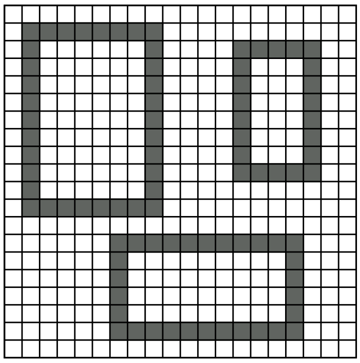
GRAPHICAL MARKOV MODELS (WS2011)
6. SEMINAR

Assignment 1. Consider the language L of all (rectangular) b/w images containing an arbitrary number of non-overlapping rectangular frames (one pixel wide).
a) Prove that L is not expressible by a locally conjunctive predicate i.e. a conjunction of predicates, each one defined on an image fragment of a fixed size.
b) Show that L can be expressed by introducing non-terminal symbols and a locally conjunctive predicate for these.



Assignment 2. Transform the *Travelling Salesman Problem* into a $(\min, +)$-problem.

Assignment 3. Consider a CSP for K-valued labellings s of the vertices of a graph $\mathcal{G} = (V, E)$

$$G(s) = \Big[\bigwedge_{i \in V} g_i(s_i)\Big] \wedge \Big[\bigwedge_{ij \in E} g_{ij}(s_i, s_j)\Big],$$

where g_{ij} and g_i are predicates of arity 2 and 1 respectively. The task is to calculate $c = \bigvee_{s \in K^V} G(s)$.
Mister X proposes the following algorithm. Edges and vertices are repeatedly visited, each time updating the functions g_{ij} and g_i respectively by

$$g_{ij}(s_i, s_j) := g_i(s_i) \wedge g_{ij}(s_i, s_j) \wedge g_j(s_j)$$
$$g_i(s_i) := g_i(s_i) \wedge \Big[\bigwedge_{j \in N_i} \Big[\bigvee_{k \in K} g_{ij}(s_i, k)\Big]\Big].$$

The algorithm stops in a fix-point g_i^*, g_{ij}^*. If all these functions are identically equal to zero then $c = 0$ is assumed. Otherwise c is assumed to be equal to 1.
a) Prove that the algorithm is not correct in general. Construct a counterexample.
b)** Prove that the algorithm is indeed correct if the arity 2 predicates g_{ij} are supermodular w.r.t. an ordering of the label set K, i.e.

$$g\big(\max(k_1, k_1'), \max(k_2, k_2')\big) \wedge g\big(\min(k_1, k_1'), \min(k_2, k_2')\big) \geqslant g(k_1, k_2) \wedge g(k_1', k_2'),$$

where the operations $\min$ and $\max$ return the greater and lower label respectively (w.r.t. the ordering of K).

Hints

– Prove, that the update rules preserve supermodularity.

– Consider $k_i^* = \max_k\{k \in K \mid g_i^*(k) = 1\}$. Prove that s^* defined by $s_i^* = k_i^*$ is a solution, i.e. $G(s^*) = 1$.