Problem 1/23

Functions $f(x)$, $g(x)$ are continuous and growing, $f(x) \in \Omega(g(x))$.
Therefore:

a) $f(x) \in O(g(x))$

b) $f(x) \in \Theta(g(x))$

c) $g(x) \in \Theta(f(x))$

d) $g(x) \in \Omega(f(x))$

e) $g(x) \in O(f(x))$

Problem 2/23

Functions f(x), g(x) are continuous and growing, $f(x) \in O(g(x))$.
Therefore:

a) $f(x) \in \Theta(g(x))$

b) $f(x) \in \Omega(g(x))$

c) $g(x) \in \Theta(f(x))$

d) $g(x) \in \Omega(f(x))$

e) $g(x) \in O(f(x))$

Problem 3/23

When function f grows asymptotically faster than function g (ie. $f(x) \notin O(g(x))$) some of the following is true:

a) if both f and g are defined in x then $f(x) > g(x)$

b) the difference $f(x) - g(x)$ is always positive

c) the difference $f(x) - g(x)$ is positive for each $x > y$,
where y is some sufficiently big real number

d) both f and g are defined only for non-negative arguments

e) none of the previous

Problem 4/23

When the rate of growth of function f is same as that of function g (ie. $f(x) \in \Theta(g(x))$) some of the following is true:

a) if bot f and g are defined in x than $f(x) = g(x)$

b) none of the ratios $f(x)/g(x)$ and $g(x)/f(x)$ converges to 0 with x increasing to infinity

c) the difference $f(x) - g(x)$ is positive for each $x > y$, where y is some sufficiently big real number

d) both f and g are defined only for non-negative arguments

e) none of the previous

Problem 5/23

Two functions f(x) and g(x) continuous and growing on the whole **R** satisfy $f(x) < g(x)$ for each $x \in$ **R**. This also means that:

a) $f(x) \notin \Omega(g(x))$

b) $f(x) \notin O(g(x))$

c) it is possible that $f(x) \in \Omega(g(x))$

d) $g(x) \notin \Theta(f(x))$

e) f(x) grows asymptotically more slowly than g(x)

Problem 6/23

Two functions f(x) and g(x) continuous and growing on the whole **R** satisfy $f(x) \notin \Omega(g(x))$, $f(x) \notin \Theta(g(x))$. Therefore:

a) $g(x) \in O(f(x))$

b) $g(x) \in \Theta(f(x))$

c) $f(x) < g(x)$ for each $x \in$ **R**

d) $f(x) \leq g(x)$ for each $x \in$ **R**

e) there may exist $y \in$ **R** such that $f(y) > g(y)$

Problem 7/23

Algorithm A processes all elements of a 2D array of size $n \times n$.

Processing of an element consists of a subroutine call which asymptotic complexity is $\Theta(\log_2(n))$. Asymptotic complexity of A is therefore:

a) $\Theta(n \cdot \log_2(n))$

b) $\Theta(n^2)$

c) $\Theta(n^3)$

d) $\Theta(n^2+\log_2(n))$

e) $\Theta(n^2 \cdot \log_2(n))$

Problem 8/23

Mark the stament which is not true.

a) $x \cdot \log_2(x) \in O(x^2 - x)$

b) $x \cdot \log_2(x) \in O(x^2 - \log_2(x))$

c) $x \cdot \log_2(x) \in \Omega(x^2 - \log_2(x))$

d) $x \cdot \log_2(x) \in \Omega(x + \log_2(x))$

e) $x \cdot \log_2(x) \in \Theta(x \cdot \log_2(x^2))$

Problem 9/23

Algorithm A processes all elements of a 1D array of size n.

Processing of an element with index k consists of a subroutine call which asymptotic complexity is $\Theta(k + n)$. Asymptotic complexity of A is therefore:

a) $\Theta(k+n)$

b) $\Theta((k+n)\cdot n)$

c) $\Theta(k^2+n)$

d) $\Theta(n^2)$

e) $\Theta(n^3)$

Problem 10/23

Fill the empty spaces (........) with symbols O or Θ or Ω to obtain a true statment. If more symbols can be used at particular position write them all. If no symbol can be used, cross out the empty space.

a) $x^2 \cdot 2^x \in$ ....$((\ln(x^2))^2 + 2^x)$

b) $(\ln(x^2))^2 + 2^x \in$ ....$(x^2 + \ln(x^2))$

c) $2^x \cdot (\ln(x))^{-1} \notin$ ....$(2^x \cdot (\ln(x^2))^{-1})$

Problem 11/23

Fill the empty spaces (........) with symbols O or Θ or Ω to obtain a true statment. If more symbols can be used at particular position write them all. If no symbol can be used, cross out the empty space.

a) $x^2 \cdot \ln(x^2) \in$ ....$(x^2 + \ln(x))$

b) $x^3 + \ln(x^2) \in$ ....$(x^3 + 2^x)$

c) $x^3 \cdot \ln(x^2) \notin$ ....$(\ln(x^2) + 2^x)$

Problem 12/23

Find particular real functions f(x), g(x), h(x) which satisfy all three following conditions:

$$f(x) \notin O(g(x)),\ \ g(x) \notin \Theta(h(x)),\ \ h(x) \notin \Omega(f(x))$$

If no such three functions exist, explain why.

Problem 13/23

Find particular real functions f(x), g(x), h(x) which satisfy all three following conditions:

$$f(x) \notin O(g(x)), \quad g(x) \notin \Omega(h(x)), \quad h(x) \notin \Theta(f(x))$$

If no such three functions exist, explain why.

Problem 14/23

There are M rows and N columns in matrix T. We need exactly c operations to process an element at position [*r*][*c*] ($0 \le r < M$, $0 \le c < N$).

Each operation has constant asymptotic complexity. What is the asymptotic complexity of processing the whole matrix?

Problem 15/23

There are M rows and N columns in matrix T. We need exactly c+r operations to process an element at position [*r*][*c*] $(0 \le r < M,\ 0 \le c < N)$.

Each operation has constant asymptotic complexity. What is the asymptotic complexity of processing the whole matrix?

Exploit the formula:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 16/23

Consider the standard elementary school algorithm which is used to multiply two integers by hand. Suppose that adding or multiplying two digits is an operation of constant complexity.

Determine the asymptotic complexity of multiplying two integers M and N written in usual decimal representation.

```
Example                     9803
                          x 347
M =  9803                -------
N =  347                  68621
                         39212
                        29409
                       ---------
                        3401641
```

Problem 17/23

Determine the asymptotic complexity of the code with respect to the value of N.

```
//a = array[0..N-1] of int;
for(i = 0; i < N; i++)
  a[i] = N;
for (i = 0; i < N; i++)
  while (a[i] > 0) {
     print(a[i]);
     a[i] = a[i]/2;    // integer division
  }
```

Problem 18/23

Determine the asymptotic complexity of the code with respect to the value of N.

```
//a = array[0..N-1] of int;
for(i = 0; i < N; i++)
  a[i] = i;
for (i = 0; i < N; i++)
  while (a[i] > 0) {
     print(a[i]);
     a[i] = a[i]/2;    // integer division
  }
```

Problem 19/23

Determine the asymptotic complexity of the code with respect to the value of N.

```
//a = array[0..N-1] of int;
for(i = 0; i < N; i++)
  a[i] = 1;
for (i = 1; i < N; i++)
  while (a[i] <= 2*a[i-1]) {
     print(a[i]);
     a[i] = a[i]+1;
  }
```

Problem 20/23

A. What is the asymptotic complexity of matrix multiplication of two matrices of size N x N?

B. What is the asymptotic complexity of Gaussova elimination algorithm applied to the system of N equations with N variables?

C. What is the asymptotic complexity of computing determinant of a N x N matrix using the definition of the determinant?

Can a determinant be computed more effectively with lower asymptotic complexity?

D. What is the asymptotic complexity of solving a system of N linear equations with N variables using Cramer's rule?

Problem 21/23

There are N points labeled 1, 2, ..., N which are irregularly positioned on the perimeter of a given circle. The task is to compute the number of such triangles which vertices are in the labeled points and which do not contain in their interior the circle center.

Suggest an algorithm and determine its asymptotic complexity.

Solve an analogous problem with convex quadrilaterals instead of triangles.

Problem 22/23

The task is to print all such positive integer smaller than N which binary representation contains exactly three 1's.

What is the asymptotic complexity of an effective algorithm?

We do not consider an algorithm linear in N to be effective.

Problem 23/23

Describe how to find the value

$$\log(\log(N^{(N!)}))$$

when $N = 10^7$.

How long will it take to your personal computer to compute the value? The base of logarithm is 10.

Do not use approximations like Stirling's formula etc.