Reinforcement learning II

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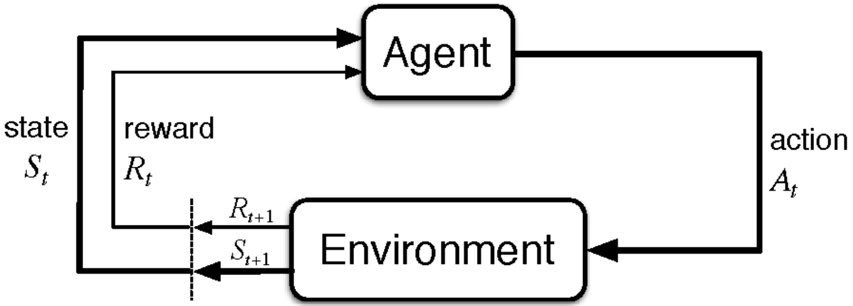
Recap: Reinforcement Learning



[1]

- Feedback in form of Rewards
- Learn to act so as to maximize sum of expected rewards.
- In `kuimaze` package, `env.step(action)` is the method.

[1]Scheme from [2]

Learning to control flippers

htt

What are states? What could be rewards?

From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- A set of states $s \in \mathcal{S}$ (map)
- A set of actions per state. $a \in \mathcal{A}$
- A transition model $p(s'|s,a)$ (robot)
- A reward function $r(s,a,s')$ (map, robot)

Looking for the optimal policy $\pi(s)$. We can plan/search before the robot enters the environment.

On-line problem:

- Transition p and reward r functions not known.
- Agent/robot must act and learn from experience.

For MDPs, we know p, r for all possible states and actions.

(Transition) Model-based learning

The main idea: Do something and:

- Learn an approximate model from experiences.
- Solve as if the model were correct.

Learning MDP model:

- Try s, a, observe s', count s, a, s'.
- Normalize to get and estimate of $p(s'|s, a)$
- Discover each $r(s, a, s')$ when experienced.

Solve the learned MDP.

Model-free learning

- r, p not known.
- Move around, observe
- And learn on the way.
- **Goal:** learn the state value $v(s)$ or (better) q-value $q(s, a)$ functions.

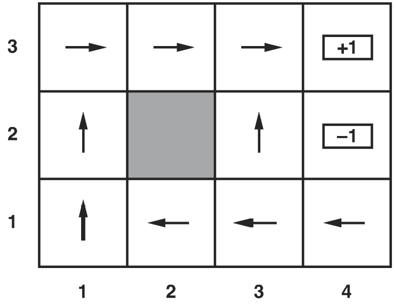


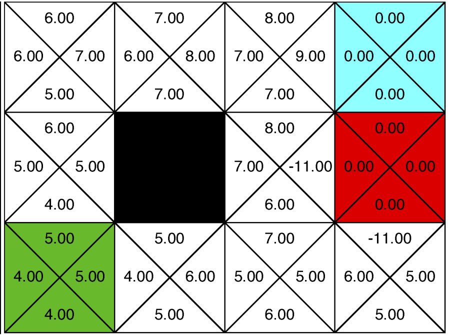
Image from [1]

Executing policies - training, then learning from the observations. We want to do the policy evaluation but the necessary model is not known.

Recap: $V-$ and $Q-$ values, converged ...

$\gamma = 1$, rewards $-1, +10, -10$, and no confusion - deterministic robot

7.00	8.00	9.00	10.00
6.00		8.00	-10.00
5.00	6.00	7.00	6.00



$$
\begin{aligned}
V(S_t) &= R_{t+1} + V(S_{t+1}) \\
Q(S_t, A_t) &= R_{t+1} + \max_a Q(S_{t+1}, a)
\end{aligned}
$$

$\gamma = 1$, Rewards $-1, +10, -10$, and no confusion - deterministic robot/agent. Rewards associated with leaving the state. Q values close next to terminal state includes the actual reward and the transition cost steping in, or better, leaving the last living state.

$Q(s, a)$ - expected sum of rewards having taken the action and acting according to the (optimal) policy.

How would the (q)values change if $\gamma = 0.9$?

Model-free TD learning, updating after each transition

- Observe, experience environment through learning episodes, collecting:

$$S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}, R_{t+2}, \ldots$$

- Update by mimicking Bellman updates after each transition $(S_t, A_t, R_{t+1}, S_{t+1})$

Think about $S_t - A_t - S_{t+1} - A_{t+1} - S_{t+2}$ tree with associated rewards.
Episode starts in a start state and ends in a terminal state.

Recap: Bellman optimality equations for $v(s)$ and $q(s,a)$

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\begin{aligned}
v(s) &= \max_a \sum_{s'} p(s'|s,a)\left[r(s,a,s') + \gamma\, v(s')\right] \\
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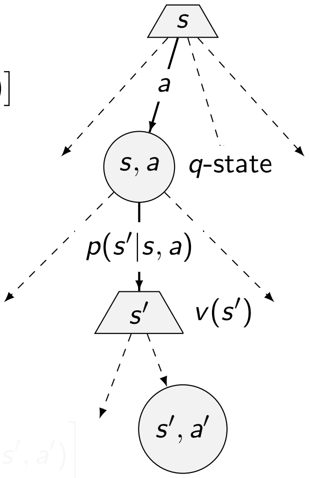

The value of a q-state (s,a):

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The tree continues from s' through a' and so on until it terminates

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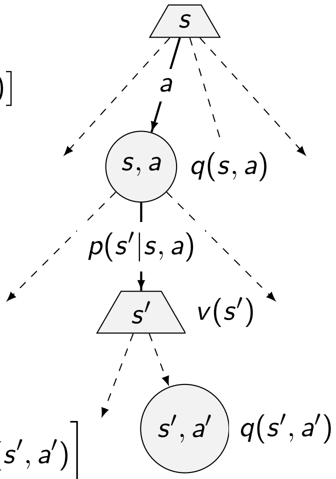

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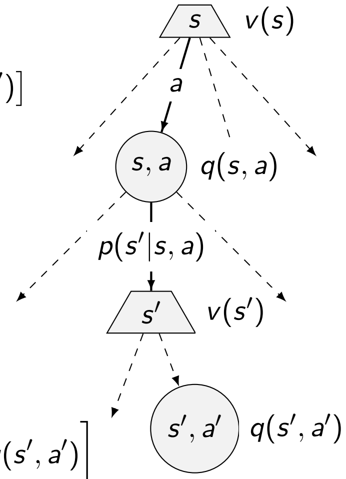

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Q-learning

Learn Q values as the robot/agent goes (temporal difference). If some Q quantity not known, initialize.

- time t, at S_t
- take $A_t \in \mathcal{A}(S_t)$, observe R_{t+1}, S_{t+1}
- compute trial/sample estimate at time t
  trial $= R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- α temporal difference update
  $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$
- $S_t \leftarrow S_{t+1}$ and repeat (unless S_t is terminal)

In each step Q approximates the optimal q^* function.

There are alternatives how to compute the trial. SARSA method takes $Q(S_t, A_t)$ directly, not the max. Hence we need 5-tuples $S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}$

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Q-learning: algorithm

step size $0 < \alpha \leq 1$
initialize $Q(s, a)$ for all $s \in \mathcal{S}, a \in \mathcal{S}(s)$
repeat episodes:
    initialize S
    for for each step of episode: **do**
        choose A from S
        take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
        $S \leftarrow S'$
    end for until S is terminal
until Time is up, ...

How to select A_t in S_t?

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How to select A_t in S_t?

- time t, at S_t
- take A_t derived from Q , observe R_{t+1}, S_{t+1}
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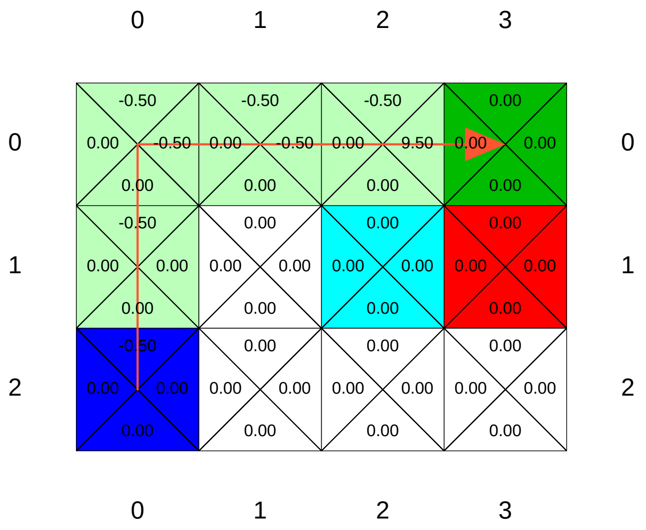
$\ldots A_t$ derived from Q

What about keeping optimality, taking max?

$$A_t = \arg\max_a Q(S_t, a)$$

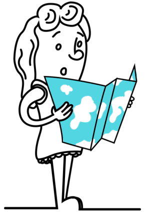
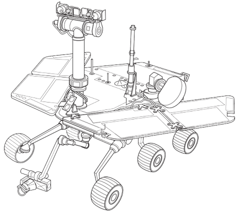
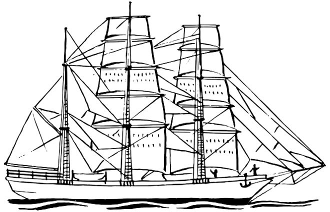
see the demo run of `rl_agents.py`.

Two good goal states



Discuss the on-line demo with two good goal states. $\gamma = 1, \alpha = 0.5$, Living reward -1, $R(1,2) = 10, R(3,0) = 20, R(1,1) = -10$. Taking the action, corresponding the max Q. If equal options, than in the $0,1,2,3$ action order. 50 training episodes. What happened?

Exploration vs Exploitation

- Drive the known road or try a new one?
- Go to the university menza or try a nearby restaurant?
- Use the SW (operating system) I know or try new one?
- Go to bussiness or study a demanding program?
- ...

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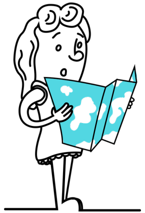

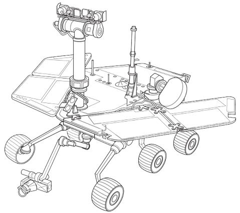

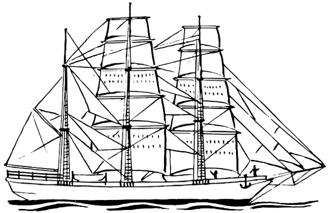

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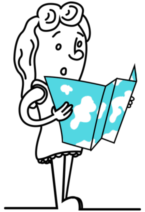 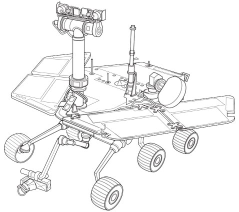 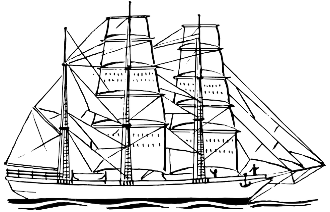

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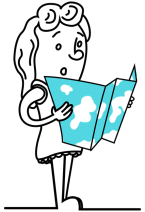 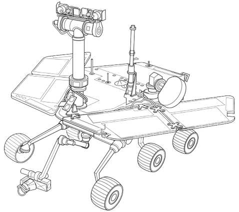 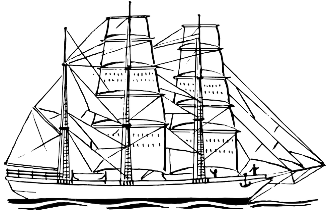

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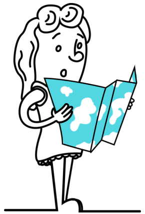

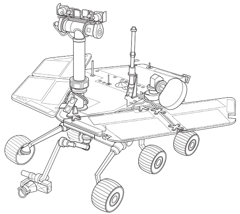

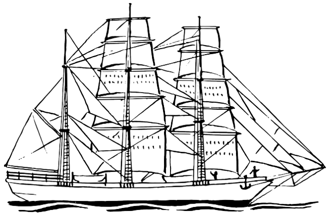

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How to explore?

Random (ϵ-greedy):

- Flip a coin every step.
- With probability ϵ, act randomly.
- With probability $1 - \epsilon$, use the policy.

Problems with randomness?

- Keeps exploring forever.
- Should we keep ϵ fixed (over learning)?
- ϵ same everywhere?

We can think about lowering ϵ as the learning progresses.

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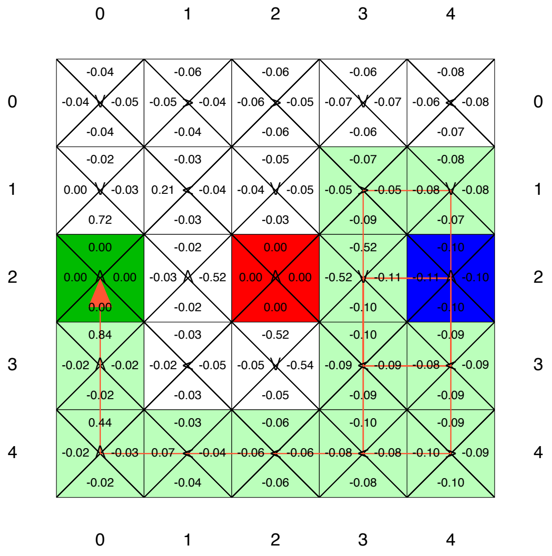
How to evaluate result, when to stop learning?



Run the found policy, discuss some traps, ...

Exploration function $f(u, n)$

- Regular trial/sample estimate: trial $= R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- If (S_t, a) not yet tried, then perhaps too pesimistic.
- trial $= R_{t+1} + \gamma \max_a f(Q(S_{t+1}, a), N(S_{t+1}, a))$

where $f(u, n)$

$$\begin{aligned} f(u,n) &= R^+ \text{ if } n < N_e \\ &= u \text{ otherwise} \end{aligned}$$

where

- R^+ is an optimistic estimate of the best possible reward obtainable in any state
- N_e fixed parameter
- The function $f(u, n)$ should be increasing in u and decreasing in n.

Will have the effect of making the agent try each actionstate pair at least N_e times

Exploration function $f(u, n)$

- Regular trial/sample estimate: trial $= R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$
- If (S_t, a) not yet tried, then perhaps too pesimistic.
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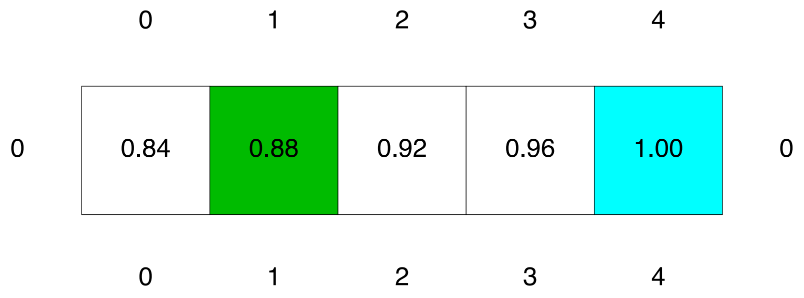
Going beyond tables - generalizing across states



We were talking about $v-$ and $q-$ *functions* but what was the representation? (look-up) Tables. Looking at $v(s)$, we need a table for each of the state!

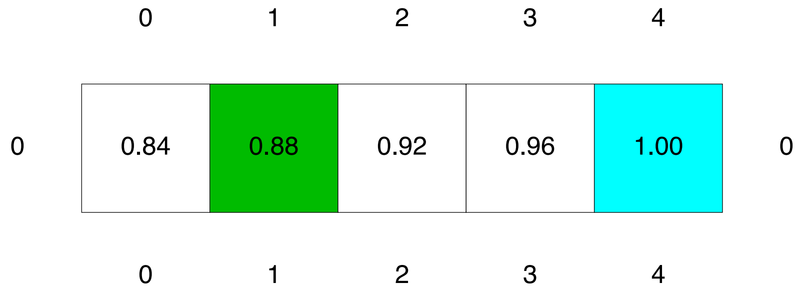
This world is small, but think bigger!

Going beyond tables - generalizing across states

	0	1	2	3	4	
0	0.84	0.80	0.76	0.72	0.68	0
1	0.88	0.84	0.80	0.76	0.72	1
2	0.92	0.88	0.84	0.80	0.76	2
3	0.96	0.92	0.88	0.84	0.80	3
4	1.00	0.96	0.92	0.88	0.84	4
	0	1	2	3	4	

Looking a $V(s)$, we need a table for each of the state! This world is small, but think bigger!

v(s) not as table but as an approximation function



$$\hat{v}(s, \mathbf{w}) = w_0 + w_1 s$$

What are w_0, w_1 equal to?
Instead of the complete table, only 2 parameters to learn $\mathbf{w} = [w_0, w_1]^\top$

What are w_0, w_1 equal to?, we can start from left, target is the true $v(s = 0) = 0.84$, next target is $v(s = 1) = 0.88$, ...

Note about notation. Bold lower cases are used to denote vectors.

Vectors are always considered oriented columnwise unless explicitly stated otherwise.

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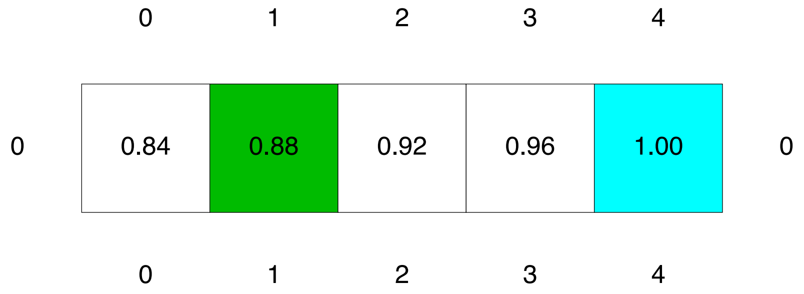

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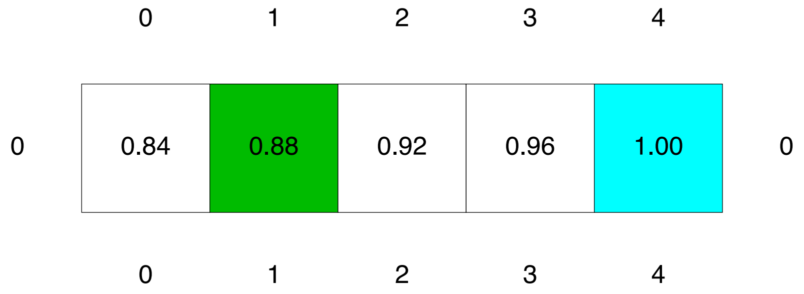

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Linear value functions

7.00	8.00	9.00	10.00
6.00		8.00	-10.00
5.00	6.00	7.00	6.00

$$
\begin{aligned}
\hat{v}(s, \mathbf{w}) &= w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \cdots + w_n f_n(s) \\
\hat{q}(s, a, \mathbf{w}) &= w_1 f_1(s, a) + w_2 f_2(s, a) + w_3 f_3(s, a) + \cdots + w_n f_n(s, a)
\end{aligned}
$$

What could be the f functions for the grid world?

Obviously, when data are available, we can fit. How to do it on-line?

Learning $\mathbf{w}$ by Stochastic Gradient Descent (SGD)

- assume $\hat{v}(s, \mathbf{w})$ differentiable in all states
- we update $\mathbf{w}$ in discrete time steps t
- in each step S_t we observe a new example of (true) $v^\pi(S_t)$
- $\hat{v}(S_t, \mathbf{w})$ is an approximator $\rightarrow$ error $v^\pi(S_t) - \hat{v}(S_t, \mathbf{w}_t)$

$$\begin{aligned}\mathbf{w}_{t+1} &\doteq \mathbf{w}_t - \frac{1}{2}\alpha\nabla\Big[v^\pi(S_t) - \hat{v}(S_t, \mathbf{w}_t)\Big]^2 \\ &= \mathbf{w}_t + \alpha\Big[v^\pi(S_t) - \hat{v}(S_t, \mathbf{w}_t)\Big]\nabla\hat{v}(S_t, \mathbf{w}_t)\end{aligned}$$

$$\nabla f(\mathbf{w}) \doteq \left[\frac{\partial f(\mathbf{w})}{\partial w_1}, \frac{\partial f(\mathbf{w})}{\partial w_2}, \ldots, \frac{\partial f(\mathbf{w})}{\partial w_d}\right]^\top$$

Gradient descent - all samples are known, Stochastic GD - update after each sample
$\hat{v}(s, \mathbf{w})$ could be quite complex, e.g. a Multi Layer Perceptron (MLP), Deep Network, and $\mathbf{w}$ represents the weights. See, e.g.

- https://skymind.ai/wiki/deep-reinforcement-learning
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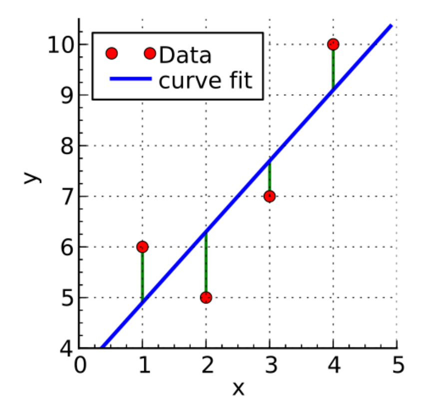
Approximate Q-learning (of a linear combination)

$$\hat{q}(s, a, \mathbf{w}) = w_1 f_1(s, a) + w_2 f_2(s, a) + w_3 f_3(s, a) + \cdots + w_n f_n(s, a)$$

- transition $= S_t, A_t, R_{t+1}, S_{t+1}$
- trial $R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \mathbf{w}_t)$
- diff $= \left[R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \mathbf{w}_t)\right] - \hat{q}(S_t, A_t, \mathbf{w}_t)$
- Update: $\mathbf{w} = [w_1, w_2, \cdots, w_d]^\top$
  $w_i \leftarrow w_i + \alpha\,[\text{diff}]\, f_i(S_t, A_t)$

How is it possible at all? On-line least squares!



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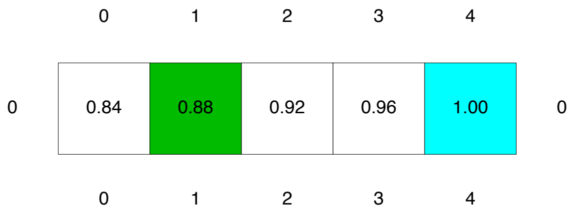
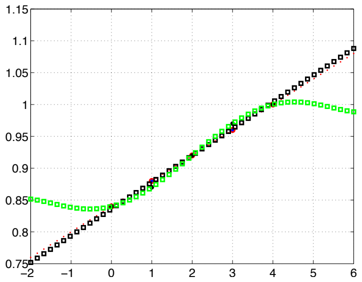
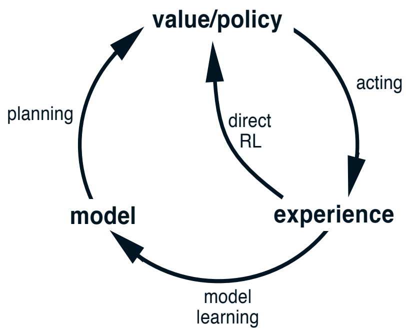
Overfitting

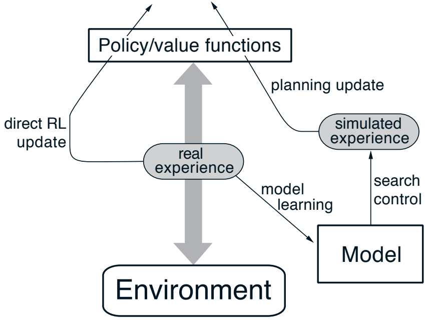




See the `fitdemo.m` run, higher degree polynomials perfectly fits, but poorly generalizes outside the range

Going beyond - Dyna-Q integration planning, acting, learning





[2]

[2] Schemes from [2]

References

Further reading: Chapter 21 of [1]. More detailed discussion in [2] Chapters 6 and 9. You can read about strategies for exploratory moves at various places, Tensor Flow related[3]. More RL URLs at the course pages[4].

[1] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

[2] Richard S. Sutton and Andrew G. Barto.
Reinforcement Learning; an Introduction.
MIT Press, 2nd edition, 2018.
http://www.incompleteideas.net/book/the-book-2nd.html.

[3] https://medium.com/emergent-future/simple-reinforcement-learning-with-tensorflow-part-7-action-selection-stra

[4] https://cw.fel.cvut.cz/wiki/courses/b3b33kui/cviceni/program_po_tydnech/tyden_09#reinforcement_learning_plus