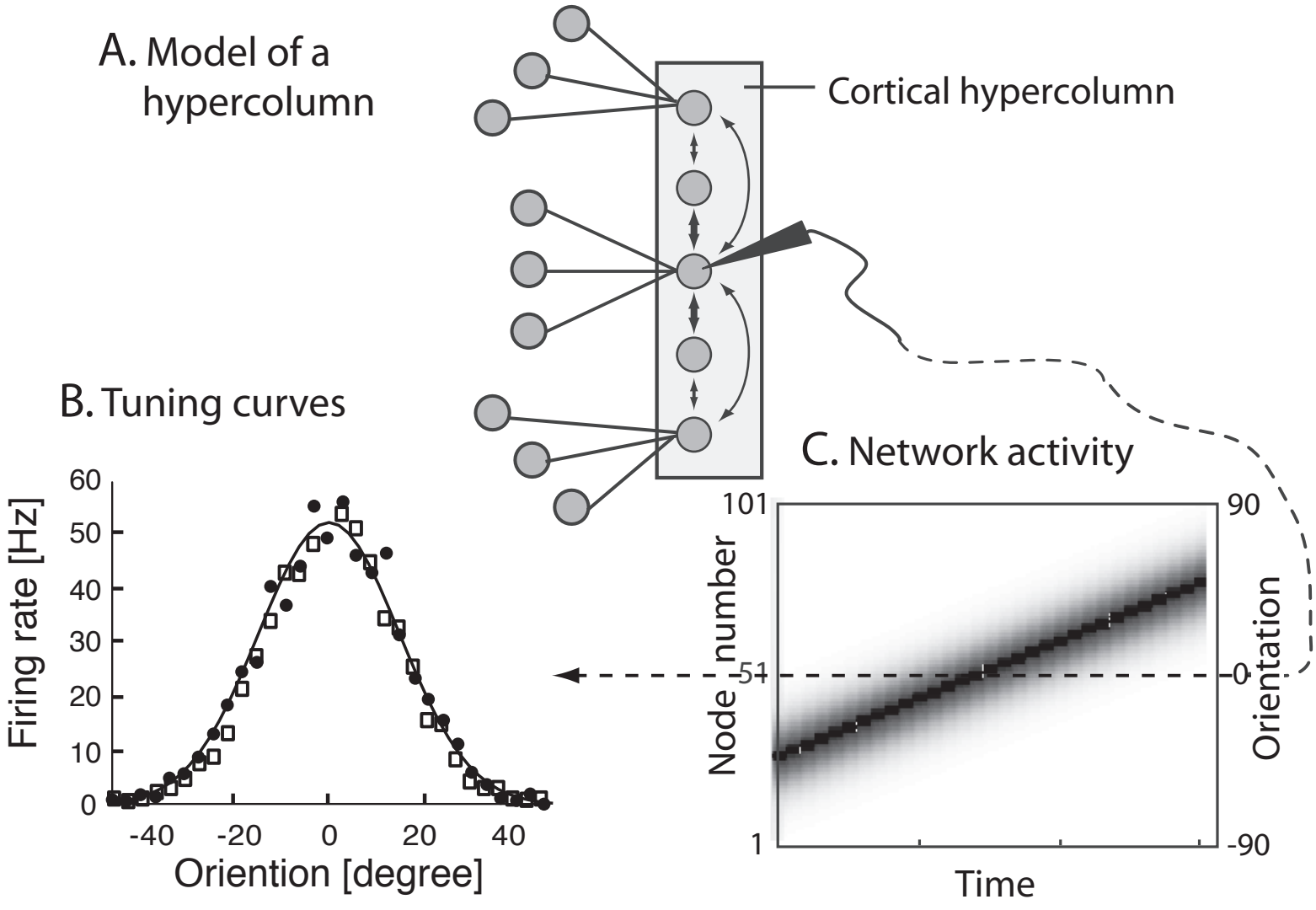
Neuroinformatics

May 3, 2018

Dynamical neural fields (DNF)

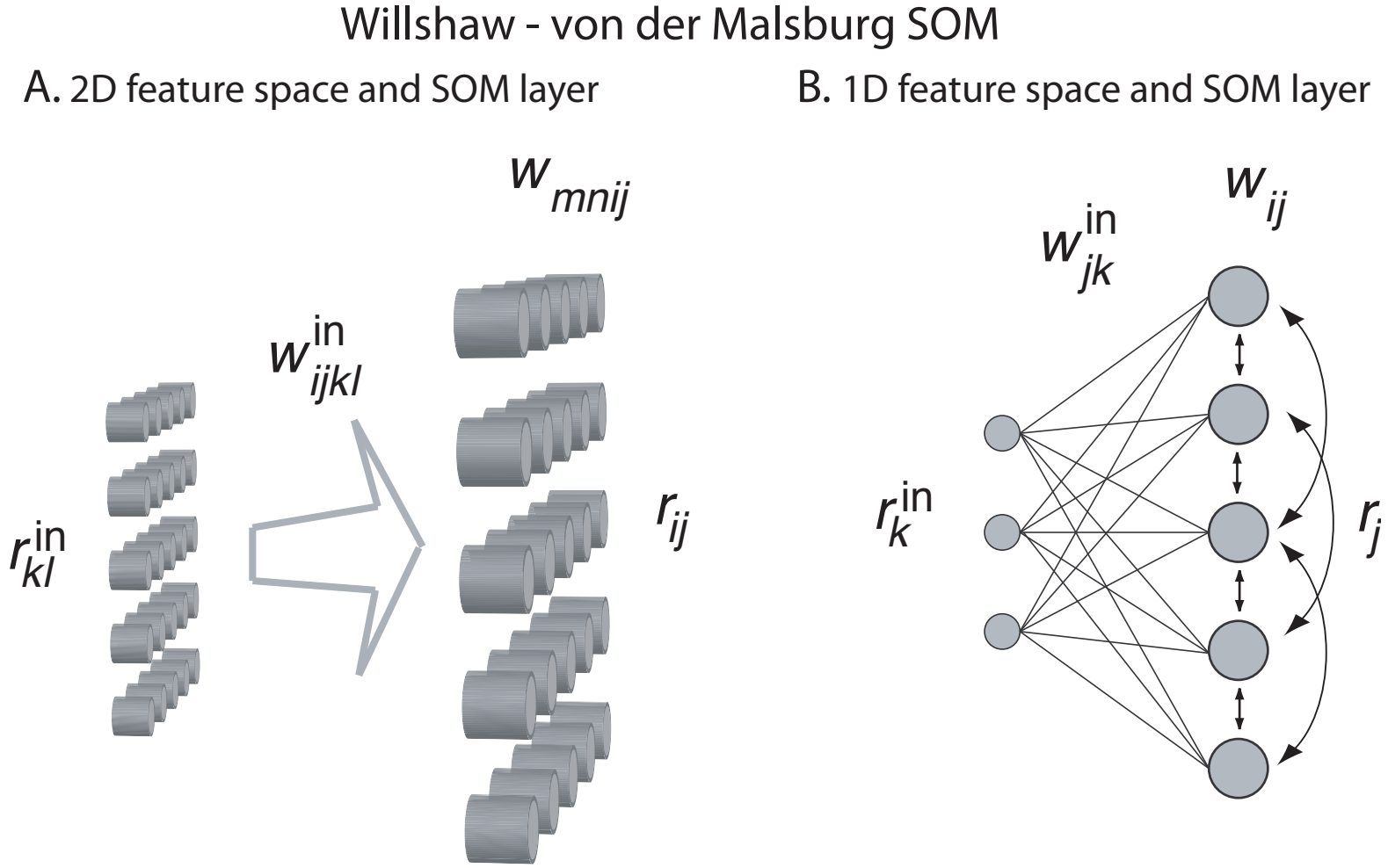
Motivation for SOM and DNF - Tuning Curves



Self-organizing maps (SOMs)

- The development of SOM as a neural model is motivated by the topographical nature of cortical maps.
- Visual, tactile, and acoustic inputs are mapped in a topographical manner. Visual: retinotopy (position in visual field), orientation, spatial frequency, direction, ocular dominance, etc. Tactile: somatotopy (position on skin,thumb and SMS) Acoustic: tonotopy (frequency)
- Self-organizing maps (SOM) is based on competitive learning, where output neurons compete with each other to be activated (Kohonen, 1982)
- The output neuron that activates is called the winner-takes-all neuron
- Lateral inhibition is one way to implement competition for map formation (von der Malsburg 1973)
- In SOM, neurons are placed on a lattice, on which a meaningful coordinate system for different features is created (feature map).
- The lattice thus forms a topographic map where the spatial location on the lattice is indicative of the input features.

SOM -von der Malsburg 1973



Network equations

Update rule of (recurrent) cortical network:

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + \frac{1}{N}\sum_j w_{ij} r_j(t) + \frac{1}{M}\sum_k w^{\mathrm{in}}_{ik} r^{\mathrm{in}}_k(t)$$

Activation function: $r_j(t) = \frac{1}{1+e^{\beta(u_j(t)-\alpha)}}$.

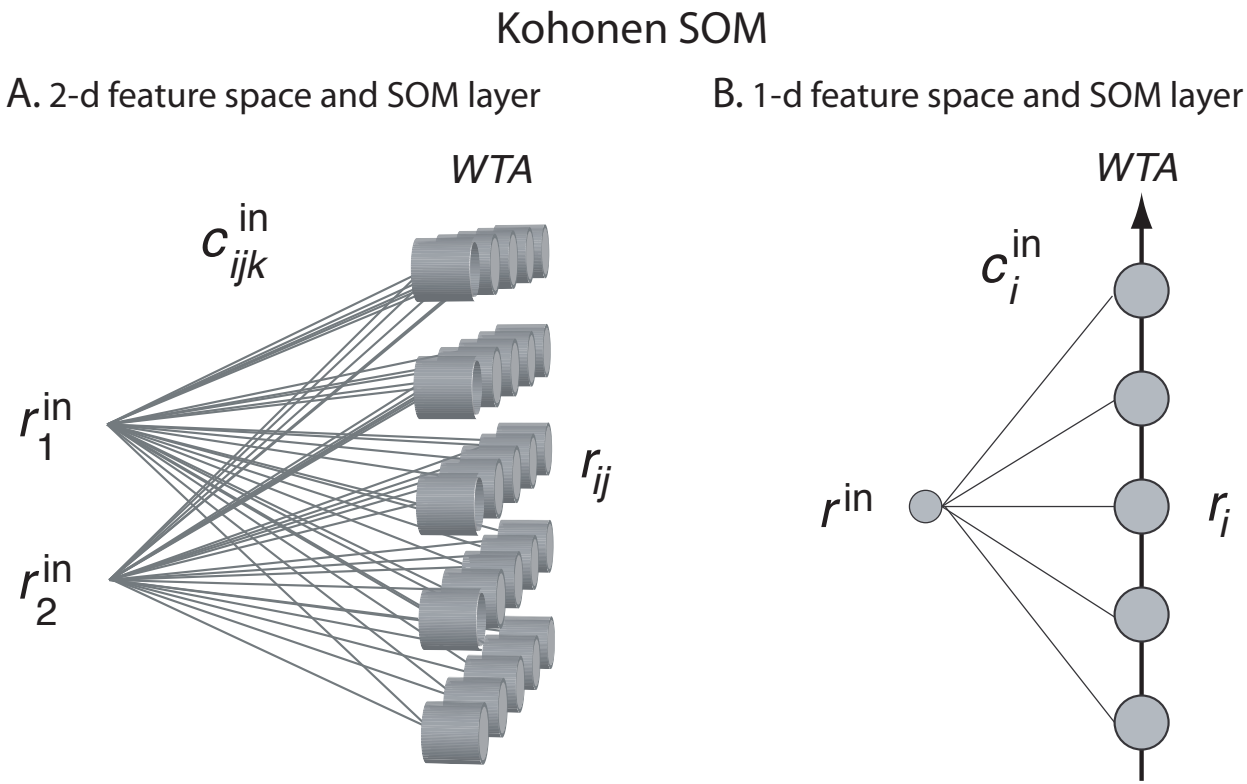
Lateral weight matrix: $w_{ij} \propto r_i r_j$

$$= A_{\mathrm{w}} \left(\mathrm{e}^{-((i-j)*\Delta x)^2/2\sigma^2} - C \right)$$

Input weight matrix: $w^{\mathrm{in}}_{ij} \propto r_i r^{\mathrm{in}}_j$

Kohonen - Shortcut

- Willshaw-von der Malsburg model: input neurons arranged in 2D lattice, output in 2D lattice. Lateral excitation/inhibition (Mexican hat) gives rise to soft competition. Normalized Hebbian learning. Biological motivation.
- Kohonen model: input of any dimension, output neurons in 1D, 2D, or 3D lattice. Relaxed winner-takes-all (neighborhood). Competetive learning rule. Computational motivation.



Kohonen model

- cortical sheet activation, σ_r^2 width of activated area, activation fce resembels tuning curves, radial-basis networks

$$r_{ij} = \exp(-\sum_k (c_{ijk} - r_k^{in})^2 / 2\sigma_r^2)$$

- strength connection around the winning node r_{ij}^*, WTA rule - winner takes all

$$\Delta c_{ijk} = \epsilon r_{ij}^* (r_{in} - c_{ijk})$$

- ML approach (Matlab implementation):
  $w^i(q) = w^i(q-1) + \alpha(p(q) - w^i(q))$, i are lying in neighborhood $N(i)_d = \{j, d_{ij} < d\}$

SOM Algorithm

1. Randomly initialize weight vectors w_i
2. Randomly sample input vector x
3. Find Best Matching Unit (BMU)

$$i(x) = \arg\min_j ||x - w_j||$$

4. Update weight vectors, where $h(j, i(x))$ is neighborhood function of BMU

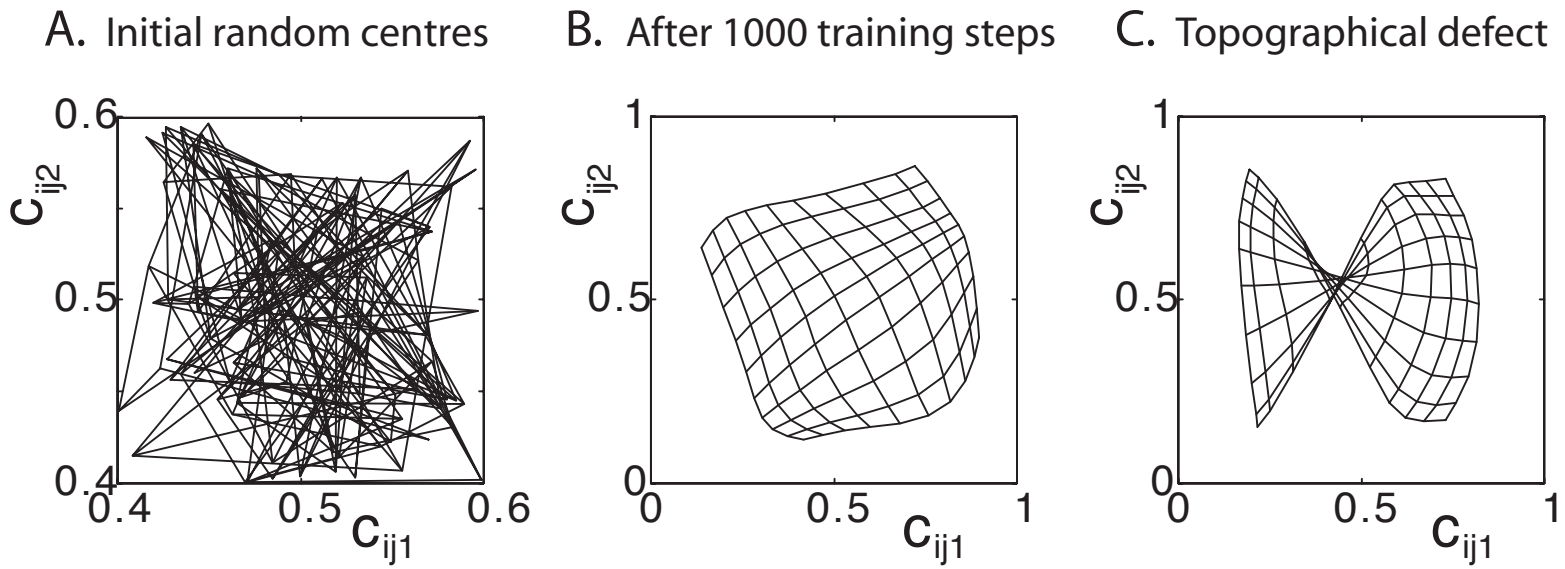
$$w_j = w_j + \epsilon h(j, i(x))(x - w_j)$$

5. Repeat steps 2-4

som.m

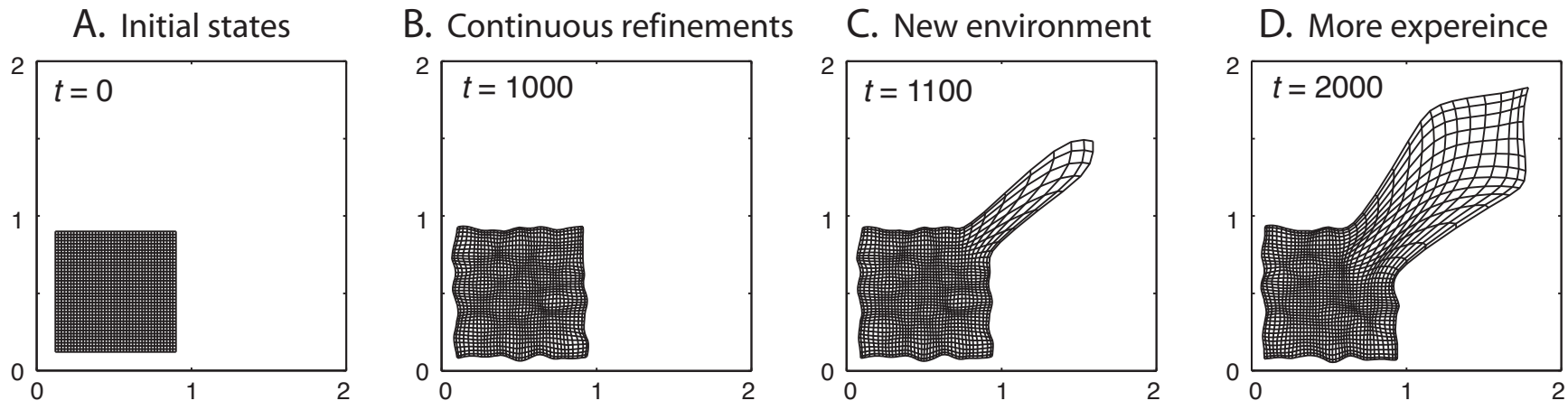
```
1   %% Two dimensional self-organizing feature map al la Kohonen
2    clear; nn=10; lambda=0.2; sig=2; sig2=1/(2*sig^2);
3    [X,Y]=meshgrid(1:nn,1:nn); ntrial=0;
4
5    % Initial centres of prefered features:
6    c1=0.5-.1*(2*rand(nn)-1);
7    c2=0.5-.1*(2*rand(nn)-1);
8
9   %% training session
10   while(true)
11      if(mod(ntrial,100)==0) % Plot grid of feature centres
12          clf; hold on; axis square; axis([0 1 0 1]);
13          plot(c1,c2,'k'); plot(c1',c2','k');
14          tstring=[int2str(ntrial) ' examples']; title(tstring);
15          waitforbuttonpress;
16      end
17      r_in=[rand;rand];
18      r=exp(-(c1-r_in(1)).^2-(c2-r_in(2)).^2);
19      [rmax,x_winner]=max(max(r)); [rmax,y_winner]=max(max(r'));
20      r=exp(-((X-x_winner).^2+(Y-y_winner).^2)*sig2);
21      c1=c1+lambda*r.*(r_in(1)-c1);
22      c2=c2+lambda*r.*(r_in(2)-c2);
23      ntrial=ntrial+1;
24   end
```

SOM simulation



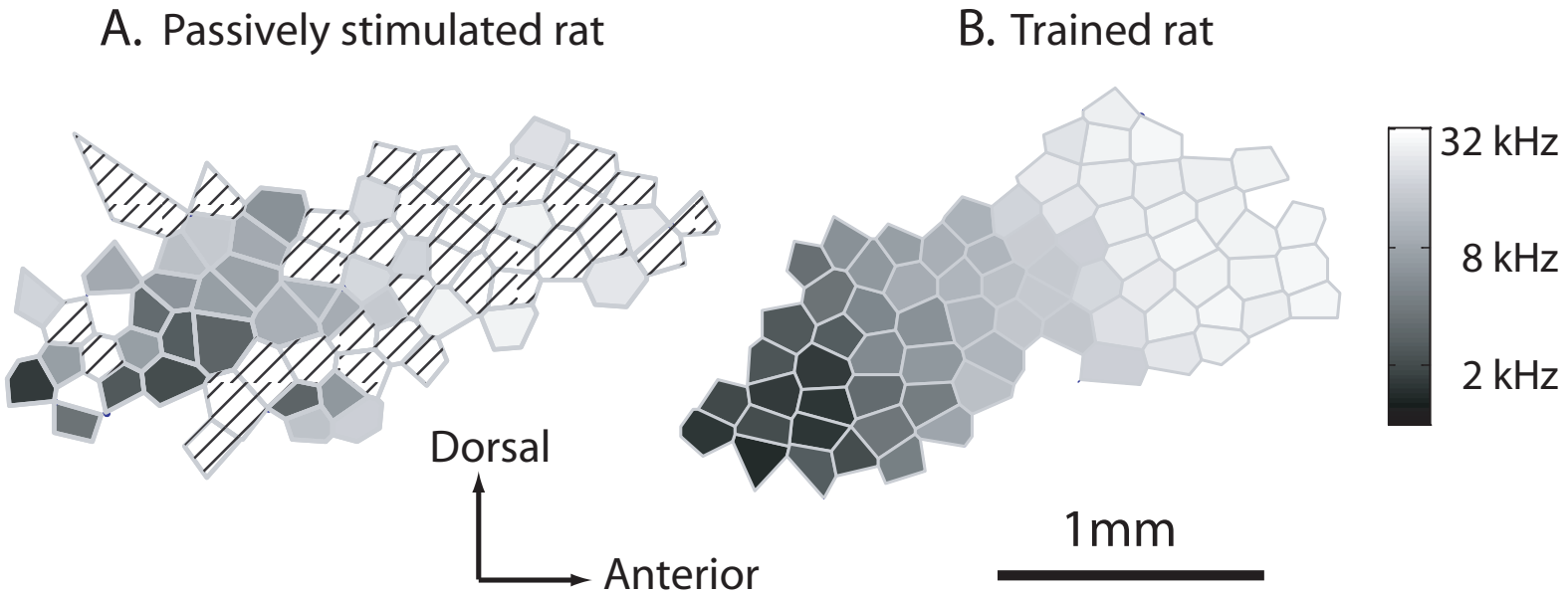
Another example

- Simulating development processes
- SOM can represent new domains, representation less fine-grained compared to initial domain
- Early in life exposed to broad feature space (learning languages)



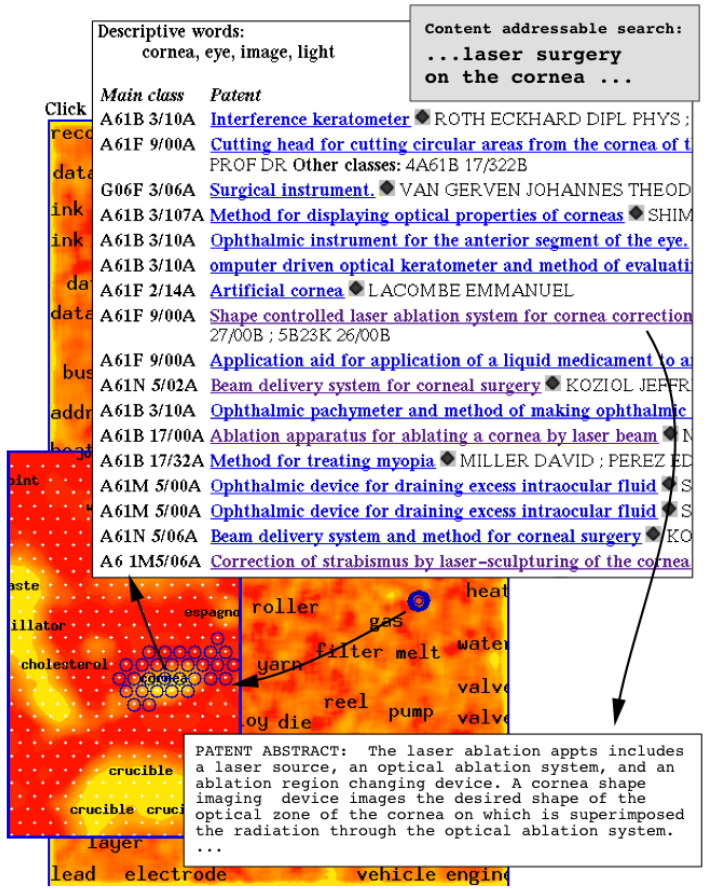
Representational plasticity - Zhou and Merzenich, PNAS 2007

- rat pups raised in noisy environment ← severely impaired tonotopicity (tones representations) in primary auditory cortex - A1
- no recovery after stimulation with sounds of different frequencies
- stimulation by discrimination task with food reward ← rats were able to recover tonotopic maps
- traditionally SOM maps are driven by data: bottom - up approach
- top-down processing explains those experimental results (reinforcement learning)

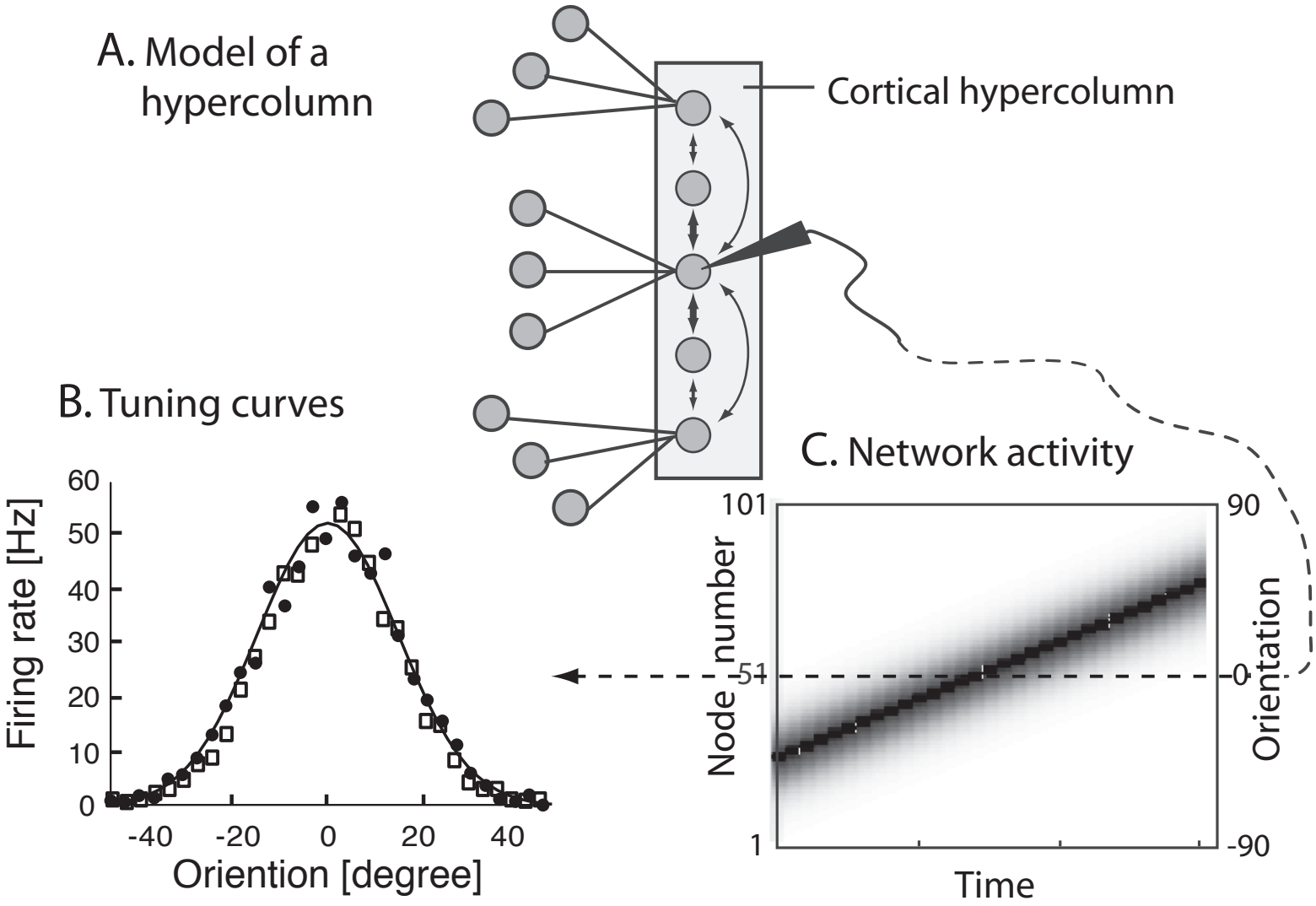


WEBSOM - Self-Organizing Maps for Internet Exploration

- find information on laser surgery on the cornea of eye, `http://websom.hut.fi`
- best matching locations marked with circles
- sparse feature vector, each raw representing single document, each term relative frequency of predefined entries (e.g. 50 000 words)



Tuning Curves



Dynamic Neural Field Theory

Field dynamics:

$$\tau \frac{\partial \mathbf{u}(\mathbf{x},t)}{\partial t} = -\mathbf{u}(\mathbf{x},t) + \int_{\mathbf{y}} \mathbf{w}(\mathbf{x},\mathbf{y})\mathbf{r}(\mathbf{y},t)\mathrm{d}\mathbf{y} + I^{\mathrm{ext}}(\mathbf{x},t)$$

$$\mathbf{r}(\mathbf{x},t) = g(\mathbf{u}(\mathbf{x},t)),$$

Continuous version of equations above with discretization:

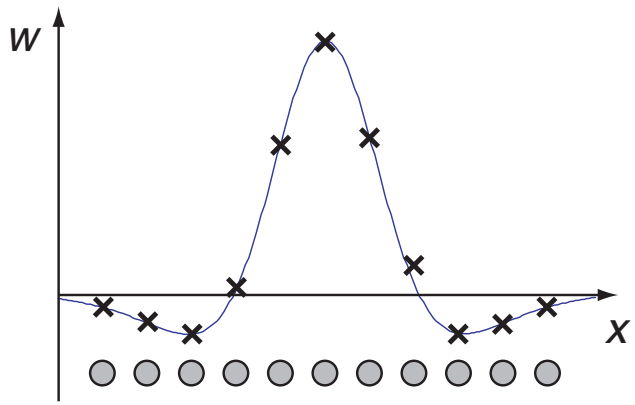
$$x \to i\Delta x \quad \text{and} \quad \int \mathrm{d}x \to \Delta x \sum$$

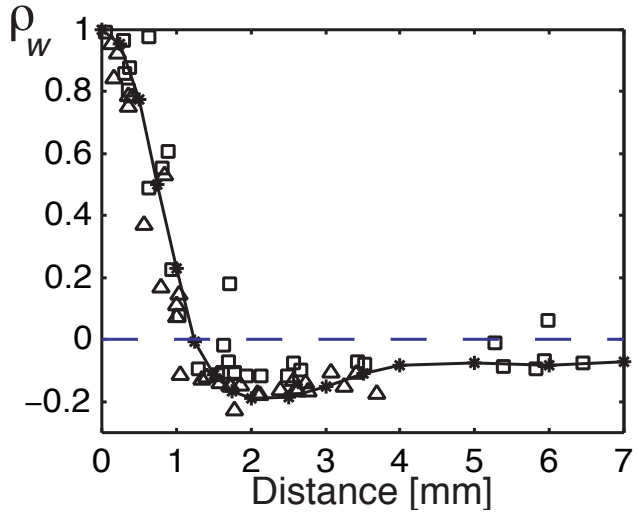
Main assumption: Short-distance excitation and long-distance inhabitation

Learning in cortical sheet - Lateral weight kernel

$$
\begin{aligned}
r(x - x^p) &= e^{(x-x_p)^2/2\sigma_r^2} \\
|x - x^p| &= \min(|x - x^p|, 2\pi - |x - x^p|) \\
\mathbf{w}^{\mathrm{E}}(|x - y|) &= \int_0^{2\pi} r(x - x^p) r(y - x^p) dx^p \\
\mathbf{w}^{\mathrm{E}}(|x - y|) &= A_{\mathrm{w}} \mathrm{e}^{-(x-y)^2/4{\sigma_r}^2} \\
\mathbf{w}(|x - y|) &= A_{\mathrm{w}} \mathrm{e}^{-(x-y)^2/4{\sigma_r}^2} - C) \\
\mathbf{w}_{ij} \propto r_i r_j &= A_{\mathrm{w}} \left(\mathrm{e}^{-((i-j)*\Delta x)^2/2\sigma^2} - C \right)
\end{aligned}
\tag{1}
$$

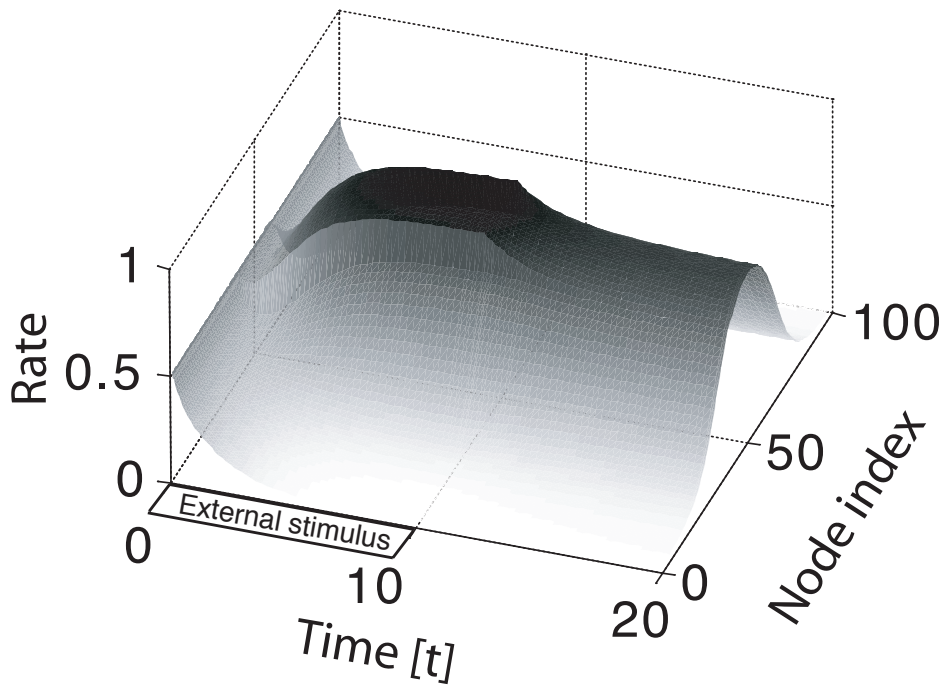
Can be learned from Gaussian response curves of individual nodes

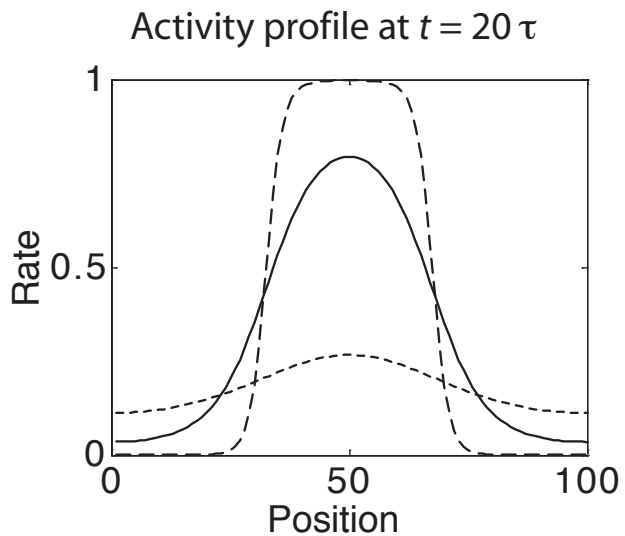




Self-sustained activity packet

- growing activity: $C << E$, whole map is active, undesirable
- decaying activity: $C >> E$, decaying after removal of external input
- memory activity: stability even when external input is removed !
- simulation: string external stimulus: nodes 40-50, excitatory weights to nearby nodes, active nodes: activity packets, buble or bump $\leftarrow$ continuos attractor neural networks $\leftarrow$ working memory, $A_w = 4, C = 0.5$





dnf.m

```
1   %% Dynamic Neural Field Model (1D)
2    clear; clf; hold on;
3    nn = 100; dx=2*pi/nn; sig = 2*pi/10; C=0.5;
4
5   %% Training weight matrix
6    for loc=1:nn;
7        i=(1:nn)'; dis= min(abs(i-loc),nn-abs(i-loc));
8        pat(:,loc)=exp(-(dis*dx).^2/(2*sig^2));
9    end
10   w=pat*pat'; w=w/w(1,1); w=4*(w-C);
11  %% Update with localised input
12   tall = []; rall = [];
13   I_ext=zeros(nn,1); I_ext(nn/2-floor(nn/10):nn/2+floor(nn/10))=1;
14   [t,u]=ode45('rnn_ode',[0 10],zeros(1,nn),[],nn,dx,w,I_ext);
15   r=1./(1+exp(-u)); tall=[tall;t]; rall=[rall;r];
16  %% Update without input
17   I_ext=zeros(nn,1);
18   [t,u]=ode45('rnn_ode',[10 20],u(size(u,1),:),[],nn,dx,w,I_ext);
19   r=1./(1+exp(-u)); tall=[tall;t]; rall=[rall;r];
20  %% Plotting results
21   surf(tall',1:nn,rall','linestyle','none'); view(0,90);
```

rnn_ode.m

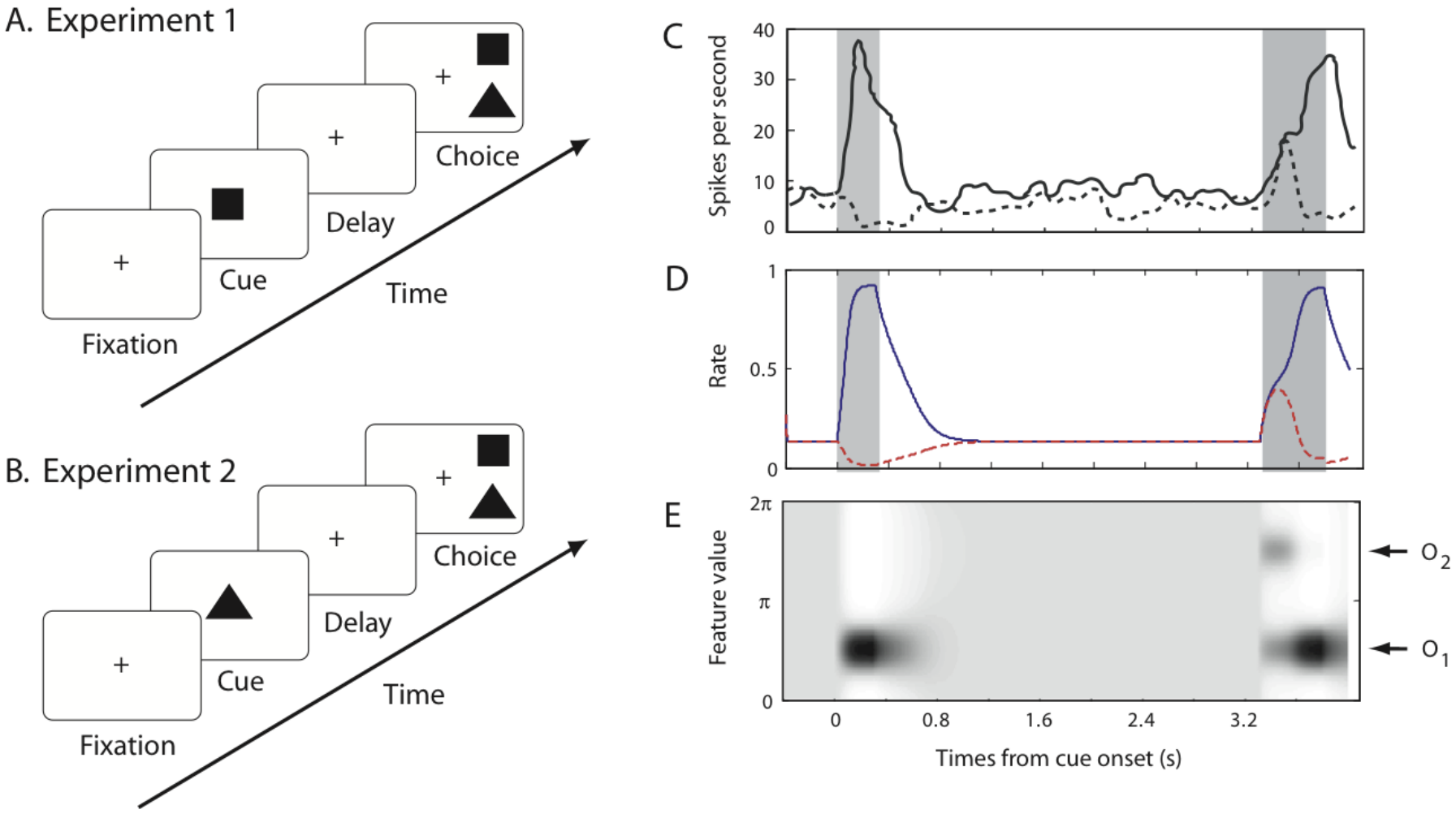
```
1   function udot=rnn_ode(t,u,flag,nn,dx,w,I_ext)
2   % odefile for recurrent network
3    tau_inv = 1.;       % inverse of membrane time constant
4    r=1./(1+exp(-u));
5    sum=w*r*dx;
6    udot=tau_inv*(-u+sum+I_ext);
7   return
```

Update rule of (recurrent) cortical network:

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + \frac{1}{N}\sum_j w_{ij} r_j(t) + \frac{1}{M}\sum_k w_{ik}^{\mathrm{in}} r_k^{\mathrm{in}}(t)$$

Activation function: $r_j(t) = \frac{1}{1+e^{\beta(u_j(t)-\alpha)}}$.

DNF example - Chelazzi, Nature, 1993

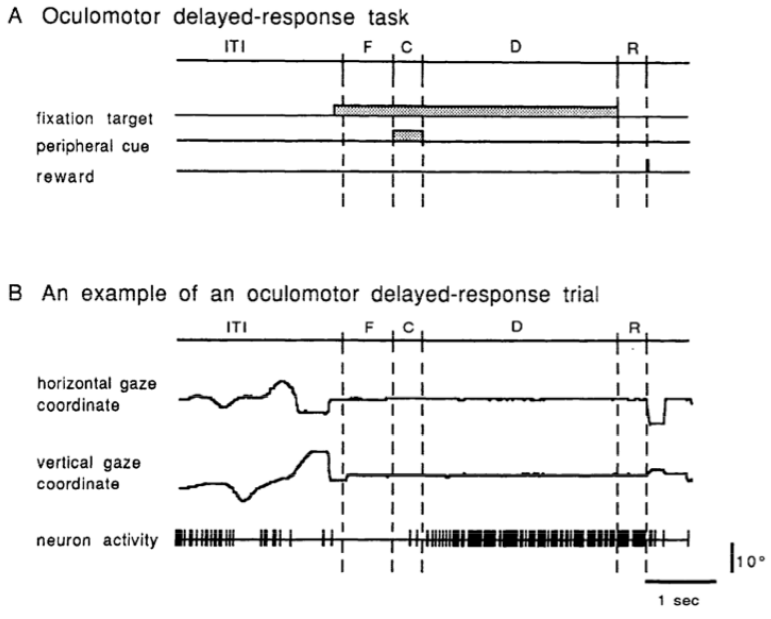


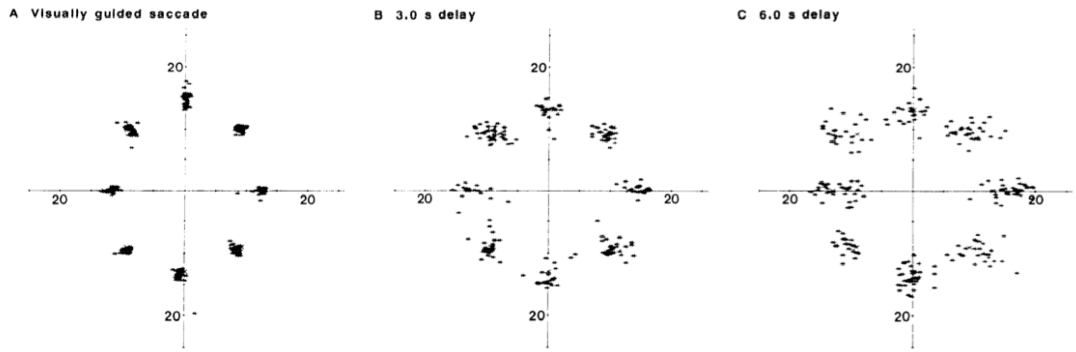
DNF example - Chelazzi, Nature, 1993, Matlab code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 % 1-d Continuous Attractor Neural Network with Hebbian learning
 % two gaussian signal: decision network
 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 clear;  close all;
 nn = 100; dx=2*pi/nn; % number of nodes and resolution in deg
 %weight matrices
    sig = 2*pi/20;
    w_sym=hebb(nn,sig,dx);
    w_inh=0.07;%use 0.04, 7,6,3; 3*(sqrt(2*pi)*sig)?2/nn;
    w=500*(w_sym-w_inh);
%inputs
    perc=0.01; Is=11;
    Ia=(1+0.5*perc)*Is;
    Ib=(1-0.5*perc)*Is;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%   Experiment    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    param=0;
%%%% no external input
     u0 = zeros(nn,1)-10;
     I_ext=zeros(nn,1);
     tspan=[0,40];
     [t,u]=ode45('rnn_ode_u',tspan,u0,[],nn,dx,w,I_ext);
     r=f1(u);
%%%% external cue
    u0 = u(size(t,1),:);
    I_ext=zeros(nn,1);
    loc1=pi/2;%+pi/16;
    loc2=3*pi/2;%-pi/16;
    I_ext=I_ext+in_signal_pbc(loc1,Is,sqrt(2)*sig,nn,dx);
    tspan=[40 70];
    [t2,u]=ode45('rnn_ode_u',tspan,u0,[],nn,dx,w,I_ext);
    r=[r;f1(u)];
t=[t;t2];
%%%% no external input
    u0 = u(size(t2,1),:);
    I_ext=zeros(nn,1);
    param=0;
    tspan=[70,370];
    [t2,u]=ode45('rnn_ode_u',tspan,u0,[],nn,dx,w,I_ext);
    r=[r;f1(u)];
t=[t;t2];
```

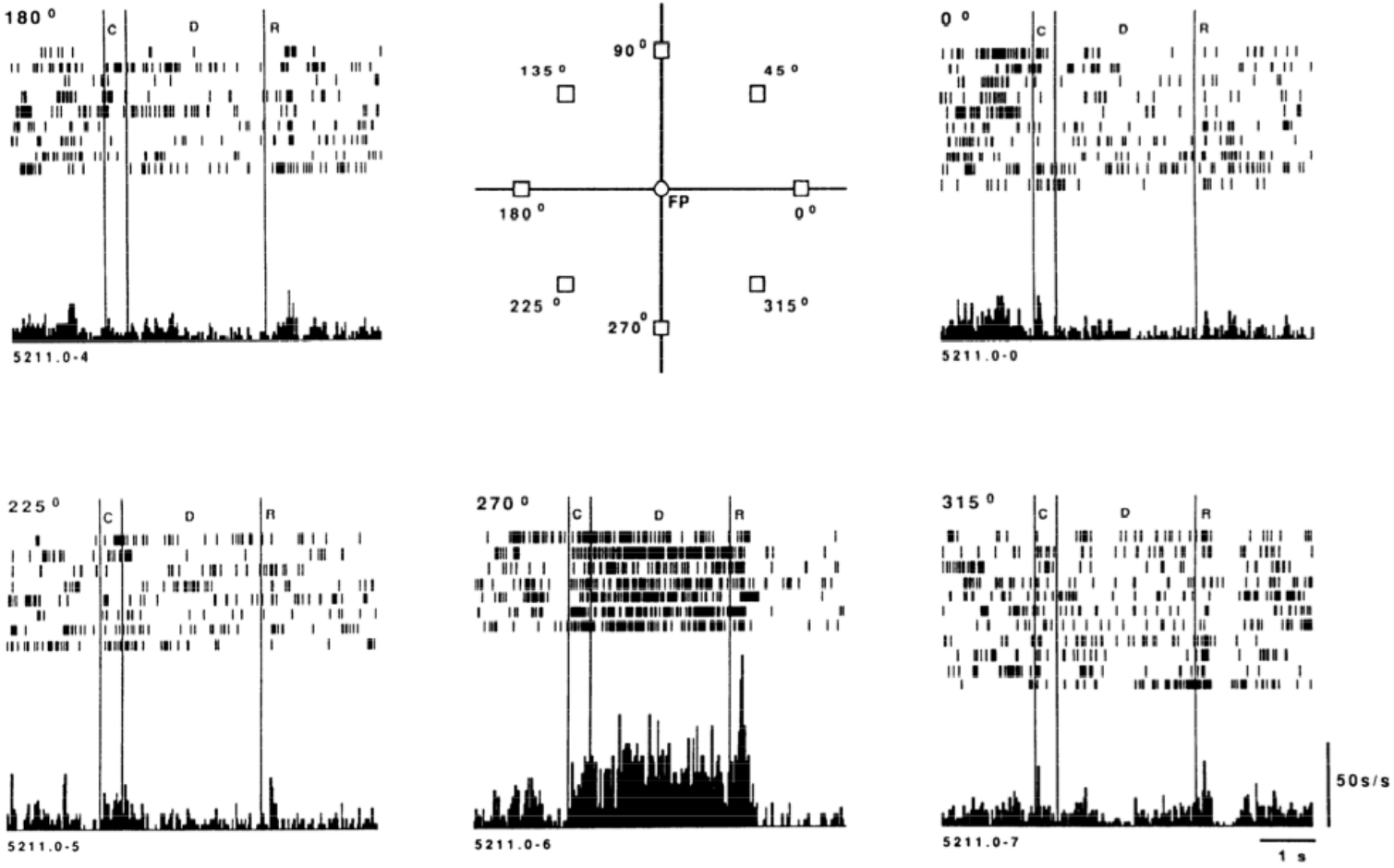
Working memory by ongoing firing - sustained DNF buble

- F- fixation period (0.75s), C-cue period (0.5s), D - delay period (3-6 s), R - response period (0.5s) $\rightarrow$ reward





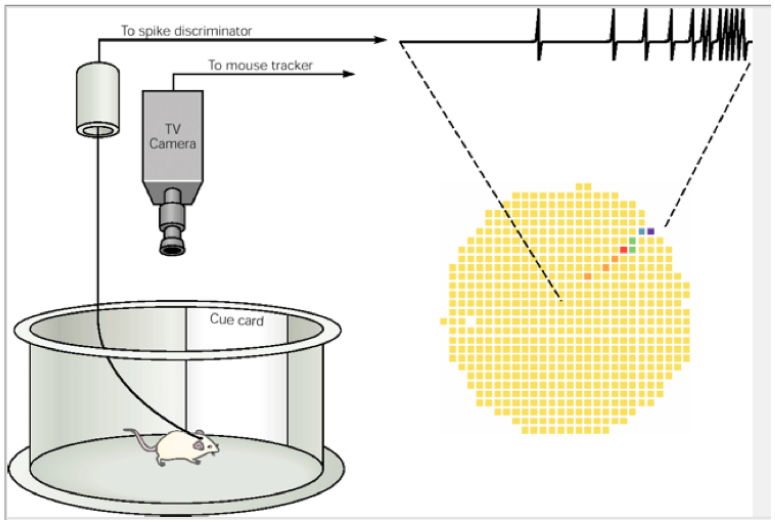
Directional delay period activity

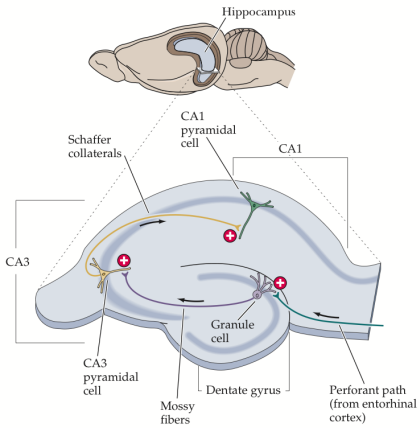


S. Funahashi, C.J. Bruce and P.S. Goldman-Rakic, Mnemonic coding of visual space in the monkeys dorsolateral prefrontal cortex, J Neurophysiol 61:331349, 1989

Place cells

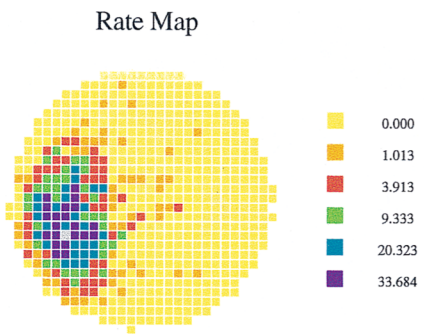
- Place cells are neurons in the hippocampus that exhibit a high rate of firing whenever an animal is in a specific location (pyramidal cells in CA1,CA4)
- On initial exposure to a new environment, place fields become established within minutes. The place fields of cells tend to be stable over repeated exposures to the same environment.
- Remapping - In a different environment, however, a cell may have a completely different place field or no place field at all

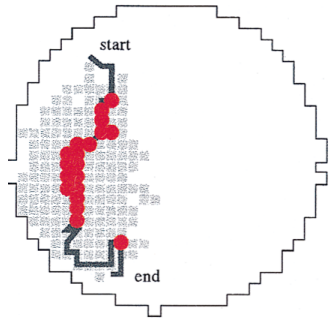


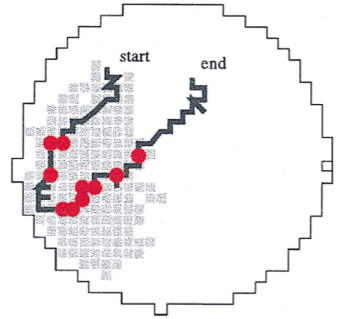


Place cells - 16 mins experiment

- colored circular region is an overhead view of a 76 cm diameter cylinder, each small square region (pixel) is about 2.5 cm squared, firing rate $\rightarrow$ total number of spikes fired in the pixel divided by the total time spent in the pixel.
- hungry rat ran around for 16 min chasing small food pellets, the black line indicates the rat's path and the red dots the locations at which action potentials were fired, action potentials were fired all along the second path even though the rat turned and ran out of the field in the direction opposite to its entry; this is an indication that the firing is not directionally selective.
- `http://www.youtube.com/watch?v=PGHRDcPKio8`

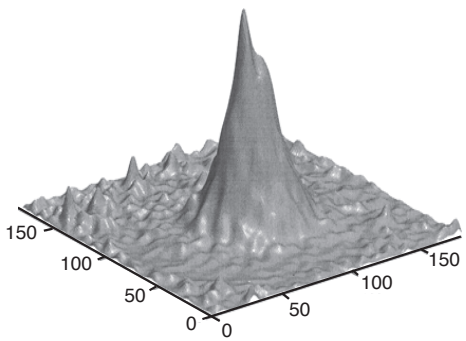


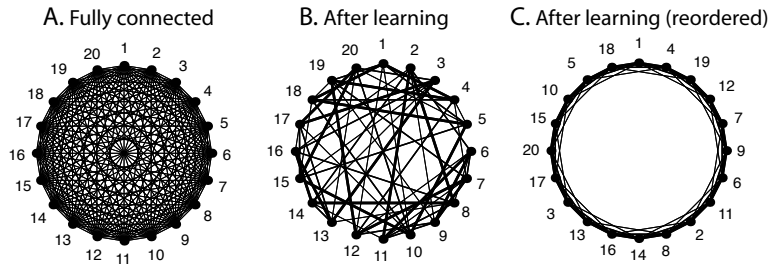




Place cells - is there any topography ?

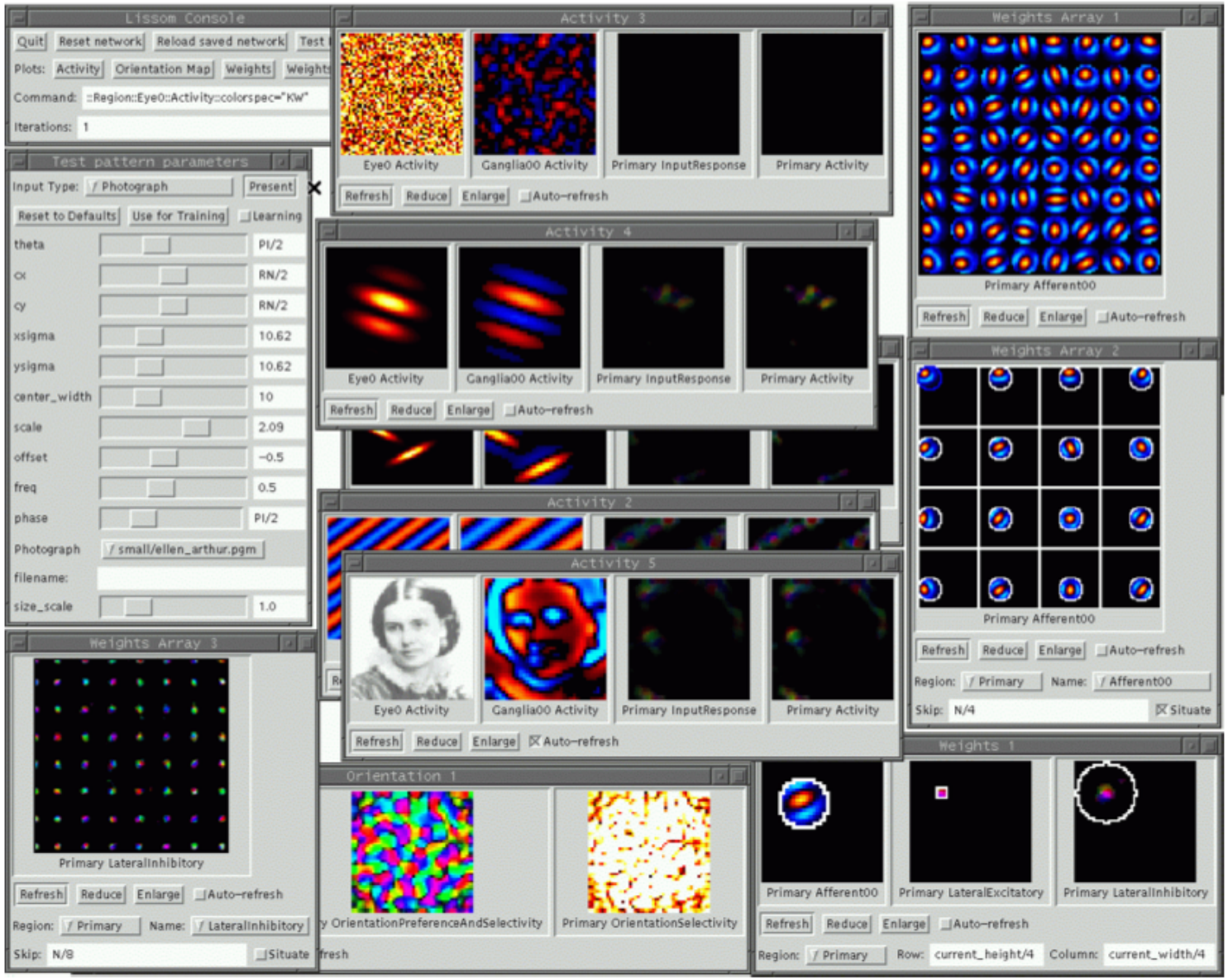
- no specific topography found with respect to neuron's maximal response to a particular place
- rearranging plot - neurons firing maximally in response to adjacent location $\rightarrow$ plot neurons adjacent to each other
- direction head cells $\rightarrow$ recurrent AAN simulation, high dimensionality - (i) before learning - equal weights for all nodes (ii) training- each node assigned (Gaussian profile) to preferred direction where fires maximally, competitive Hebb's rule (iii) strongly connected nodes adjacent to each other
- dimensionality was reduced to 1D model, networks self-organized to reflect the dimensionality of feature space





Topographica - general simulator for cortical maps

http://topographica.org/



Further Readings

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