Sequential decisions under uncertainty
Policy iteration

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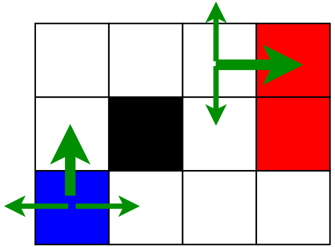
Unreliable actions in observable grid world

- Walls block movement – agent/robot stays in place.
- Actions do not always go as planned.
- Agent receives rewards each time step:
  - Small "living" reward/penalty.
  - Big rewards/penalties at the end.
- Goal: maximize sum of (discounted) rewards



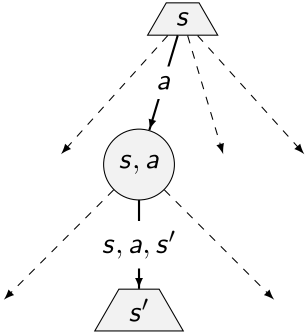
MDPs recap

Markov decision processes (MDPs):

- Set of states $\mathcal{S}$
- Set of actions $\mathcal{A}$
- Transitions $p(s'|s, a)$ or $T(s, a, s')$
- Rewards $r(s, a, s')$; and discount γ

MDP quantities:

- Policy $\pi(s) : \mathcal{S} \to \mathcal{A}$
- Utility – sum of (discounted) rewards.
- Values – expected future utility from a state (max-node), $v(s)$
- Q-Values – expected future utility from a q-state (chance-node), $q(s, a)$



Q-values – like values but given that I have commited to do action a from state s.

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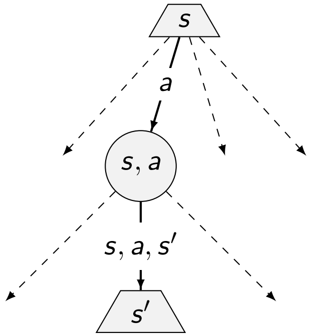


Q-values – like values but given that I have commited to do action a from state s.

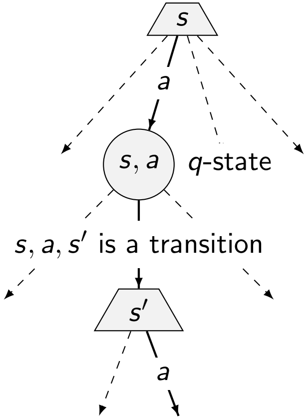
Optimal quantities

- The optimal policy: $\pi^*(s)$ – optimal action from state s
- Expected utility/return of a policy.

$$U^\pi(S_t) = \mathrm{E}^\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}\right]$$

Best policy π^* maximizes above.

- The value of a state s: $v^*(s)$ – expected utility starting in s and acting optimally.
- The value of a q-state (s, a): $q^*(s, a)$ - expected utility having taken a from state s and acting optimally thereafter.



Remember: Discounted return G_t
Returns are successive steps related to each other

$$\begin{aligned} G_t &= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + \cdots \\ &= R_{t+1} + \gamma(R_{t+2} + \gamma^1 R_{t+3} + \gamma^2 R_{t+4} + \cdots) \\ &= R_{t+1} + \gamma G_{t+1} \end{aligned}$$

$G_t \doteq \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ including the possibility that $T = \infty$ or $\gamma = 1$, but not both.

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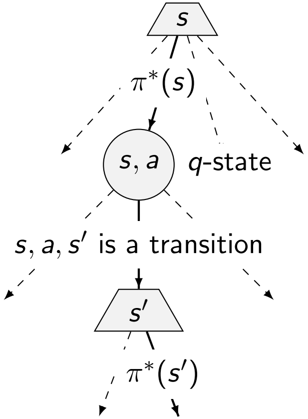

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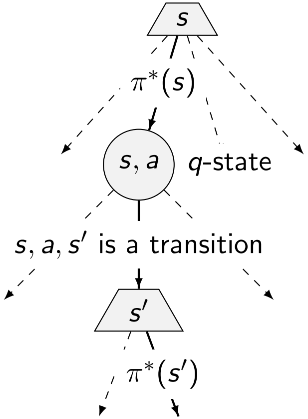

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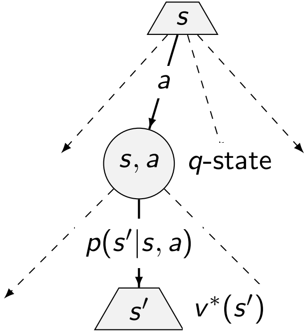
V^* and Q^*

The value of a q-state (s, a):

$$q^*(s, a) = \sum_{s'} p(s'|a, s) \left[r(s, a, s') + \gamma \, v^*(s') \right]$$



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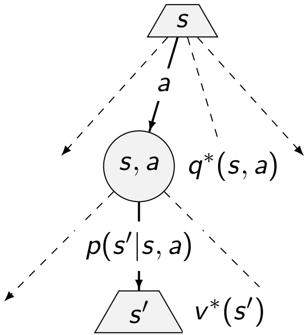

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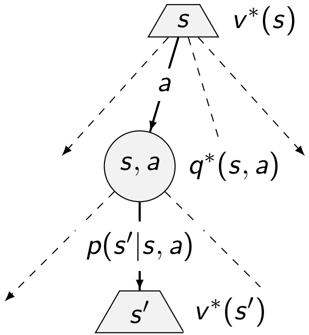

Maze: v^* vs. q^*

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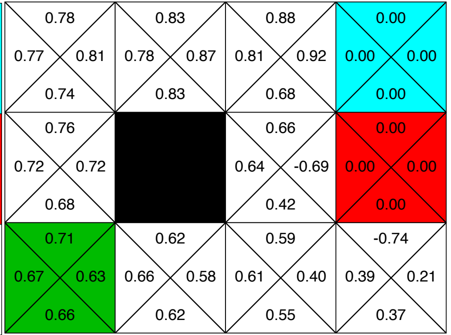
0.76		0.66	-1.00
0.71	0.66	0.61	0.39



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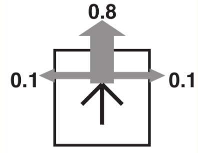
This is the $R = -0.04$ for nonterminal states maze (AIMA Fig. 17.3).



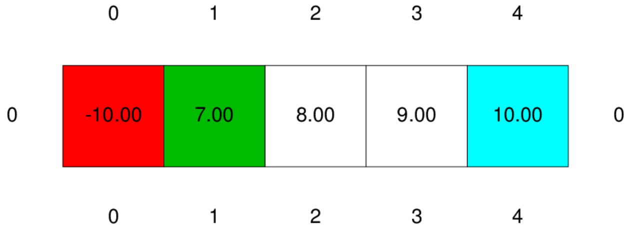
, $\gamma = 1$

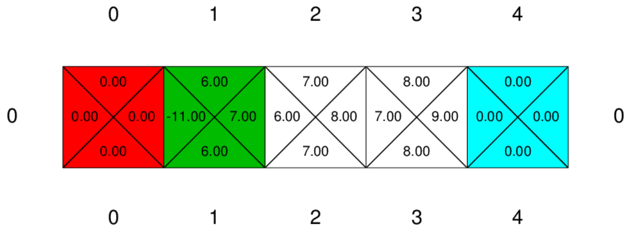
Note that the Value of a state takes into account a number of things:

- the policy – which action will chosen in s
- the fact that the goal may be far away and
  - there will be a number of living penalties incured before reaching it
  - the final reward will be discounted
- the transition probabilities

Q-values - useful for choosing the best action – getting the policy.

Maze: v^* vs. q^*



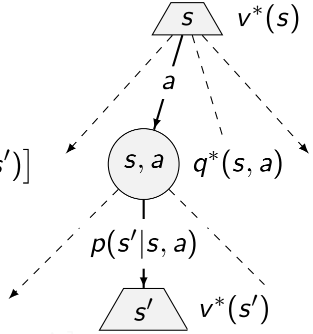


$A = \{\leftarrow, \rightarrow\}$
$P(action - succeeds - as - planned) = 0.8,$
$P(reverse - direction - of - movement - than - commanded) = 0.2$

Value iteration

- Bellman equations **characterize** the optimal values

$$v^*(s) = \max_{a \in A(s)} \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma v^*(s') \right]$$



- Value iteration **computes** them:

$$V_{k+1}(s) \leftarrow \max_{a \in A(s)} \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_k(s') \right]$$

Value iteration is a fixed point solution method.

Bellman equations:

1. Take correct first action (1 ply of Expectimax)
2. Keep being optimal (recursion $v^*(s')$)

Recall that we may simplify equations by marginalizing rewards if all $r(s, a, s')$ are the same.

$$r(s) = \sum_{s'} p(s'|a,s) r(s,a,s')$$

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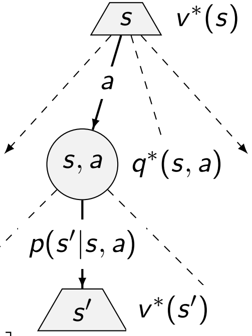

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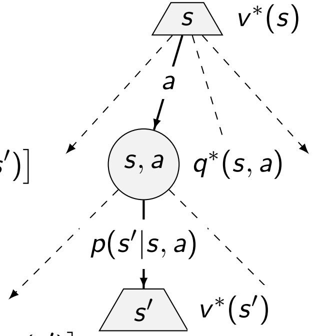

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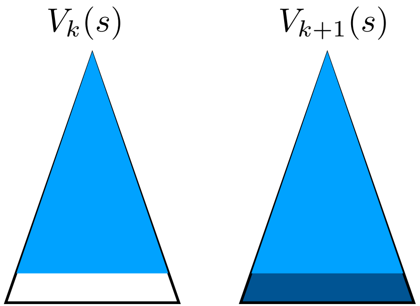
Convergence

$$V_{k+1}(s) \leftarrow \max_{a \in A(s)} \sum_{s'} p(s'|s,a)\left[r(s,a,s') + \gamma V_k(s')\right]$$

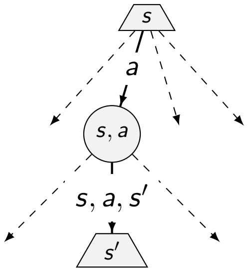
- Thinking about special cases: deterministic world, $\gamma = 0$, $\gamma = 1$.
- For all s, $V_k(s)$ and $V_{k+1}(s)$ can be seen as expectimax search trees of depth k and $k+1$



We will show it on the blackboard during the lecture

From Values to Policy

Policy extraction - computing actions from Values



- Assume we have $v^*(s)$
- What is the optimal action?
- We need a one-step expectimax:

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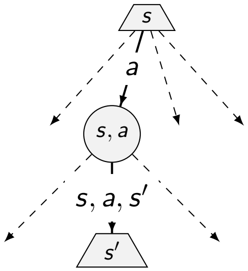

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Policy extraction - computing actions from q-Values

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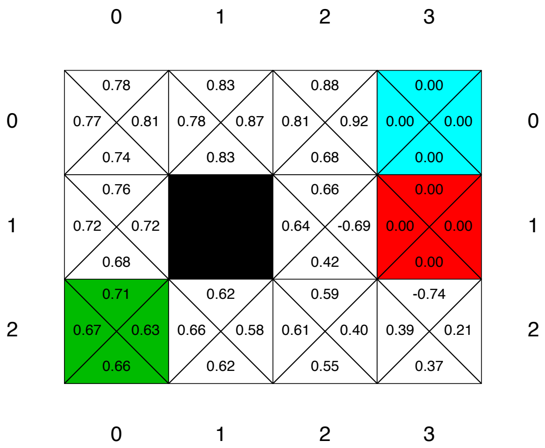

Actions are easier to extract from q-values.

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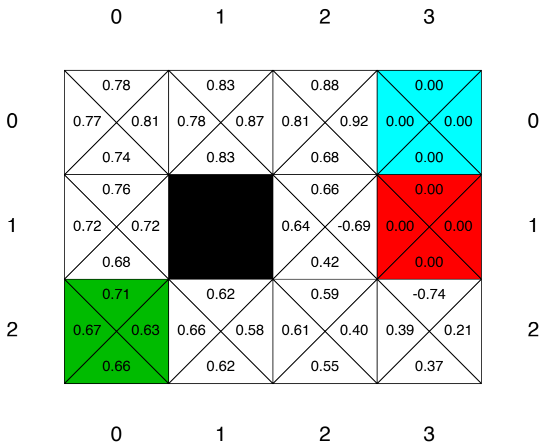

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What is wrong with the Value iteration?

$$V_{k+1}(s) \leftarrow \max_{a \in \mathcal{A}(s)} \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V_k(s') \right]$$

- What is complexity of one iteration - over all S states?
- Does the "max" change often?
- When the does the policy converge?
- Can we compute the policy directly?

Complexity: $O(S^2 * A)$ per iteration
For every state (LHS), there can be up to $\sharp S$ also on RHS – if every other state was reachable from the current state.
In addition, all actions from every state need to be considered.
$Max(A)$ **does not** change often.
Policy often converges long before the values.

Run "AIMA Fig. 17.2 / 17.3 demo" with $R = -0.04$
mdp_agents.py, value iteration with $eps = 0.03$, $discount = 0.999999$

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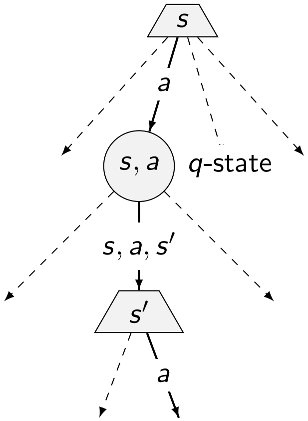
Policy evaluation

- Assume $\pi(s)$ given.
- How to evaluate (compare)?

Remember last week's quizz?

Fixed policy, do what π says



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  ...
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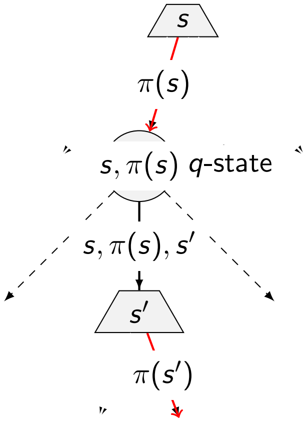

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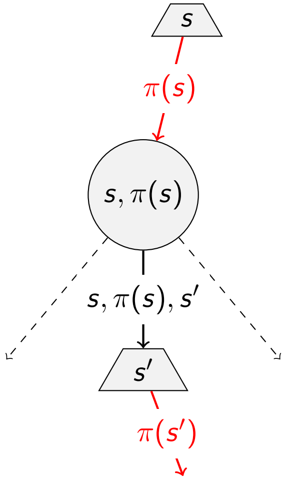

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Recall that $v^{\pi}(s)$ quantity contains all the future – expected discounted sum of rewards – executing policy from the state s onwards.

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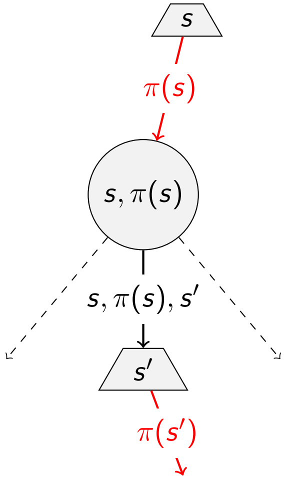

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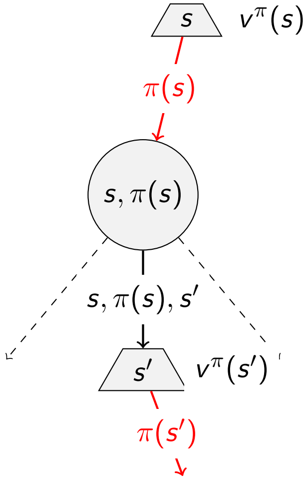

- Expectimax trees "max" over all actions …
- Fixed π for each state $\to$ no "max" operator!

$$v^\pi(s) = \sum_{s'} p(s' \mid s, \pi(s)) \left[r(s, \pi(s), s') + \gamma v^\pi(s') \right]$$

Recall that $v^\pi(s)$ quantity contains all the future – expected discounted sum of rewards – executing policy from the state s onwards.

How to compute $v^\pi(s)$?

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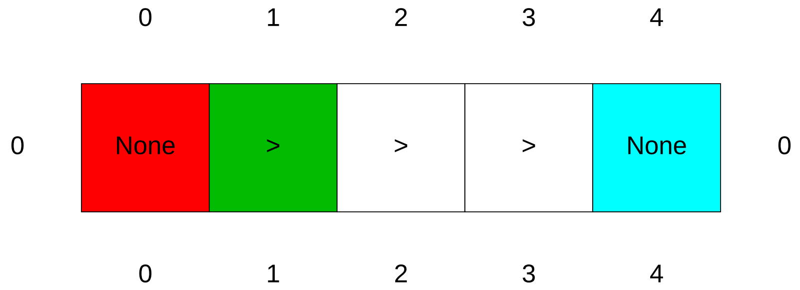

- by iteration
- solving set of equations

Policy iteration

- Start with a random policy.
- Step 1: Evaluate it.
- Step 2: Improve it.
- Repeat steps until policy converges.

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Policy iteration

- Policy π evaluation. Solve equations or iterate until convergence.

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} p(s' \mid s, \pi(s)) \left[r(s, \pi(s), s') + \gamma V_k^{\pi_i}(s') \right]$$

- Policy improvement. Look-ahead and keep optimality. Policy extraction from fixed values.

$$\pi_{i+1}(s) = \arg\max_{a \in \mathcal{A}(s)} \sum_{s'} p(s' \mid s, a) \left[r(s, a, s') + \gamma V_k^{\pi_i}(s') \right]$$

A few demo runs of `mdp_agents.py`.
Note that the value is taken from "old policy" on RHS.

Policy iteration algorithm

function POLICY-ITERATION(env) **returns:** policy π
  input: env - MDP problem
  $\pi(s) \leftarrow$ random $a \in A(s)$ in all states
  $V(s) \leftarrow 0$ in all states
  repeat $\triangleright$ iterate values until no change in policy
    $V \leftarrow$ POLICY-EVALUATION(π, V, env)
    unchanged $\leftarrow$ True
    for each state s **in** S **do**
      if $\max_{a \in A(s)} \sum_{s'} P(s'|a,s)V(s') > \sum_{s'} P(s'|s,\pi(s))V(s')$ **then**
        $\pi(s) \leftarrow \arg\max_{a \in A(s)} \sum_{s'} P(s'|a,s)V(s')$
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    end for
  until unchanged
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Policy vs. Value iteration

- Value iteration.
  - Iteration updates values and policy. Although policy implicitly – extracted from values
  - No track of policy.
- Policy iteration.
  - Update utilities is fast – only one action per state.
  - New policy from values (slower)
  - New policy is better or done.
- Both methods belong to Dynamic programming realm.

Complexity (of one iteration step):
Value iteration: $O(S^2 * A)$
For every state (LHS), there can be up to $\sharp S$ also on RHS – if every other state was reachable from the current state.
In addition, all actions from every state need to be considered.
$Max(A)$ **does not** change often.
Policy often converges long before the values.
Policy evaluation: $O(S^3)$ (after AIMA, pg. 657)
The Bellman equations are *linear* because the max operator is gone.
For $\sharp S$ states, we have $\sharp S$ equations, which can be solved exactly in time $O(S^3)$ using standard linear algebra methods.
For small state spaces - ok.
For large state spaces - may be prohibitive $\rightarrow$ *modified policy iteration* with only a certain number of simplified Bellman update.

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References

Further reading: Chapter 17 of [1] however, policy iteration is quite compact there. More detailed discussion can be found in chapter Dynamic programming in [2] with slightly different notation, though. This lecture has been also greatly inspired by the 9th lecture of CS 188 at `http://ai.berkeley.edu` as it convincingly motivates policy search and offers an alternative convergence proof of the value iteration method.

[1] Stuart Russell and Peter Norvig.
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Prentice Hall, 3rd edition, 2010.
`http://aima.cs.berkeley.edu/`.

[2] Richard S. Sutton and Andrew G. Barto.
Reinforcement Learning; an Introduction.
MIT Press, 2nd edition, 2018.
`http://www.incompleteideas.net/book/the-book-2nd.html`.

Bandits