Classifiers, intro, Naïve Bayes, evaluation

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thanks to Daniel Novák and Filip Železný, Ondřej Drbohlav

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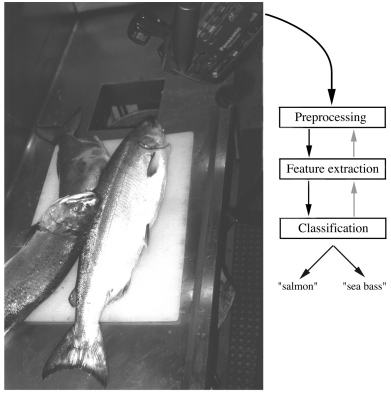
Center for Machine Perception (CMP)

May 12, 2019

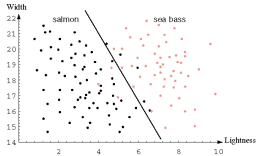
Classification example: What's the fish?

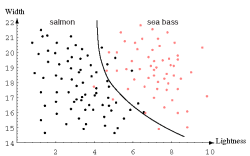


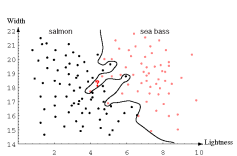
- Factory for fish processing
- 2 classes $s_{1,2}$:
  - salmon
  - sea bass
- Features $\vec{x}$: length, width, lightness etc. from a camera

Classification example:
http://robotics.fel.cvut.cz/cras/darpa-subt/

Fish classification in feature space







- Linear, quadratic, k-nearest neighbor classifier
- Feature frequency per class shown using histograms
- Classification errors due to histogram overlap

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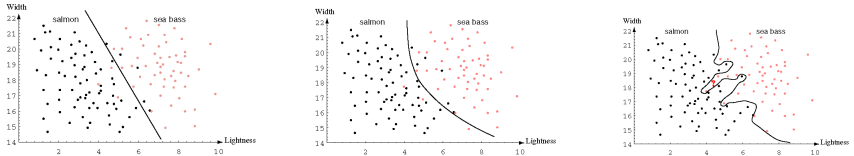

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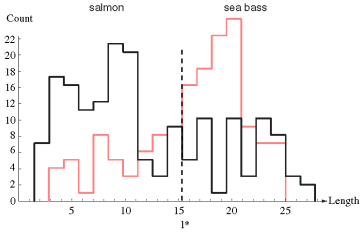

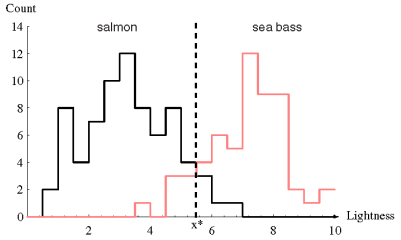

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Fish – classification using probability

$$posterior = \frac{likelihood \times prior}{evidence}$$

- Notation for classification problem
  - Classes $s_j \in S$ (e.g., salmon, sea bass)
  - Features $x_i \in X$ or feature vectors ($\vec{x_i}$) (also called attributes)
- Optimal classification of $\vec{x}$:(?)

$$\delta^*(\vec{x}) = \arg\max_j P(s_j|\vec{x})$$

- We thus choose the most probable class for a given feature vector
- Both likelihood and prior are taken into account – recall Bayes rule:

$$P(s_j|\vec{x}) = \frac{P(\vec{x}|s_j)P(s_j)}{P(\vec{x})}$$

Bayesian classification is a special case of statistical decision theory:

- Attribute vector $\vec{x} = (x_1, x_2, \dots)$: pixels 1, 2, ...
- **State set $\mathcal{S}$ = decision set $\mathcal{D} = \{0, 1, \dots 9\}$.**
- State = actual class, Decision = recognized class
- Loss function:

$$l(s,d) = \begin{cases} 0, & d = s \\ 1, & d \neq s \end{cases}$$

$$\delta^*(\vec{x}) = \arg\min_d \sum_s \underbrace{l(s,d)}_{0 \text{ if } d=s} P(s|\vec{x}) = \arg\min_d \sum_{s \neq d} P(s|\vec{x})$$

Obviously $\sum_s P(s|\vec{x}) = 1$, then:

$$P(d|\vec{x}) + \sum_{s \neq d} P(s|\vec{x}) = 1$$

Inserting into above:

$$\delta^*(\vec{x}) = \arg\min_d [1 - P(d|\vec{x})] = \arg\max_d P(d|\vec{x})$$

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Bayes classification in practice

- Usually we are not given $P(s|\vec{x})$
- It has to be estimated from already classified examples – training data
- For discrete $\vec{x}$, training examples $(\vec{x}_1, s_1), (\vec{x}_2, s_2), \ldots (\vec{x}_l, s_l)$
  - so-called i.i.d (independent, identically distributed) multiset
  - every $(\vec{x}_i, s)$ is drawn independently from $P(\vec{x}, s)$
- Without knowing anything about the distribution, a non-parametric estimate:

$$P(s|\vec{x}) \approx \frac{\#\text{ examples where } \vec{x}_i = \vec{x} \text{ and } s_i = s}{\#\text{ examples where } \vec{x}_i = \vec{x}}$$

- Hard in practice:
  - To reliably estimate $P(s|\vec{x})$, the number of examples grows exponentially with the number of elements of $\vec{x}$.
    - e.g. with the number of pixels in images
    - curse of dimensionality
    - denominator often 0

Why hard? Way too many various $\vec{x}$. Think about simple binary 10×10 image - $\vec{x}$ contains $0, 1$, position matters. What is the total number of unique images? Think binary, 1×8 binary image?
What is the difference between set and multiset?
Reminder about math notation. In literature, vectors are mostly denoted by bold lower case **x**. In lectures, we use $\vec{x}$ to match notation used on blackboard. It is difficult to write bold with a chalk.

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Naïve Bayes classification

- For efficient classification we must thus rely on additional assumptions.
- In the exceptional case of statistical independence between components of $\vec{x}$ for each class s it holds

$$P(\vec{x}|s) = P(x[1]|s) \cdot P(x[2]|s) \cdot \ldots$$

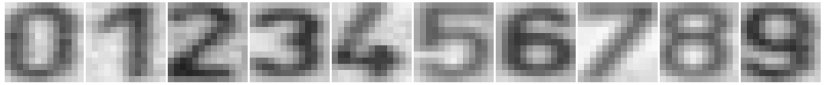
- Use simple Bayes law and maximize:

$$P(s|\vec{x}) = \frac{P(\vec{x}|s)P(s)}{P(\vec{x})} = \frac{P(s)}{P(\vec{x})} P(x[1]|s) \cdot P(x[2]|s) \cdot \ldots =$$

- No combinatorial curse in estimating $P(s)$ and $P(x[i]|s)$ separately for each i and s.
- No need to estimate $P(\vec{x})$. (Why?)
- $P(s)$ may be provided apriori.
- naïve = when used despite statistical dependence

Why naïve at all? Consider $N-$ dimensional space, $8 - bit$ values. Instead of problem 8^N we have $8 \times N$ problem.
Think about statistical independence. Example1: person's weight and height. Are they independent? Example2: pixel values in images.

Example: Digit recognition



- Input: 8-bit image 13×13, pixel intensities $0 - 255$.
- Output: Digit $0 - 9$. Decision about the class, classification.
- Features: Pixel intensities ...

Collect data, ...

- $P(\vec{x})$. What is the dimension of $\vec{x}$? How many possible images?
- Learn $P(\vec{x}|s)$ per each class (digit).
- Classify $s^* = \text{argmax}_s P(s|\vec{x})$.

We can create many more features than just pixel intensities. But first things first.

We are assuming all errors are equally important - minimizing the number of wrong decisions.

Dimension of $\vec{x}$ is $13 \times 13 = 169$. There are 255^{169} possible images.

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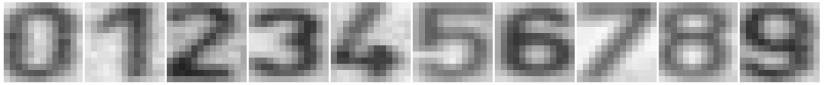

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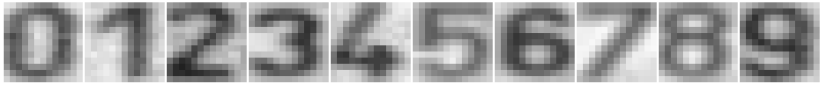

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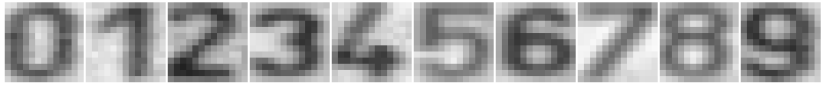

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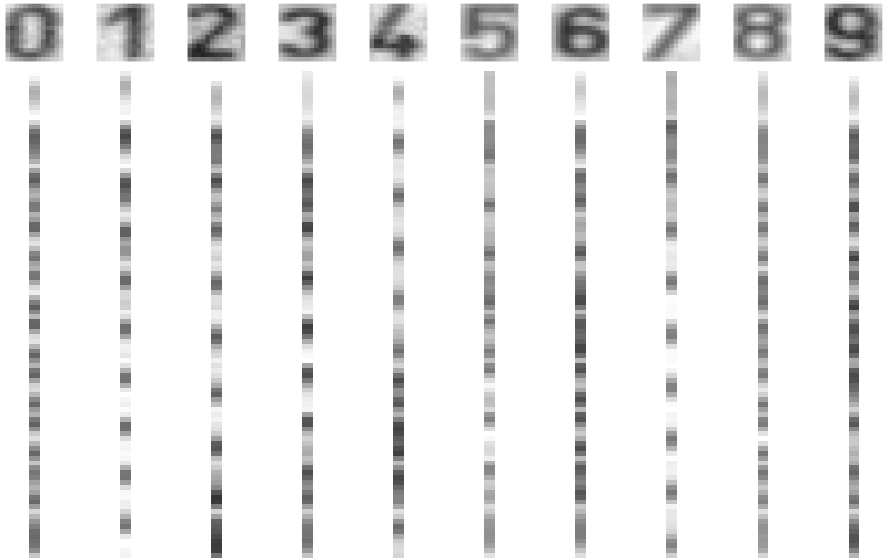
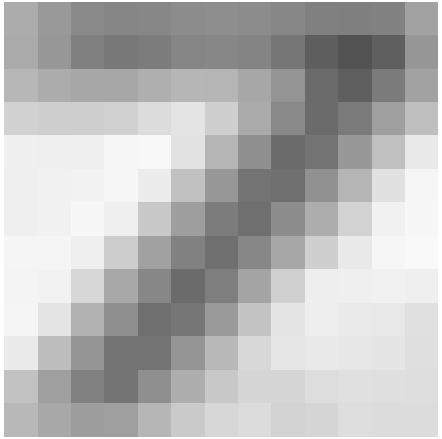
From images to $\vec{x}$

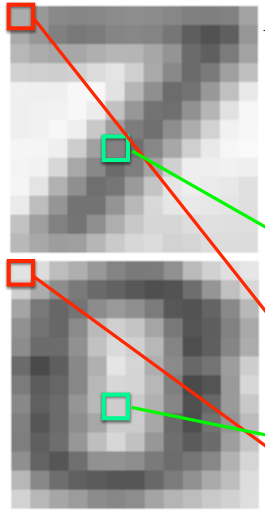


Conditional probabilities



- Apriori digit probabilities $P(s_k)$
- Likelihoods for pixels.
  $P(x_{u,v} = I_i | s_k)$

Conditional probabilities



s_k	$P(s_k)$
1	0.1
2	0.1
3	0.1
4	0.1
5	0.1
6	0.1
7	0.1
8	0.1
9	0.1
0	0.1

$P(x_{0,0} = 25|s_k)$

s_k	$P(x_{0,0} = 25\|s_k)$
1	0.0009
2	0.05
3	0.00006
4	0.0005
5	0.07
6	0.0007
7	0.1
8	0.0006
9	0
0	0

$P(x_{7,5} = 25|s_k)$

s_k	$P(x_{7,5} = 25\|s_k)$
1	0.0009
2	0.05
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4	0.0005
5	0.004
6	0.07
7	0.1
8	0.006
9	0.08
0	0.005

Generalization and overfiting

- Data: training, validation, testing. Wanted classifier performs well on what data?
- Overfitting: too close to training, poor on testing

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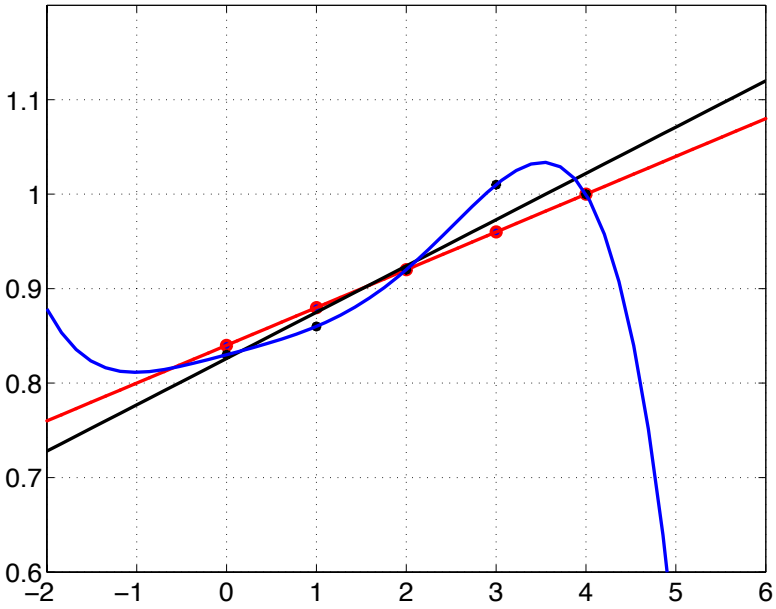
Overfiting



see the `overfit.m` demo

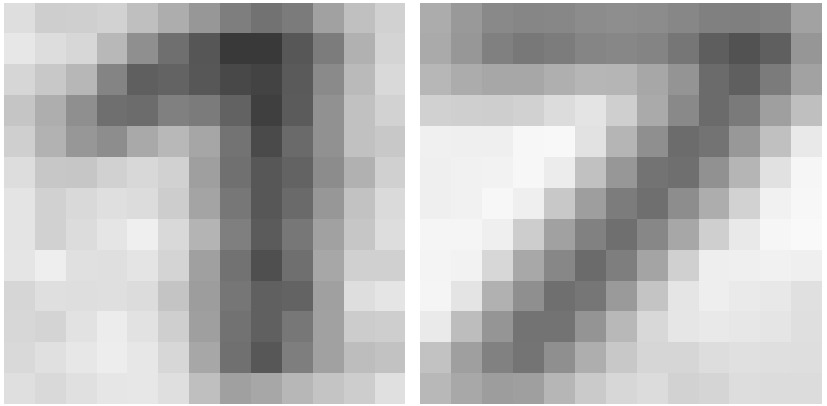
Think about the problem of classifying numerals. Some $P(x_{u,v} = I \mid s) = 0$. What about an example:

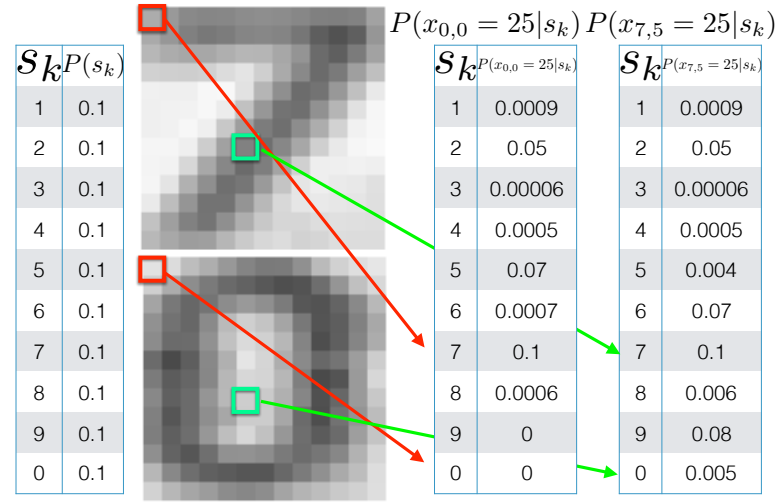
$$\begin{aligned} P(x_{0,0} = 100 \mid s = 7) &= 0.05 \\ P(x_{0,0} = 101 \mid s = 7) &= 0 \\ P(x_{0,0} = 102 \mid s = 7) &= 0.06 \end{aligned}$$

A new (not in training) query image with $x_{0,0} = 101$. How would you classify?

Unseen events

Images 13×13, intensities $0 - 255$, 100 exemplars per each class.





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7	0.1
8	0.006
9	0.08
0	0.005

Laplace smoothing ("additive smoothing")

$$P(x) = \frac{\text{count}(x)}{\text{total samples}}$$

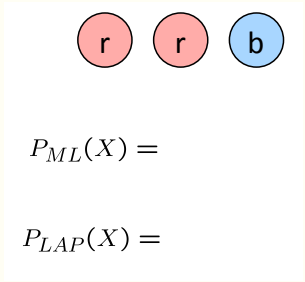
Problem: $\text{count}(x) = 0$

Pretend you see the sample one more time.

$$P_{LAP}(x) = \frac{c(x)+1}{\sum_x [c(x)+1]}$$

$$P_{LAP}(x) = \frac{c(x)+1}{N+|X|}$$

where N is the number of observations; $|X|$ is the number of possible values X can take.



originally:

- $P(red) = 2/3$
- $P(blue) = 1/3$

after Laplace smoothing – adding one red ball and blue ball to the actual observations:

- $P_{LAP}(red) = (2+1)/(2+1+1+1) = 3/5$
- $P_{LAP}(blue) = (1+1)/(2+1+1+1) = 2/5$

this slide: courtesy of P. Abeel, http://ai.berkeley.edu. 21st lecture of CS 188.

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$P_{ML}(X) =$

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after Laplace smoothing – adding one red ball and blue ball to the actual observations:

- $P_{LAP}(red) = (2+1)/(2+1+1+1) = 3/5$
- $P_{LAP}(blue) = (1+1)/(2+1+1+1) = 2/5$

this slide: courtesy of P. Abeel, http://ai.berkeley.edu. 21st lecture of CS 188.

Laplace smoothing ("additive smoothing")

$$P(x) = \frac{\text{count}(x)}{\text{total samples}}$$

Problem: $\text{count}(x) = 0$
Pretend you see the sample one more time.

$$P_{\text{LAP}}(x) = \frac{c(x)+1}{\sum_x [c(x)+1]}$$

$$P_{\text{LAP}}(x) = \frac{c(x)+1}{N+|X|}$$

where N is the number of observations; $|X|$ is the number of possible values X can take.

$P_{ML}(X) =$

$P_{LAP}(X) =$

originally:

- $P(red) = 2/3$
- $P(blue) = 1/3$

after Laplace smoothing – adding one red ball and blue ball to the actual observations:

- $P_{LAP}(red) = (2+1)/(2+1+1+1) = 3/5$
- $P_{LAP}(blue) = (1+1)/(2+1+1+1) = 2/5$

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Laplace smoothing - as a hyperparameter k

Pretend you see every sample k extra times:

$$P_{\text{LAP}}(x) = \frac{c(x) + k}{\sum_x [c(x) + k]}$$

$$P_{\text{LAP}}(x) = \frac{c(x) + k}{N + k|X|}$$

For conditional, smooth each condition independently

$$P_{\text{LAP}}(x|s) = \frac{c(x, s) + k}{c(s) + k|X|}$$

hyperparameter would be tuned along with your classifier
For $k = 100$ and blue and red, you would get:

- $P_{LAP}(red) = (2 + 100)/(3 + 100 * 2) = 102/203$
- $P_{LAP}(blue) = (1 + 100)/(3 + 100 * 2) = 101/203$

In this case, smoothing ("prior") would dominate over the observations - shifting estimate from empirical to uniform.
In the digit recognition from pixels example: 255 intensity values;
$13 \times 13 = 169$ pixels: Applying Laplace smoothing with $k = 1$ to $P(x)$ (prior probability of a particular pixel will take an intensity value i):
$P(x_{u,v} = i) = (c(x) + 1)/(N + 255)$
Conditional: relevant for the Naïve Bayes case.

Product of many small numbers ...

$$P(s|\vec{x}) = \frac{P(\vec{x}|s)P(s)}{P(\vec{x})} = \frac{P(s)}{P(\vec{x})} P(x[1]|s) \cdot P(x[2]|s) \cdot \ldots$$

$P(\vec{x})$ not needed, ...

$$\log(P(x[1]|s)P(x[2]|s)\cdots) = \log(P(x[1]|s)) + \log(P(x[2]|s)) + \cdots$$

just try

- `prod(rand(1,100))` and `prod(rand(1,10000))` in Matlab.
- `prod(rand(1,100)) == 0` and `prod(rand(1,10000)) == 0` in Matlab.

Hitting the limit of number representation.
What is the way out?
$P(\vec{x})$ not needed – does not depend on the class.
Laws of logarithms...

Product of many small numbers ...

$$P(s|\vec{x}) = \frac{P(\vec{x}|s)P(s)}{P(\vec{x})} = \frac{P(s)}{P(\vec{x})} P(x[1]|s) \cdot P(x[2]|s) \cdot \dots$$

$P(\vec{x})$ not needed, ...

$$\log(P(x[1]|s)P(x[2]|s)\cdots) = \log(P(x[1]|s)) + \log(P(x[2]|s)) + \cdots$$

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Hitting the limit of number representation.
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Laws of logarithms...

Training and testing

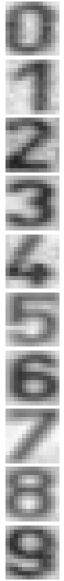
Data labeled instances.

- Training set
- Held-out (validation) set
- Testing set.

Features : Attribute-value pairs.

Learning cycle:

- **Learn** parameters (e.g. probabilities) on training set.
- **Tune** hyperparameters on held-out (validation) set.
- **Evaluate** performance on testing set.



How to evaluate a classifier? Confusion table

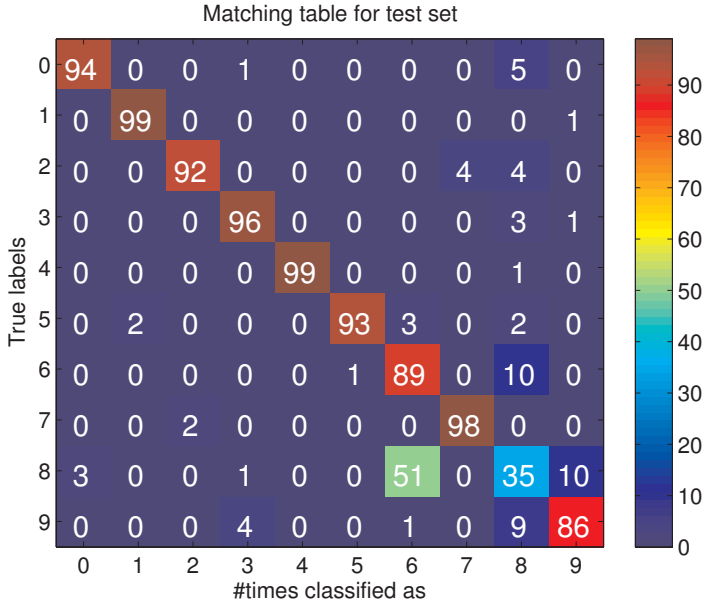


Figure from [5]

A result for a one particular classifer and its setting (parameters), one particular testing set

Precision and Recall, and ...

Consider digit detection (is there a digit?) or SPAM/HAM classification.

Recall :

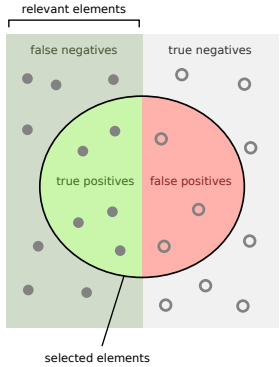
- How many relevant items are selected?
- Are we missing some items?
- Also called: **True positive rate** (TPR), sensitivity, hit rate ...

Precision

- How many selected items are relevant?
- Also called: Positive predictive value

False positive rate (FPR)

- Probability of false alarm



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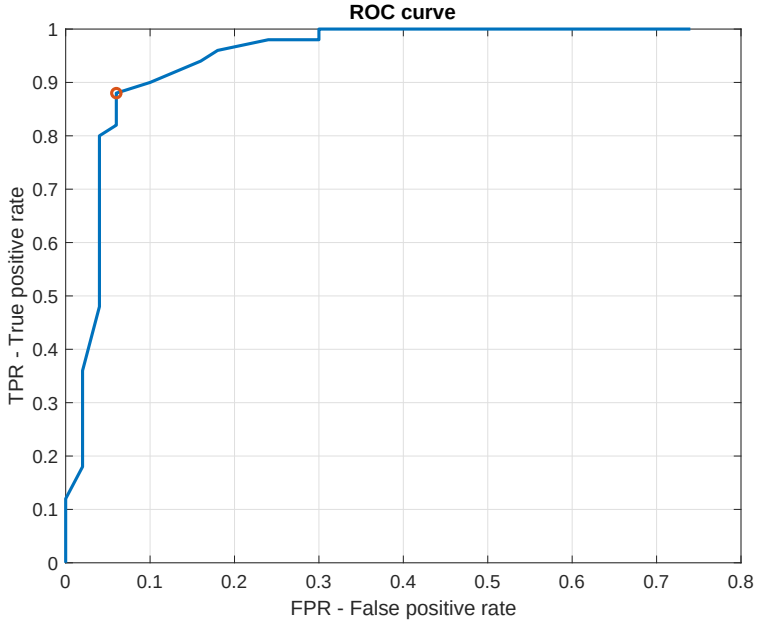
$$\text{TPR} = \frac{\text{TP}}{P} = \frac{\text{TP}}{\text{TP} + \text{FN}}$$

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$$

$$\text{FPR} = \frac{\text{FP}}{N} = \frac{\text{FP}}{\text{FP} + \text{TN}}$$

Think about TPR vs FPR graph, what is the best classifier?

ROC – Receiver operating characteristics curve



- How do you slide along the curve?
- What is the meaning of the diagonal?
- What would be the shape of the curve for the ideal/worst classifier?
- How would you compare various curve and select the best classifier?
- Think/read about other ways to evaluate/visualise classification results.

References I

Further reading: Chapter 13 and 14 of [4]. Books [1] and [2] are classical textbooks in the field of pattern recognition and machine learning. This lecture has been also inspired by the 21st lecture of CS 188 at `http://ai.berkeley.edu` (e.g., Laplace smoothing). Many Matlab figures created with the help of [3].

[1] Christopher M. Bishop.
Pattern Recognition and Machine Learning.
Springer Science+Bussiness Media, New York, NY, 2006.
PDF freely downloadable.

[2] Richard O. Duda, Peter E. Hart, and David G. Stork.
Pattern Classification.
John Wiley & Sons, 2nd edition, 2001.

References II

[3] Votjěch Franc and Václav Hlaváč.
Statistical pattern recognition toolbox.
http://cmp.felk.cvut.cz/cmp/software/stprtool/index.html.

[4] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

[5] Tomáš Svoboda, Jan Kybic, and Hlaváč Václav.
Image Processing, Analysis and Machine Vision — A MATLAB Companion.
Thomson, Toronto, Canada, 1st edition, September 2007.
http://visionbook.felk.cvut.cz/.