

# Combinatorial Optimization

## Lab No. 3

### Integer Linear Programming

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March 4, 2019

#### Abstract

This lab is devoted to formulating problems as ILP, namely Call center scheduling problem. Taking this problem as an example, we show how to formulate problems with absolute value functions in their objectives as ILP.

## 1 Call center scheduling problem

### 1.1 Problem Description

A call center needs to create a cyclic daily schedule (i.e., numbers of the shifts at particular hours are same on every day) of the *shifts* for its employees [1]. Let  $\mathbf{d}$  be a vector of a *personnel demand* in each hour during the day, i.e.,  $d_i$  is defined  $\forall i = 0, 1, 2, \dots, 23$ . The personnel demand  $d_i$  determines the minimal number of shifts (i.e., employees who will be assigned to these shifts) between  $i$  and  $(i + 1)$  hour, e.g.,  $d_{10}$  expresses the minimal number of shifts running between 10 a.m. and 11 a.m. The objective of this problem is to obtain the cyclic daily schedule of the shifts such that the total number of the shifts used to cover the personnel demand in a day is minimized. The shift may start at arbitrary full hour and its length is set to 8 hours.

The ILP model of this problem is based on a variable  $\mathbf{x}$  such that  $x_i$  represents a number of the shifts starting at hour  $i$ .

**Lab exercise:** Formulate this problem as an ILP.

Consider an example with the following vector of the demands

$\mathbf{d} = [6, 6, 6, 6, 6, 8, 9, 12, 18, 22, 25, 21, 21, 20, 18, 21, 21, 24, 24, 18, 18, 18, 12, 8]$

Using ILP to solve this example, we find out that the minimal number of the shifts that satisfy this demand is 55 (i.e., optimal value of the objective function). The cyclic daily schedule of the used shifts is illustrated in Fig. 1. You can notice the large surplus of the shifts in comparison to the personnel demand between 4 a.m. and 5 a.m. How to minimize this difference between the personnel demand and its coverage is handled in Section 2.

## 2 A Homework Assignment

To avoid a large surplus of shifts in comparison to the personnel demand, we modify our objective to find a daily schedule of shifts such that the difference of the personnel demand and its coverage is minimized, i.e., we try to cover the personnel demand as precisely as possible

$$\min \sum_{i=0}^{23} \left| d_i - \sum_{j=i-7}^i x_j \bmod 24 \right| \quad (1)$$



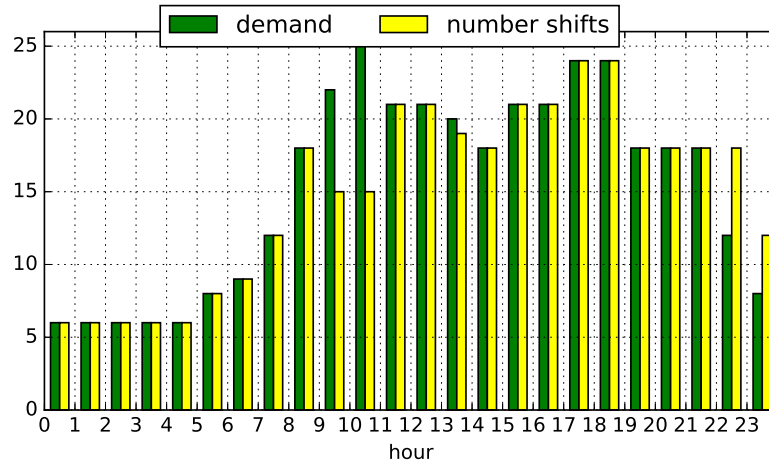


Figure 2: The coverage of the personnel demand  $\mathbf{d}$  and the final daily schedule of the shifts

**A homework assignment:** Implement a program that takes the vector of personnel demand  $\mathbf{d}$  as the input and outputs the optimal allocation of personnel for each our (values of  $x_i$  variables). Given the input, construct an ILP model solving the problem described in Eq. (1). Solve the problem and output optimal values of  $\mathbf{x}$  in the format specified below. Upload your source code to the CourseWare Upload System where it will be automatically evaluated.

## 2.1 Input/Output format

Your program will be called with two arguments: the first one is absolute path to input file and the second one is the absolute path to output file (the output file has to be created by your program).

The input file consists of a single line with 24 integers separated by space representing the vector of demands  $\mathbf{d}$ .

The output consists of two lines. The first line is an integer denoting the optimal objective value. The second line are the optimal values of  $x_0, x_1, x_2, \dots, x_{23}$  variables separated by spaces.

### 2.1.1 Example 1

Input:

```
6 6 6 6 6 8 9 12 18 22 25 21 21 20 18 21 21 24 24 18 18 18 12 8
```

Output:

```
28
0 0 0 0 6 1 2 3 6 3 0 0 6 0 0 6 6 0 0 0 6 0 0 0
```

## References

- [1] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall; United States Ed edition, 1993.