(x^2 - \log_2(x))$
c) $x \cdot \log_2(x) \in \Omega(x^2 - \log_2(x))$
d) $x \cdot \log_2(x) \in \Omega(x + \log_2(x))$
e) $x \cdot \log_2(x) \in \Theta(x \cdot \log_2(x^2))$

18.
Algorithm A performs one pass through an array with n elements. When processing an element in the position k it performas k + n operations. The operating (= asymptotic) complexity of the algorithm A is

a) $\Theta(k+n)$
b) $\Theta((k+n)\cdot n)$
c) $\Theta(k^2+n)$
d) $\Theta(n^2)$
e) $\Theta(n^3)$

19.
Fill in symbols O or Θ or Ω at the blank spaces (........) in the following relationships so to form a true statement. If multiple options, give them all. If no symbol fits, cross the blank space.

a) $x^2 \cdot 2^x \in$ ....$((\ln(x^2))^2 + 2^x)$
b) $(\ln(x^2))^2 + 2^x \in$ ....$(x^2 + \ln(x^2))$
c) $2^x \cdot (\ln(x))^{-1} \notin$ ....$(2^x \cdot (\ln(x^2))^{-1})$

20.
Fill in symbols O or Θ or Ω at the blank spaces (........) in the following relationships so to form a true statement. If multiple options, give them all. If no symbol fits, cross the blank space.

a) $x^2 \cdot \ln(x^2) \in$ ....$(x^2 + \ln(x))$
b) $x^3 + \ln(x^2) \in$ ....$(x^3 + 2^x)$
c) $x^3 \cdot \ln(x^2) \notin$ ....$(\ln(x^2) + 2^x)$

21.
Give an example of three growing real variable function f (x), g (x), and h (x), which all three of the following relationships also apply to:
$f(x) \notin O(g(x)),\ g(x) \notin \Theta(h(x)),\ h(x) \notin \Omega(f(x))$
If such three functions cannot exist, write a brief justification of why.

22.
Give an example of three growing real variable function f (x), g (x), and h (x), which all three of the following relationships also apply to:
$f(x) \notin O(g(x)),\ g(x) \notin \Omega(h(x)),\ h(x) \notin \Theta(f(x))$
If such three functions cannot exist, write a brief justification of why.

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