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ePAL - Heaps

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- 1 Binary Heap
- 2 Binomial Heap

Outline

1 Binary Heap

2 Binomial Heap

Example 1

One of the following sequences represents a heap stored in an array.
Which one?

- ① 9 5 4 6 3
- ② 5 4 2 3 9
- ③ 3 8 9 5 6
- ④ 5 1 8 9 1
- ⑤ 1 3 6 5 4



Example 2

One of the following sequences represents a heap stored in an array.
Which one?

- ① 3 2 6 7 4
- ② 9 3 1 6 4
- ③ 2 4 8 7 9
- ④ 6 3 8 7 3
- ⑤ 2 7 4 8 1

Example 3

One of the following sequences represents a heap stored in an array.
Which one?

- ① 2 5 6 4 9
- ② 2 6 5 4 9
- ③ 2 6 5 9 4
- ④ 2 5 4 9 6
- ⑤ 2 9 4 5 6

Example 4

One of the following sequences represents a heap stored in an array.
Which one?

- ① 1 4 5 3 8
- ② 1 4 3 8 5
- ③ 1 5 4 3 8
- ④ 1 5 4 8 3
- ⑤ 1 8 3 4 5

Example 5

One of the following sequences represents a heap stored in an array.
Which one?

- ① 3 9 4 7 5
- ② 3 4 9 7 5
- ③ 4 3 5 7 9
- ④ 3 5 7 4 9
- ⑤ 4 5 3 7 9



Example 6

One of the following sequences represents a heap stored in an array.
Which one?

- ① 2 9 3 8 6
- ② 3 2 6 8 9
- ③ 2 6 3 9 8
- ④ 2 9 8 6 3
- ⑤ 3 6 2 8 9

Example 7

What are the minimum and maximum numbers of elements in a heap of height h ?

Example 8

Show that in any subtree of a max-heap, the root of the subtree contains the largest value occurring anywhere in that subtree.

Example 9

Where in a max-heap might the smallest element reside, assuming that all elements are distinct?

Example 10

Is an array that is in sorted order a min-heap?

Example 11

Is the array with values [23; 17; 14; 6; 13; 10; 1; 5; 7; 12] a max-heap?

Example 12

Insert elements $[45, 13, 12, 16, 9, 5]$ in this order into a min-heap.

Example 13

Delete all minimum elements in step by step manner from the following array representing a min-heap [5, 12, 9, 45, 16, 13].

Example 14

What is the result min-heap represented as an array if the input sequence of elements is

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]

Example 15

Decrease key 9 by 6 and then 17 by 16 in the min-heap

[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

Example 16

Create a min-heap by inserting elements from the input sequence
[45, 1, 32, 27, 13, 7, 3, 16, 8, 11, 21, 9, 5, 10, 31]

Example 17

Delete keys 8, 13, 9 in this order from the min-heap
[1, 8, 3, 13, 11, 5, 7, 45, 16, 27, 21, 32, 9, 10, 31].

Example 18

Create two min-heaps from input sequences [45, 1, 32, 27, 13, 7, 3] and [16, 8, 11, 21, 9, 5, 10, 31] and then merge both heaps.

Outline

1 Binary Heap

2 Binomial Heap

Example 1

Create a binomial heap from the input sequence
[30, 10, 90, 80, 60, 70, 20, 50, 40].

Example 2

[30, 10, 90, 80, 60, 70, 20, 50, 40]

Extract a half of elements from the binomial heap created from the above input sequence (the previous example).

Example 3

Now, decrease the key of the max node (of the heap of the previous example) to that of the original min node (10).

Example 4

..... and then extract the min again. Which should be 10.

References I



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