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**GRAPHICAL MARKOV MODELS (WS2011)**  
**3. SEMINAR**

**Assignment 1.** Consider a fully specified HMM for sequence pairs  $(x, s)$  of length  $n$  given by

$$p(x, s) = p(s_1) \cdot \prod_{i=2}^n p(s_i | s_{i-1}) \cdot \prod_{i=1}^n p(x_i | s_i).$$

Let  $k^* \in K$  be a fixed, special hidden state. Let us call a subset of positions  $I \subset \{1, 2, \dots, n\}$  a *segmentation* and associate a subset of hidden state sequences with each segmentation:

$$\mathcal{S}(I) = \{s \in K^n \mid s_i = k^*, \forall i \in I \text{ and } s_i \in K \setminus k^*, \forall i \notin I\}.$$

- a)** Find an efficient algorithm for calculating  $p(x, \mathcal{S}(I))$ .  
**b)** Suppose we want to find the most probable segmentation given an observed feature sequence  $x$

$$I^* = \arg \max_I p(x, \mathcal{S}(I)).$$

Deduce an efficient algorithm for this task.

**Assignment 2.** Let  $x$  be a gray value image of size  $n \times m$ , where  $x_{ij}$  denotes the gray value of the pixel with coordinates  $(i, j)$ . The task is to segment such images into an upper and lower part by a boundary represented as a sequence of height values  $s_j \in \{1, 2, \dots, n\}$  for all  $j = 1, 2, \dots, m$ .

The prior probability for boundaries is assumed to be a homogeneous Markov chain such that  $p(s_j | s_{j-1}) = 0$  if  $|s_j - s_{j-1}| > 1$ . The appearance model for columns  $\vec{x}_j$  given the boundary value  $s_j$ , is assumed to be conditional independent

$$p(\vec{x}_j | s_j) = \prod_{i \leq s_j} p_1(x_{ij}) \cdot \prod_{i > s_j} p_2(x_{ij}),$$

where  $p_1()$  and  $p_2()$  are two pd-s for gray values.

- a)** Deduce an efficient algorithm for determining the most probable boundary.  
**b)** Suppose that the loss function  $c(s, s')$  for incorrectly recognised boundaries is defined by

$$c(s, s') = \sum_{j=1}^m (s_j - s'_j)^2.$$

Formulate the segmentation task for this case. Deduce an efficient inference algorithm.



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