Assignment 2

Definitions

Definition 1. A STRIPS **planning task** Π is specified by a tuple $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, where $\mathcal{F} = \{f_1, ..., f_n\}$ is a set of facts, $\mathcal{O} = \{o_1, ..., o_m\}$ is a set of operators, and c is a cost function mapping each operator to a non-negative real number. A **state** $s \subseteq \mathcal{F}$ is a set of facts, $s_{init} \subseteq \mathcal{F}$ is an **initial state** and $s_{goal} \subseteq \mathcal{F}$ is a **goal** specification. An **operator** o is a triple $o = \langle \mathrm{pre}(o), \mathrm{add}(o), \mathrm{del}(o) \rangle$, where $\mathrm{pre}(o) \subseteq \mathcal{F}$ is a set of preconditions, and $\mathrm{add}(o) \subseteq \mathcal{F}$ and $\mathrm{del}(o) \subseteq \mathcal{F}$ are sets of add and delete effects, respectively. All operators are well-formed, i.e., $\mathrm{add}(o) \cap \mathrm{del}(o) = \emptyset$ and $\mathrm{pre}(o) \cap \mathrm{add}(o) = \emptyset$. An operator o is **applicable** in a state s if $\mathrm{pre}(o) \subseteq s$. The **resulting state** of applying an applicable operator o in a state s is the state $\mathrm{res}(o, s) = (s \setminus \mathrm{del}(o)) \cup \mathrm{add}(o)$. A state s is a **goal state** iff $s_{goal} \subseteq s$.

A **sequence of operators** $\pi = \langle o_1, ..., o_n \rangle$ is applicable in a state s_0 if there are states $s_1, ..., s_n$ such that o_i is applicable in s_{i-1} and $s_i = \mathrm{res}(o_i, s_{i-1})$ for $1 \leq i \leq n$. The resulting state of this application is $\mathrm{res}(\pi, s_0) = s_n$ and the cost of the plan is $\mathrm{c}(\pi) = \sum_{o \in \pi} \mathrm{c}(o)$. A sequence of operators π is called a **plan** iff $s_{goal} \subseteq \pi[s_{init}]$, and an **optimal plan** is a plan with the minimal cost over all plans.

Definition 2. The **relaxation** o^+ of a STRIPS operator $o = \langle \mathrm{pre}(o), \mathrm{add}(o), \mathrm{del}(o) \rangle$ is the operator $o^+ = \langle \mathrm{pre}(o), \mathrm{add}(o), \emptyset \rangle$. The relaxation Π^+ of a STRIPS planning task $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ is the planning task $\Pi^+ = \langle \mathcal{F}, \{o^+ \mid o \in \mathcal{O}\}, s_{init}, s_{goal}, \mathrm{c} \rangle$ with $\mathrm{c}(o^+) = \mathrm{c}(o)$ for every $o \in \mathcal{O}$.

Definition 3. A **disjunctive operator landmark** $L \subseteq \mathcal{O}$ is a set of operators such that every plan contains at least one operator from L.

Definition 4. A **labeled transition system** (LTS) is a tuple $\Theta = \langle \mathcal{S}, L, T, s_I, S_\star \rangle$, where $\mathcal{S}$ is a finite set of **states**, L is a finite set of **labels** with associated cost $\mathrm{c}(l) \in \mathbb{R}_0^+$ to each label $l \in L$, $T \subseteq \mathcal{S} \times L \times \mathcal{S}$ is a set of **transitions**, $s_I \in \mathcal{S}$ is the **initial state**, and $S_\star \subseteq \mathcal{S}$ is a set of **goal states**. We write $s_1 \xrightarrow{l} s_2$ to refer to a transition from s_1 to s_2 with the label l. A sequence of labels $\langle l_1, \ldots, l_n \rangle$ is a **path** from s_0 to s_n in Θ if there exist $s_{i-1} \xrightarrow{l_i} s_i \in T$ for every $i \in \{1, \ldots, n\}$ and a **plan** is a path from s_I to one of the goals from $S_\star$.

Definition 5. The **state space** of a STRIPS planning task $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, is the LTS Θ_Π where $\mathcal{S}$ are states over $\mathcal{F}$, $s_I := s_{init}$, $s \in S_\star$ iff $s_{goal} \subseteq s$, the labels L are the operators $\mathcal{O}$ with the given costs, and $s \xrightarrow{o} s'$ is a transition in T if $\mathrm{pre}(o) \subseteq s$ and $\mathrm{res}(o, s) = s'$.

Definition 6. An **abstraction** α for a transition system Θ is a function mapping states $\mathcal{S}$ into a set of abstract states $\mathcal{S}^\alpha$. The **abstract transition system** Θ^α is defined as

$\langle \mathcal{S}^\alpha, L, T^\alpha, s_I^\alpha, S_\star^\alpha \rangle$, where $\alpha(s) \xrightarrow{o} \alpha(s') \in T^\alpha$ iff $s \xrightarrow{o} s' \in T$, $s_I^\alpha = \alpha(s_I)$, and $S_\star^\alpha = \{\alpha(s) \mid s \in S_\star\}$.

A **projection** of the state space Θ_Π to the set of facts $F \subseteq \mathcal{F}$ is an abstract transition system $\Theta_\Pi^{\alpha_F}$ with the abstraction $\alpha_F(s) = s \cap F$.

Assignments

Prove or disprove the following claims:

1. Let Π denote a STRIPS planning task and Π^+ its relaxation. If there exists a plan for Π, then there exists a plan for Π^+.

2. Let Π denote a STRIPS planning task and Π^+ its relaxation. If there exists a plan for Π^+, then there exists a plan for Π.

3. Let $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote a STRIPS planning task, $L \subseteq \mathcal{O}$ a set of operators, $\Pi_2 = \langle \mathcal{F}, \mathcal{O} \setminus L, s_{init}, s_{goal}, \mathrm{c} \rangle$, and Π_2^+ relaxation of Π_2. If there does not exist any plan for Π_2^+, then L is a disjunctive operator landmark.

4. Let $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote a STRIPS planning task, $F \subseteq \mathcal{F}$ a set of facts, and $\Theta_\Pi^{\alpha_F}$ a projection of the state space of Π to the set of facts F. If there exists a plan for Π then there exists a plan for $\Theta_\Pi^{\alpha_F}$.

5. Let $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$ denote a STRIPS planning task, $F \subseteq \mathcal{F}$ a set of facts, and $\Theta_\Pi^{\alpha_F}$ a projection of the state space of Π to the set of facts F. If there exists a plan for $\Theta_\Pi^{\alpha_F}$, then there exists a plan for Π.