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Ant Colony Optimization Algorithms

- Biological motivation
- ACO metaheuristic
- Ant System and its application to TSP and JSP

Motivation

- **NP-hard problems** – does not exist algorithm that can solve large instances of these problems to optimality
 - Discrete combinatory problems
- **Approximate methods** – can find solutions of good quality in reasonable time
- **Approximate methods**
 - **Local search/optimization**
 - Iteratively improves one solution (typically initialized at random) till it reaches some local optimum.
 - **Construction algorithms**
 - Build a solution making use of some problem-specific heuristic information
- **Ant Colony Optimization (ACO)** – extends traditional construction heuristics with an ability to exploit experience gathered during the optimization process.

Local Search

Procedure *IterativeImprovement*($s \in S$)

$s' = \text{Improve}(s)$

while $s' \neq s$ **do**

$s = s'$

$s' = \text{Improve}(s)$

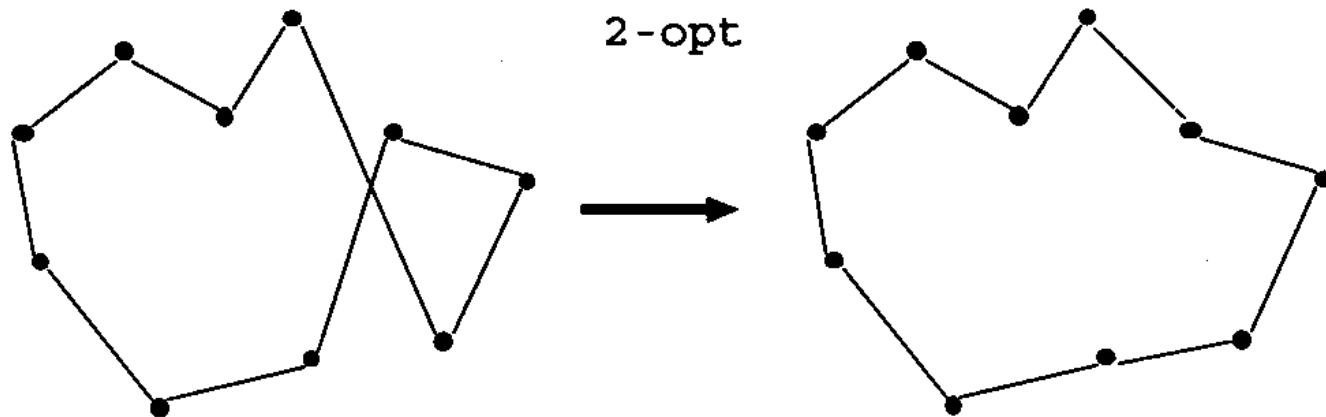
end

return s

end

Problems

- Gets stuck in local optimum
- Quality of the final solution depends on the initial solution from which the optimization starts



Construction Algorithms

- Build solutions to a problem under consideration in an incremental way starting with an empty initial solution and iteratively adding opportunely defined solution components without backtracking until a complete solution is obtained.

Procedure *GreedyConstructionHeur*

$s_p = \text{empty_solution}$

while not complete(s_p) **do**

$e = \text{GreedyComponent}(s_p)$

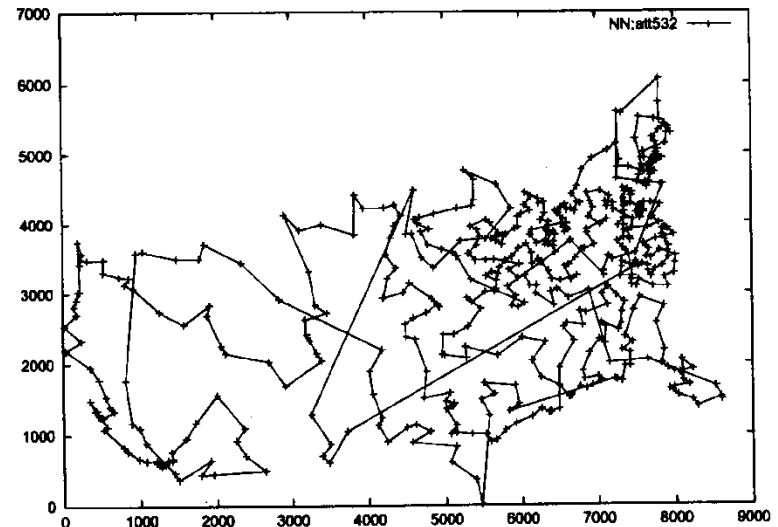
$s_p = s_p \otimes e$

end

return s_p

end

TSP: nearest neighbor heuristic



- **Pros/Cons**

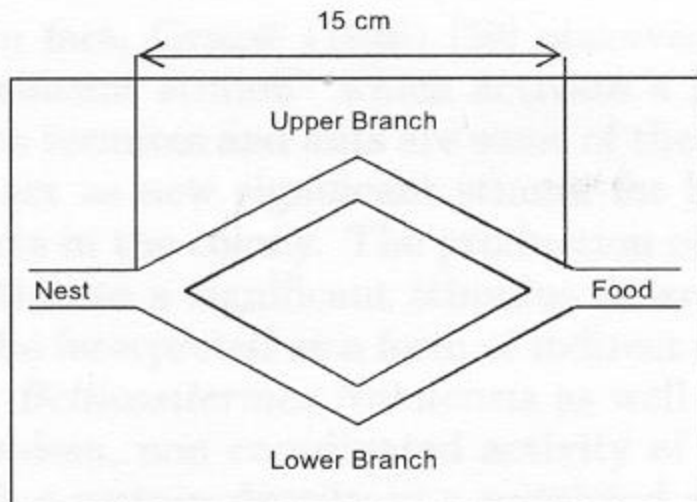
- + fast
- Solution quality may be low
- Generate only limited number of different solutions
- Decisions made at early stages reduce a set of possible steps at latter stages

Ant Algorithms: Biological Inspiration

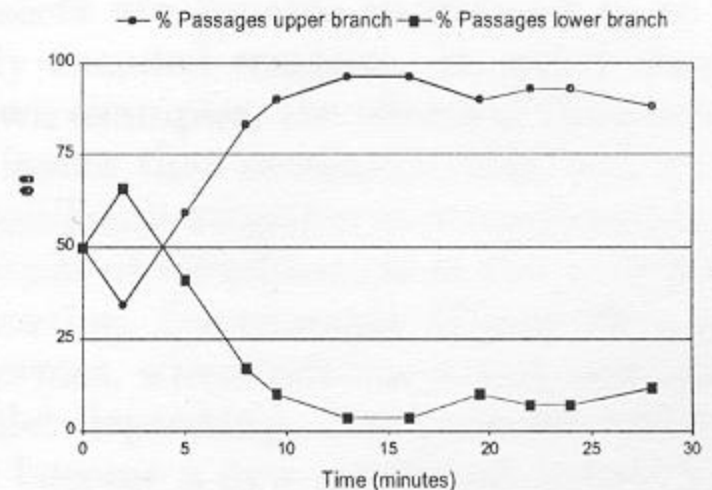
- **Inspired by behavior of an ant colony**
 - Social insects – behave towards survival of the colony
 - Simple individual behavior × complex behavior of a colony
- Ability to find the shortest path from the colony to the source of food and back using an **indirect communication via pheromone**
 - **Write** – ants lay down pheromone on their way to food
 - **Read** – ant detects pheromone (can sense different intensity) laid down by other ants and can choose a direction of the highest concentration of pheromone.
 - **Emergence** – this simple behavior applied by the whole colony can lead to emergence of the shortest path.

Experiments with Real Ants

- **Deneuborg et al.** (ants *Linepithema humile*)
- **Nest separated from food with a double-bridge**
 - Both path of the same length
 - At the beginning there is no pheromone
 - After some time one of the alternatives gets dominant due to random fluctuations



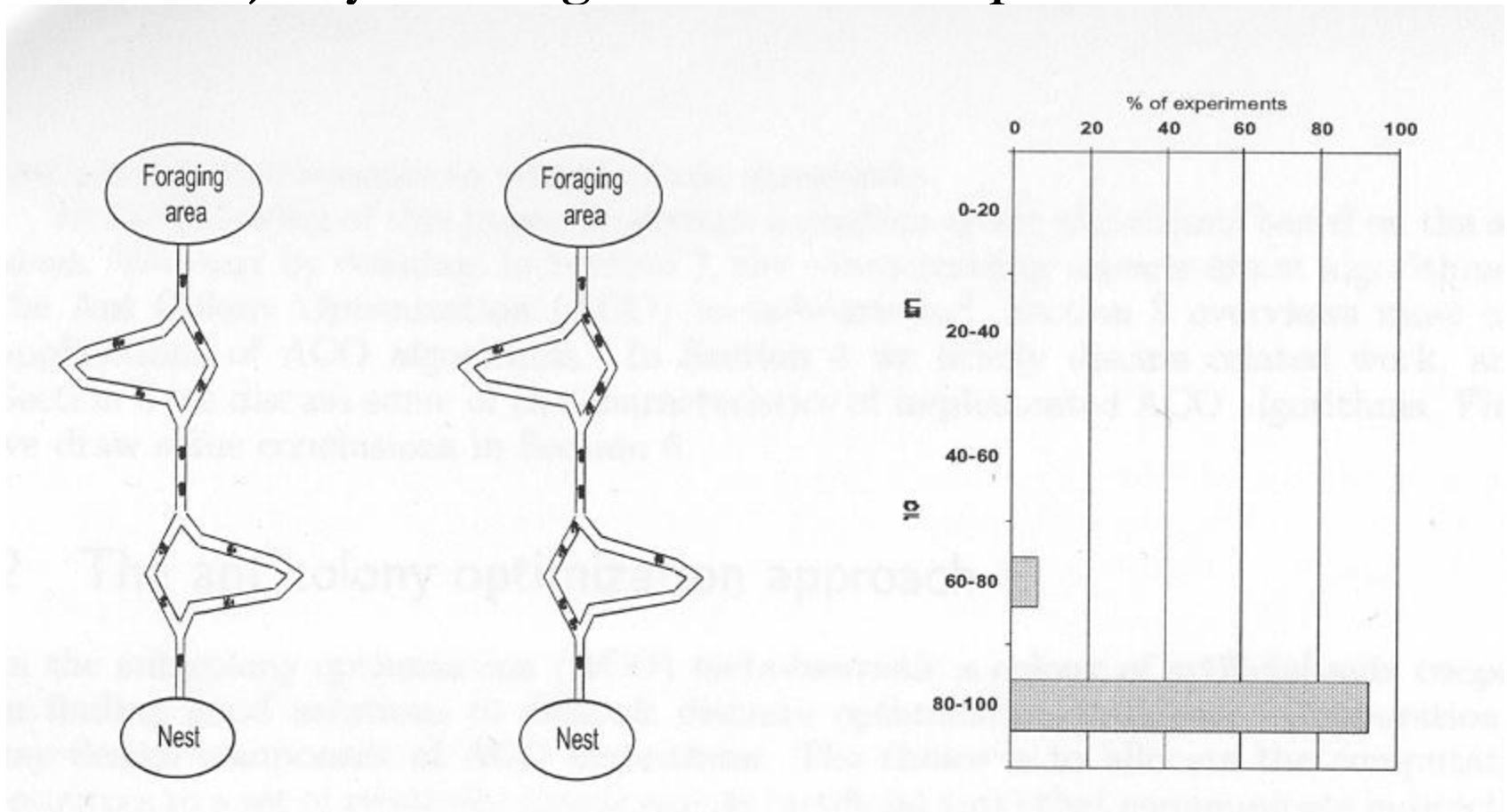
(a)



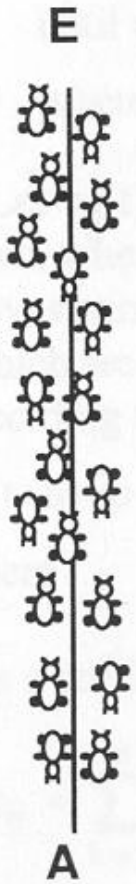
(b)

Bridges with Different Branches

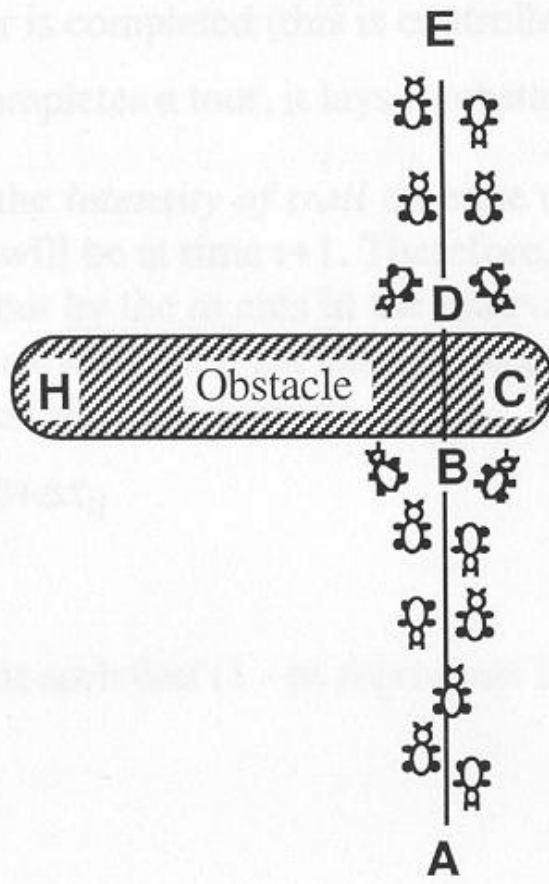
- Influence of random fluctuations is significantly reduced and majority of ants go for the shorter path in the end.



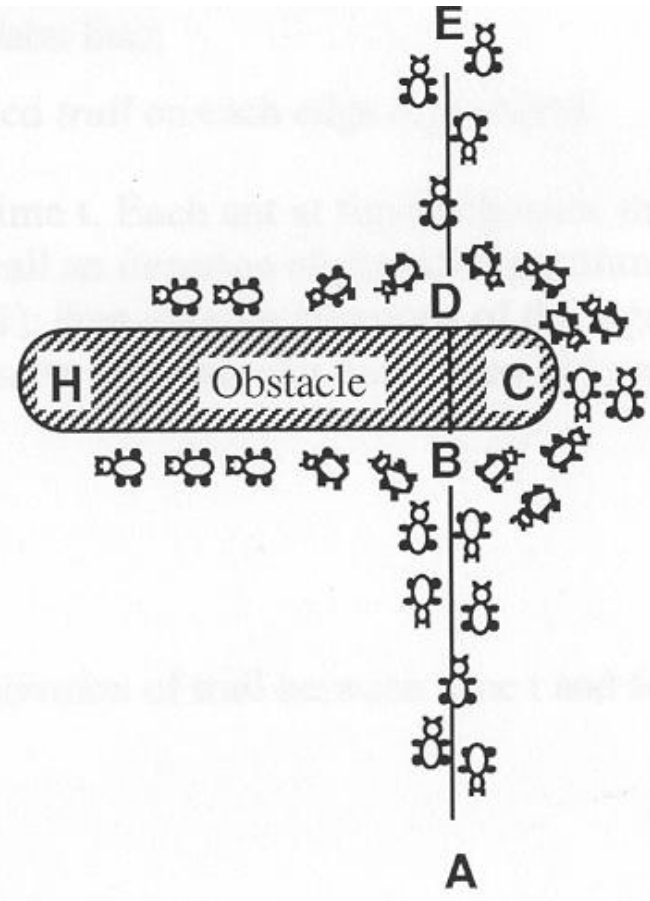
Example



a)



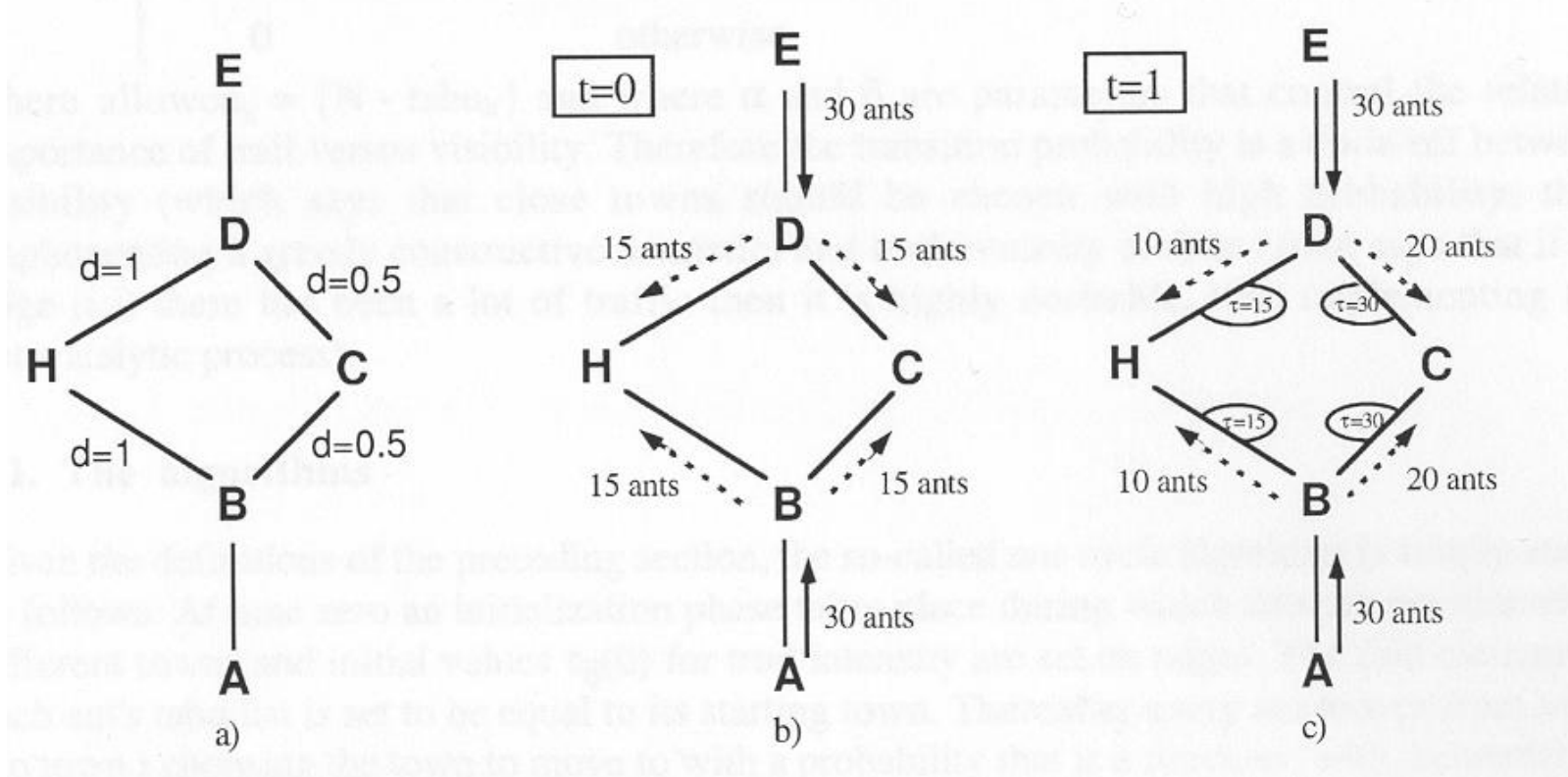
b)



c)

Example

- In each step 30 new ants go from A to B, and 30 ants from E to D
 - All ants go with the same speed 1 s^{-1}
 - Each ant deposits down 1 unit of pheromone per 1 time unit



Stigmergy

- **Stigmergie** - two individuals interact indirectly when one of them modifies the environment and the other responds to the new environment at a later time.
 - **Physically** – by depositing a pheromone the ants modify the place they have visited.
 - **Locality of information** – pheromone is “visible” only to ants that are in its close vicinity.
 - **Autocatalytic behavior** – the more ants follow a trail, the more attractive that trail becomes for being followed.
The process is thus characterized by a **positive feedback loop**, where the probability of a discrete path choice increases with the number of times the same path was chosen before
- **Pheromone evaporation** – realizes forgetting, which prevents premature convergence to suboptimal solutions.

About the Ants

- Almost blind
- Incapable of achieving complex tasks alone
- Capable of establishing shortest-route paths from their colony to feeding sources and back
- Use *stigmergic* communication via pheromone trails
- Follow existing pheromone trails with high probability

Artificial Ants

- **Similarity with real ants:**
 - Colony of cooperating ants
 - Pheromone trail and stigmergy
 - Probabilistic decision making, locality of the strategy
 - Prior information given by the problem specification
 - Local modification of states, induced by preceding ants
- **Differences from real ants:**
 - Discrete world
 - Inner states – personal memory with already performed actions
 - Ants are not completely blind
 - Amount of deposited pheromone is a function of the quality of the solution
 - Problem dependent timing of depositing the pheromone
 - Extras – local optimization, backtracking

Ant Colony Optimization Metaheuristic

- ACO can be applied to any discrete optimization problem for which some solution construction mechanism can be conceived.
- **Artificial ants are stochastic solution construction heuristics** that probabilistically build a solution by iteratively adding solution components to partial solutions by taking into account
 - heuristic information on the problem instance being solved, if available,
 - (artificial) pheromone trails which change dynamically at run-time to reflect the agents' acquired search experience.
- **Stochastic component** allows generating a large number of different solutions.

Ant System (AS) for TSP

- **Problem:** Given n cities, the goal is to find the shortest path going through all cities and visiting each exactly once.
 - Consider complete graph.
 - d_{ij} is Euclidean distance from city i to city j
- **Definition**
 - m is the number of ants
 - $\tau_{ij}(t)$ is the intensity of pheromone on the link (i, j) in time t
 - η_{ij} is visibility (heuristic information) expressed by $1/d_{ij}$
 - $(1-\rho)$ evaporation factor, ρ is constant for the whole opt. process
 - $tabu_k$ is dynamically growing vector of cities that have already been visited by the ant
 - **AS iteration** - each ant adds one city to the built route
 - **AS cycle** – composed of n iterations during which all ants complete their routes

AS: Pheromone Deposition

- $\tau_{ij}(t+n) = \rho \cdot \tau_{ij}(t) + \Delta\tau_{ij}$
- $\Delta\tau_{ij} = \sum_k \Delta\tau_{ij}^k$
- $\Delta\tau_{ij}^k = \begin{cases} Q/L_k, & \text{if } k\text{-th ant used the edge } (i, j) \\ 0, & \text{otherwise.} \end{cases}$

where

$\Delta\tau_{ij}^k$ is the amount of pheromone deposited on the edge (i, j) by k -th ant within a time interval $(t, t+n)$

Q is a constant

L_k is the length of the route constructed by k -th ant

ρ must be smaller than 1, otherwise the pheromone would accumulate unboundedly

$\tau_{ij}(0)$ is set to small positive values

AS: Probabilistic Decision Making

- $$p_{ij}^k(t) = \begin{cases} [\tau_{ij}(t)]^\alpha \cdot [\eta_{ij}]^\beta / \sum_{l \in \{N - tabu_k\}} [\tau_{ij}(t)]^\alpha \cdot [\eta_{ij}]^\beta, & \text{if } j \in \{N - tabu_k\} \\ 0, & \text{otherwise.} \end{cases}$$

where

$$l \in \{N - tabu_k\}$$

α, β define relative importance of the pheromone and the visibility

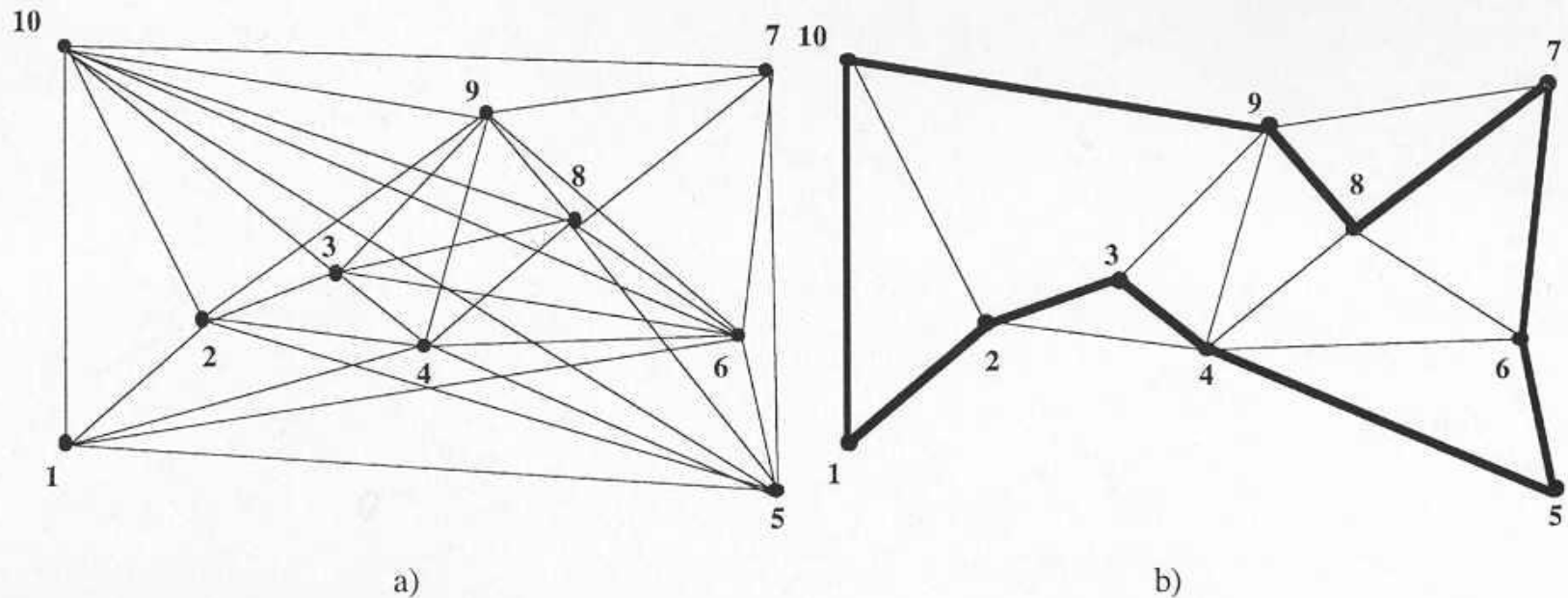
- Probability is a compromise between
 - **visibility** that prefers closer cities to more distant ones and
 - **intensity of pheromone** that prefers more frequently used edges.

AS: Cycle

- ***Ant-cycle:***
 1. Initialization
 - time: $t=0$
 - number of cycles: $NC=0$
 - pheromone: $\tau_{ij}(t)=c$
 - Initial positioning of m ants to n cities
 2. Initialization of *tabu* lists
 3. Ants' action
 - Each ant iteratively builds its route
 - Calculate length of the routes L_k for all ants $k \in (1, \dots, m)$
 - update the shortest route found
 - Calculate $\Delta\tau_{ij}^k$ and update $\tau_{ij}(t+n)$
 4. Increment discrete time
 - $t = t+n, NC = NC+1$
 5. If($NC < NC_{max}$) then goto step 2
else stop

AS: Setting ρ

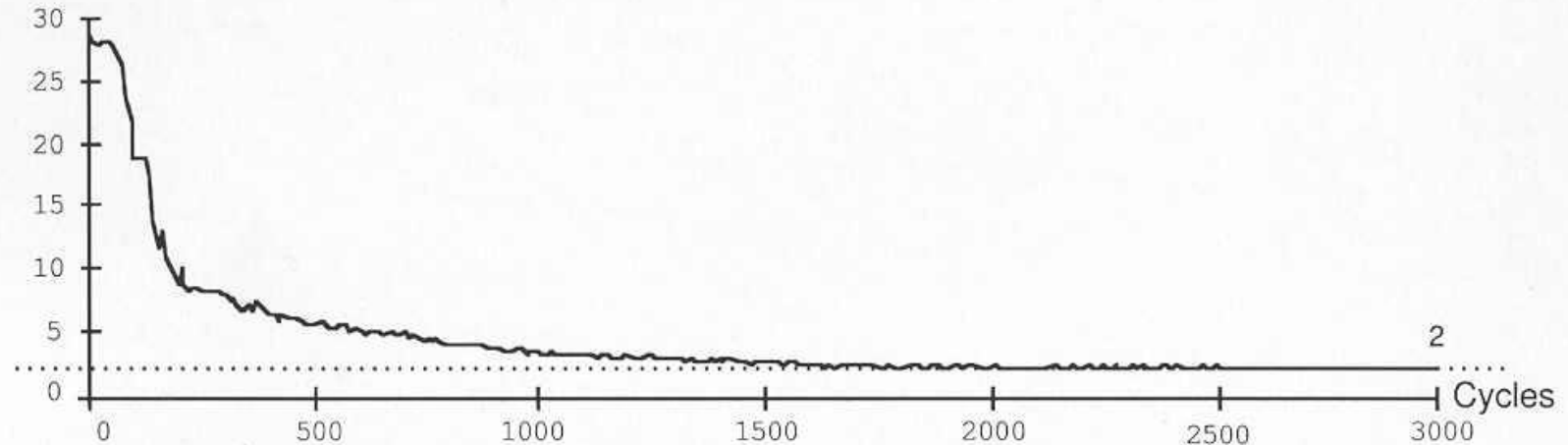
- Optimal value of ρ is 0.5
 - After greedily searching the space it is desirable to adapt global information stored in $\tau_{ij}(t)$
(it is necessary to partially forget)



AS: Meaning of α , β

- α , β (recommended setting: $\alpha=1$, $\beta=2$)
 - High values of α mean that the intensity of pheromone is dominant so that ants choose the route that was used by their predecessors.
 - Low values of α change the method to stochastic re-started greedy algorithm.

Average node branching



- **Stagnation** – branching factor is 2, all ants go the same way.

AS: Elitism

- Intensity of pheromone is strengthened on edges that lie on the shortest path out of all generated paths
 - Amount of added pheromone: $e \cdot Q/L^*$,
where e is a number of „elite“ ants and L^* is the shortest path
 - **Beware of premature convergence.**

General ACO metaheuristic

procedure *ACO metaheuristics*

ScheduleActivities

 ManageAntActivity()

 EvaporatePheromone() // forgetting

 DaemonActions() {optional} // centralized actions
 local search, elitism

end ScheduleActivities

end *ACO metaheuristics*

Steps for implementing ACO

- Choose appropriate graph representation
- Define positive feedback
- Choose constructive heuristic
- Choose a model for constraint handling (*tabu* list at TSP)

Applications of ACO algorithms

- **Static problems**
 - Traveling salesman
 - Quadratic assignment
 - Job-shop scheduling
 - Vehicle routing
 - Graph colouring
 - Shortest common supersequence
- **Dynamic problems**
 - Network routing

References

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<http://iridia.ulb.ac.be/~mdorigo/ACO/ACO.html>



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