Two-player Games

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Branislav Bošanský

bosansky@fel.cvut.cz

Two Player Games

- Important test environment for AI algorithms
- Benchmark of AI
  - Chinook (1994/96) – world champion in checkers
  - Deep Blue (1997) – beats G. Kasparov in chess (3.5 – 2.5)
  - …
  - Alpha Go (2016) – beats Lee Sedol in Go (4 – 1)
  - DeepStack (2016/2017) – beats Poker Pros (https://www.deepstack.ai/)
  - …

Game-tree Search / Adversarial Search

- until now – only the searching player acts in the environment
- there could be others:
  - Nature – stochastic environment (MDP, POMDP, …)
  - other agents – rational opponents

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- Game Theory
  - mathematical framework that describes optimal behavior of rational self-interested agents
  - Mag. OI → B4M36MAS (Multi-agent Systems)

Game-tree Search / Adversarial Search

- What are the basic games categories?
  - perfect / imperfect information
  - deterministic / stochastic
  - zero-sum / general-sum
  - finite / infinite
  - two-player / n-player
  - …

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- What is the goal?

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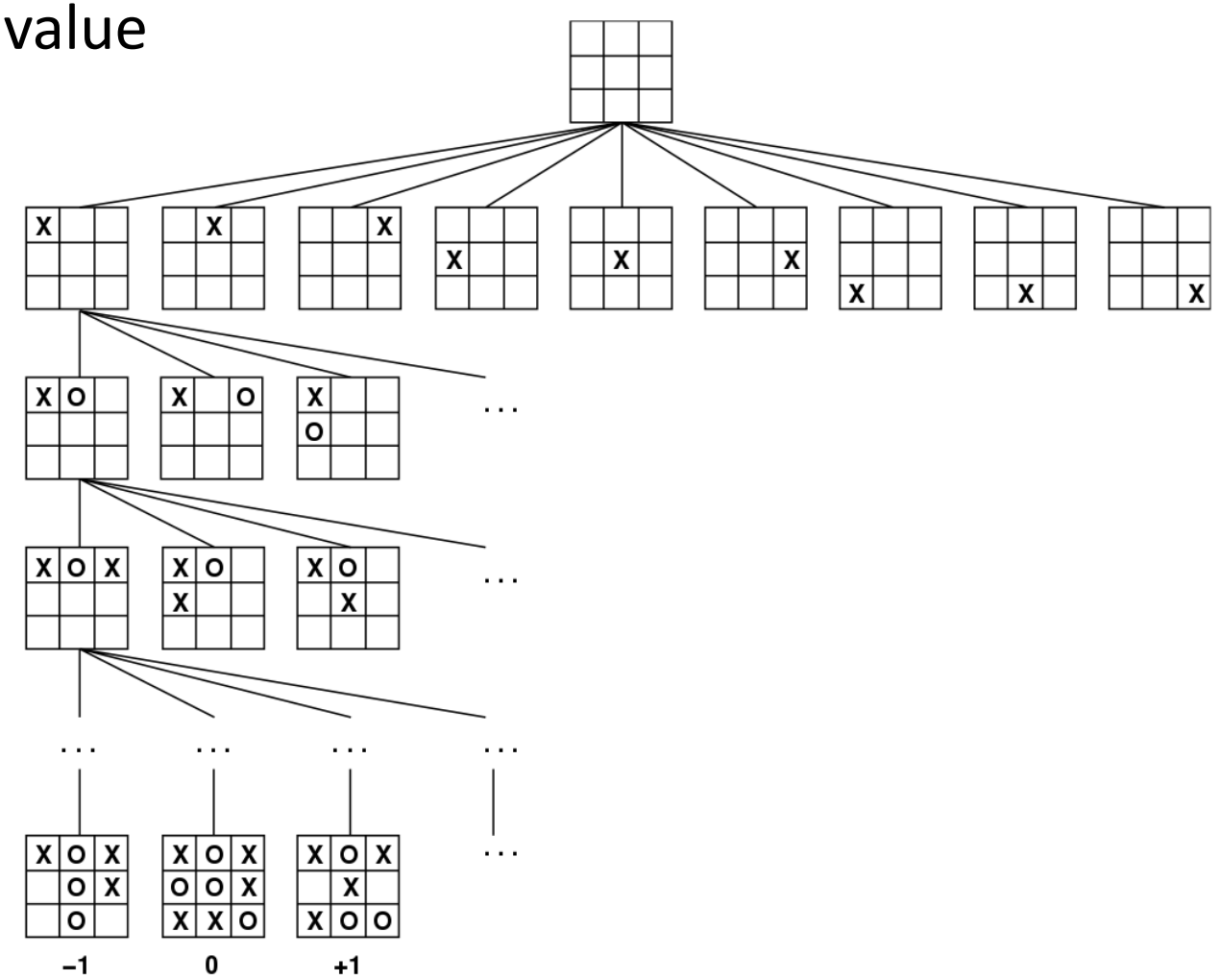
- What is the goal?
  - Finding an optimal **strategy** (i.e., what to play in which situation)

Game-tree Search / Adversarial Search

- Players are rational – each player wants to maximize her/his utility value



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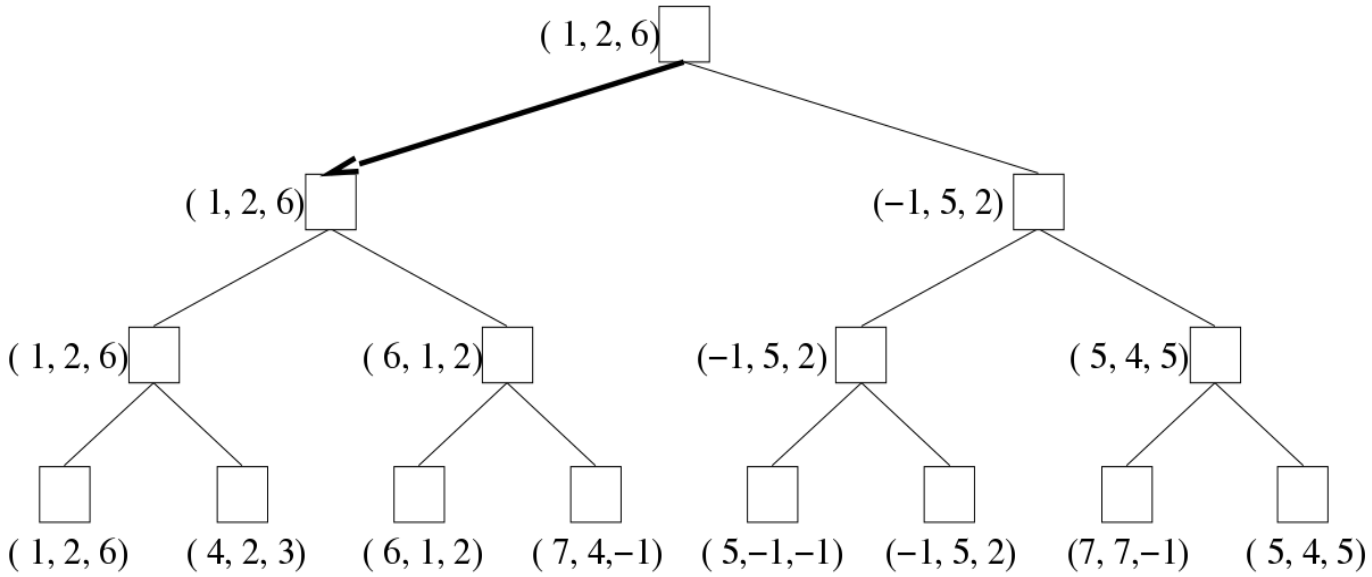

Minimax

- **function** minimax(node, Player)
- **if** (node is a terminal node) **return** utility value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- α := max(α, minimax(child, switch(Player)))
-
- **return** α
- **else**
- **for each** child of node
- β := min(β, minimax(child, switch(Player)))
-
- **return** β

Minimax in Real Games

- search space in games is typically **extremely large**
  - exponential in branching factor b^d
    - e.g., 35 in chess, up to 360 in Go, up to 45000 in Arimaa
    - No-limit heads-up poker has 10^{160} decision points
- we have to limit the depth of the search
- we need an evaluation function

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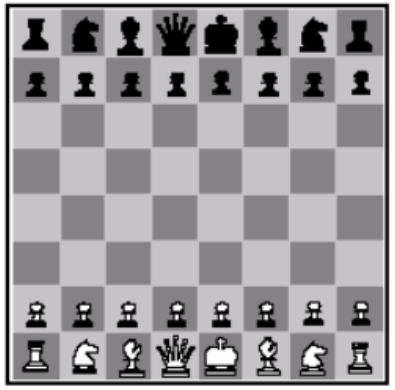
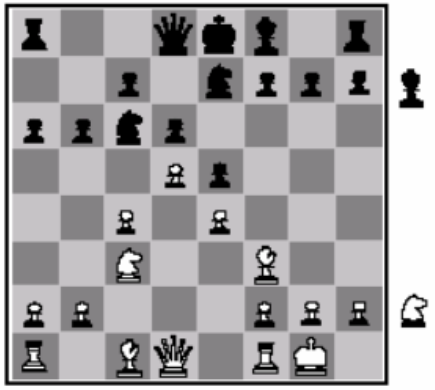
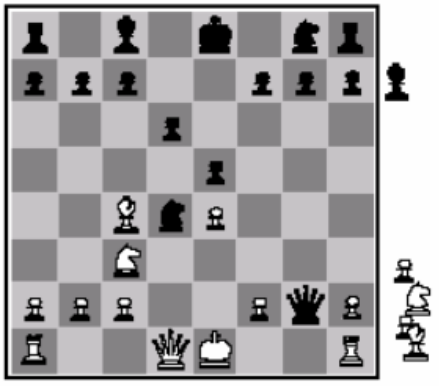


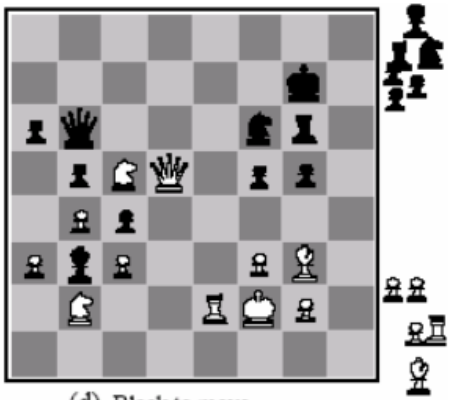
(a) White to move
Fairly even



(b) Black to move
White slightly better



(c) White to move
Black winning



(d) Black to move
White about to lose

Minimax

- **function** minimax(node, depth, Player)
- **if** (depth = 0 or node is a terminal node) **return** evaluation value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- α := max(α, minimax(child, depth-1, switch(Player)))
-
- **return** α
- **else**
- **for each** child of node
- β := min(β, minimax(child, depth-1, switch(Player)))
-
- **return** β

Minimax in Real Games - Problems

- good evaluation function
- depth?
  - horizon problem
  - iterative deepening
  - searching deeper does not always improve the results

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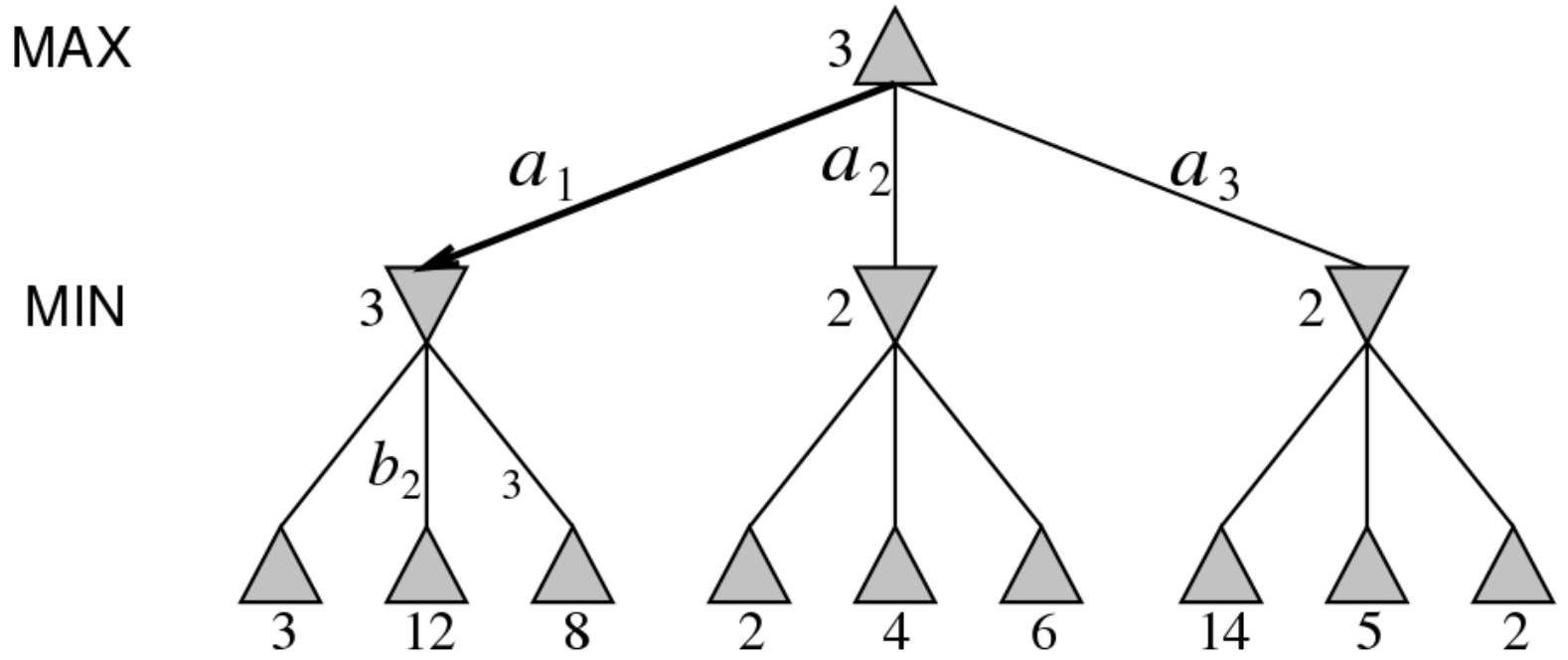
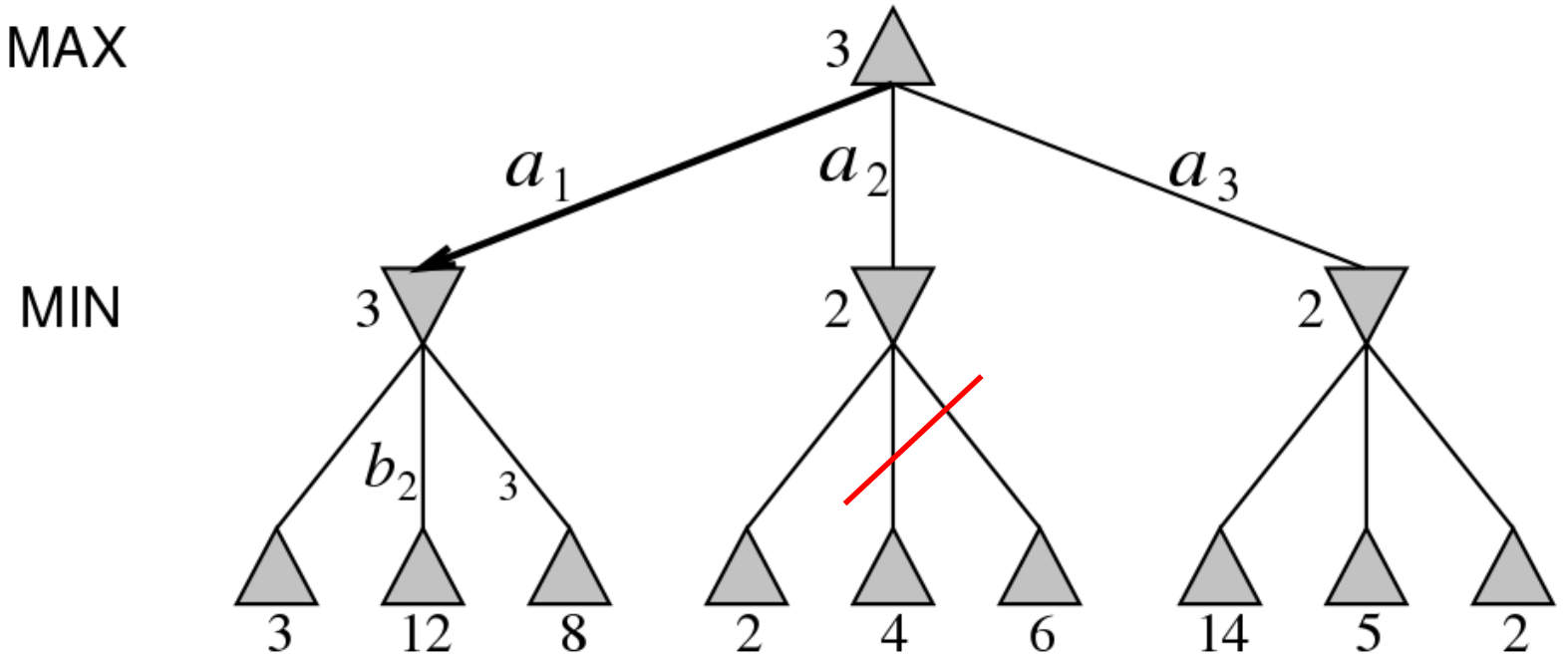
- online vs. offline decision making
  - game playing vs. equilibrium computation
  - appear also in robotics / planning / decision making

Alpha-Beta Pruning



Alpha-Beta Pruning



Alpha-Beta Pruning

- **function** alphabeta(node, depth, α, β, Player)
- **if** (depth = 0 or node is a terminal node) **return** evaluation value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- α := max(α, alphabeta(child, depth-1, α, β, switch(Player)))
- **if** (β≤α) break
- **return** α
- **else**
- **for each** child of node
- β := min(β, alphabeta(child, depth-1, α, β, switch(Player)))
- **if** (β≤α) break
- **return** β

Negamax

- **function** negamax(node, depth, α, β, Player)
- **if** (depth = 0 or node is a terminal node) **return** the heuristic value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- α := max(α, -negamax(child, depth-1, -β, -α, switch(Player)))
- **if** (β≤α) break
- **return** α
- **else**
- **for each** child of node
- β := min(β, alphabeta(child, depth-1, α, β, not(Player)))
- **if** (β≤α) break
- **return** β

Aspiration Search

- [α, β] interval – window
- alphabeta initialization [-∞, +∞]

Aspiration Search

- $[\alpha, \beta]$ interval – window
- alphabeta initialization $[-\infty, +\infty]$
- what if we use $[\alpha_0, \beta_0]$
  - x = alphabeta(node, depth, α_0, β_0,player)
  - $\alpha_0 \leq x \leq \beta_0$ - we found a solution
  - $x \leq \alpha_0$ - failing low (run again with $[-\infty, x]$)
  - $x \geq \beta_0$ - failing high (run again with $[x, +\infty]$)

NegaScout – Main Idea

- enhancement of alpha-beta algorithm
- assume some heuristic that determines move ordering
  - the algorithm assumes that the first action is the best one
  - after evaluating the first action, the algorithm checks whether the remaining actions are worse
  - the “test” is performed via null-window search

Scout –Test

- what we really need at that moment is a bound (not the precise value)

Scout –Test

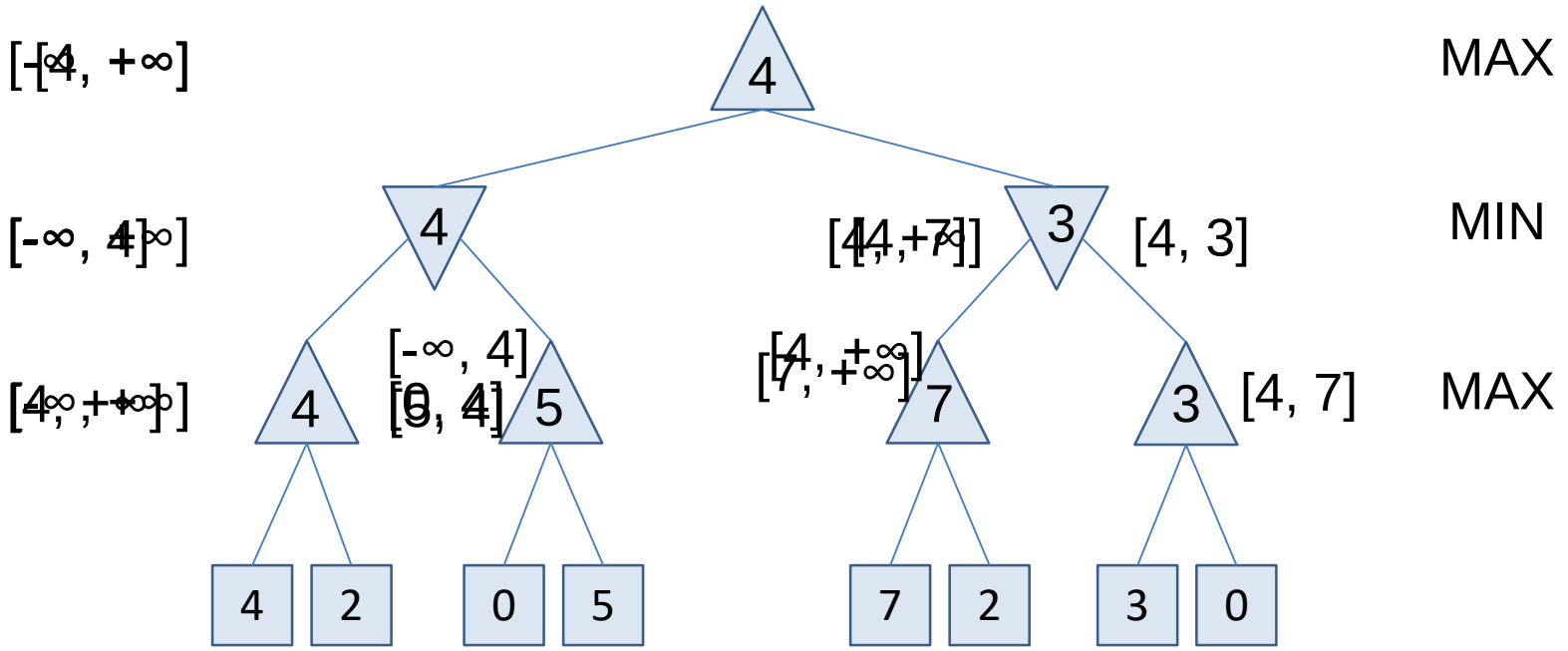
- what we really need at that moment is a bound (not the precise value)
- Remember Aspiration Search?
  - $x \leq \alpha_0$ - failing low (we know, that solution is ≤ x)
  - $x \geq \beta_0$ - failing high (we know, that solution is ≥ x)
- What if we use a null-window [α, α+1] (or [α,α])?
  - we obtain a bound …

NegaScout

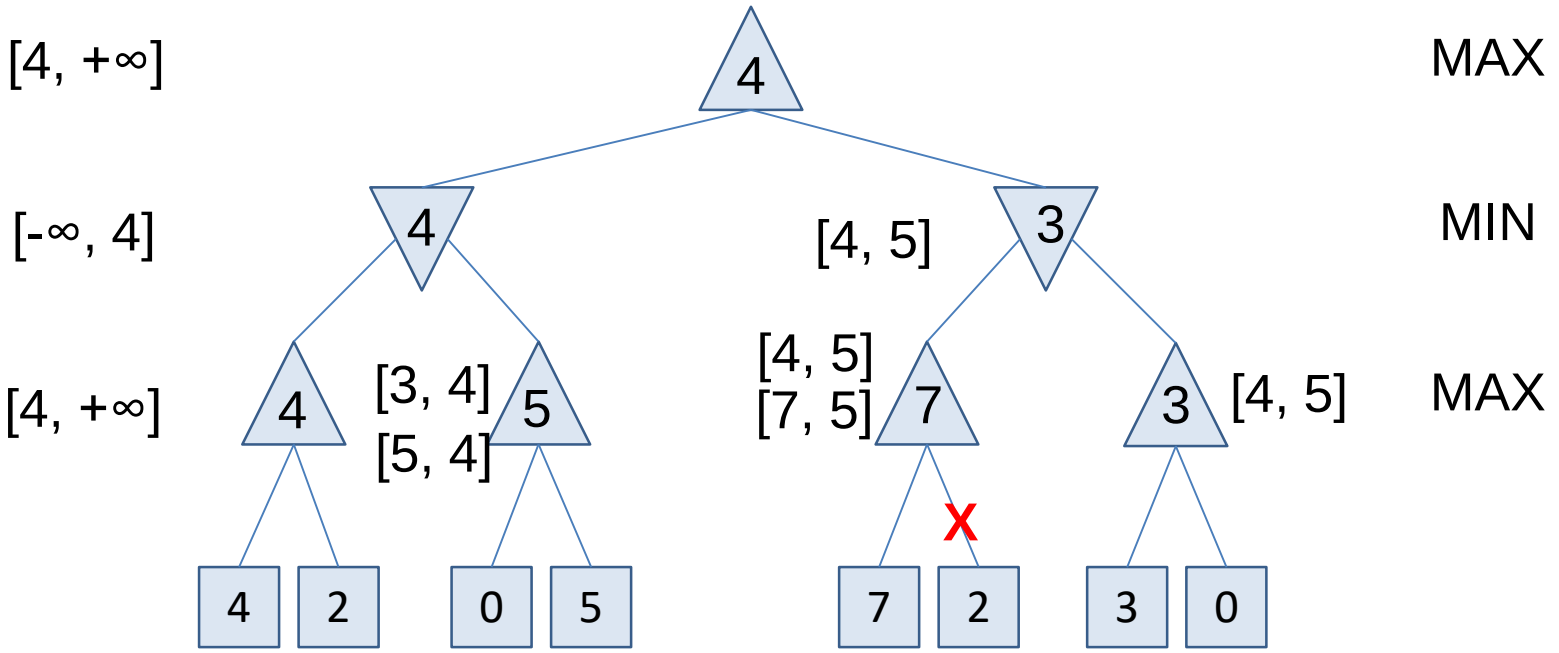
function negascout(node, depth, α, β, Player)

- **if** ((depth = 0) or (node is a terminal node)) **return** eval(node)
- b := β
- **for each** child of node
- v := -negascout(child, depth-1, -b, -α, switch(Player)))
- **if** ((α < v) and (child is not the first child))
- v := -negascout(child, depth-1, -β, -α, switch(Player)))
- α := max(α, v)
- **if** (β≤α) break
- b := α + 1
- **return** α

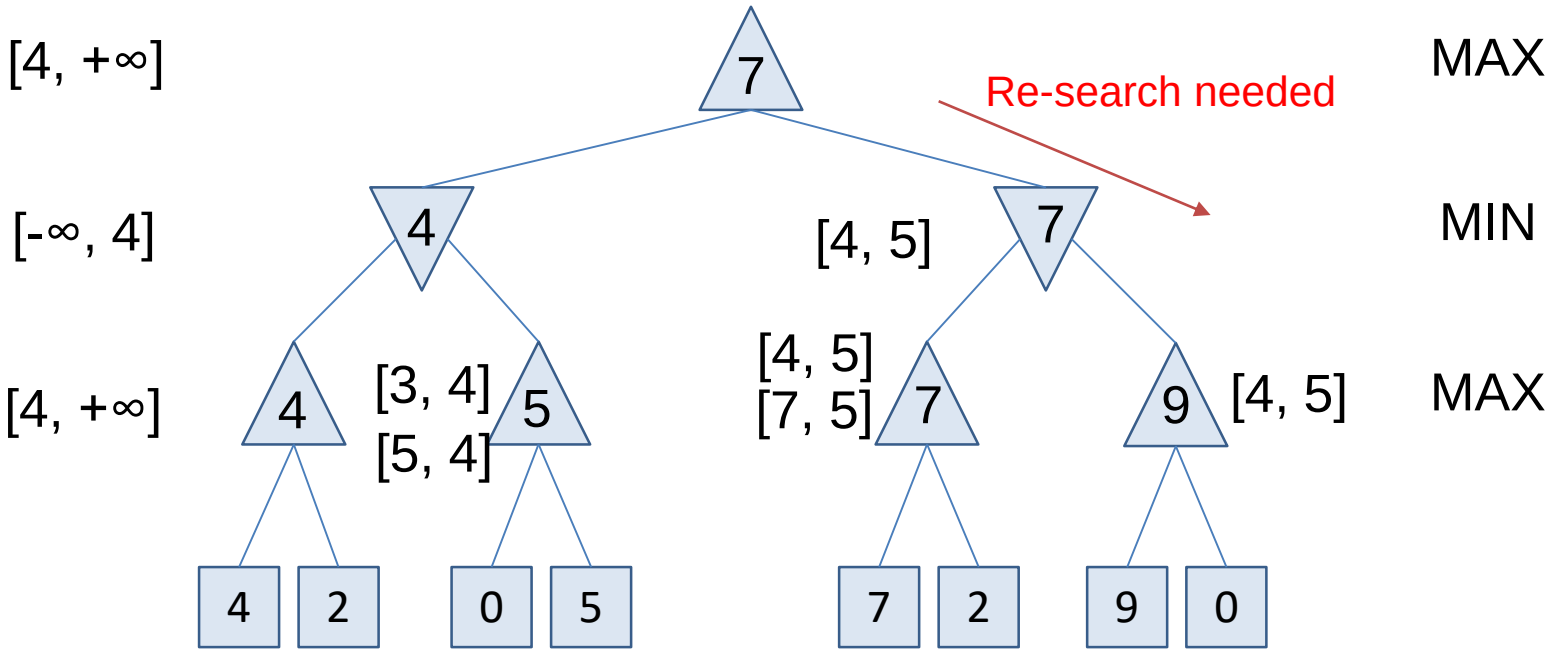
Alpha-Beta vs. Negascout



Alpha-Beta vs. Negascout



Alpha-Beta vs. Negascout

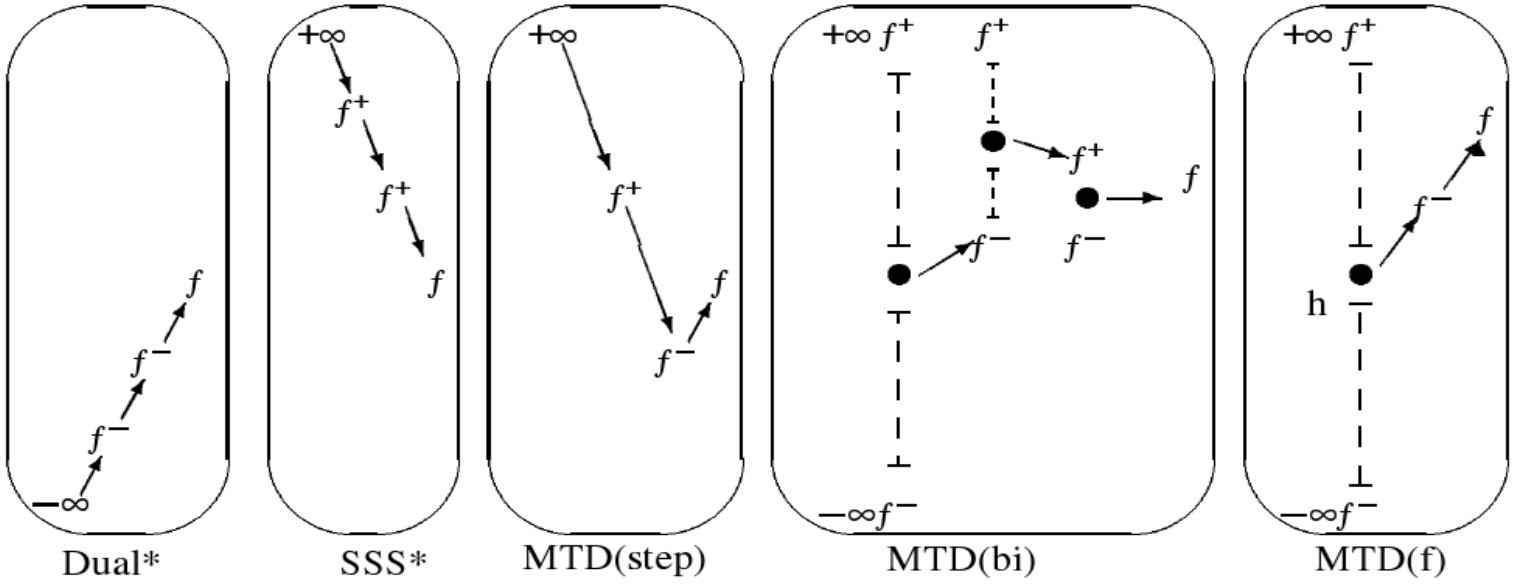


NegaScout

- also termed Principal Variation Search (PVS)
- dominates alpha-beta
  - never evaluates more different nodes than alpha-beta
  - can evaluate some nodes more than once
- depends on the move ordering
- can benefit from transposition tables
- generally 10-20% faster compared to alpha-beta

MTD

- Memory-enhanced Test Driver

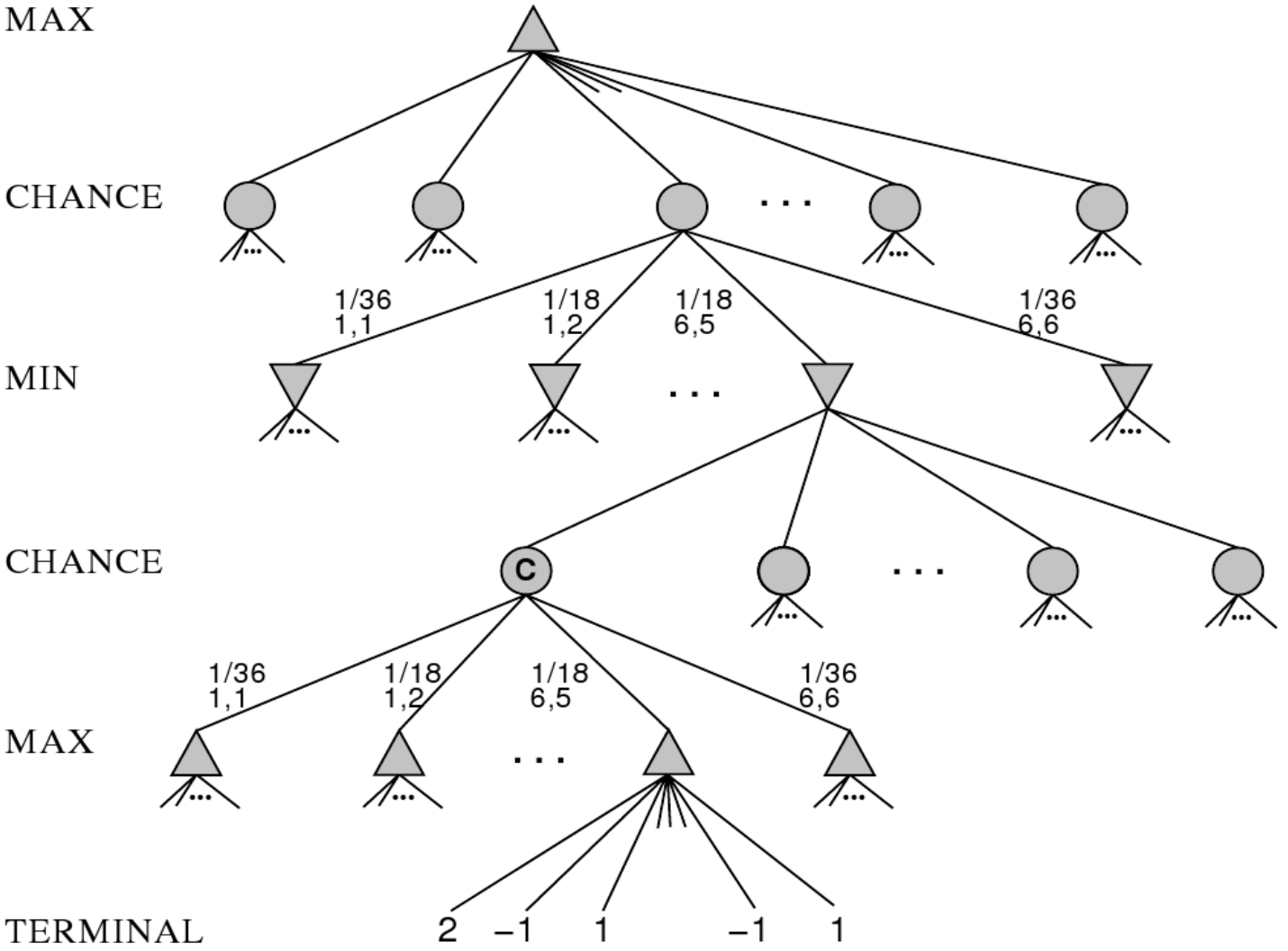


- Best-first fixed-depth minimax algorithms. Plaat et. al. , In *Artificial Intelligence,* Volume 87, Issues 1-2, November 1996, Pages 255-293

Further Topics

- more advanced algorithms
  - Monte-Carlo Tree Search (leading algorithm for many games)
  - further enhanced with many heuristics and techniques
- more complex games
  - games with uncertainty
    - chance (Nature player), calculating expected utilities
    - imperfect information (players cannot distinguish certain states)

Other Games - Chance nodes



Games and Game Theory in AIC

- more fundamental research
  - general algorithms for solving sequential games with imperfect information
  - implementation of domain independent algorithms

Invitation

Artificial Intelligence Goes All-In: Computers Playing Poker

Lecture by prof. Michael Bowling, head of Computer Poker Research Group at University of Alberta.

Prof. Michael Bowling

- World-famous expert on AI and reinforcement learning
- Led many outstanding computer poker results:
  - Polaris, beating pros in heads-up limit poker
  - Cepheus, playing optimally heads-up limit poker
  - **DeepStack, beating pros in heads-up no-limit**
  - Two publications on poker in prestigious Science
- Proposed Atari games as a benchmark for AI
- Won one of the first RoboCup challenges

March 30, 2017 at 16:00

Auditorium KN:E-107, FEL CTU,
Karlovo nám. 13, Prague 2

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COMPUTER SCIENCE

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