Computed tomography (CT)

Jiří Hozman, Jan Kybic

2005–2019

Základní uspořádání systému CT

CT history

1917 mathematical theory (Radon)

1956 tomography reconstruction in radioastronomy (Bracewell)

1963 CT reconstruction theory

1971 CT principles demonstrated (Hounsfield)

1972 first working CT for humans (EMI, London, Hounsfield)

1973 PET

1974 Ultrasound tomography

1982 SPECT

1985 Helical CT

1998 Multislice CT, 0.5 s/frame

Johann Radon

(matematik)

*** 16.12.1887 Děčín, ČR**
† 25.5.1956 Vídeň, Rakousko

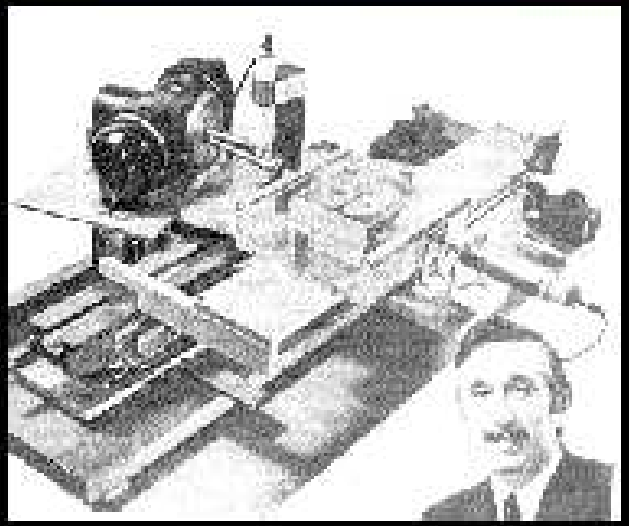
1917 - "Uber die Bestimmung von Funktionen durch ihre Integral-werte langs gewisser Mannigfaltigkeiten", *Berichte Sachsische Akademie der Wissenschaften. Leipzig, Math.-Phis. Kl.*, v.69, pp. 262-267. V této práci pan Radon matematicky vyřešil rekonstrukci prostorového obrazu na základě znalosti jeho projekcí.

Sir Godfrey Newbold Hounsfield

1919-2004

Nottinghamshire, samouk, nenavštěvoval univerzitu
Nobelova cena 1979

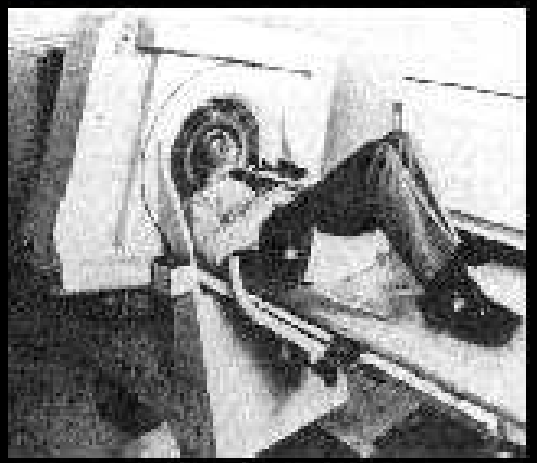
Sir Godfrey Hounsfield
Nobel Prize in Medicine, 1979



[2]

Gamma Ray Source: 28,000 measurements, 9 day collection, 2.5 hour recon, 2hour display.

X-ray source reduced collection to 9 hours. Clinical model took 18 sec



[1]

EMI-1, 1971:
Atkinson Morley Hospital, England

Allan M. Cormack

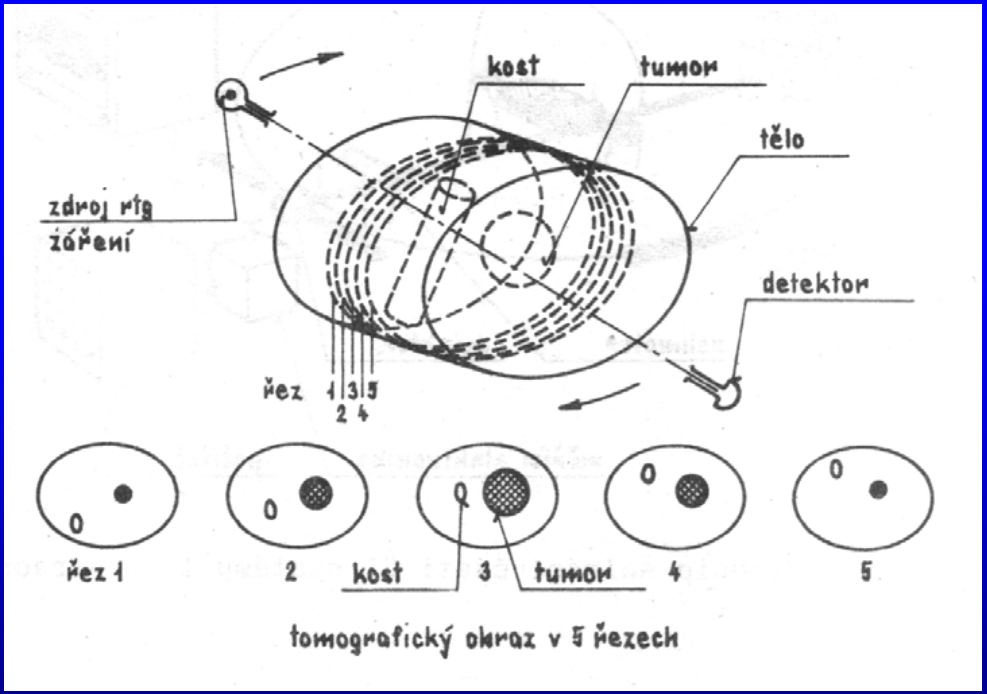
1924-1998, narozen v Johannesburgu

Tomography modalities

- x-rays — CT
- gamma rays — PET, SPECT
- light — optical tomography
- RF waves — MRI
- DC — electric impedance tomography
- ultrasound — ultrasound tomography

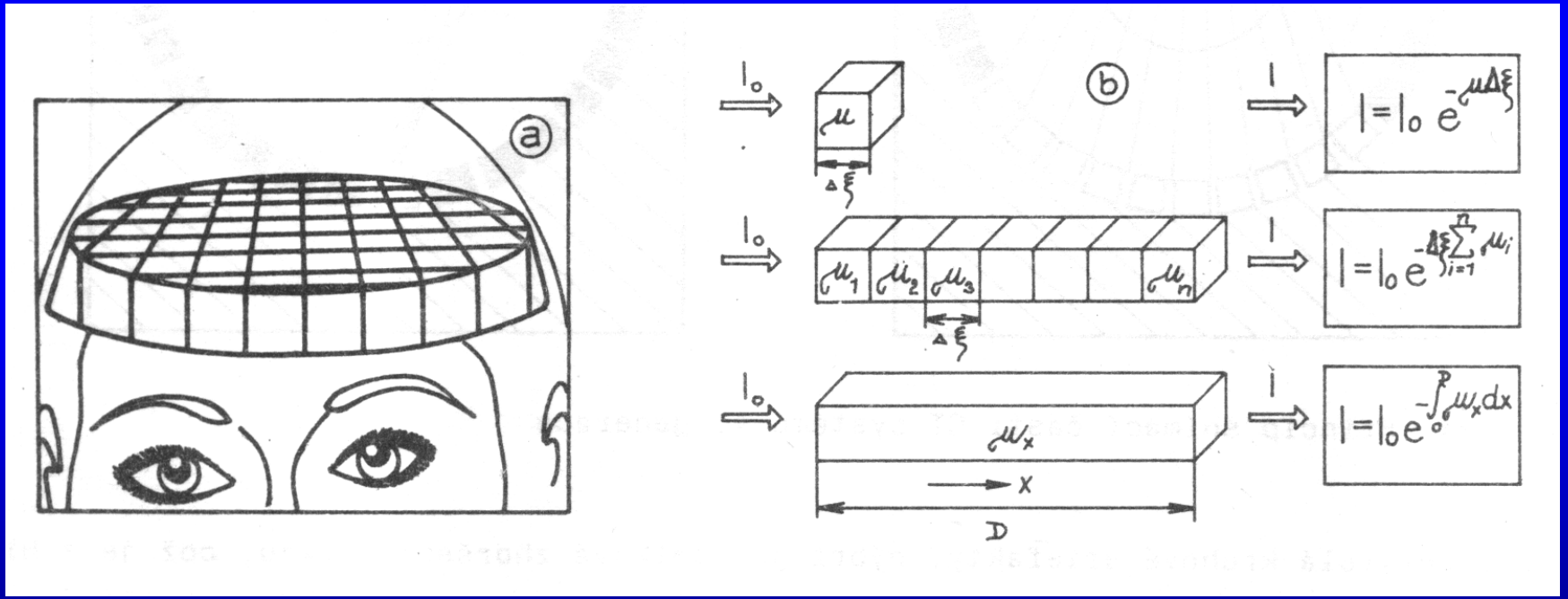
Základní princip CT

CT vytváří obraz těla pacienta jako sérii tomografických sekcí (řezů). Každý řez je vytvořen matematickou rekonstrukcí předmětu ze znalosti průmětů (projekcí) předmětu do různých směrů.

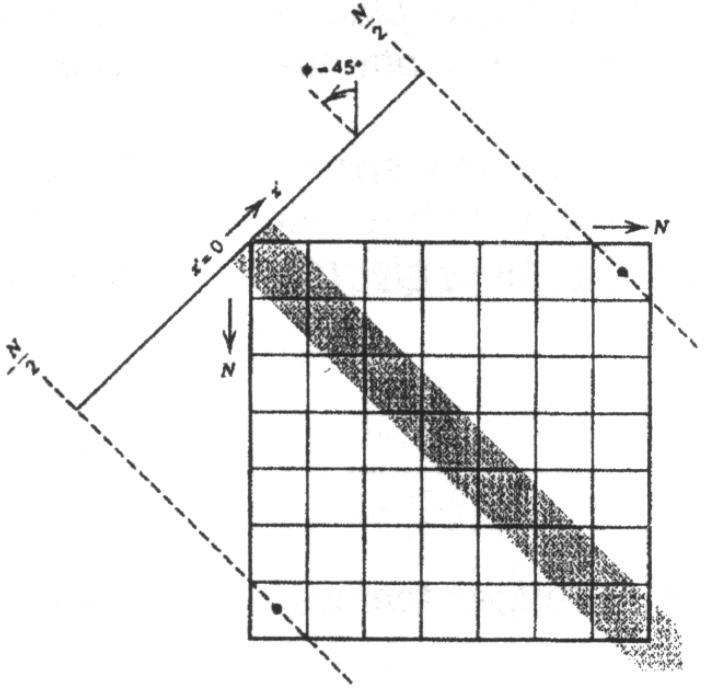


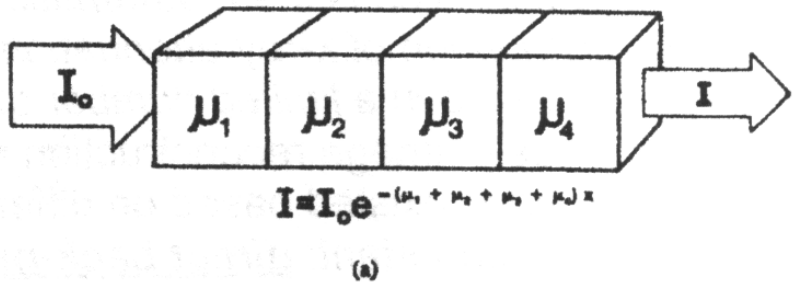
Základní princip CT

Jednotlivé řezy objektu musí být rozděleny do sítě malých objemových elementů (voxels) se čtvercovou základnou a s konstantní hodnotou útlumu.

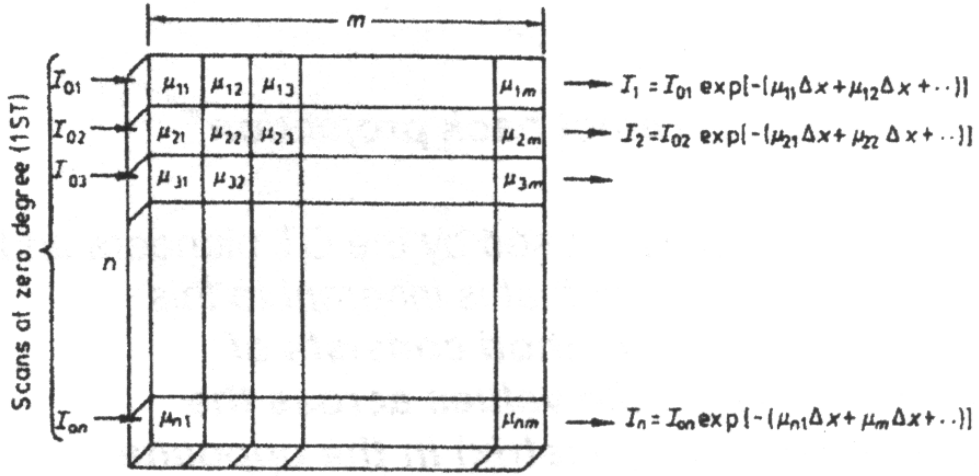


Základní fyzikální princip CT

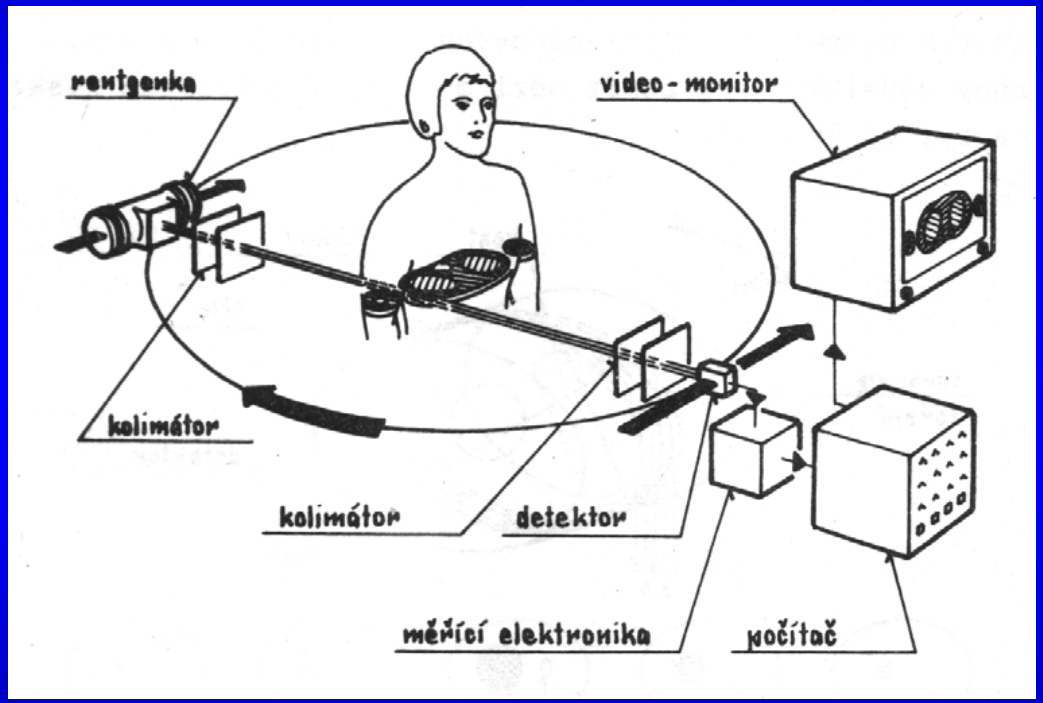


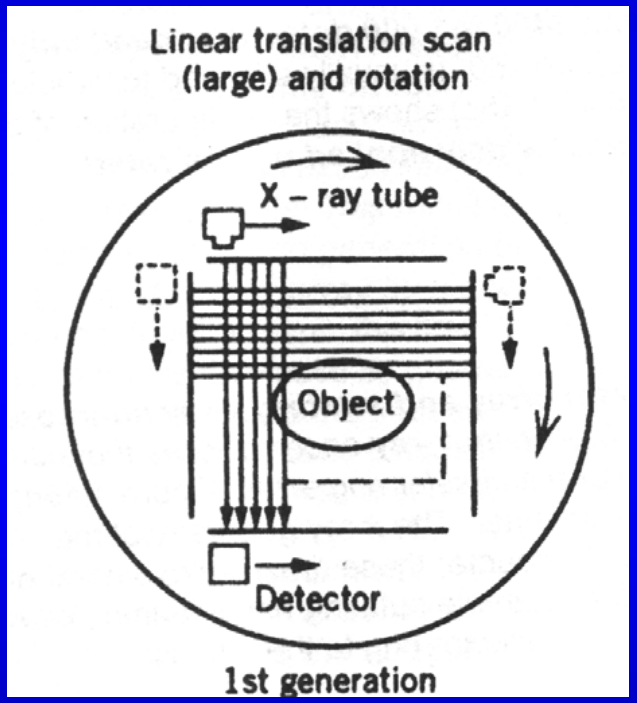


(a)

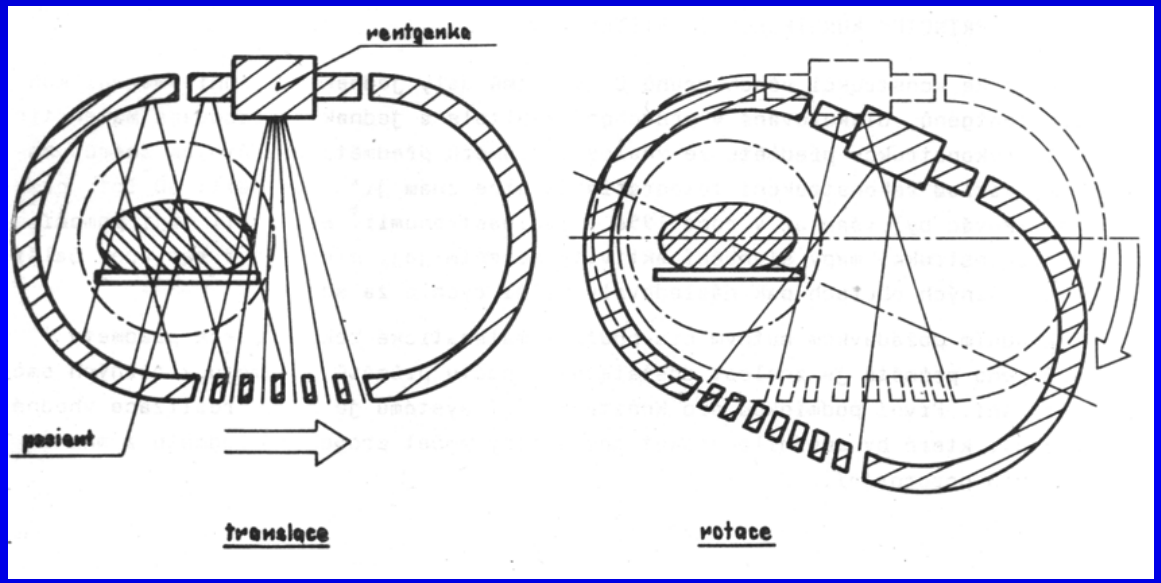


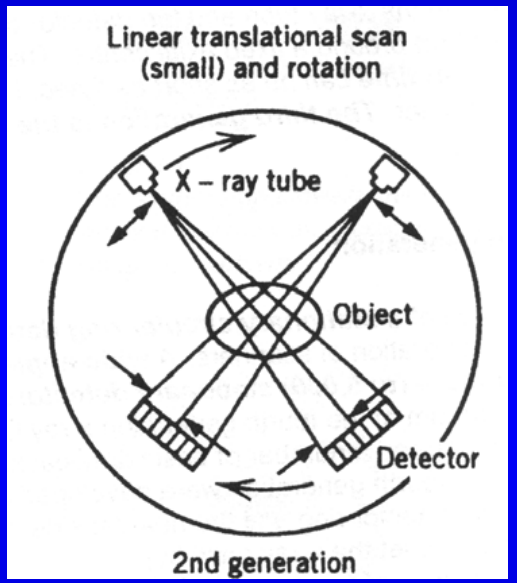
CT systémy 1. generace



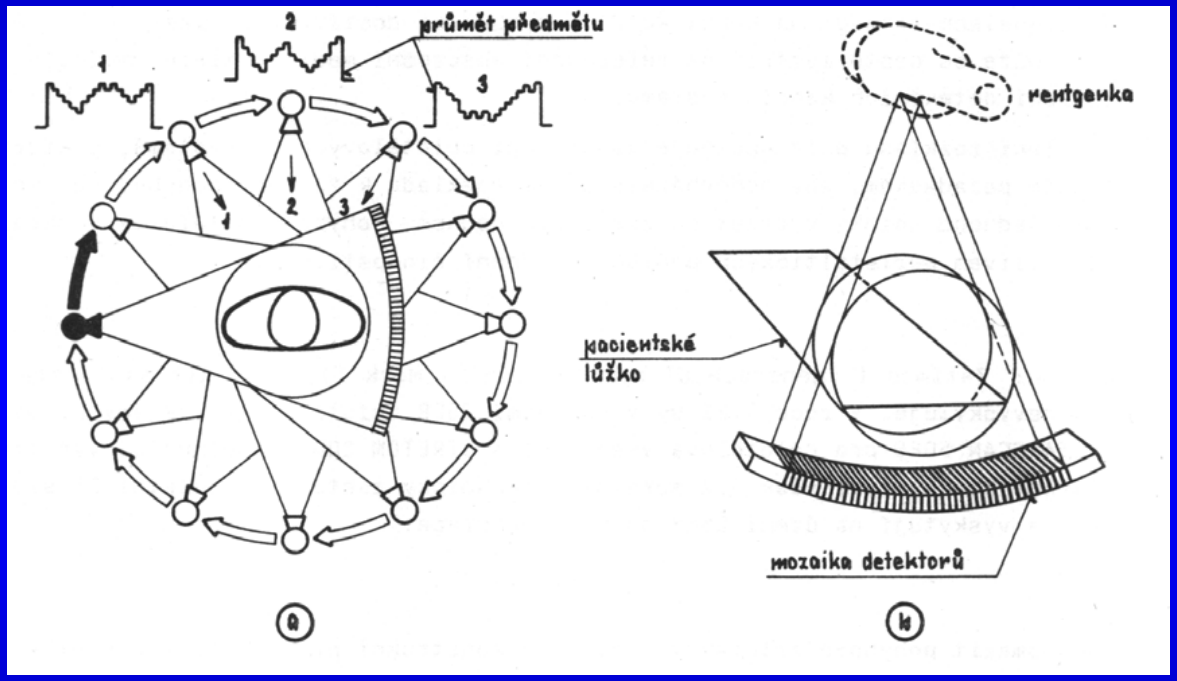


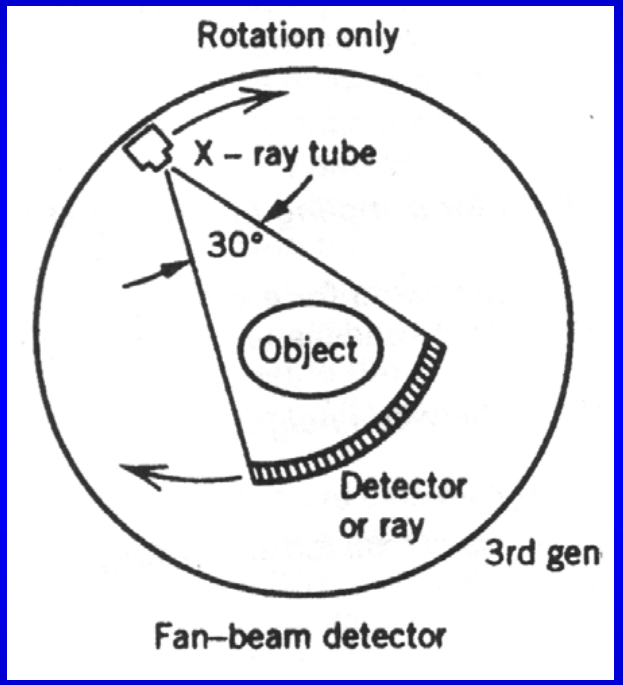
CT systémy 2. generace





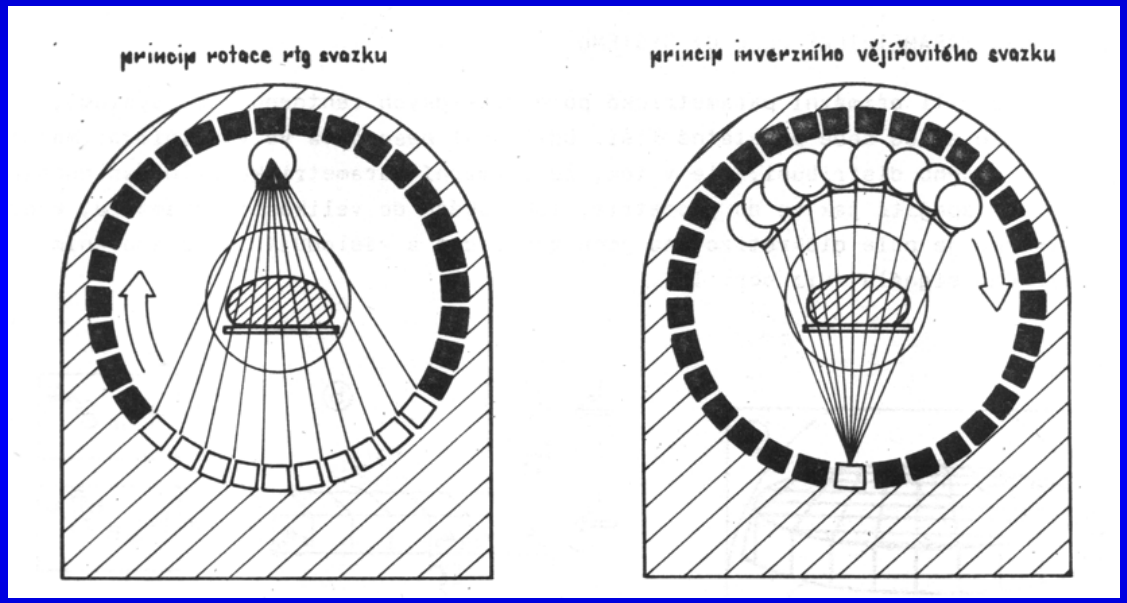
CT systémy 3. generace

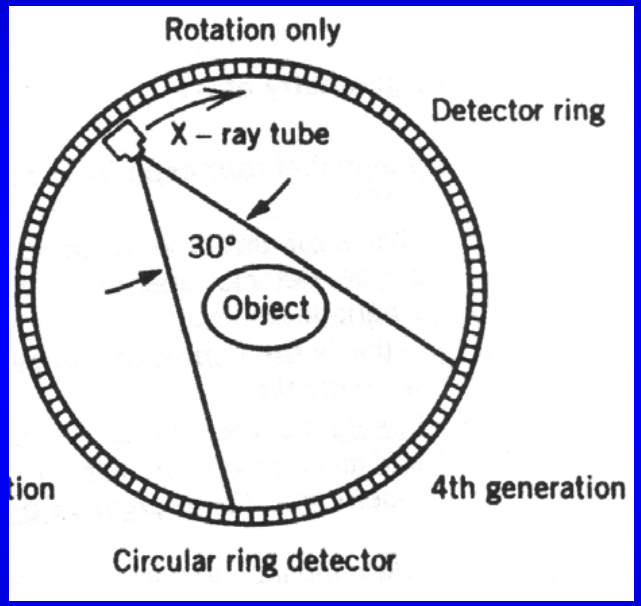




asi nejčastěji používané

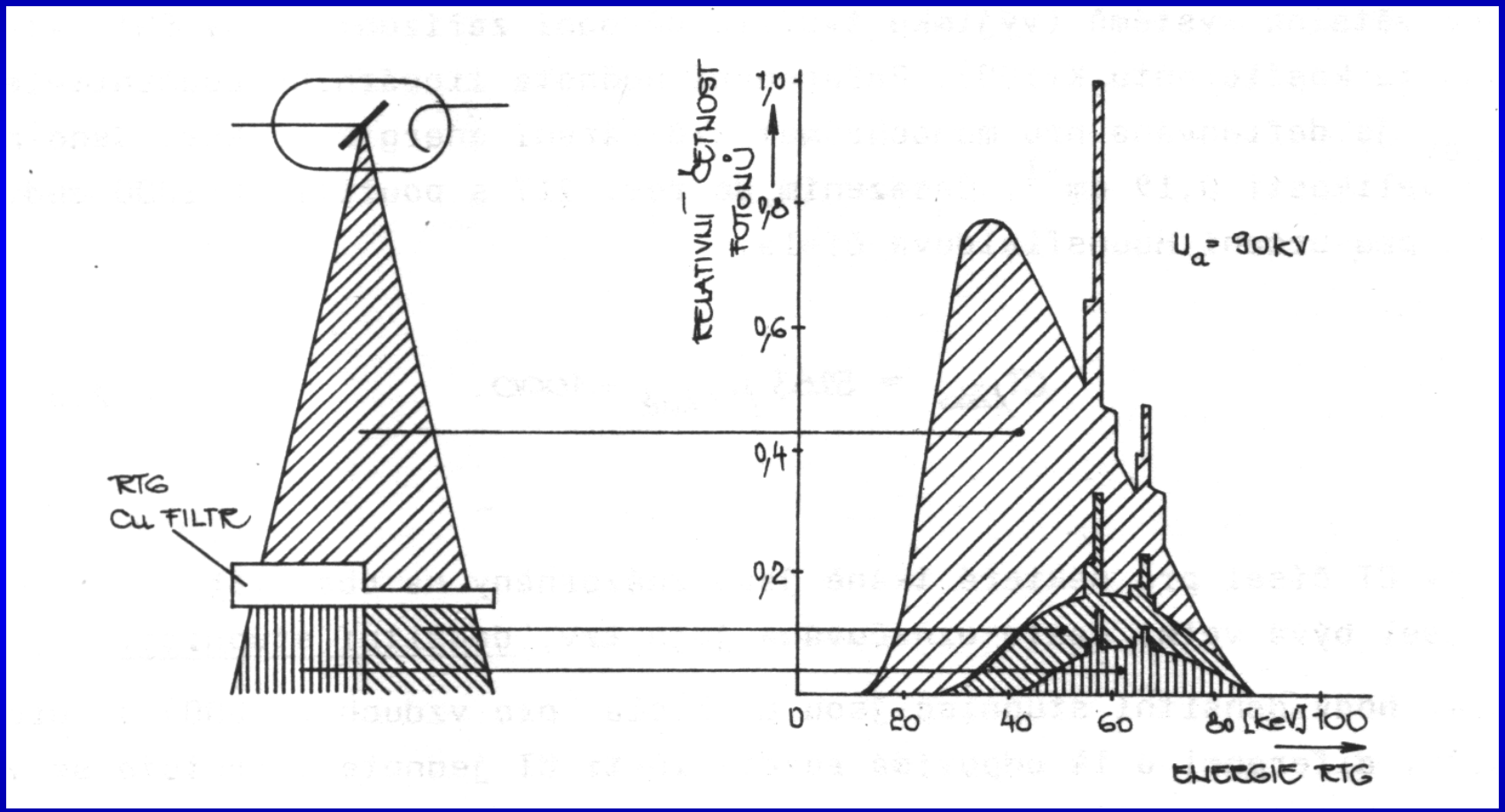
CT systémy 4. generace



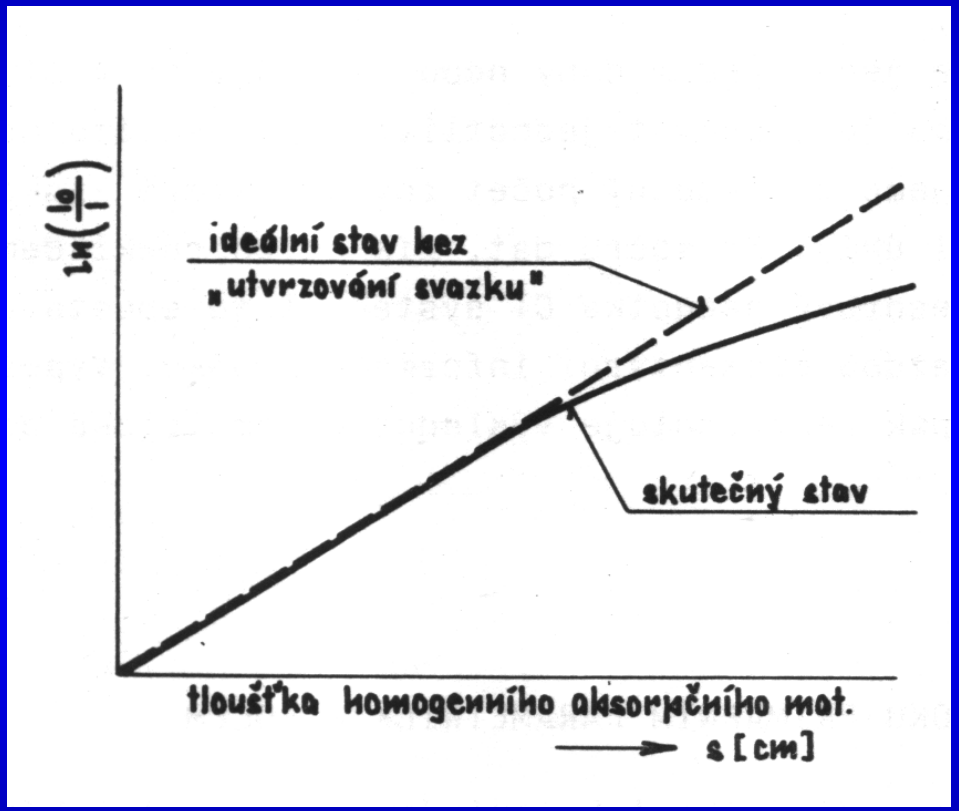


Rotuje jen zdroj, detektory stabilni

„Utvrzování svazku“ (beam hardening)



„Utvrzování svazku“ (beam hardening)



CT číslo - Haunsfieldovo číslo (HU)

Je vyjádřením kvantitativního hodnocení absorbčních vlastností tkáně.

$$CT = K \cdot \frac{\mu_{tkáně} - \mu_{vody}}{\mu_{vody}}$$

$$K = 1000$$

$$\mu_{vody} = 0{,}19\ cm^{-1}$$

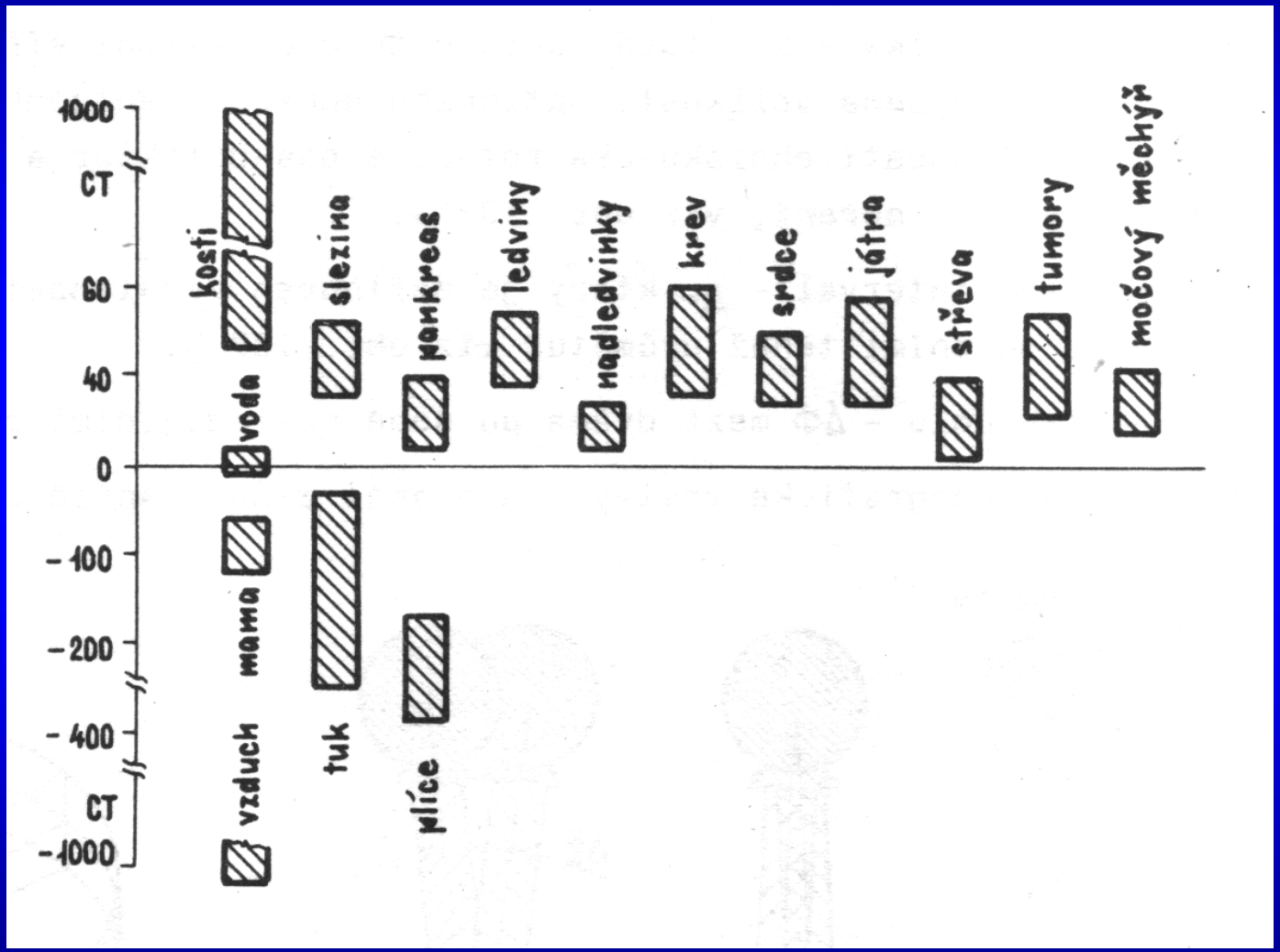
Měřeno monochromatickým zářením 73 keV.

$$CT = 5263\ \mu_{tkáně} - 1000$$

stupnice CT čísel = denzitní stupnice

rozsah od -1000 až zhruba +1000, pro vzduch -1000, pro vodu 0

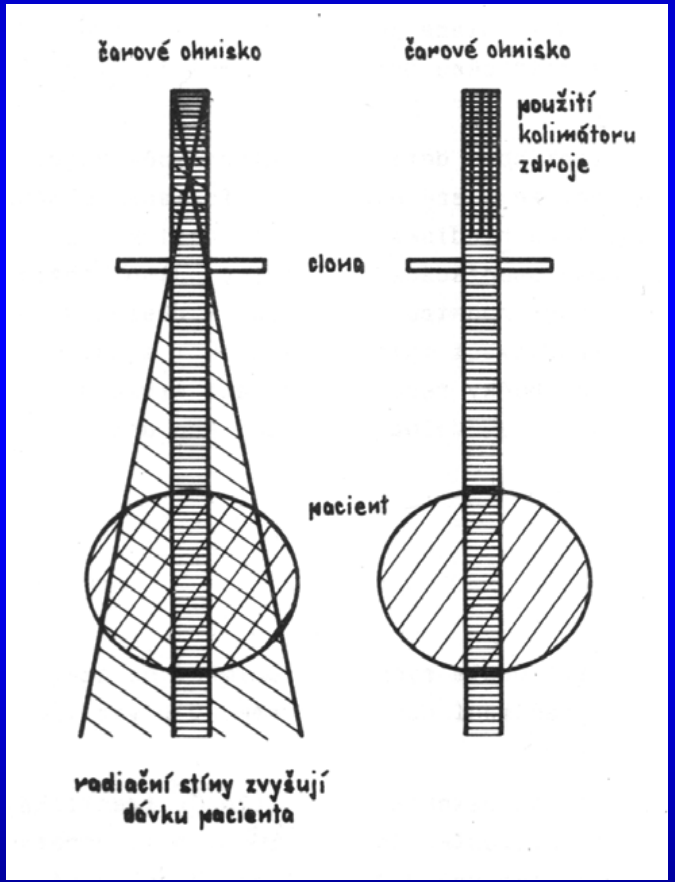
CT číslo - Haunsfieldovo číslo (HU)



Generace, zpracování a detekce radiačního signálu CT systémů

CT systémy 2. generace
(několik detektorů)

- pomalé a rychlé systémy
- sendvičový a lamelový kolimátor



Generace, zpracování a detekce radiačního signálu CT systémů

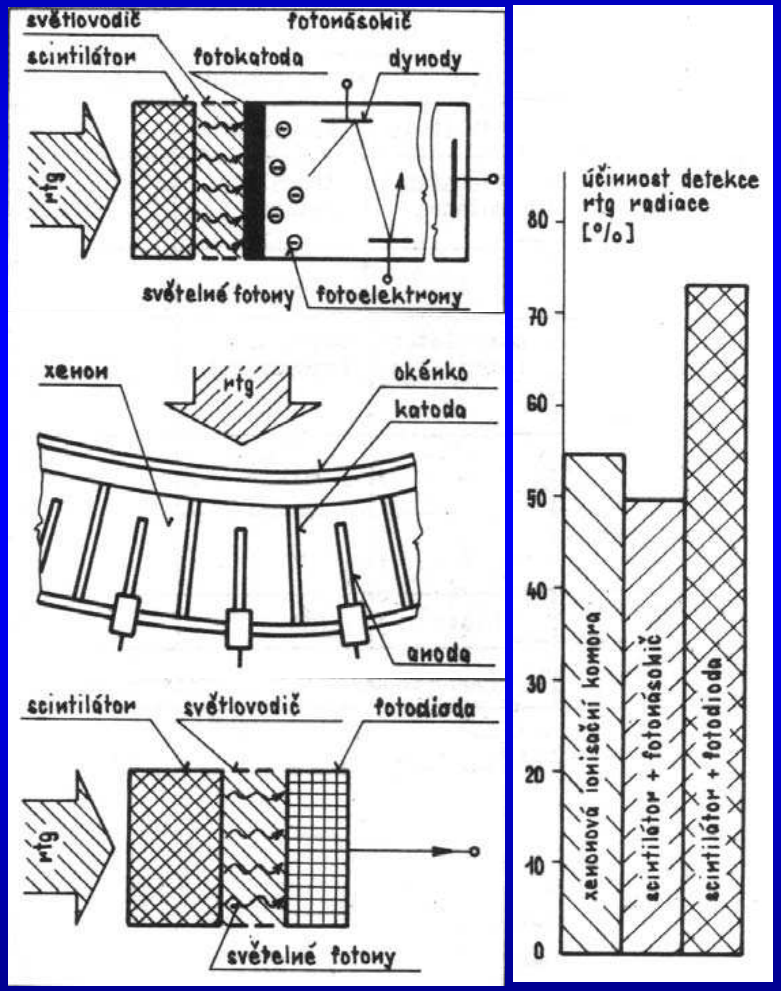
Detektory

scintilační detektor (krystal) + fotonásobič

ionizační komory plněné plynem (xenon)

scintilační detektor (krystal) + fotodioda (fototranzistor)

Flat-panel detector (FPD)
Thin-film transistor (TFT) array



Základní principy rekonstrukce obrazu

$o(\xi,\eta)$ denzitní funkce = předmětová funkce

lin. součinitel zeslabení

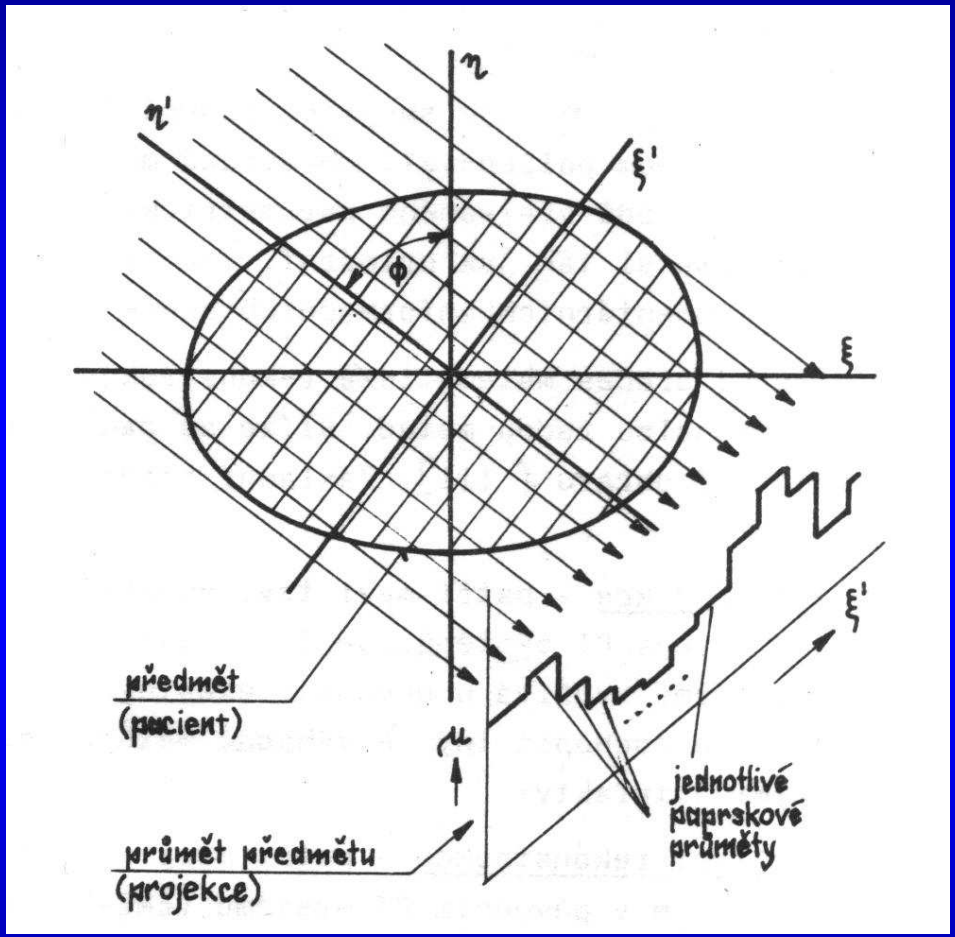
(ξ,η) původní souř.

Φ snímací úhel

ξ' rotovaná souř.

η' rotovaná souř.

$p(\xi',\Phi)$ paprskový součet či průmět



Základní principy rekonstrukce obrazu

$$p(\xi',\Phi)=\int o(\xi,\eta)\,d\eta' \qquad I=I_0\exp\left[-\int \quad (\xi,\eta)d\eta\right]$$

$$o(\xi,\eta)\approx \quad (\xi,\eta)$$

$$p(\xi',\Phi)=-\ln\frac{I_0}{I}$$

$$\xi'=\xi.\cos\Phi+\eta.\sin\Phi \qquad \xi=\xi'.\cos\Phi-\eta.\sin\Phi$$

$$\eta'=-\xi.\sin\Phi+\eta.\cos\Phi \qquad \eta=\xi'.\sin\Phi+\eta.\cos\Phi$$

Radon transform

Projection in polar coordinates:

$$P_\varphi(\xi') = \mathscr{R}\big[o(\xi,\eta)\big]$$
$$P_\varphi(\xi') = \int_L o(\xi,\eta)\mathrm{d}l$$

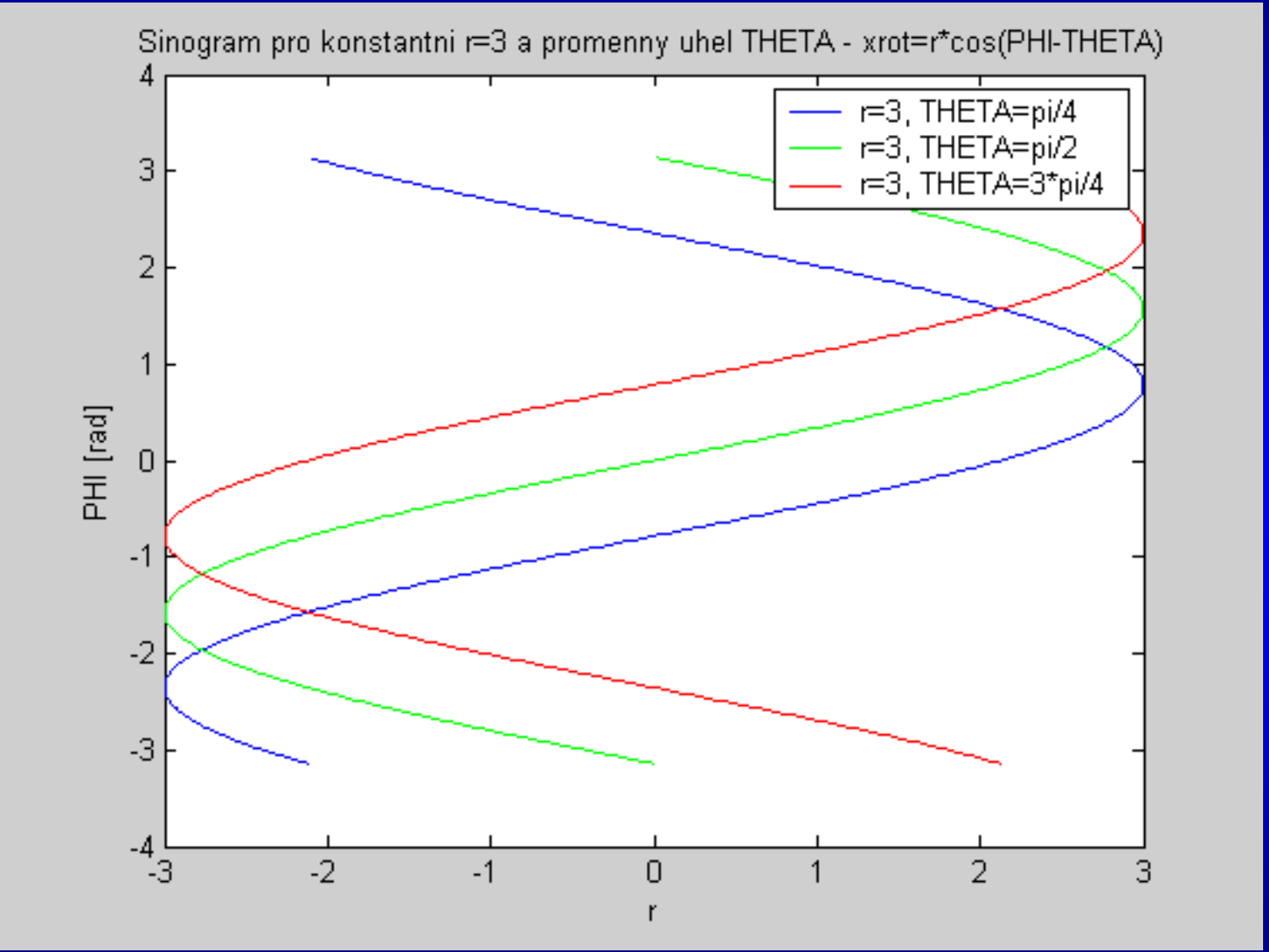
along the line L defined by φ a ξ':

$$\xi' = \xi\cos\varphi + \eta\sin\varphi$$

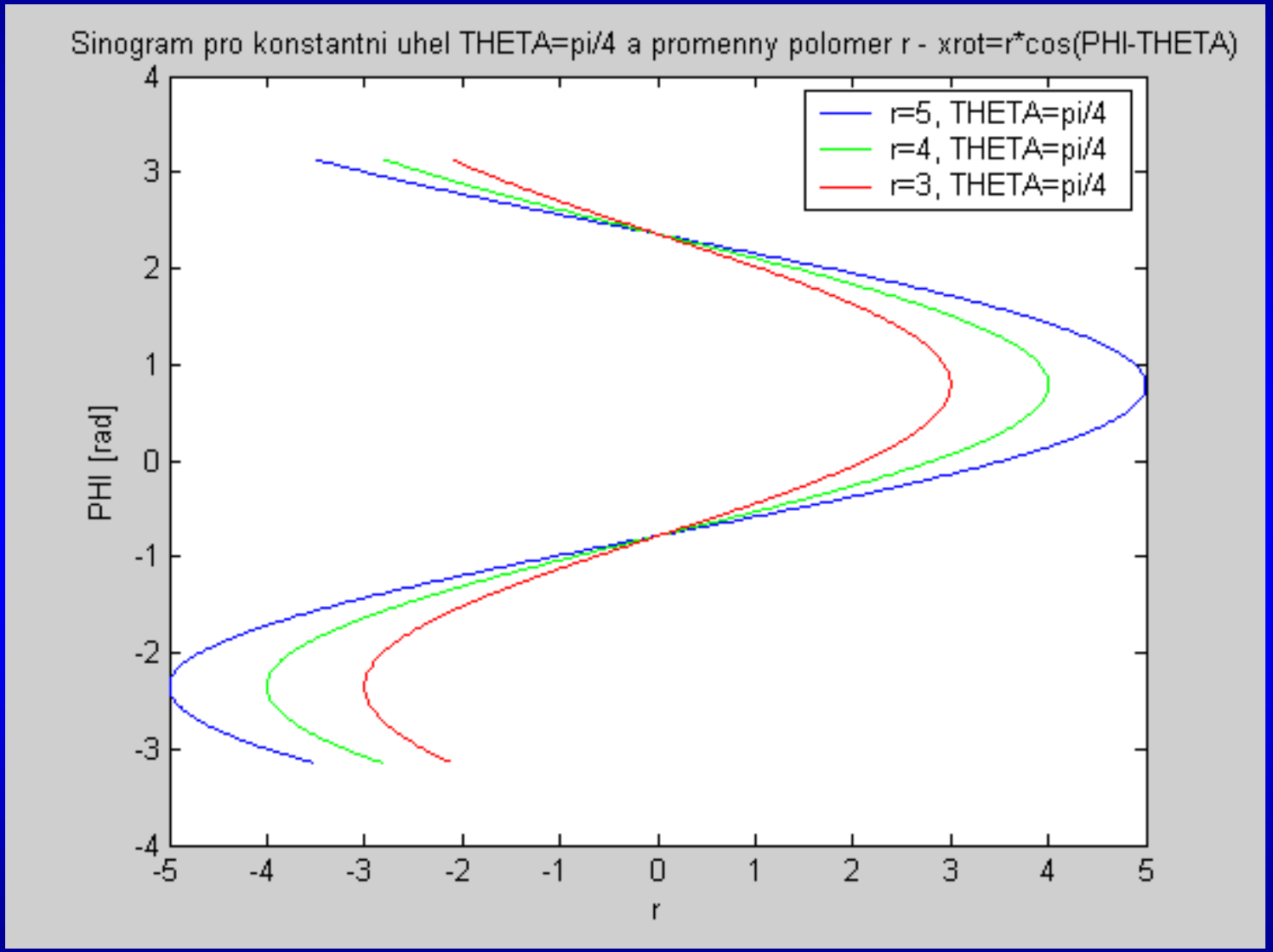
Equivalently

$$P_\varphi(\xi') = \int o(\xi'\cos\varphi - \eta'\sin\varphi, \xi'\sin\varphi + \eta'\cos\varphi)\mathrm{d}\eta'$$

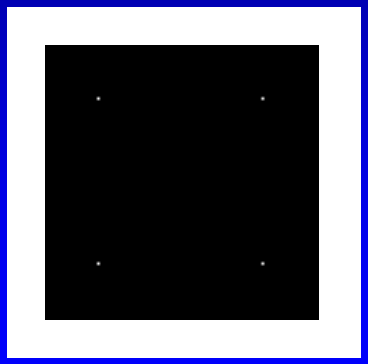
Radonova transformace

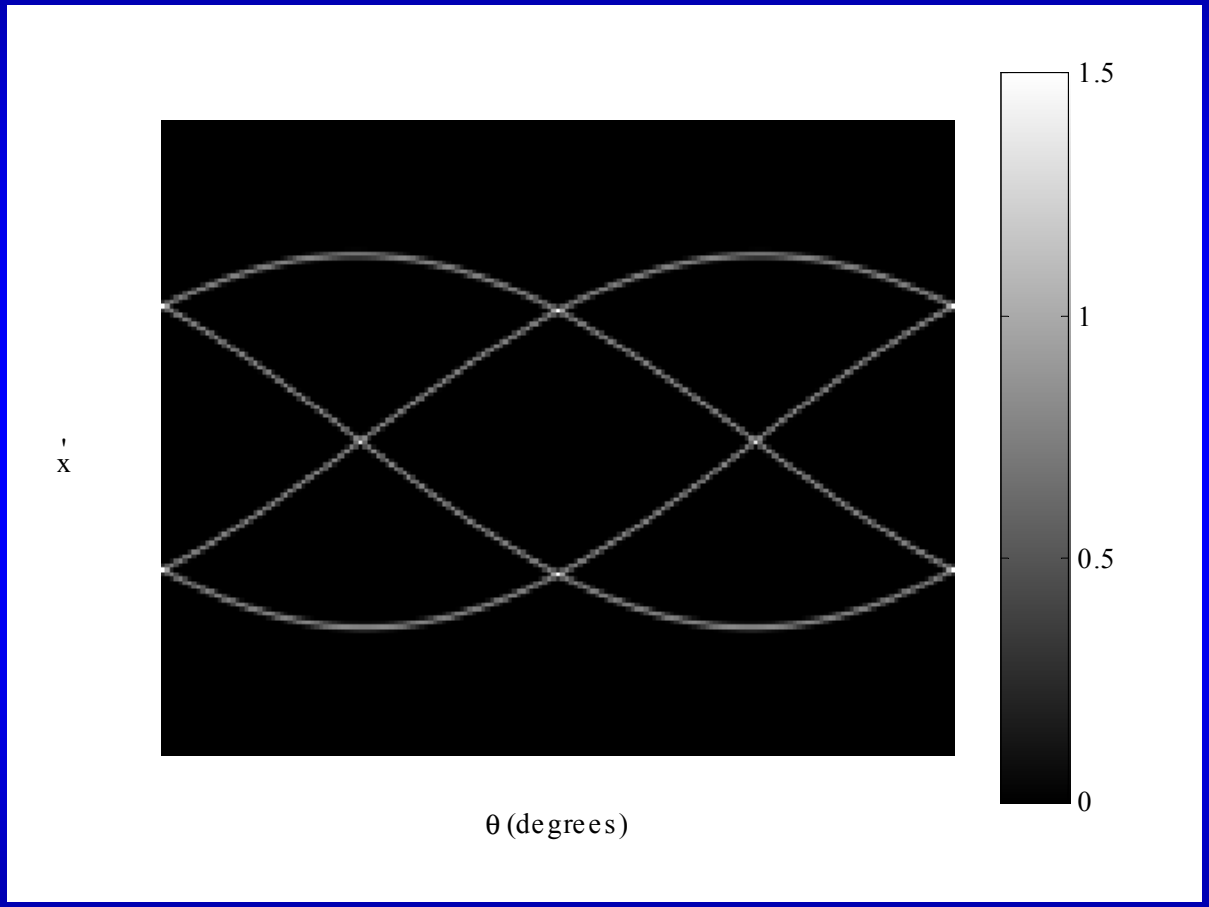


Radonova transformace

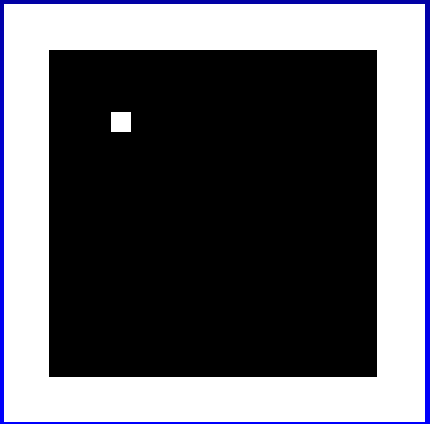


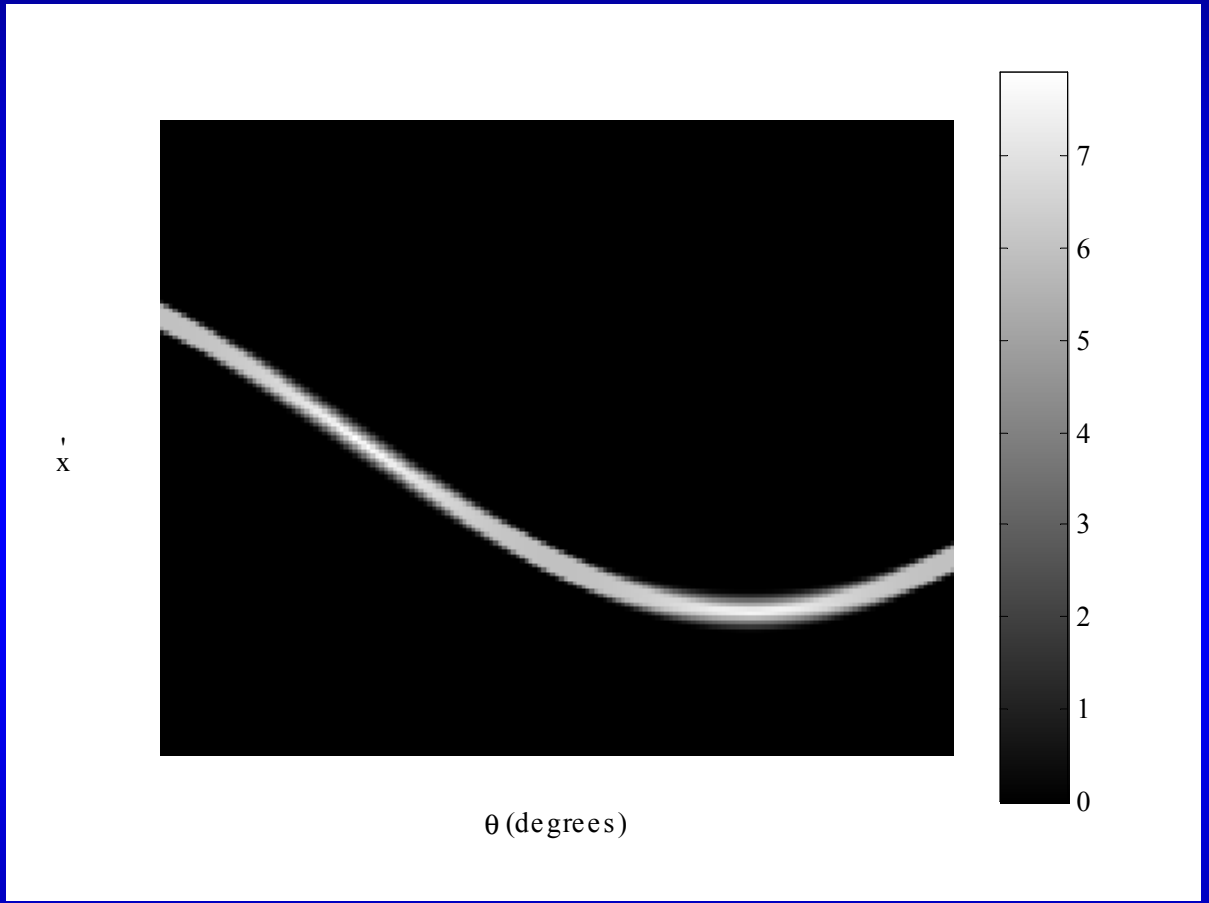
Radonova transformace



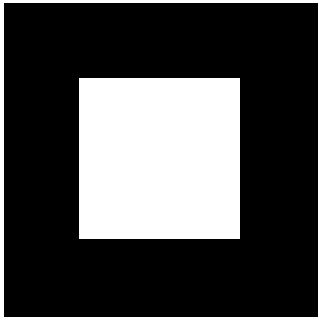


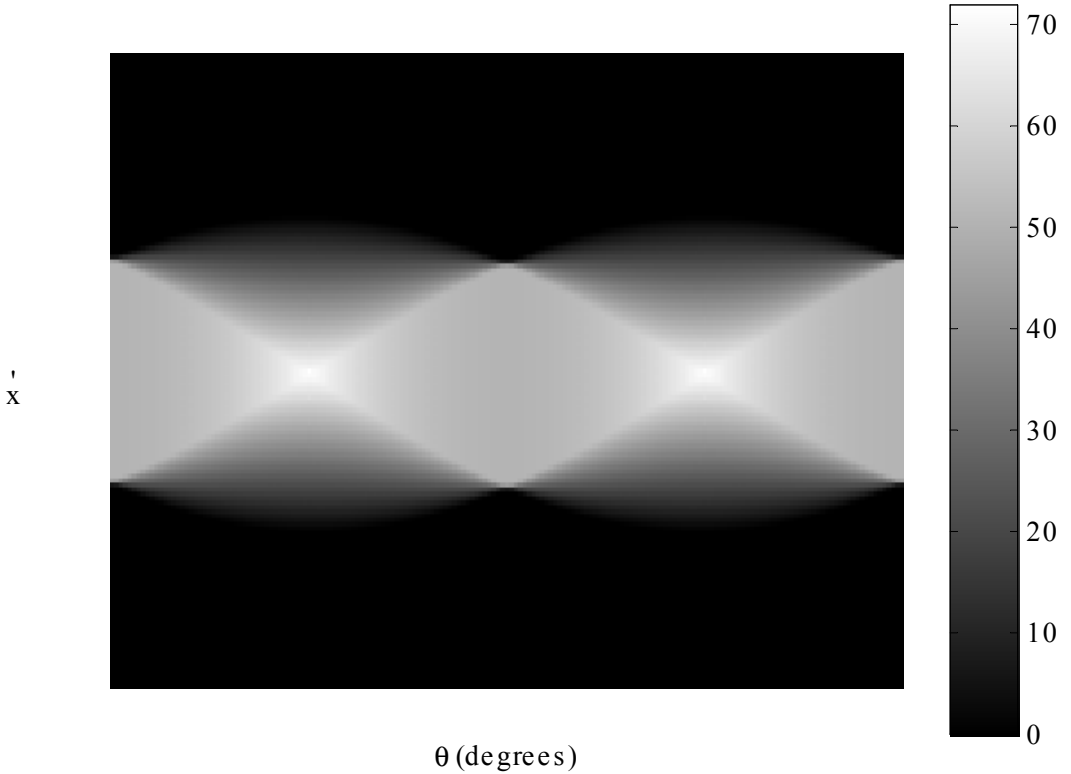
Radonova transformace



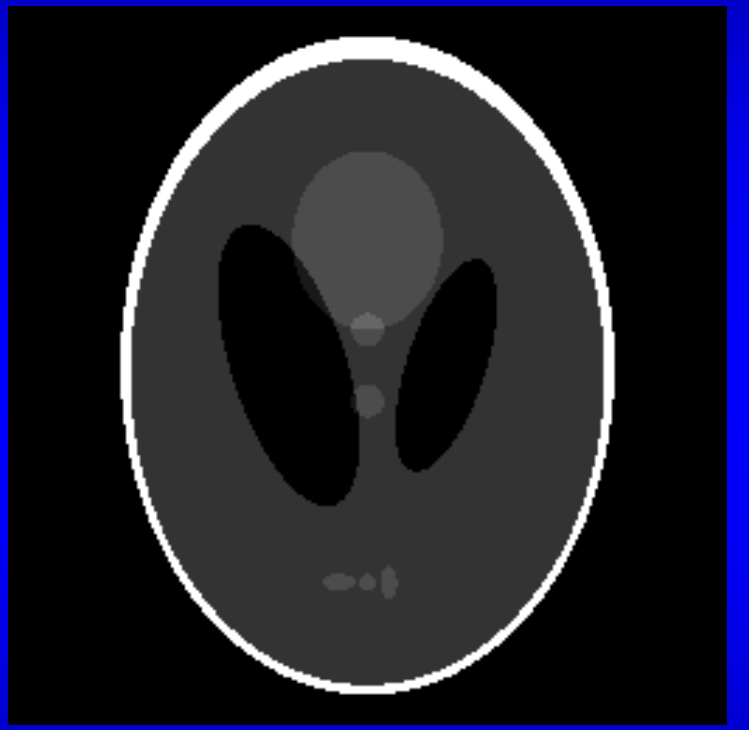


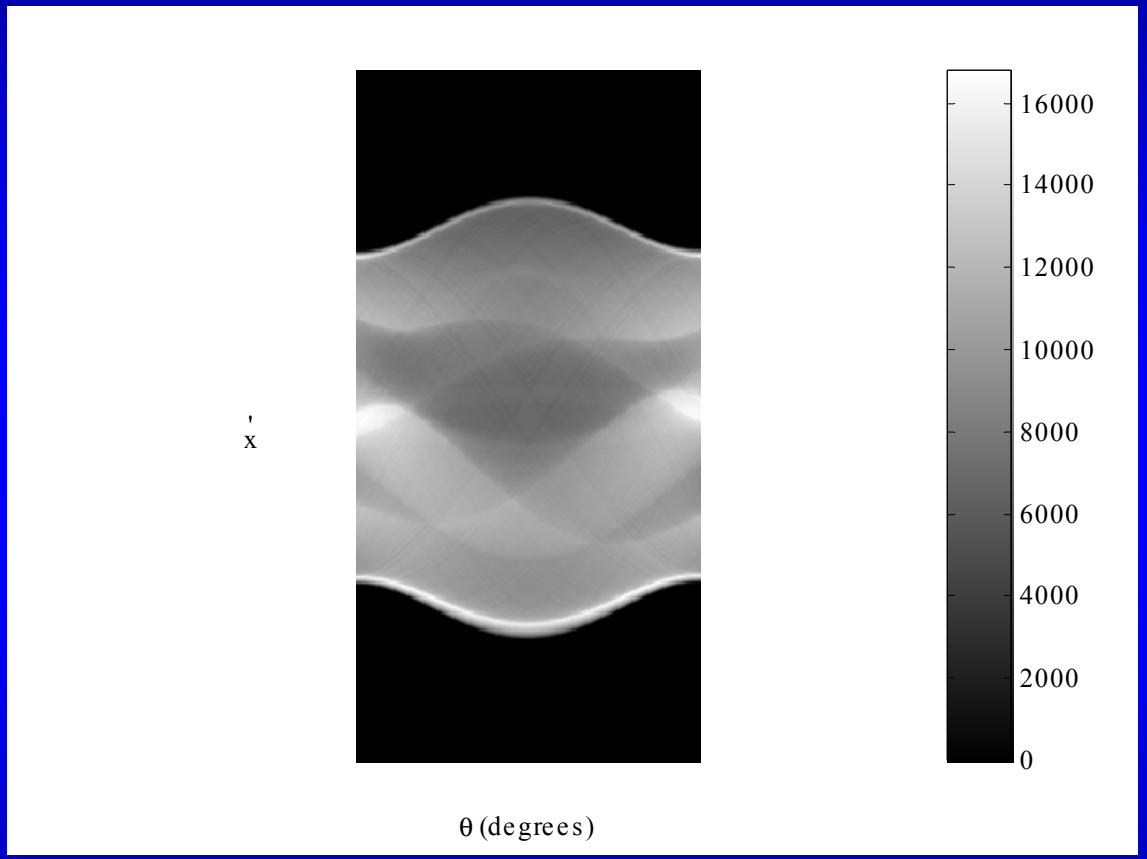
Radonova transformace





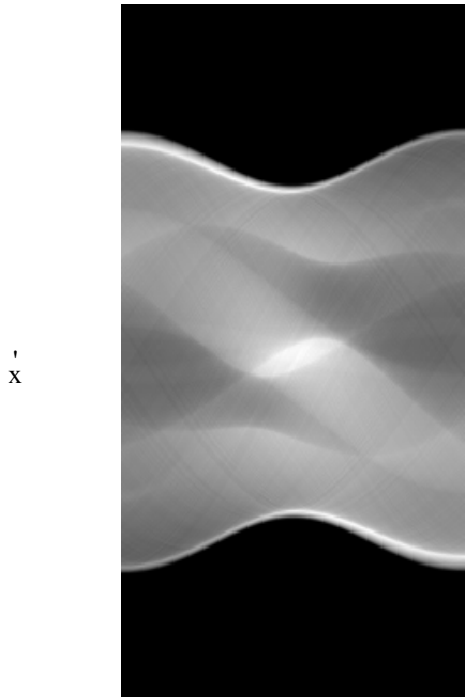
Radonova transformace

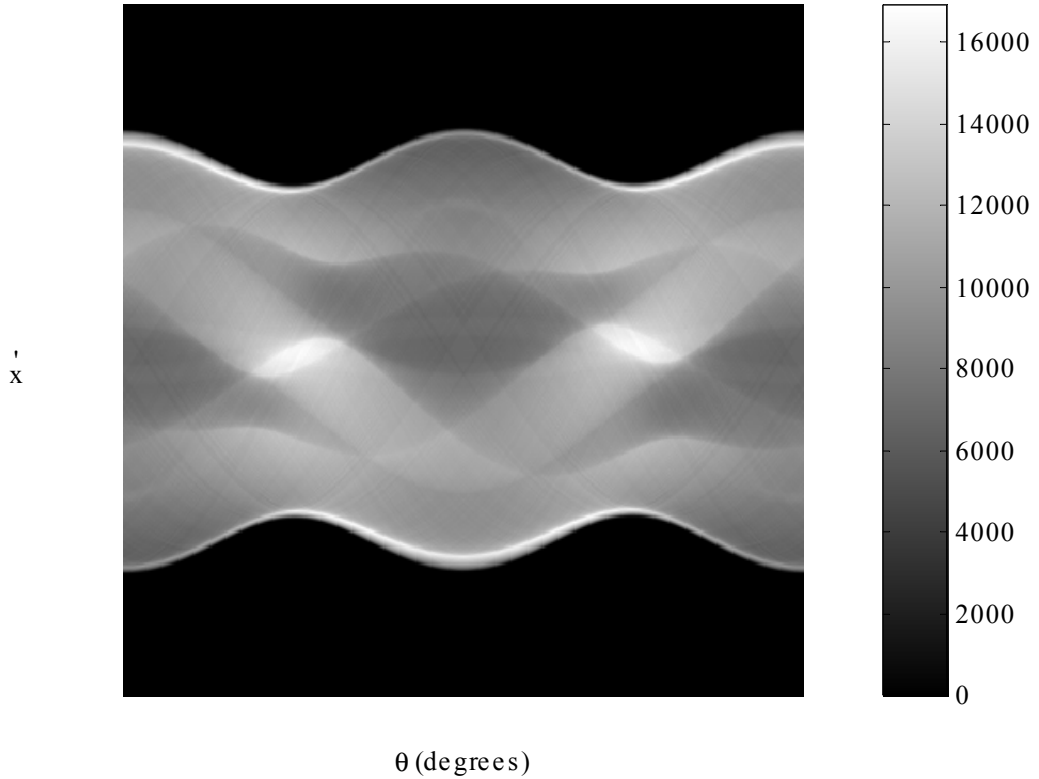




Shepp-Logan fantom

Radonova transformace





Periodicity RT vůči úhlu

Reconstruction methods

- *Backprojection*
- *Fourier reconstruction*
- Filtered backprojection
- Algebraic reconstruction (iterative)

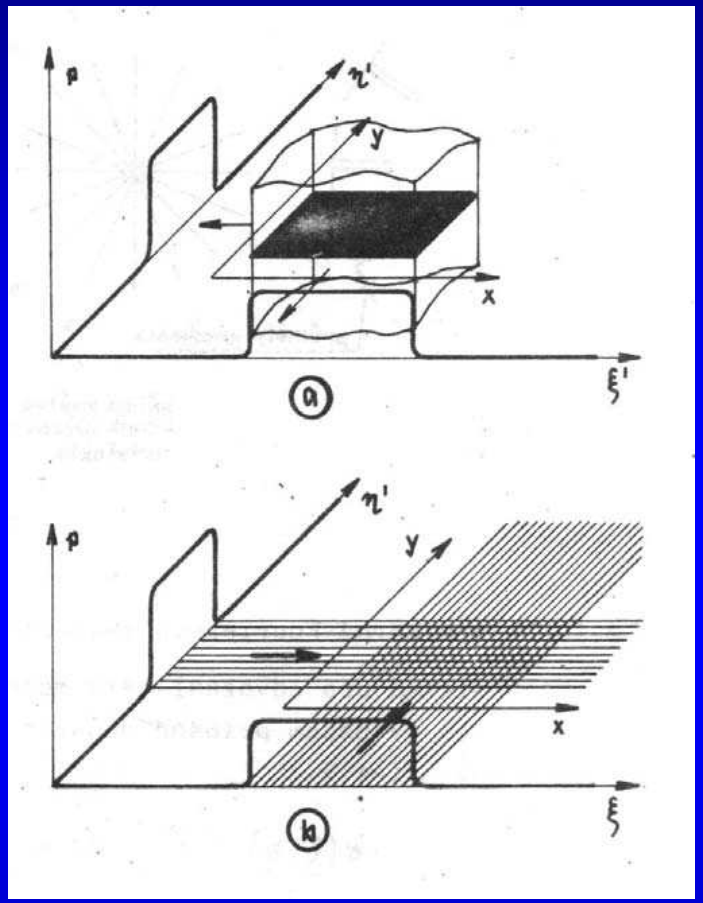
Přímá zpětná projekce

$$\mathrm{i}(x,y) = \sum_{j=1}^{m} \mathrm{p}(\xi', \Phi_j)\Delta\Phi$$

Φ_j **j-tý projekční úhel**

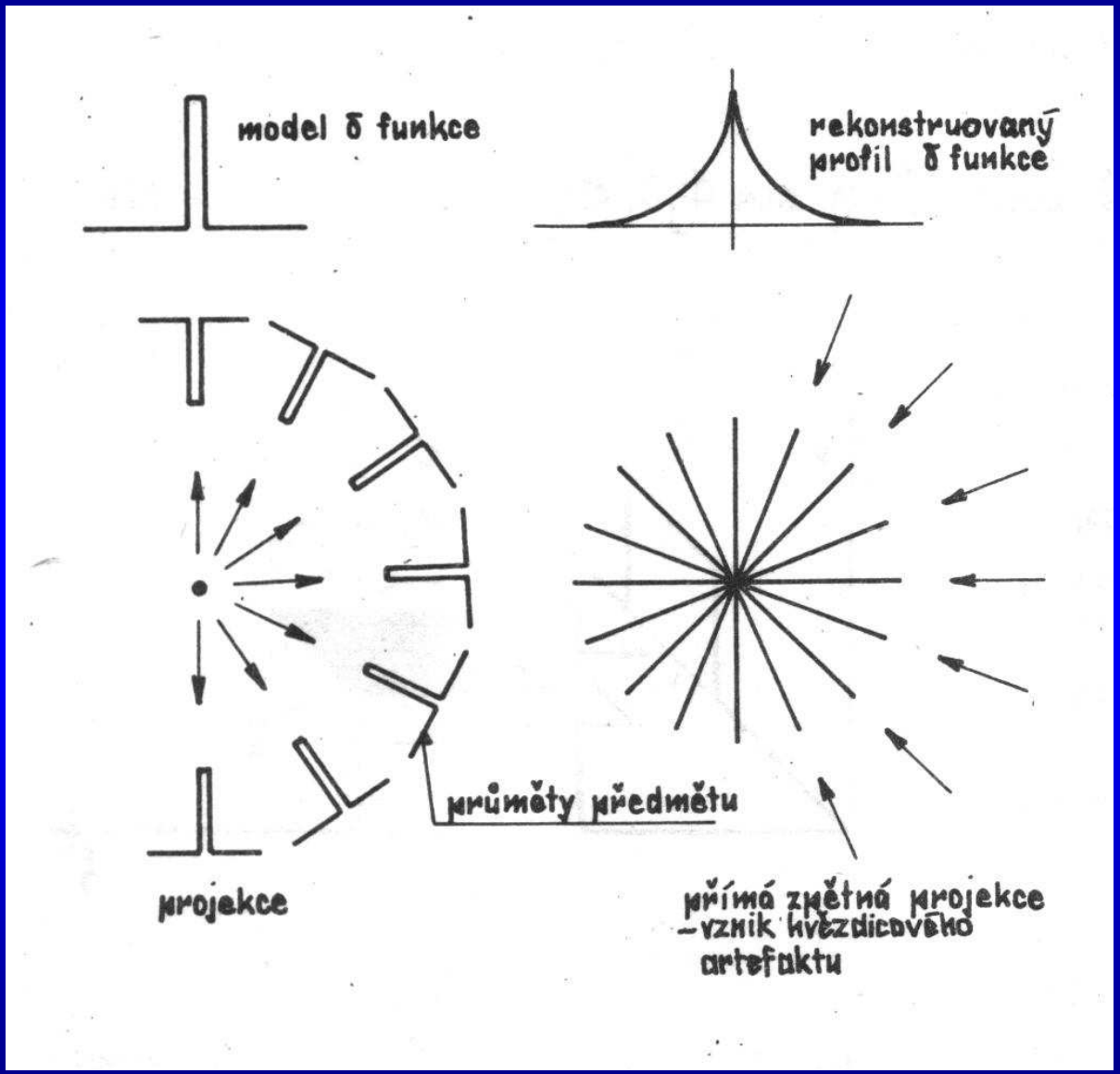
$\Delta\Phi$ **úhlový přírůstek mezi projekcemi**

m **počet projekcí**



$$\mathrm{i}(x,y) = \sum_{j=1}^{m} \mathrm{p}\left(\left(x.\cos\Phi_j + y.\sin\Phi_j\right), \Phi_j\right)\Delta\Phi$$

Přímá zpětná projekce - hvězdicový artefakt



Central slice theorem

Projection Theorem, Věta o centrálním řezu)

$$P_\varphi(\xi') = \int o(\xi' \cos\varphi - \eta' \sin\varphi, \xi' \sin\varphi + \eta' \cos\varphi)\mathrm{d}\eta'$$

Fourier transform of the Radon transform by ξ':

$$\begin{aligned}\mathscr{F}\left\{\mathscr{R}\left[o(\xi,\eta)\right]\right\} = \mathscr{F}\left\{P_\varphi(\xi')\right\} = \hat{P}_\varphi(\omega) &= \int P_\varphi(\xi')\mathrm{e}^{-2\pi j\omega\xi'}\,\mathrm{d}\xi' \\ &= \iint o(\xi' \cos\varphi - \eta' \sin\varphi, \xi' \sin\varphi + \eta' \cos\varphi)\mathrm{e}^{-2\pi j\omega\xi'}\,\mathrm{d}\xi'\mathrm{d}\eta'\end{aligned}$$

Substitution $(\xi', \eta') \to (\xi, \eta)$:

$$\hat{P}_\varphi(\omega) = \int o(\xi,\eta)\mathrm{e}^{-2\pi j\omega(\xi\cos\varphi+\eta\sin\varphi)}\mathrm{d}\xi\mathrm{d}\eta$$

Central slice theorem

$$\hat{P}_\varphi(\omega) = \int o(\xi,\eta)\mathrm{e}^{-2\pi j\omega(\xi\cos\varphi+\eta\sin\varphi)}\mathrm{d}\xi\mathrm{d}\eta$$

Denote $u = \omega\cos\varphi \quad v = \omega\sin\varphi$

$$\hat{P}(u,v) = \int o(\xi,\eta)\mathrm{e}^{-2\pi j(\xi u+\eta v)}\mathrm{d}\xi\mathrm{d}\eta$$

and therefore

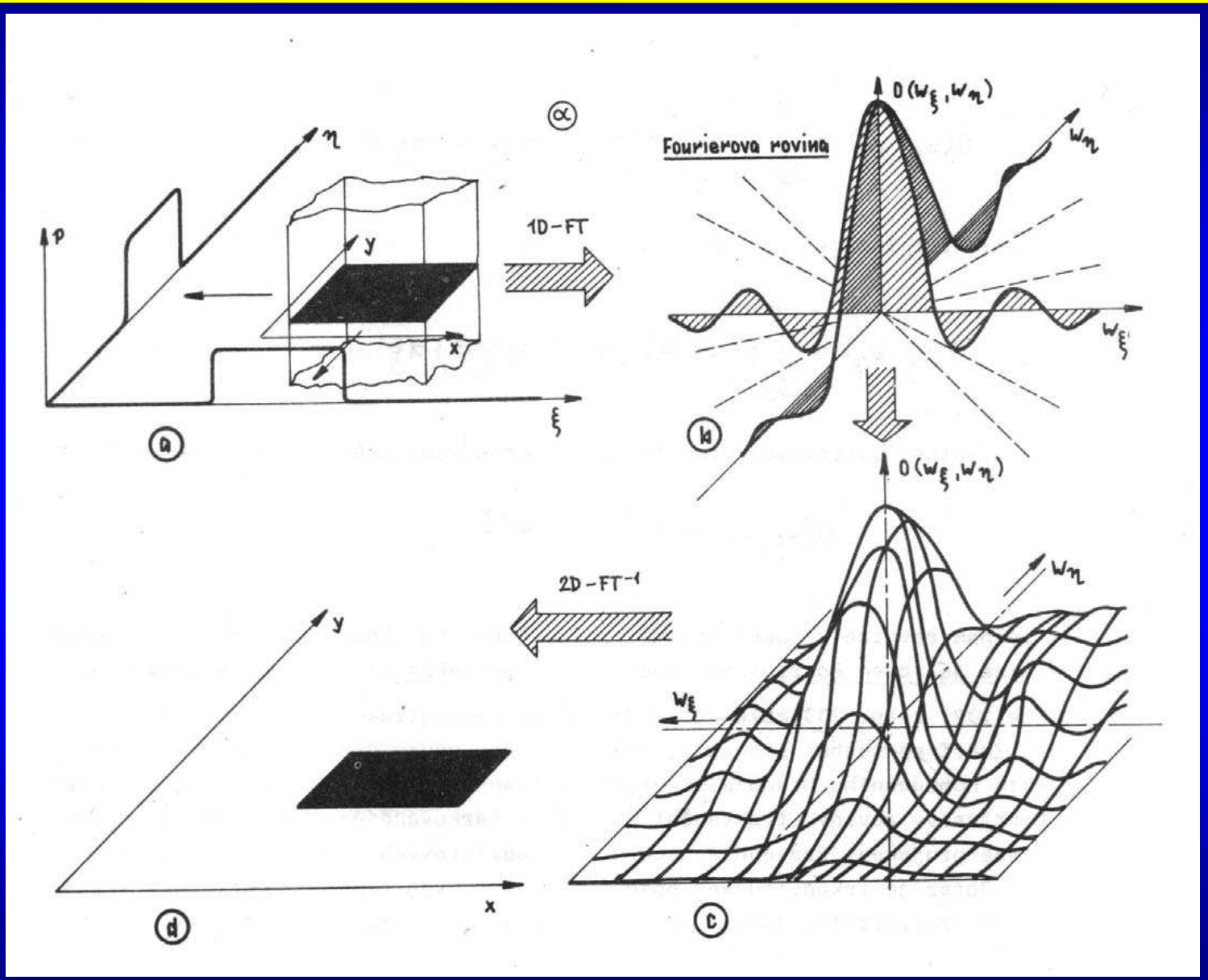
$$\hat{P}(u,v) = \mathscr{F}\{o(\xi,\eta)\}$$
$$\hat{P}_\varphi(\omega) = \mathscr{F}\{o(\xi,\eta)\}(\omega\cos\varphi,\omega\sin\varphi) = \hat{o}(\omega\cos\varphi,\omega\sin\varphi)$$

Central slice theorem

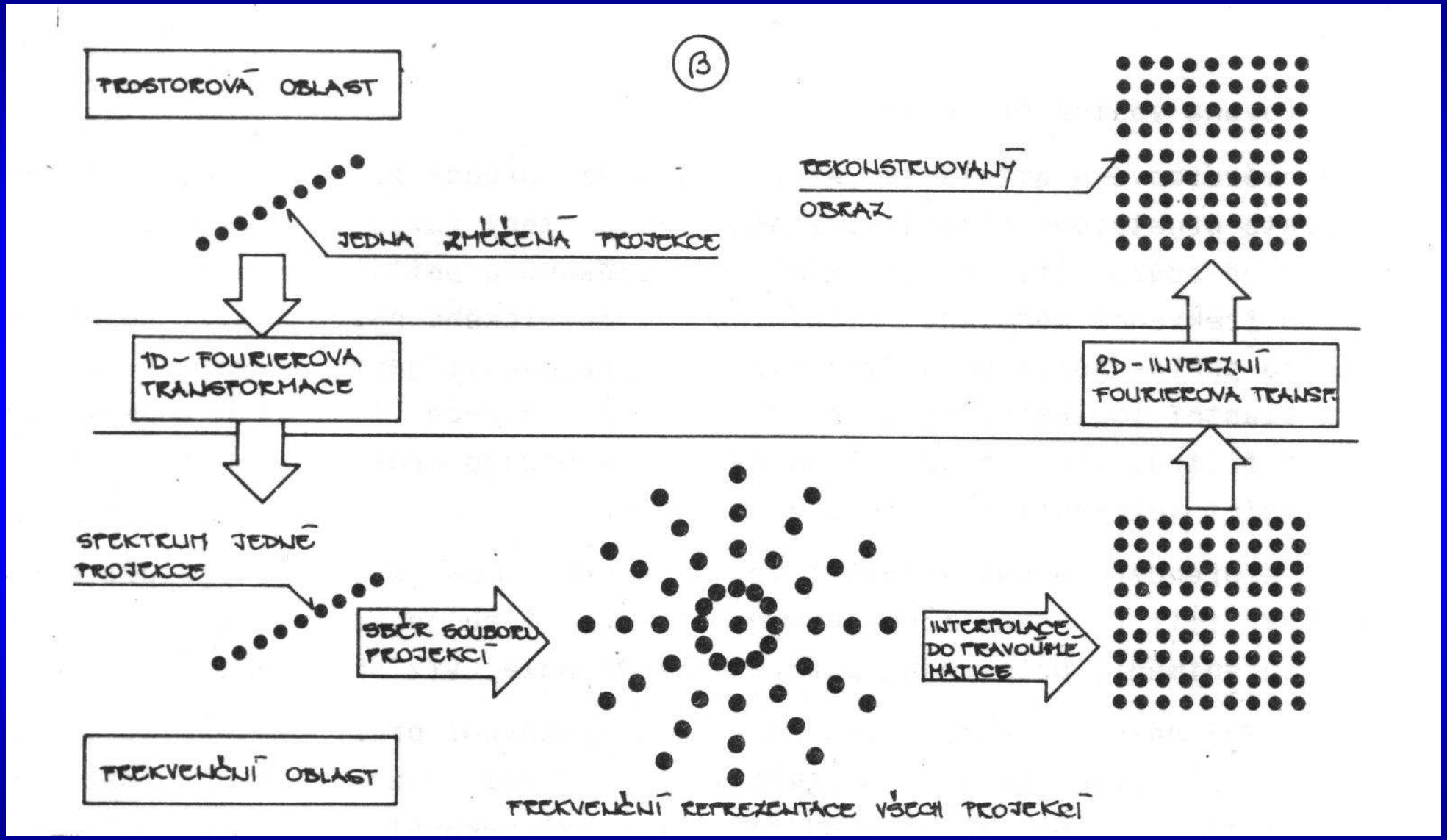
$$\hat{P}(u, v) = \mathscr{F}\{o(\xi, \eta)\}$$
$$\hat{P}_\varphi(\omega) = \mathscr{F}\{o(\xi, \eta)\}(\omega \cos\varphi, \omega \sin\varphi) = \hat{o}(\omega \cos\varphi, \omega \sin\varphi)$$

Slice of the 2D Fourier transform of the image o at angle φ is the 1D Fourier transform of the projection P_φ of the same image o.

Analytická rekonstrukce - 2D FT



Analytická rekonstrukce - 2D FT



Inverse Radonova transform

From the Fourier slice theorem:

$$\hat{P}(u,v) = \mathscr{F}\{o(\xi,\eta)\}$$

$$o(\xi,\eta) = \mathscr{F}^{-1}\left\{\hat{P}(u,v)\right\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{P}(u,v)\mathrm{e}^{2\pi j(\xi u+\eta v)}\mathrm{d}u\mathrm{d}v$$

Polar coordinates $u = \omega\cos\varphi, \quad v = \omega\sin\varphi$:

$$o(\xi,\eta) = \int_{0}^{\pi}\int_{-\infty}^{\infty} \hat{P}_{\varphi}(\omega)\mathrm{e}^{2\pi j\omega(\xi\cos\varphi+\eta\sin\varphi)}|\omega|\mathrm{d}\omega\mathrm{d}\varphi$$

where $|\omega|$ is the Jacobian (determinant).

Inverse Radonova transform

$$o(\xi,\eta) = \int_0^{\pi}\int_{-\infty}^{\infty} \hat{P}_\varphi(\omega)\mathrm{e}^{2\pi j\omega(\xi\cos\varphi+\eta\sin\varphi)}|\omega|\mathrm{d}\omega\mathrm{d}\varphi$$

can be written as

$$o(\xi,\eta) = \int_0^{\pi} Q_\varphi(\underbrace{\xi\cos\varphi+\eta\sin\varphi}_{\xi'})\mathrm{d}\varphi$$

$$Q_\varphi(\xi') = \int_{-\infty}^{\infty} \hat{P}_\varphi(\omega)\mathrm{e}^{2\pi j\omega\xi'}|\omega|\mathrm{d}\omega$$

where $Q_\varphi(\xi')$ is a modified projection

Inverse Radonova transform

$$o(\xi,\eta) = \int_0^{\pi} Q_\varphi(\xi')\mathrm{d}\varphi$$

$$Q_\varphi(\xi') = \int_{-\infty}^{\infty} \hat{P}_\varphi(\omega)\mathrm{e}^{2\pi j\omega\xi'}|\omega|\mathrm{d}\omega$$

$$Q_\varphi(\xi') = \mathscr{F}^{-1}\left\{|\omega|\hat{P}_\varphi(\omega)\right\} = \mathscr{F}^{-1}\left\{|\omega|\right\} * P_\varphi(\xi')$$

defining the exact inverse Radon transform

$$P_\varphi(\xi') = \mathscr{R}\left[o(\xi,\eta)\right]$$
$$o(\xi,\eta) = \mathscr{R}^{-1}\left[P_\varphi(\xi')\right]$$

Filtered backprojection

Filtrovaná zpětná projekce

- Filter all projections $P_\varphi(\xi')$ for all φ, get modified projections $Q_\varphi(\xi')$
- Backprojected modified projections and sum

$$o(\xi,\eta) = \int_0^\pi Q_\varphi(\xi')\mathrm{d}\varphi$$

$$Q_\varphi(\xi') = h(t) * P_\varphi(\xi') = \mathscr{F}^{-1}\left\{H(\omega)\right\} * P_\varphi(\xi')$$

$$H(\omega) = |\omega|$$

Practical implementation of filtered backprojection

- **Problem:** Ideal filter $H(\omega) = |\omega|$ amplifies noise
- **Solution 1:** Make $\hat{P}_\varphi(\omega)$ frequency limited.
  Ramakrishnan-Lakshiminaryanan $\longrightarrow$ Ram-Lak filter:

$$H(\omega) = \begin{cases} |\omega| & \text{if } |\omega| \leq \Omega \\ 0 & \textit{otherwise} \end{cases}$$

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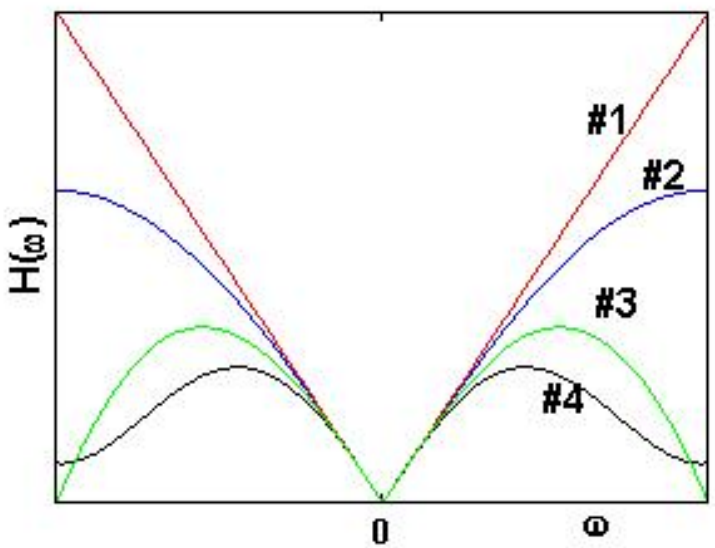
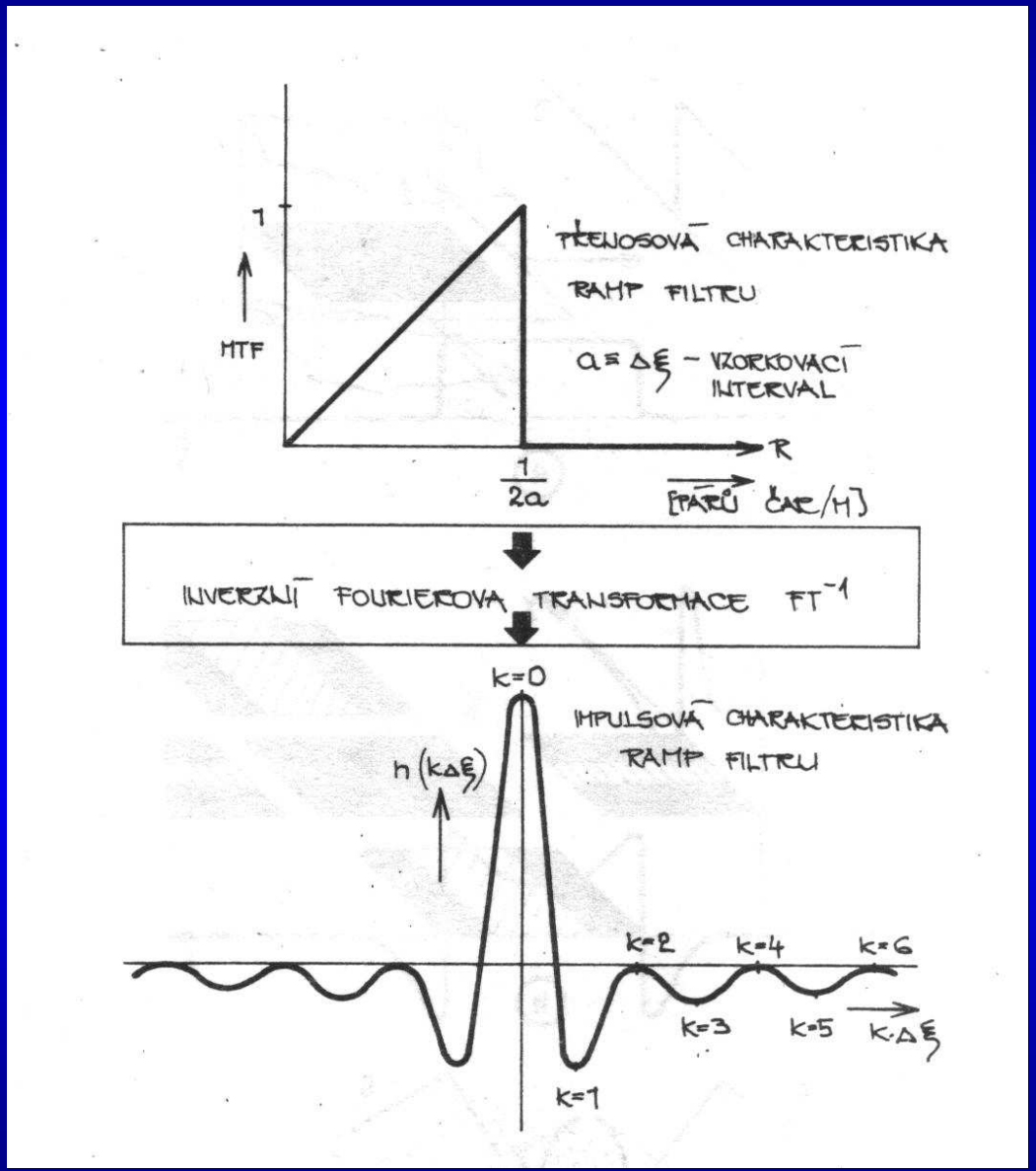
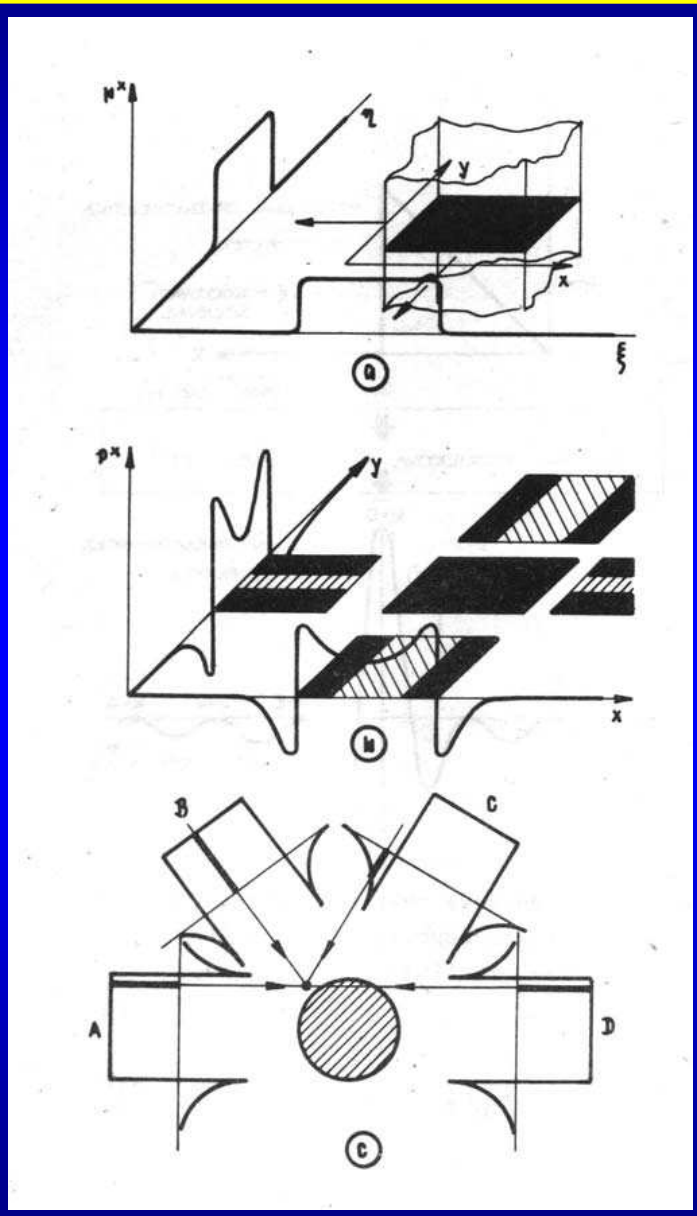
$$H(\omega) = \begin{cases} |\omega| & \text{if } |\omega| \leq \Omega \\ 0 & \textit{otherwise} \end{cases}$$

- Ram-Lak filter causes artefacts (Gibbs). Many solutions (Hamming filter, Shepp-Logan filter). Typically Hamming has better SNR but lower resolution.

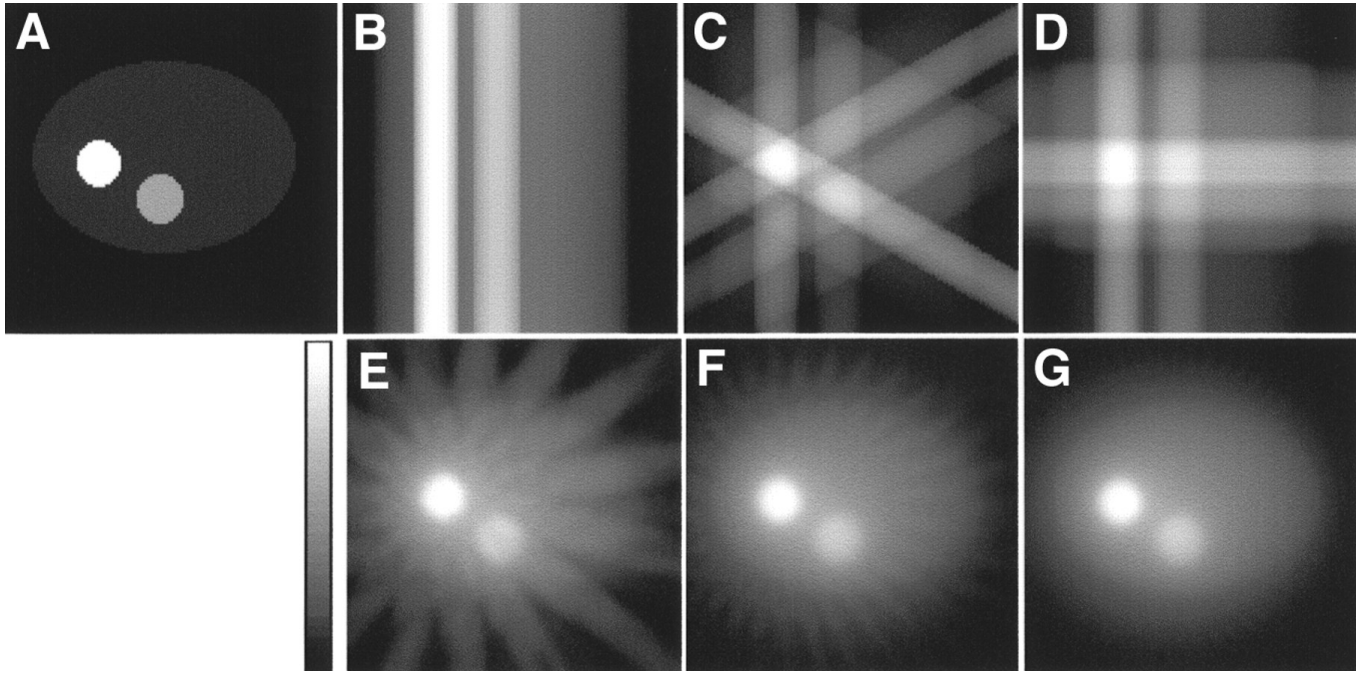


Analytická rekonstrukce - filtrovaná ZP





Filtered backprojection



original image, 1,3, 4, 16, 32, a 64 projections

Algebraic rekonstruction

- setup equations, often linear

$$p_i = \sum_j w_{ij} f_j$$

  where f_j are pixel values, p_i are projections
- We know p_i and w_{ij}, solve for f_i
- Many unknowns, iterative method
  - Compare measured projections and simulations
  - Correct pixel values to decrease the difference
  - Iterate until convergence

Algebraic rekonstruction — advantages over FBP

- Better modeling of the physics — attenuation, resolution, noise
- Better handling of limited acquisition — restricted region, restricted angles
- Can use an image model
- Less apparent artifacts

Iterativní rekonstrukce - ART

ART – Algebraic Reconstruction Technique je jedním z mnoha použitých algoritmů, které se používají do současnosti. Existují dva základní typy ART:

- aditivní

$$\widehat{f}_{ij}^{\,l} = \widehat{f}_{ij}^{\,l-1} + \frac{g_j - \sum_{i=1}^{N} \widehat{f}_{ij}^{\,l-1}}{N}$$

- multiplikativní

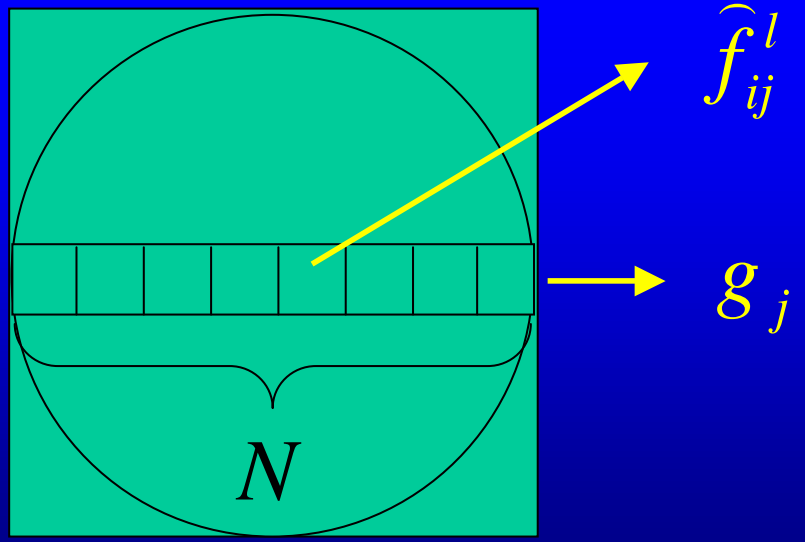
$$\widehat{f}_{ij}^{\,l} = \frac{g_j}{\sum_{i=1}^{N} \widehat{f}_{ij}^{\,l-1}} \widehat{f}_{ij}^{\,l-1}$$

Iterativní rekonstrukce – ART pokračování

kde: $\widehat{f}_{ij}^{\,l}$ - **odhad hodnoty *i*-tého voxelu podél *j*-tého paprsku během *l*-té iterace,**

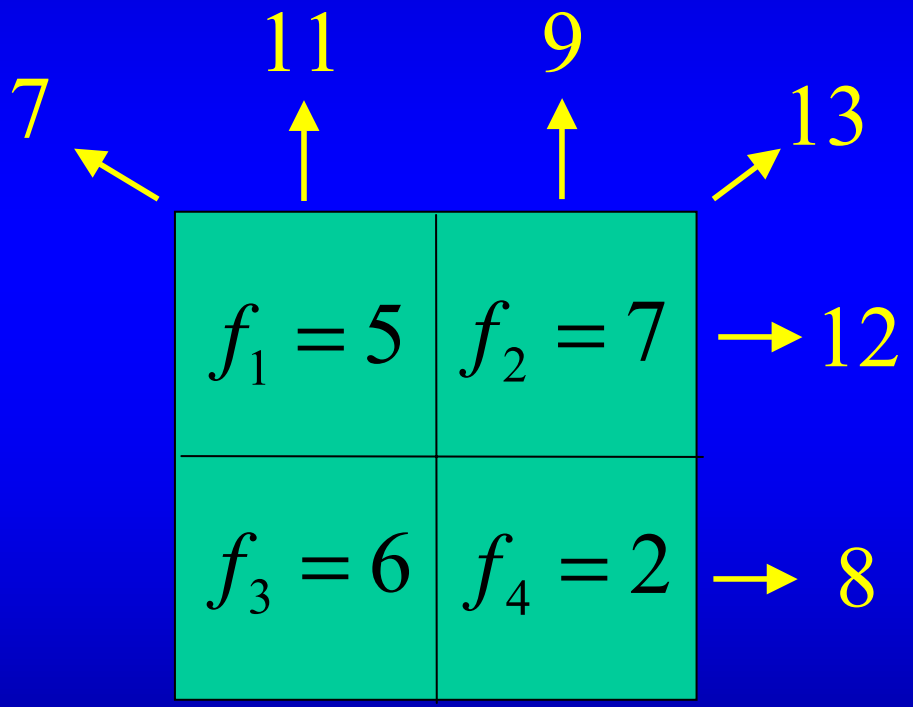
g_j - **skutečný paprskový součet (data) podél *j*-tého paprsku,**

N - **počet objemových elementů (voxelů) podél *j*-tého paprsku,**



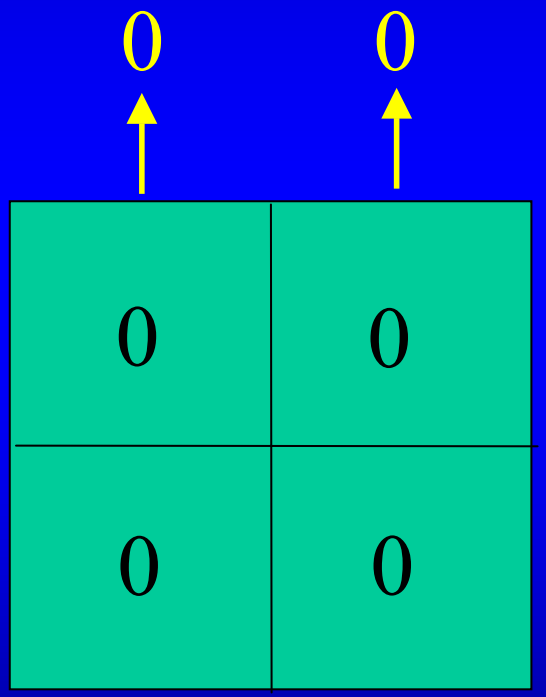
Iterativní rekonstrukce – ART aditivní - příklad

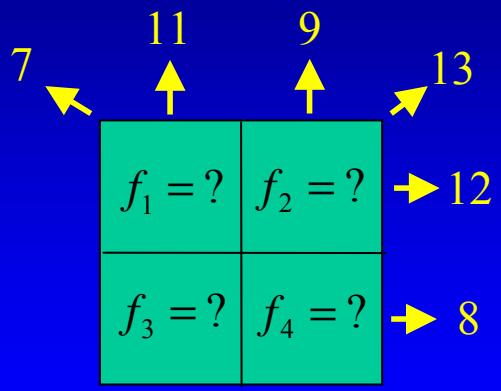
- skutečná naměřená data (projekce a paprskové součty)



Iterativní rekonstrukce – ART př. – pokrač.

1/3
vertikální paprsky

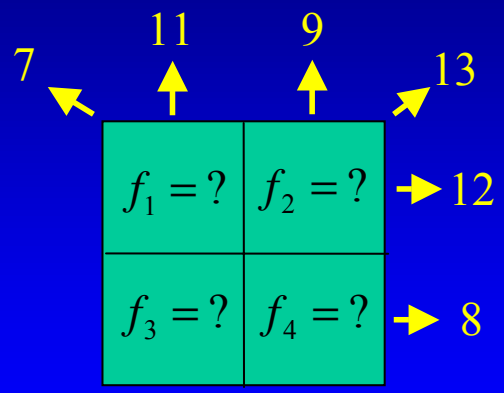




$$\widehat{f}_1^{1/3} = \widehat{f}_3^{1/3} = 0 + \frac{11-0}{2} = 5{,}5$$

$$\widehat{f}_2^{1/3} = \widehat{f}_4^{1/3} = 0 + \frac{9-0}{2} = 4{,}5$$

Iterativní rekonstrukce – ART př. – pokrač.



2/3
horizontální paprsky

5,5	4,5	→ 10
5,5	4,5	→ 10

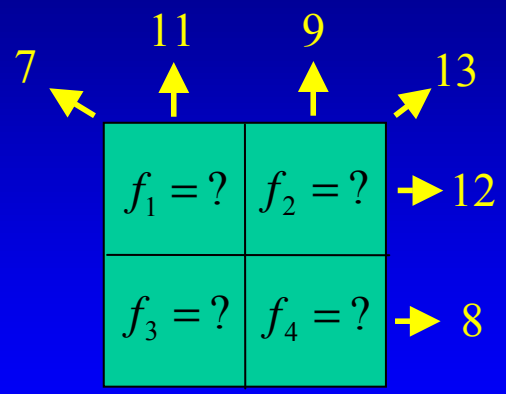
$$\widehat{f}_1^{2/3} = 5{,}5 + \frac{12-10}{2} = 6{,}5$$

$$\widehat{f}_2^{2/3} = 4{,}5 + \frac{12-10}{2} = 5{,}5$$

$$\widehat{f}_3^{2/3} = 5{,}5 + \frac{8-10}{2} = 4{,}5$$

$$\widehat{f}_4^{2/3} = 4{,}5 + \frac{8-10}{2} = 3{,}5$$

Iterativní rekonstrukce – ART př. – pokrač.



3/3=1

diagonální paprsky

10

10

6,5	5,5
4,5	3,5

$$\hat{f}_1^1 = 6{,}5 + \frac{7-10}{2} = 5$$

$$\hat{f}_2^1 = 5{,}5 + \frac{13-10}{2} = 7$$

$$\hat{f}_3^1 = 4{,}5 + \frac{13-10}{2} = 6$$

$$\hat{f}_4^1 = 3{,}5 + \frac{7-10}{2} = 2$$

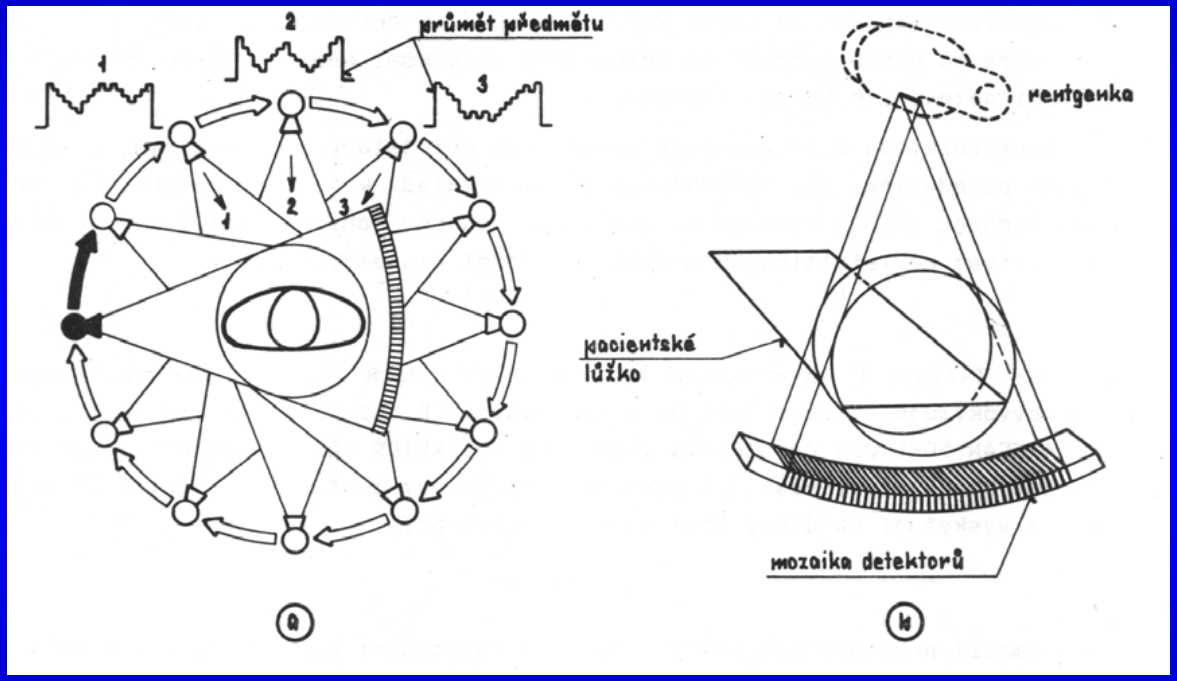
viz orig. data

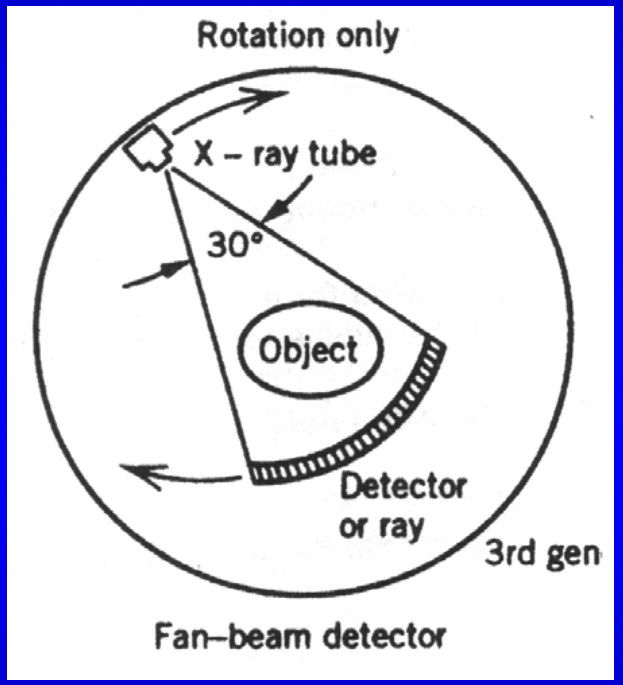
Konvergence

Electric processing — corrections

- Offset correction (zero signal at rest)
- Normalization correction (x-ray source intensity fluctuation)
- Sensitivity correction (inhomogeneous detectors and amplifiers)
- Geometric correction
- Beam hardening correction
- Cosine correction

CT systémy 3. generace





asi nejčastěji používané

Fan-beam reconstruction

- Rays not parallel, not a Radon transform.
- Rebinning

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Fan-beam reconstruction

- Rays not parallel, not a Radon transform.
- Rebinning
- Modified filtered backprojection (quadratic cosine correction, $\cos\theta$).
- Algebraic reconstruction. Best quality but slow.

3D computed tomography

- Technical challenges: power, cooling
- Rotation method (slice by slice)
- Spiral/helix method

Spiral method

- Acceleration: $10\,\mathrm{min} \rightarrow 1\,\mathrm{min}$

Spiral method

- Acceleration: 10 min $\rightarrow$ 1 min
- *Pitch*:

$$P = \Delta l / d$$

Δl bed shift per rotation, d slice thickness.
Normally $0 < P < 2$. Overlap for $P < 1$. Typically $P = 1.5$.

Spiral method

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- *Pitch*:

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- Interpolation in z axis

Spiral method

- Acceleration: 10 min $\rightarrow$ 1 min
- *Pitch*:

$$P = \Delta l / d$$

  Δl bed shift per rotation, d slice thickness.
  Normally $0 < P < 2$. Overlap for $P < 1$. Typically $P = 1.5$.
- Interpolation in z axis
- Interpolation *wide* — 1 turn. Less noise, larger effective slice thickness.

Spiral method

- Acceleration: 10 min $\rightarrow$ 1 min
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- Interpolation in z axis
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- Interpolation *Slim* — 1/2 turn, symmetry. More noise, smaller effective slice thickness.

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- Interpolation *wide* — 1 turn. Less noise, larger effective slice thickness.
- Interpolation *Slim* — 1/2 turn, symmetry. More noise, smaller effective slice thickness.
- Multislice acquisition — acceleration.

Radiation dose

- Absorbed dose D. 1 Gy (gray) = 1 J/kg Before 1 Gy = 100 rad
- Effective dose equivalent (dávkový ekvivalent)
  H_E [Sv] (sievert)

$$H_E = \sum_i w_i H_i$$

$H = cD$. Quality factor c is 1 for X-rays and γ rays, 10 for neutrons, 20 for α particles.

Coefficient w is organ dependent: male/female glands 0.2, lungs 0.12, breast 0.1, stomach 0.12, thyroid gland 0.05, skin 0.01.
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  Before 1 Sv = 100 rem
- Sum the doses

Radiation dose

- Medical limit (USA) is 50 mSv/year (=limit for a person working with radiation in CR), corresponding to 1000 chest X-rays, or 15 head CTs, or 5 whole body CTs (1 CT$\approx$ 10 mSv).
- low-dose CT$\approx 2 \sim 5$ mSv, PET$\approx$ 25 mSv
- In CR radioactive background about 3 mSv/year (mainly radon), similar to USA. In Colorado (altitude $1500 \sim 4000$ m) about 4.5 mSv/year. Mean dose from medical imaging 0.3 mSv/year, about 3 long flights.

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- cancer related death 20 %. 1 CT=10 mSv — relative increase by $10^{-3} \sim 10^{-4}$

CT image quality

- Parameters:
  - Resolution (0.5 mm)
  - Contrast (δH, about $5 - 10$ H.j.)
  - Detection threshold (about 1 mm at $\Delta H = 200$, 5 mm at $\Delta H = 5$).
  - Noise (SNR)
- Artifacts
  - Scanner defects, malfunctions, operator error
  - Metal parts (shadows)
  - Motion artifacts
  - Partial volume

Artifact examples

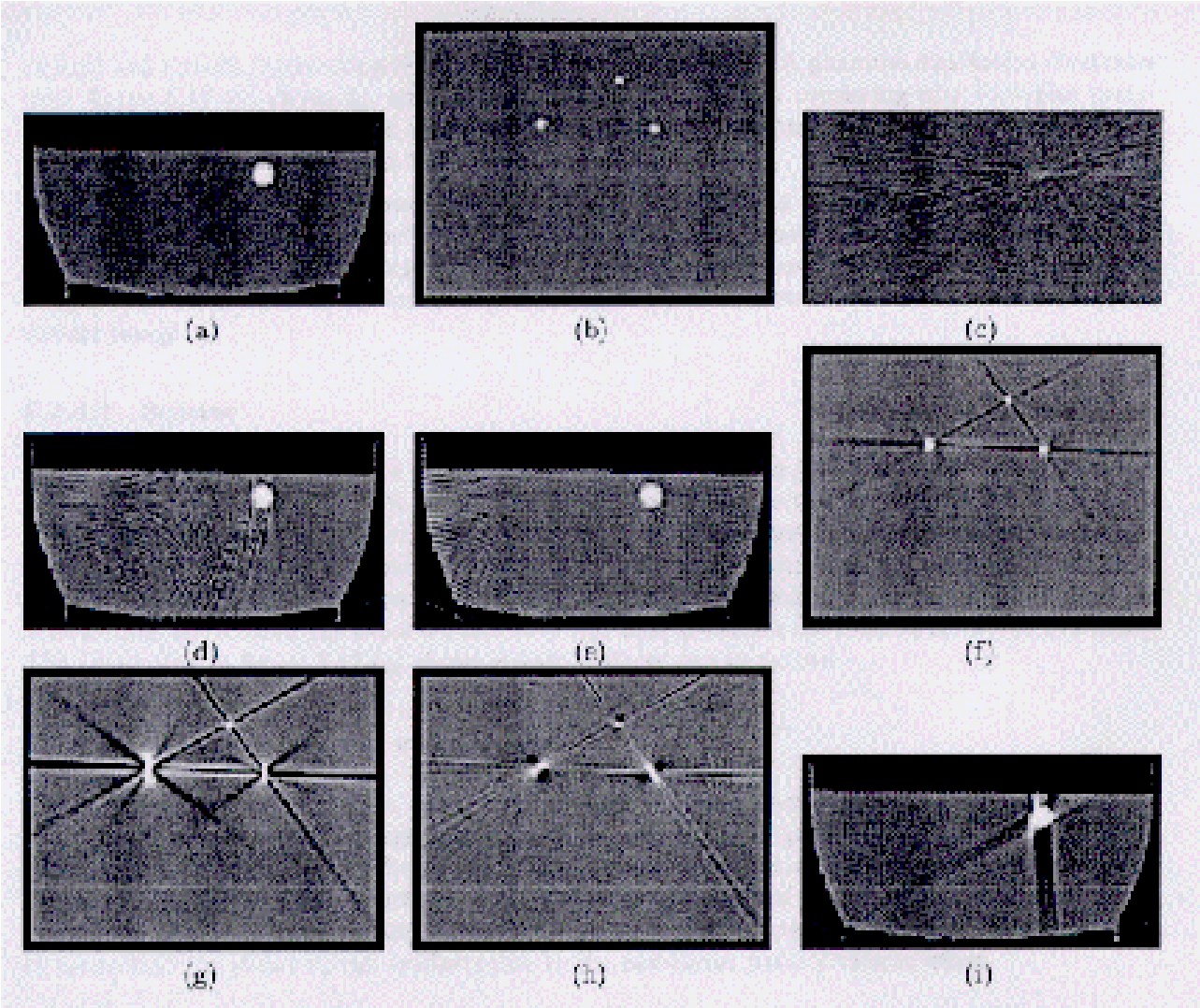
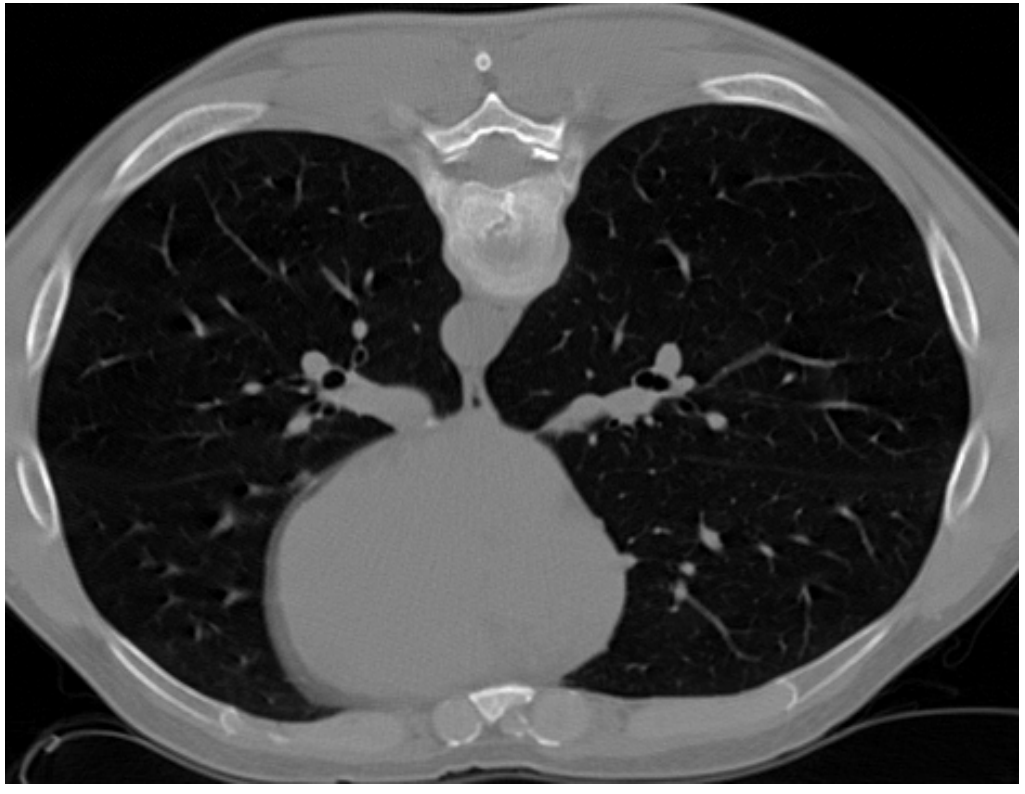


Figure 2.19 Example of image artifacts: (a) test phantom, (b) second phantom, (c) noise, (d) detector under-sampling, (e) view under-sampling, (f) beam hardening, (g) scatter, (h) nonlinear partial volume effect, and (i) object motion. (unpublished results)

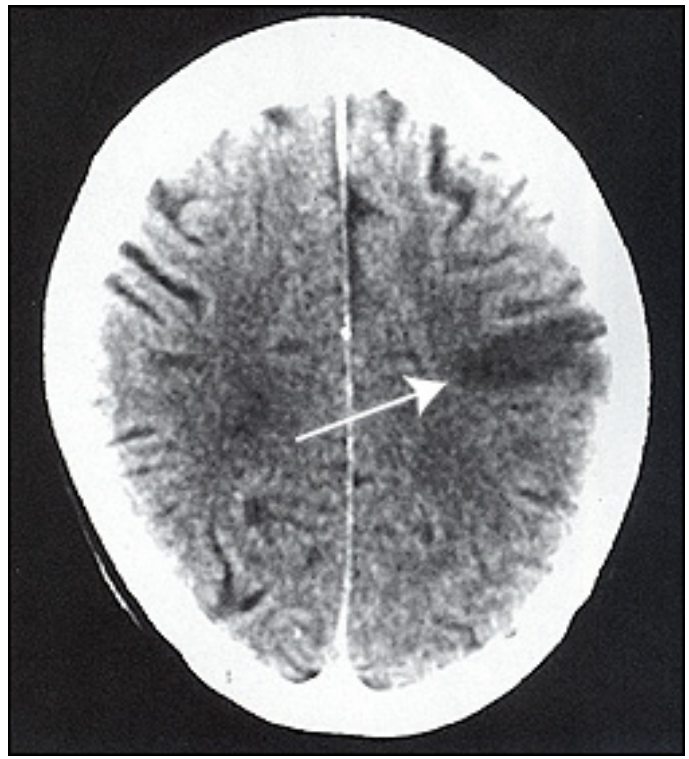
Clinical applications

- Lungs



Clinical applications

- Lungs
- Head



Clinical applications

- Lungs
- Head
- Abdomen

Computed tomography, conclusions

- Excellent spatial resolution
- 3D image
- Fast acquisition
- Weak soft tissue contract
- Reconstruction algorithm
- Radiation dose