Blind (Uninformed) Search

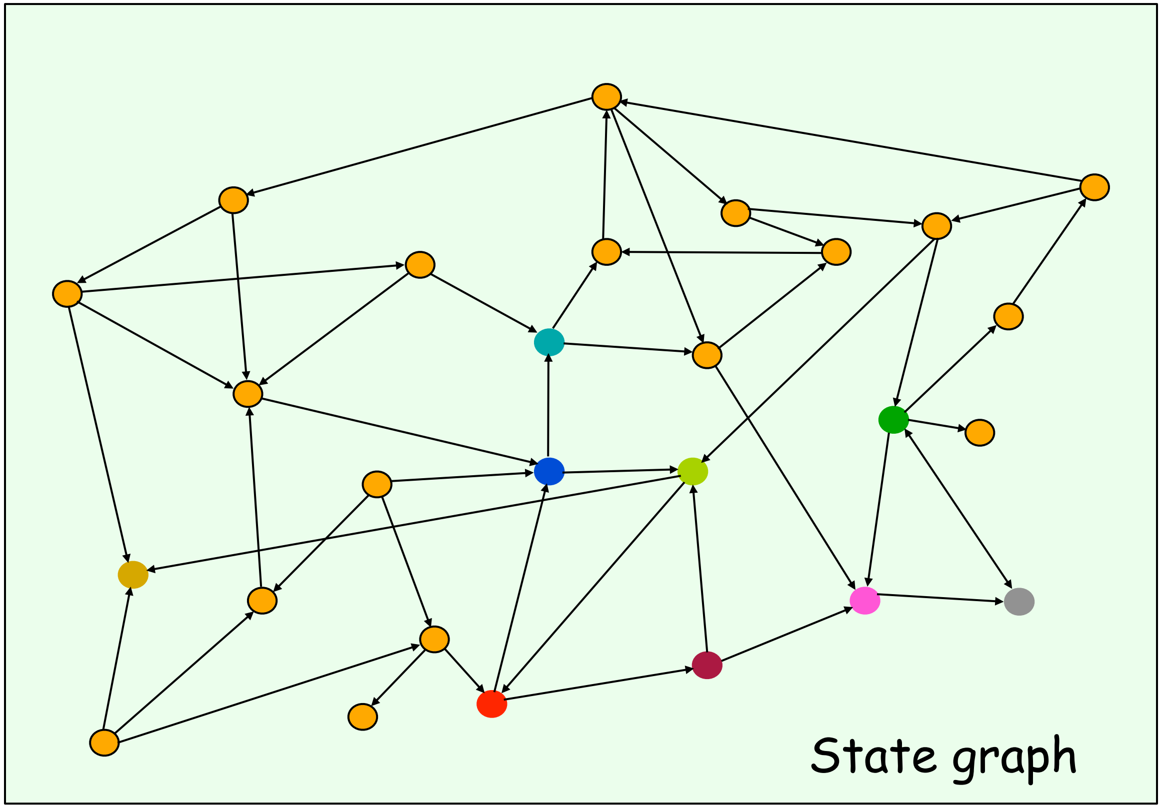
(Where we systematically explore alternatives)

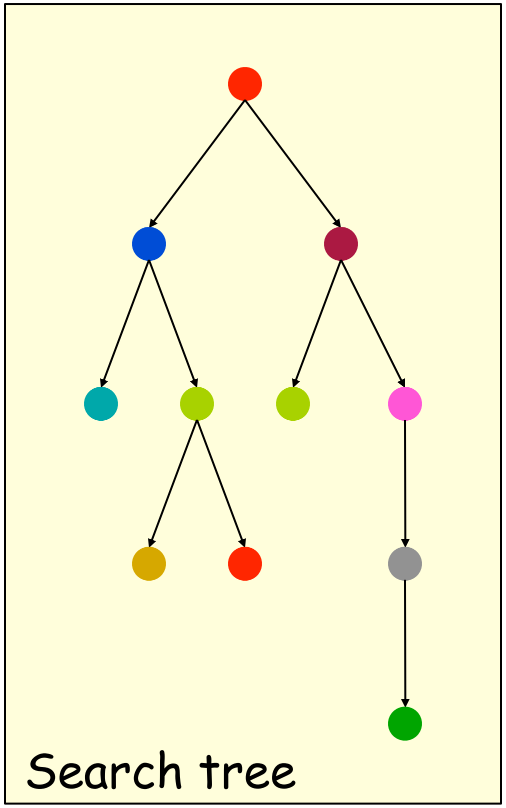
R&N: Chap. 3, Sect. 3.3–5

Simple Problem-Solving-Agent Agent Algorithm

1. s_0 ← sense/read initial state
2. GOAL? ← select/read goal test
3. Succ ← read successor function
4. solution ← **search**(s_0, GOAL?, Succ)
5. perform(solution)

Search Tree





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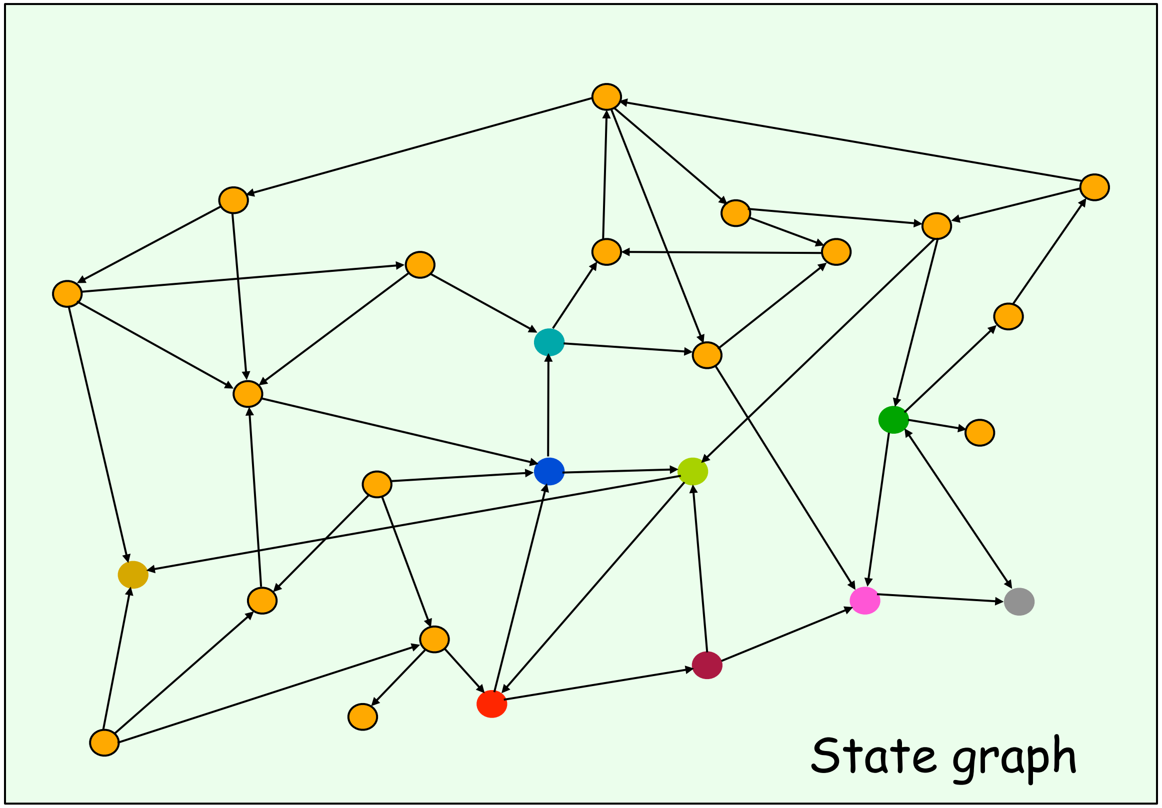
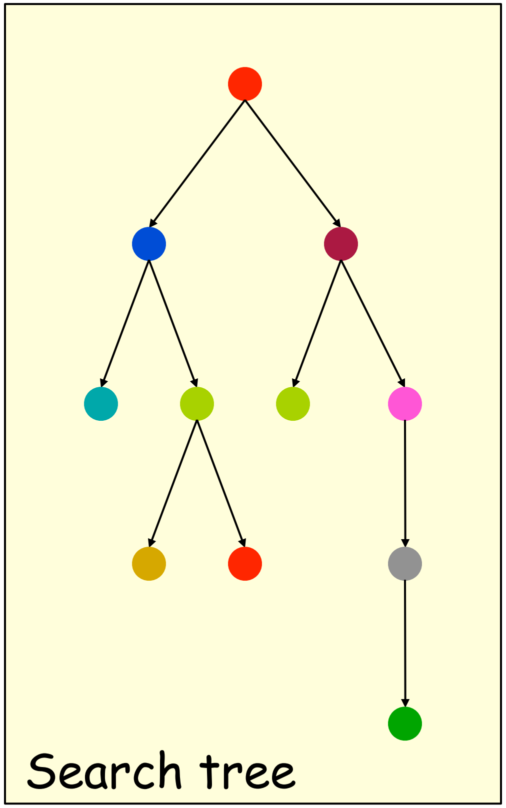
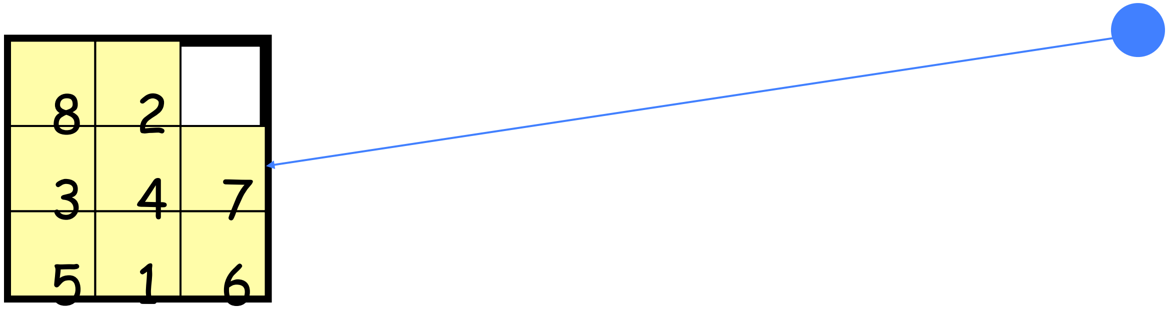




Note that some states may
be visited multiple times

Search Nodes and States



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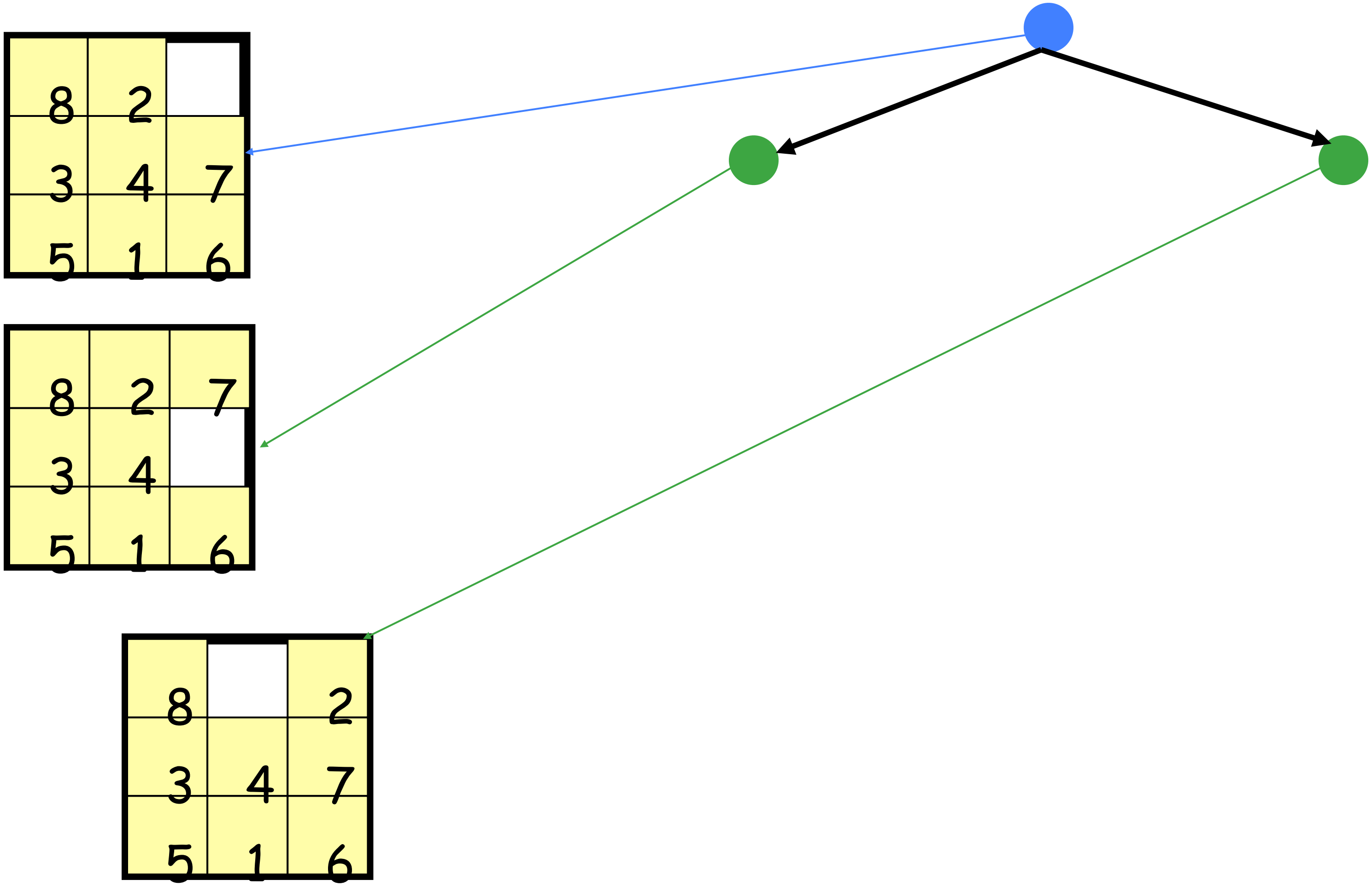

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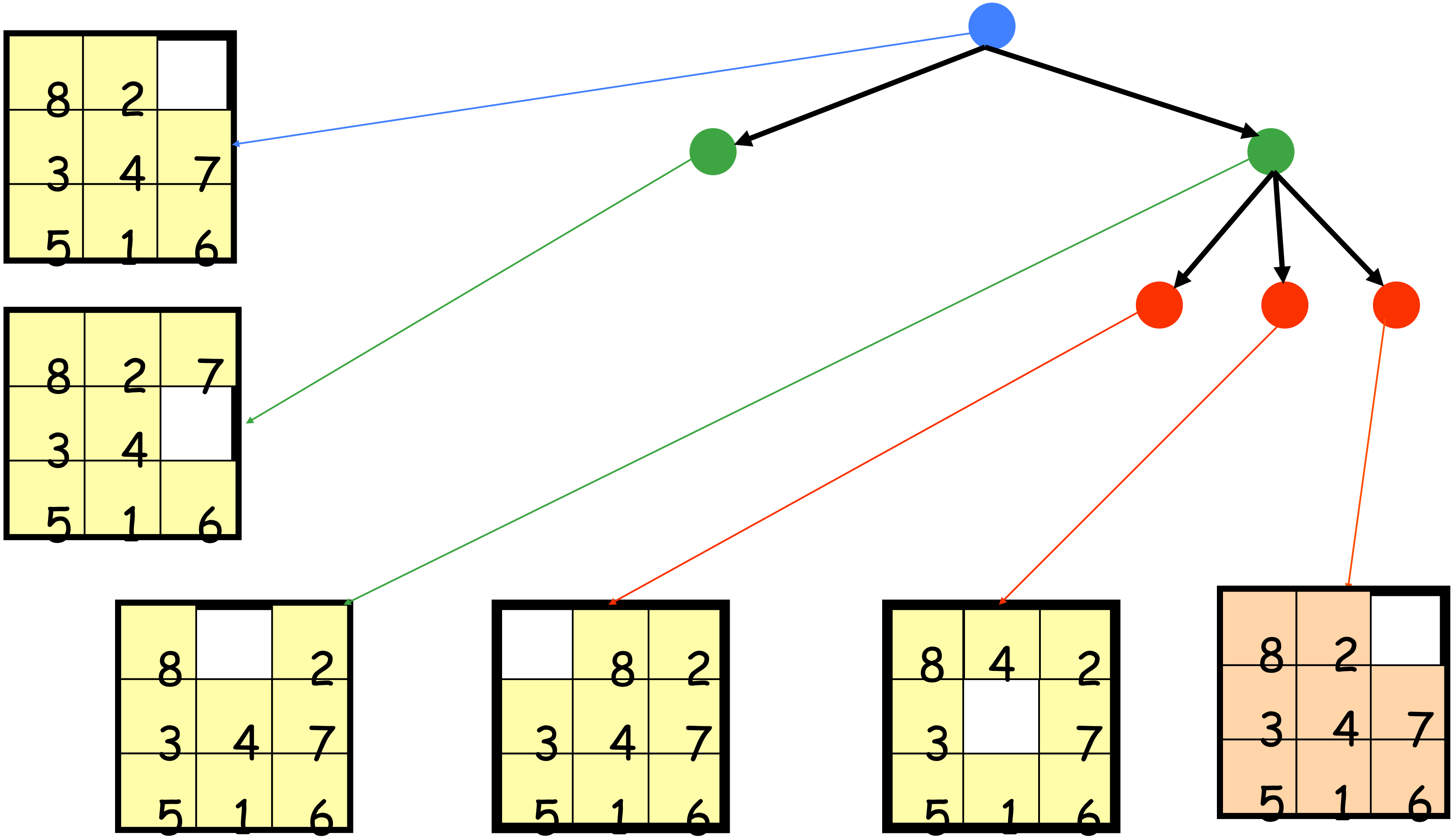

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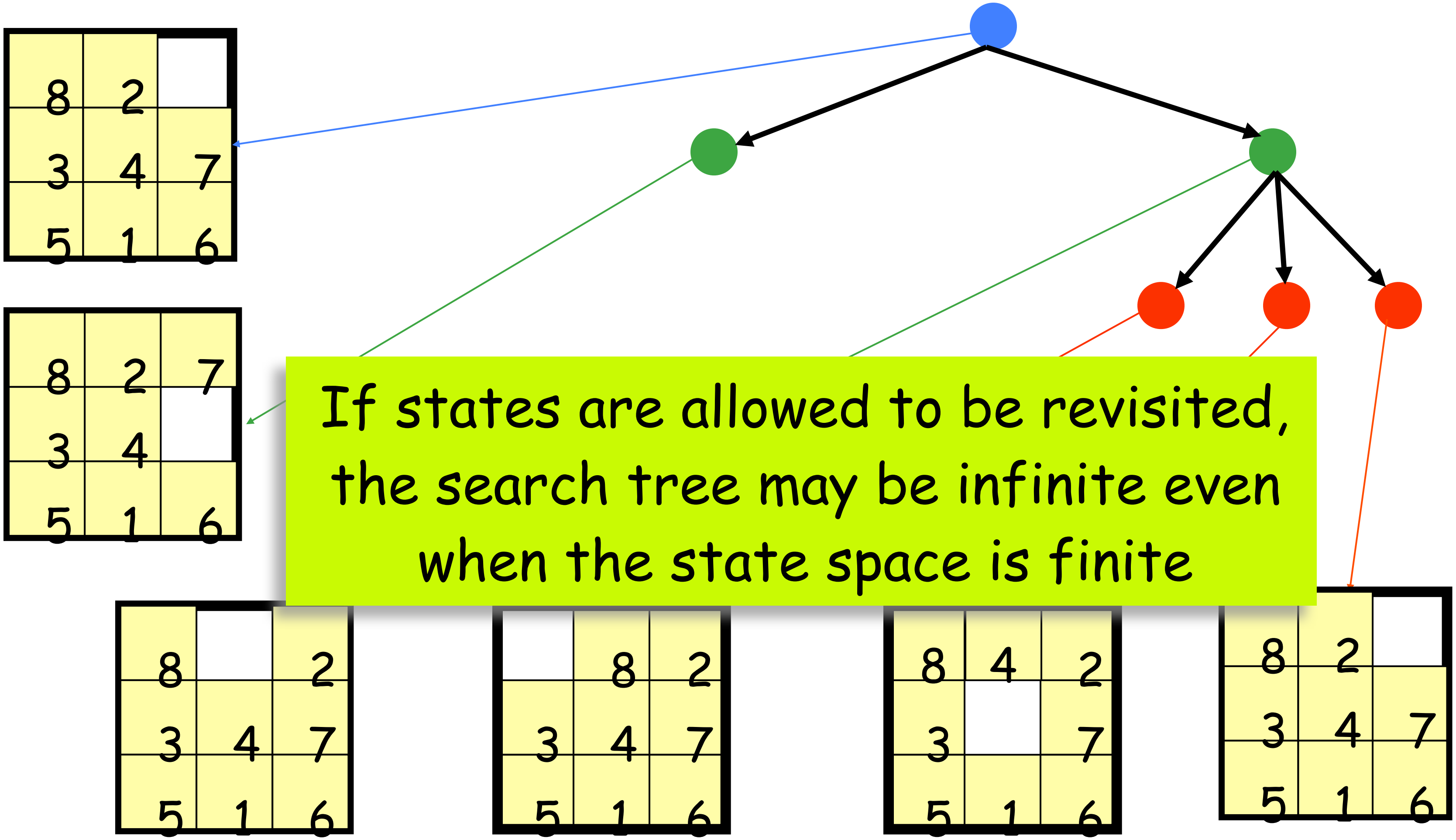
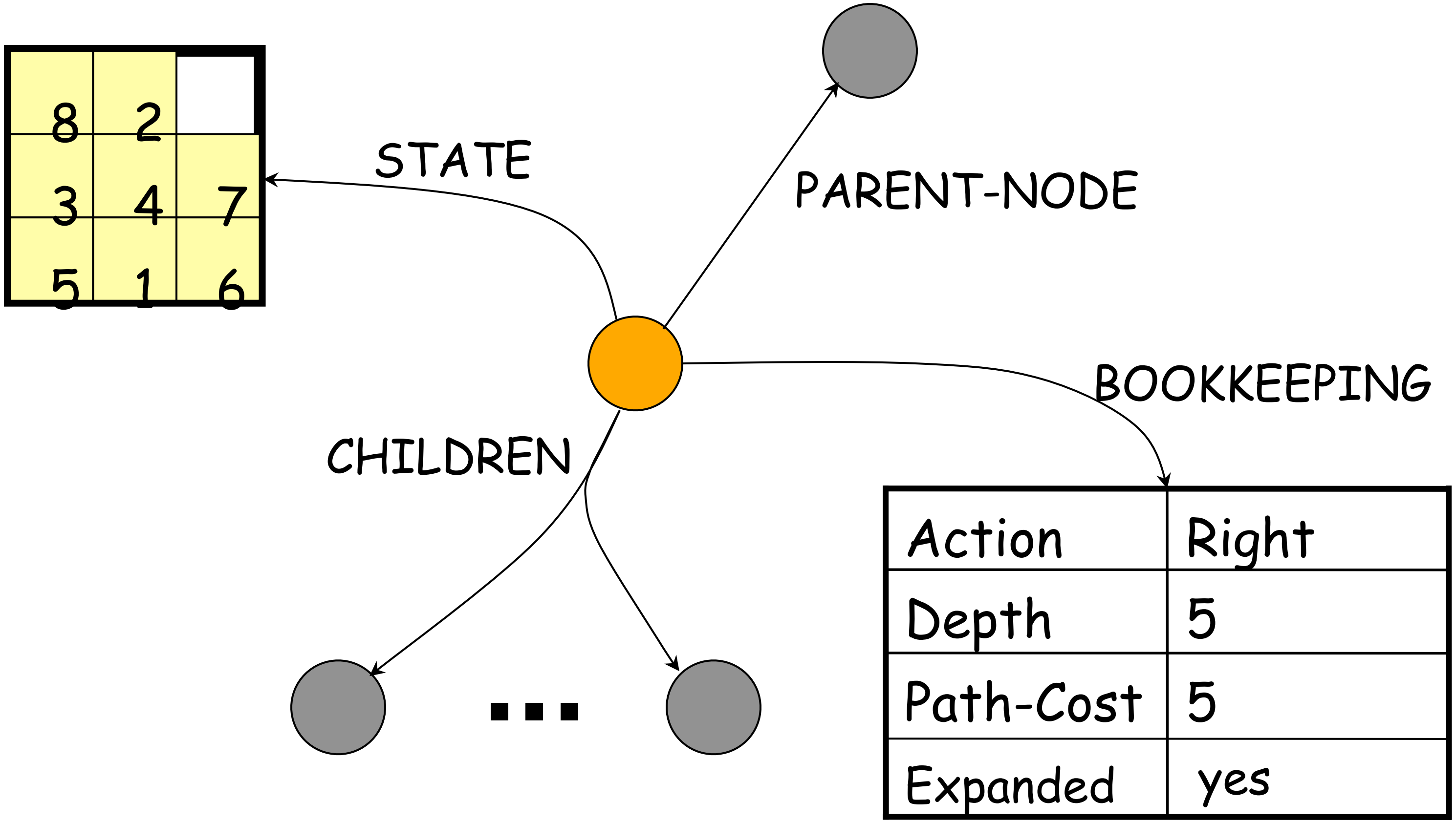


If states are allowed to be revisited, the search tree may be infinite even when the state space is finite

Data Structure of a Node



8	2	
3	4	7
5	1	6

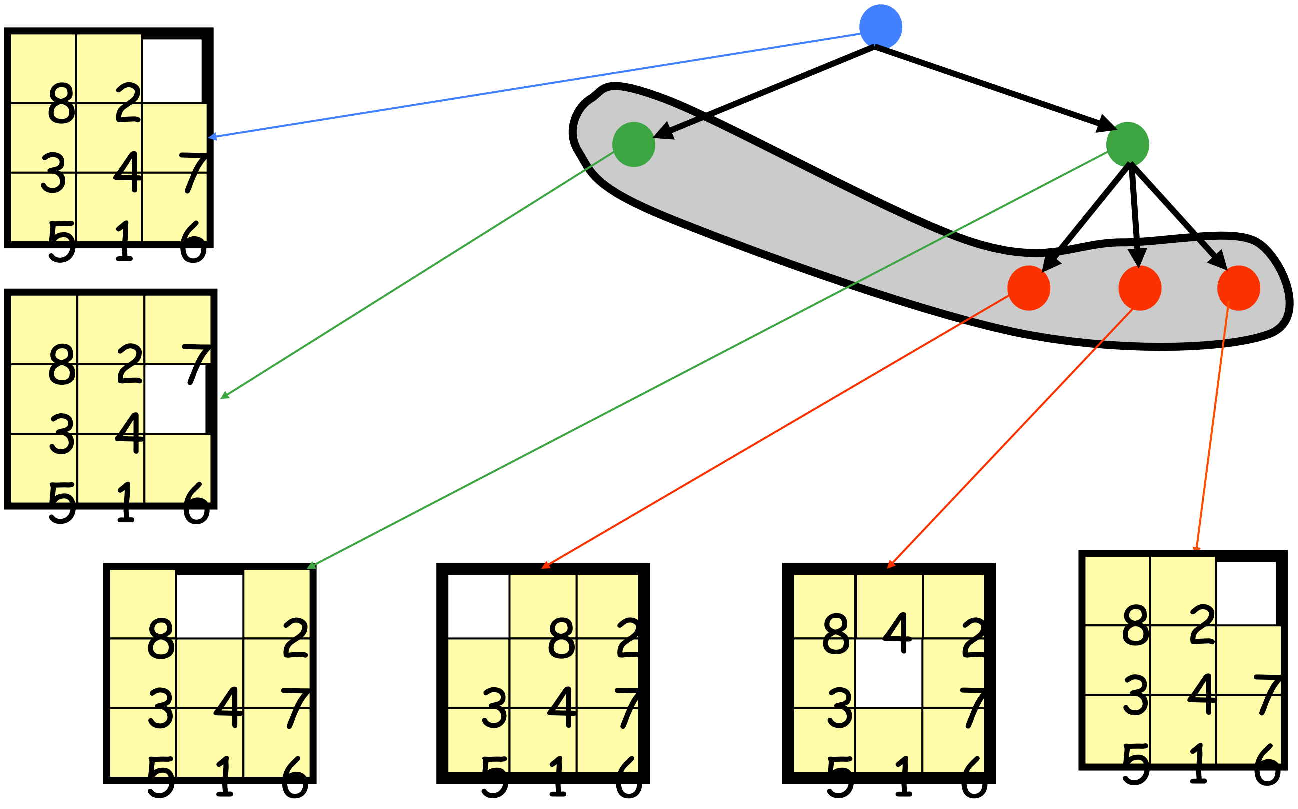
Action	Right
Depth	5
Path-Cost	5
Expanded	yes

Depth of a node N
= length of path from root to N

(depth of the root = 0)

Open List (OL) of Search Tree

- The OL is the set of all search nodes that haven’t been expanded yet



Search Strategy

- The OL is the set of all search nodes that haven't been expanded yet
- The OL is implemented as a priority queue
  - INSERT(node, OL)
  - REMOVE(OL)
- The ordering of the nodes in OL defines the search strategy

Search Algorithm #1

```
SEARCH#1
1. If GOAL?(initial-state) then return initial-state
2. INSERT(initial-node,OL)
3. Repeat:
    a. If empty(OL) then return failure
    b. N ← REMOVE(OL)
    c. s ← STATE(N)
    d. For every state s’ in SUCCESSORS(s)
        i.   Create a new node N’ as a child of N
        ii.  If GOAL?(s’) then return path or goal state
        iii. INSERT(N’,OL)
```

Performance Measures

- Completeness

  A search algorithm is complete if it finds a solution whenever one exists

  [What about the case when no solution exists?]
- Optimality

  A search algorithm is optimal if it returns a minimum-cost path whenever a solution exists
- Complexity

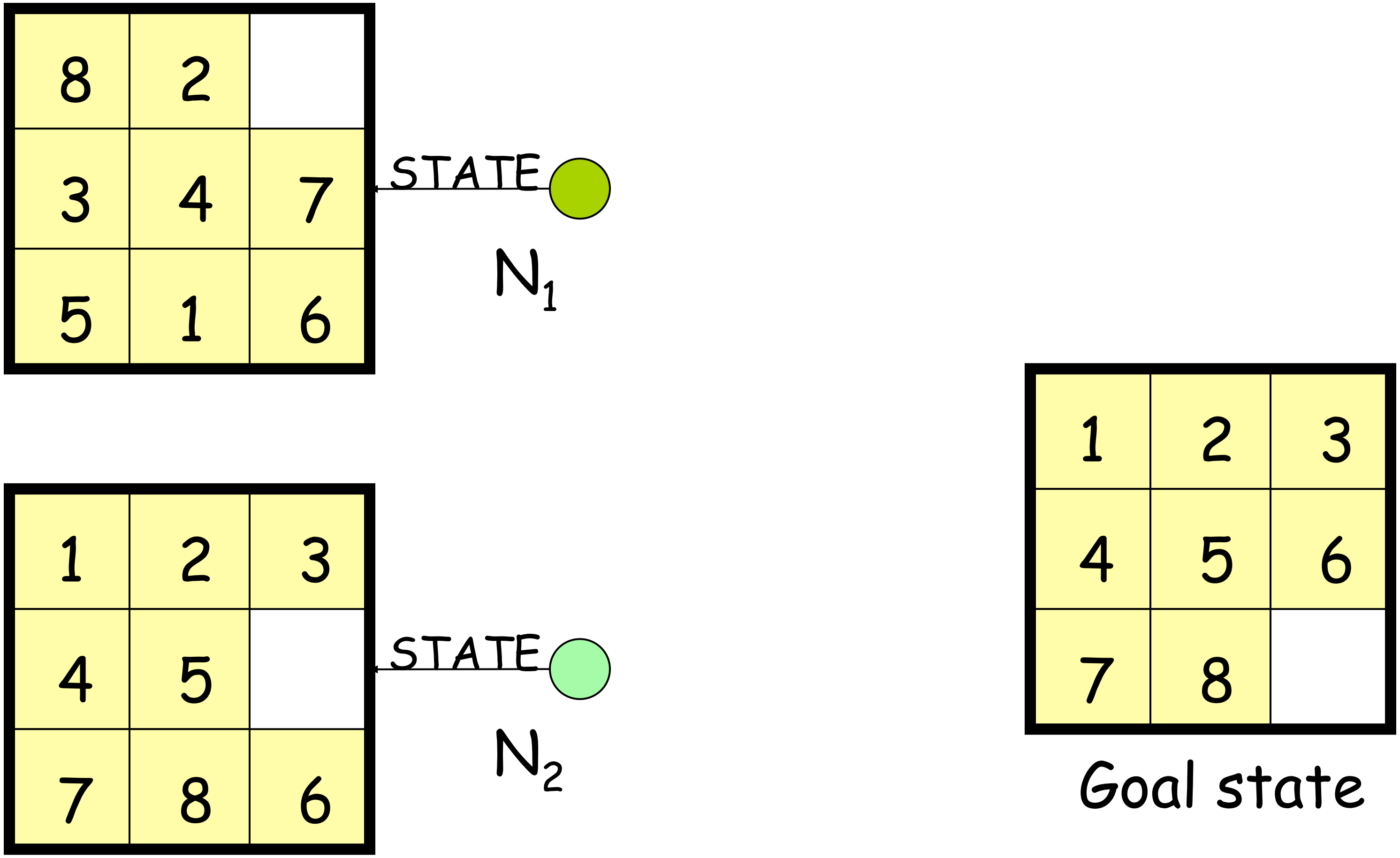
  It measures the time and amount of memory required by the algorithm

Blind vs. Heuristic Strategies

- Blind (or un-informed) strategies do not exploit state descriptions to order OL. They only exploit the positions of the nodes in the search tree

- Heuristic (or informed) strategies exploit state descriptions to order OL (the most "promising" nodes are placed at the beginning of OL)

Example



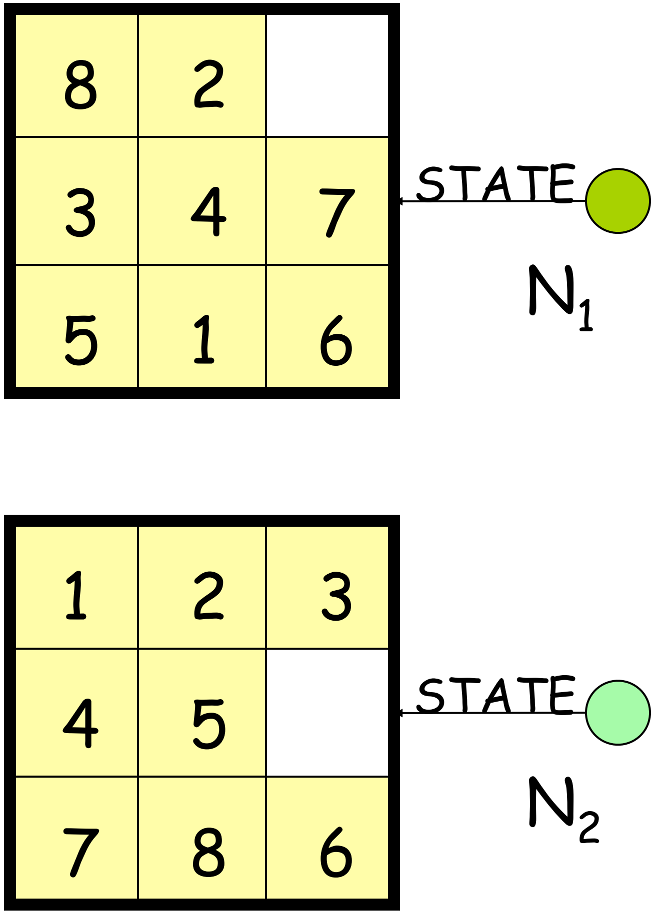
8	2	
3	4	7
5	1	6

1	2	3
4	5	
7	8	6

1	2	3
4	5	6
7	8	

Example

For a blind strategy, N_1 and N_2 are just two nodes (at some position in the search tree)



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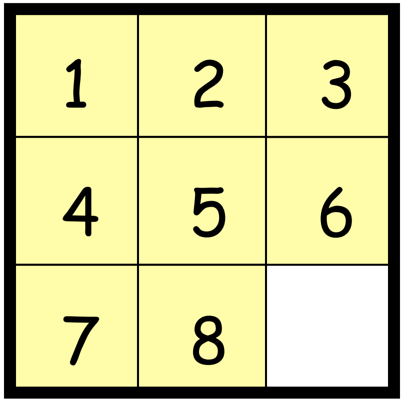
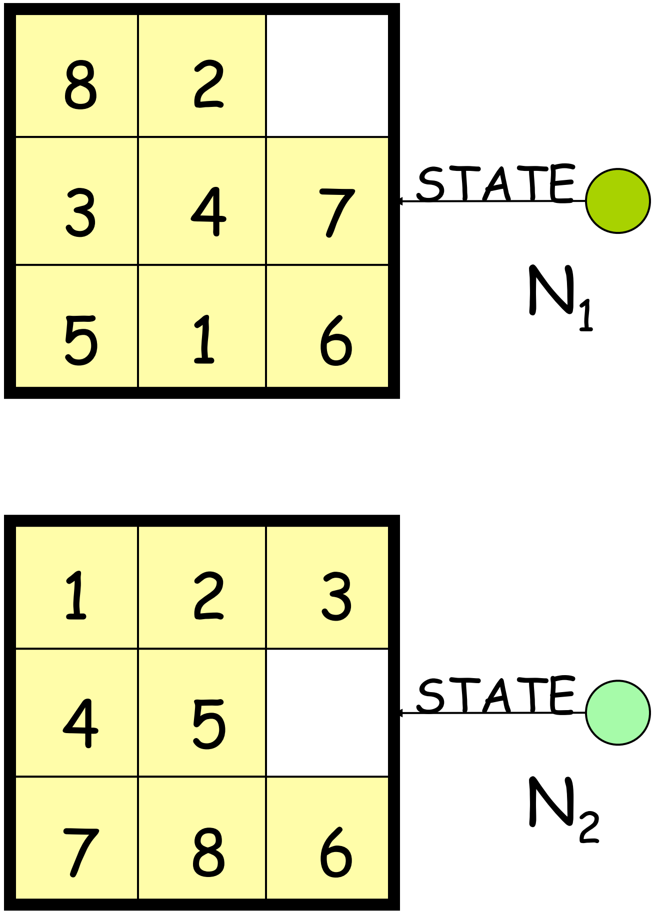


1	2	3
4	5	6
7	8	

Goal state

Example

For a heuristic strategy counting the number of misplaced tiles, N_2 is more promising than N_1



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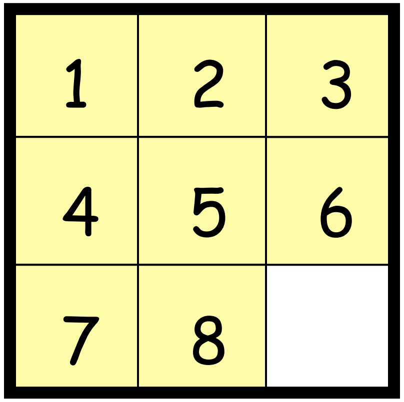

1	2	3
4	5	6
7	8	

Goal state

Remark

- Some search problems, such as the (n^2-1)-puzzle, are NP-hard
- One can't expect to solve all instances of such problems in less than exponential time (in n)
- One may still strive to solve each instance as efficiently as possible

This is the purpose of the search strategy

Blind Strategies

Blind Strategies

Arc cost = 1

Blind Strategies

- Breadth-first
  - Bidirectional

} Arc cost = 1

Blind Strategies

- Breadth-first
  - Bidirectional
- Depth-first
  - Depth-limited
  - Iterative deepening

Arc cost = 1

Blind Strategies

- Breadth-first
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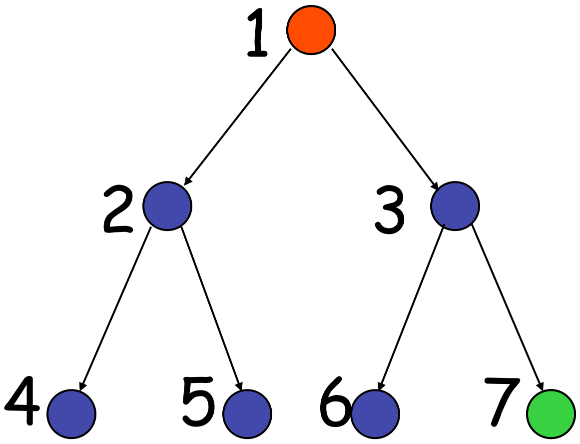
Arc cost = 1

- Uniform-Cost
  (variant of breadth-first)

Arc cost = c(action) $\geq \varepsilon > 0$

Breadth-First Strategy

New nodes are inserted at the end of OL



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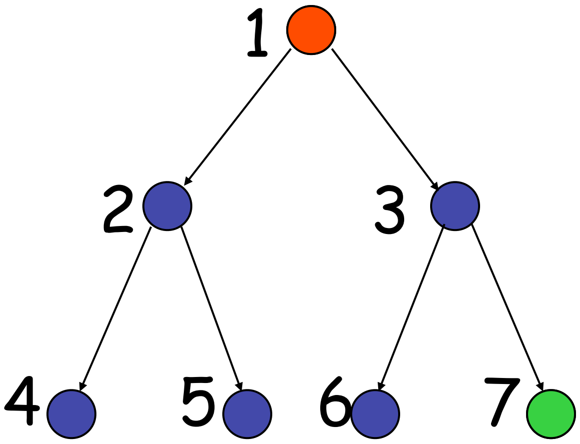

OL = (1)

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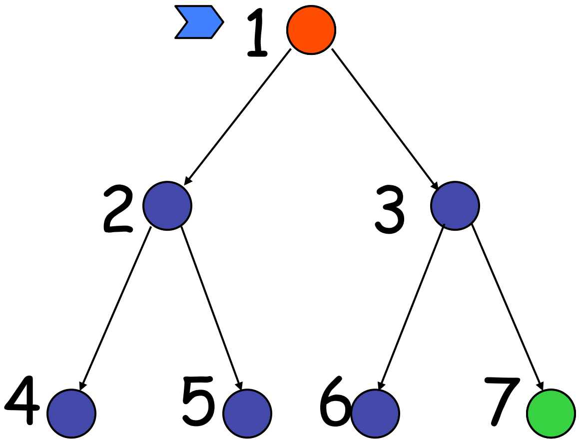

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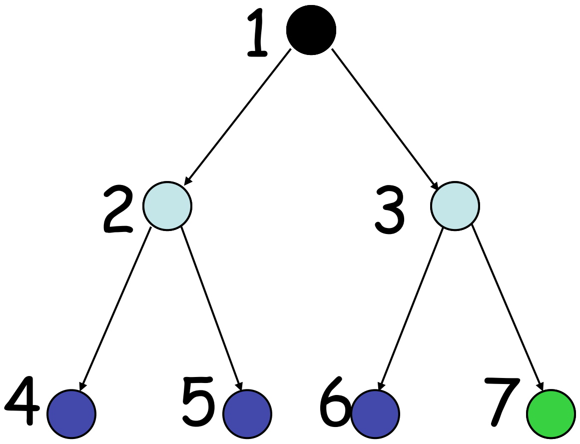
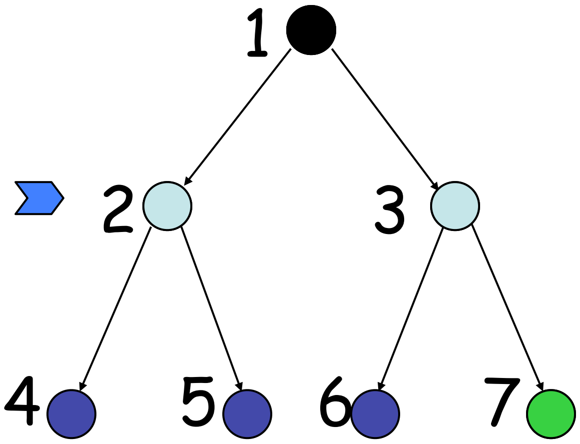


OL = (2, 3)

Breadth-First Strategy

New nodes are inserted at the end of OL



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Breadth-First Strategy

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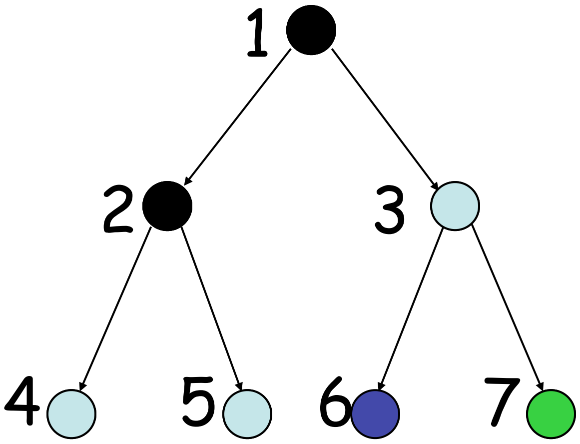
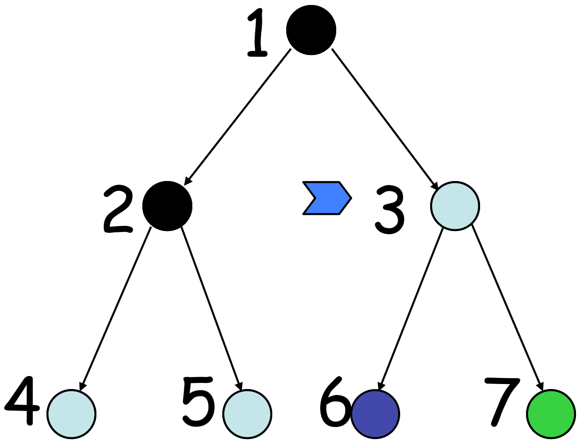


OL = (3, 4, 5)

Breadth-First Strategy

New nodes are inserted at the end of OL



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Breadth-First Strategy

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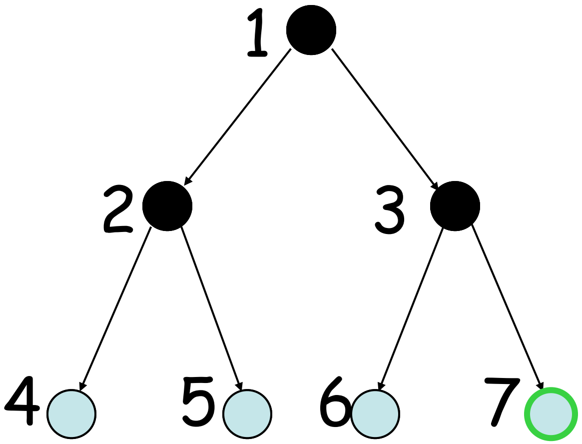

OL = (4, 5, 6, 7)

Important Parameters

1) Maximum number of successors of any state

→ branching factor b of the search tree

2) Minimal length (≠ cost) of a path between the initial and a goal state

→ depth d of the shallowest goal node in the search tree

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete? Not complete?
  - Optimal? Not optimal?

Evaluation

Evaluation

- b: branching factor

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Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:

  ???

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:
  $1 + b + b^2 + \ldots + b^d \quad = \quad ???$

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:
  $1 + b + b^2 + \ldots + b^d \;=\; (b^{d+1}-1)/(b-1) \;=\; O(b^d)$
- → Time and space complexity is $O(b^d)$

Big O Notation

$g(n) = O(f(n))$ if there exist two positive constants a and N such that:

for all $n > N$: $g(n) \leq af(n)$

Time and Memory Requirements

d	# Nodes	Time	Memory
2	111	.01 msec	11 Kbytes
4	11,111	1 msec	1 Mbyte
6	~10^6	1 sec	100 Mb
8	~10^8	100 sec	10 Gbytes
10	~10^{10}	2.8 hours	1 Tbyte
12	~10^{12}	11.6 days	100 Tbytes
14	~10^{14}	3.2 years	10,000 Tbytes

Assumptions: b = 10; 1,000,000 nodes/sec; 100bytes/node

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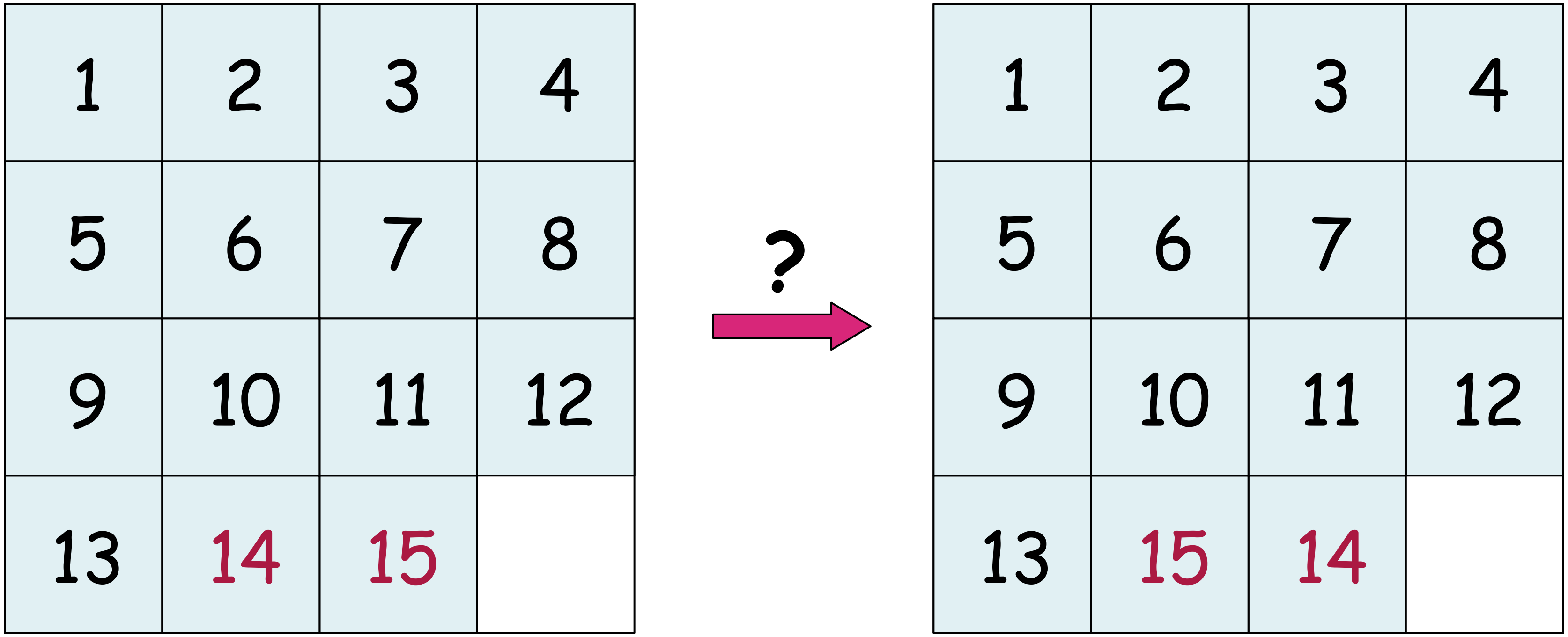
Assumptions: b = 10; 1,000,000 nodes/sec; 100bytes/node

Remark

If a problem has no solution, breadth-first may run for ever (if the state space is infinite or states can be revisited arbitrary many times)



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

Bidirectional Strategy

Bidirectional Strategy

2 fringe queues: FRINGE1 and FRINGE2

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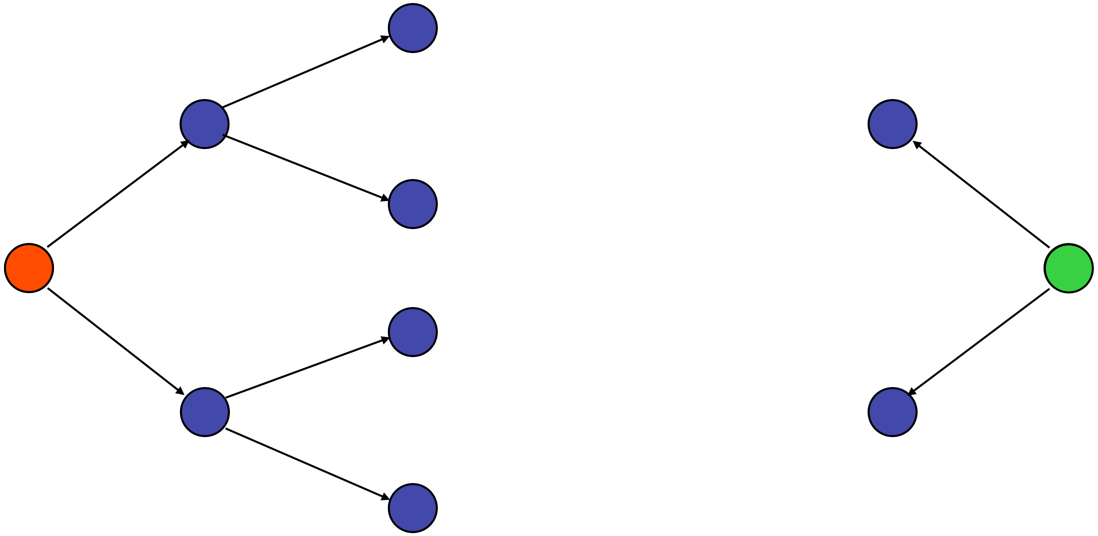

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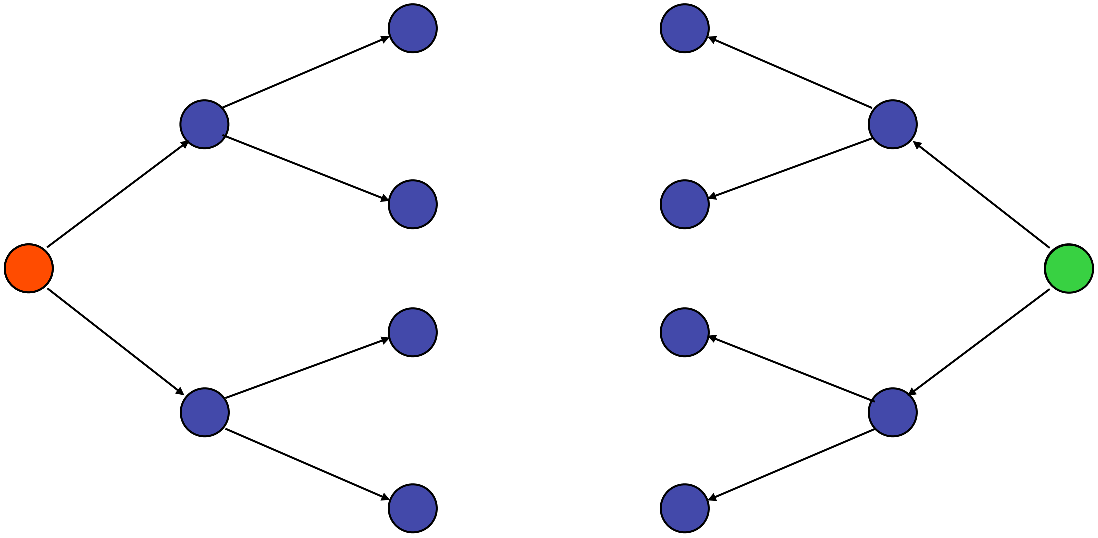

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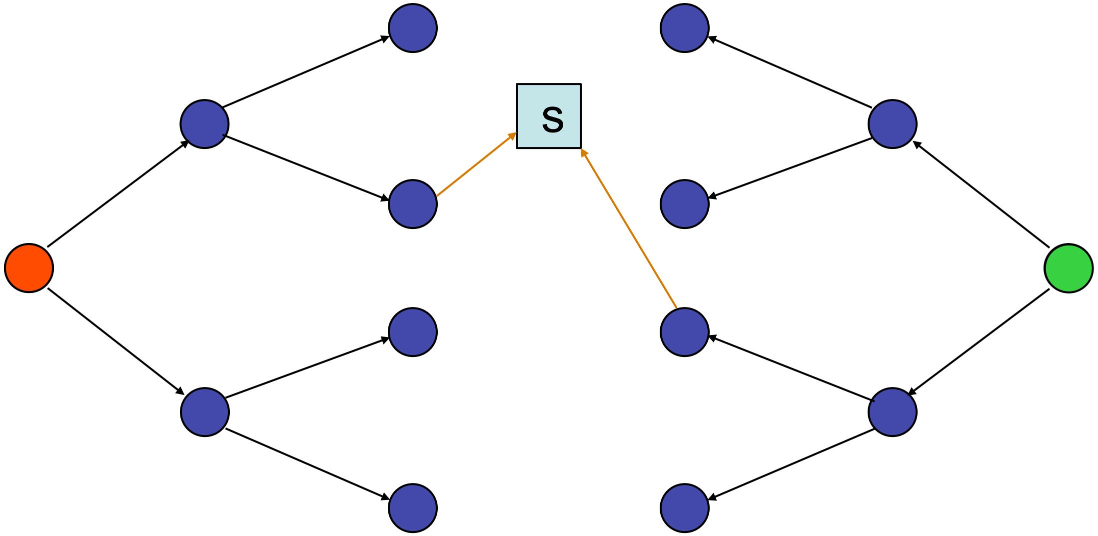

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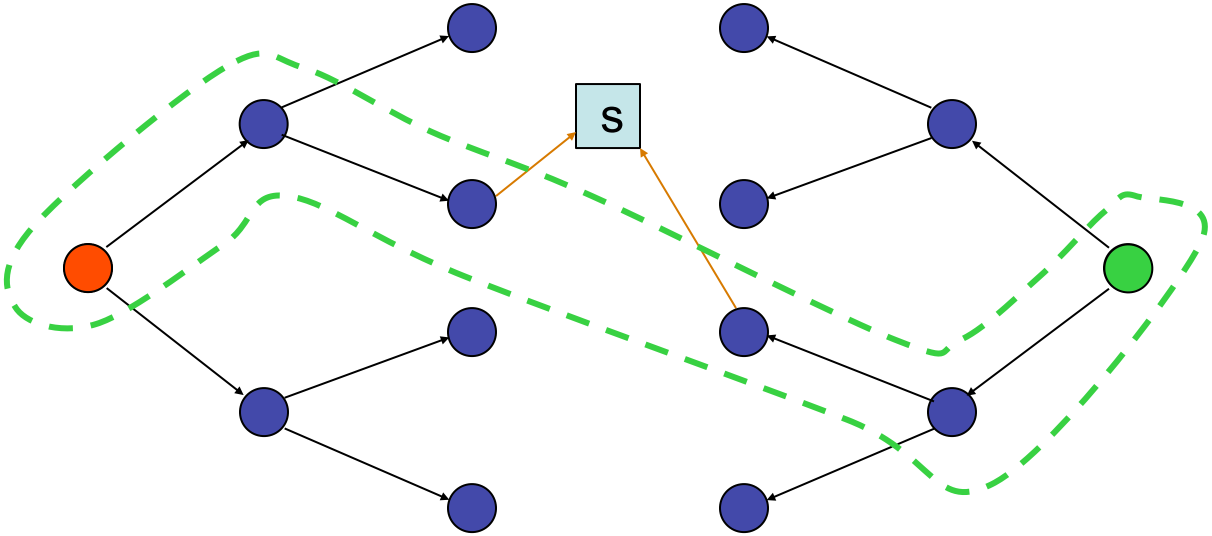

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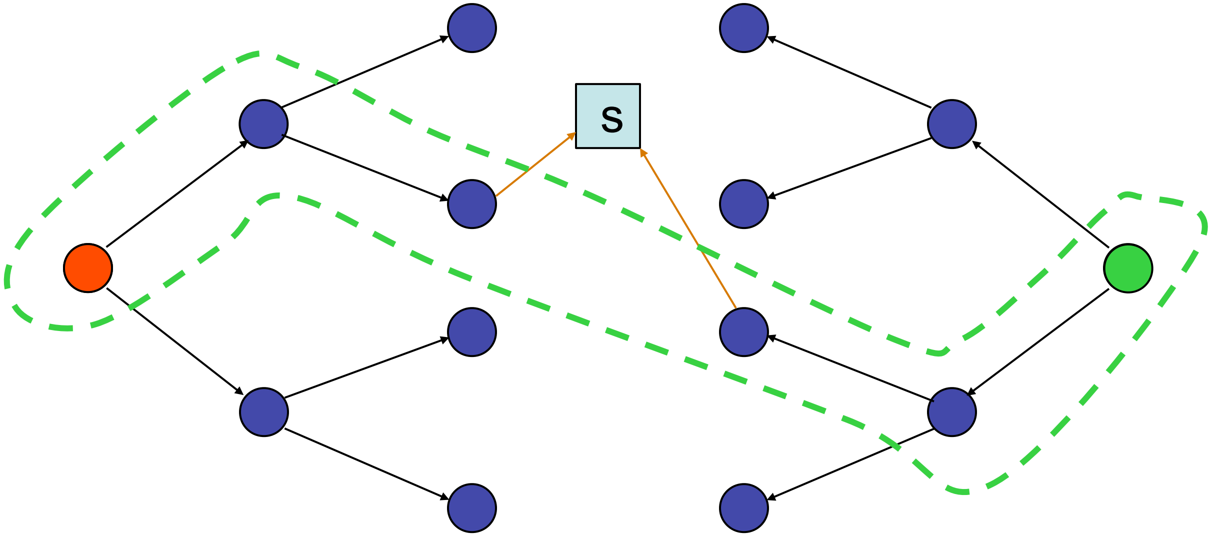

Time and space complexity is $O(b^{d/2}) << O(b^d)$ if both trees have the same branching factor b

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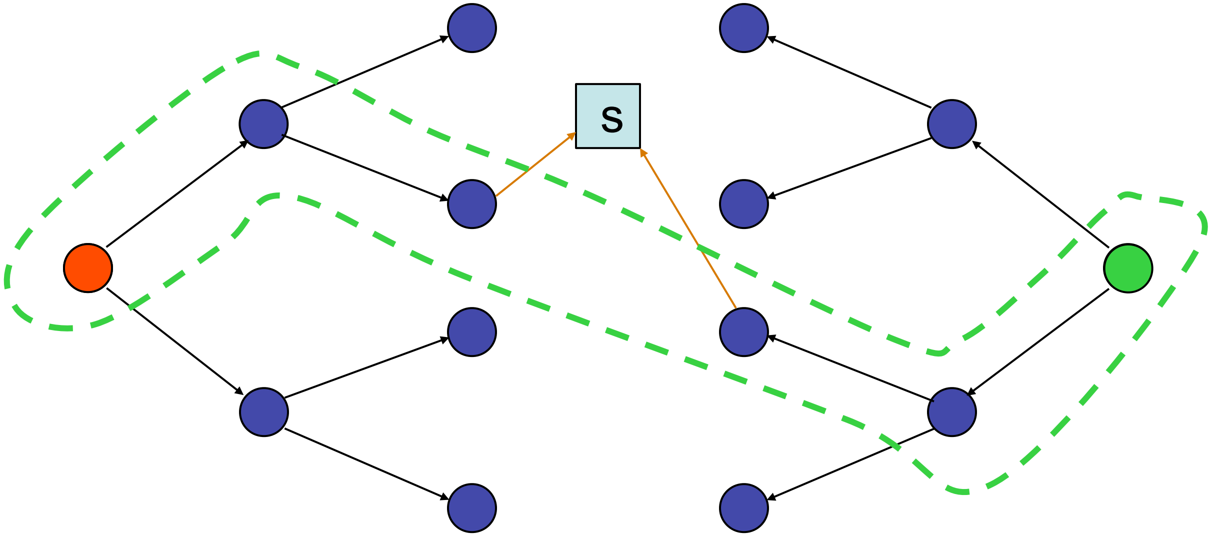
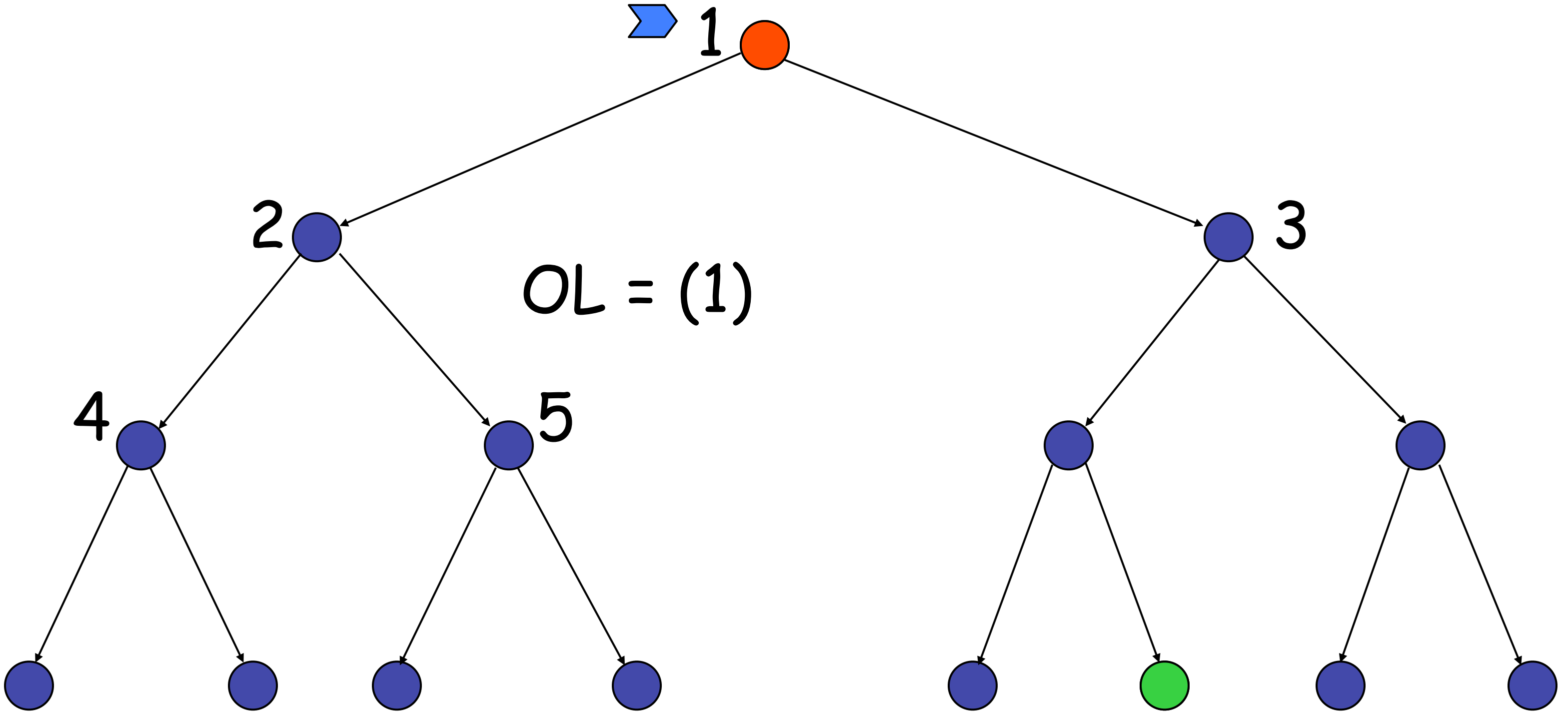


Time and space complexity is $O(b^{d/2}) << O(b^d)$ if both trees have the same branching factor b

Question: What happens if the branching factor is different in each direction?

Depth-First Strategy

New nodes are inserted at the front of OL



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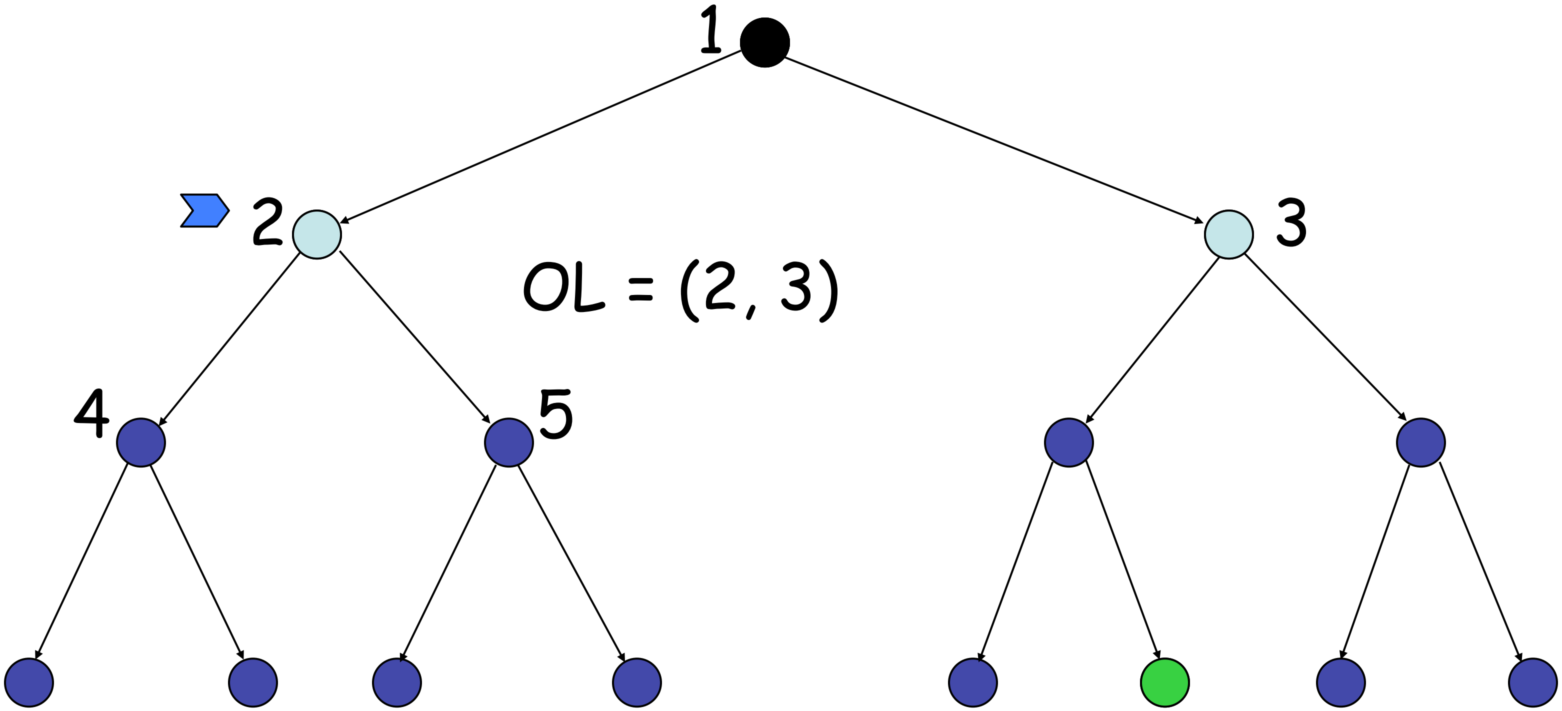

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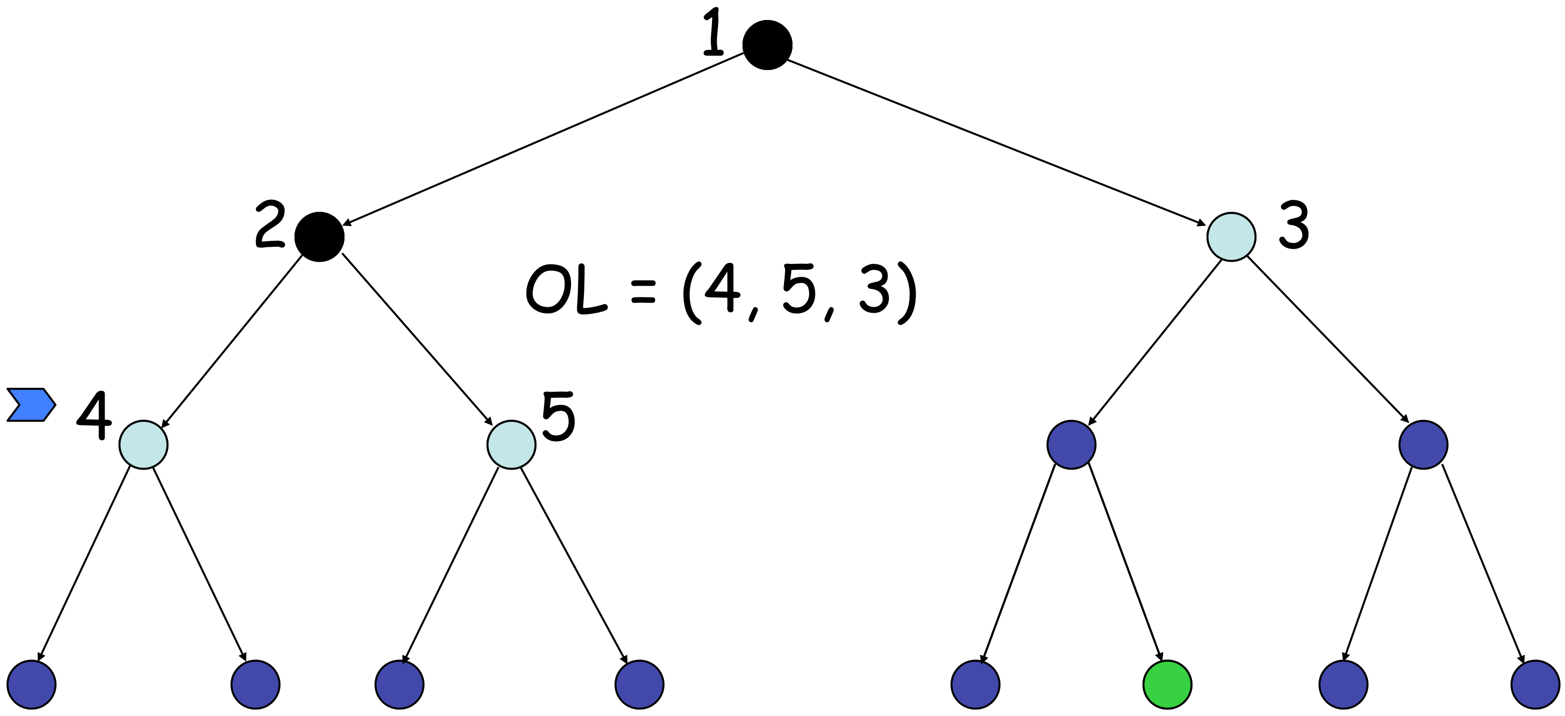

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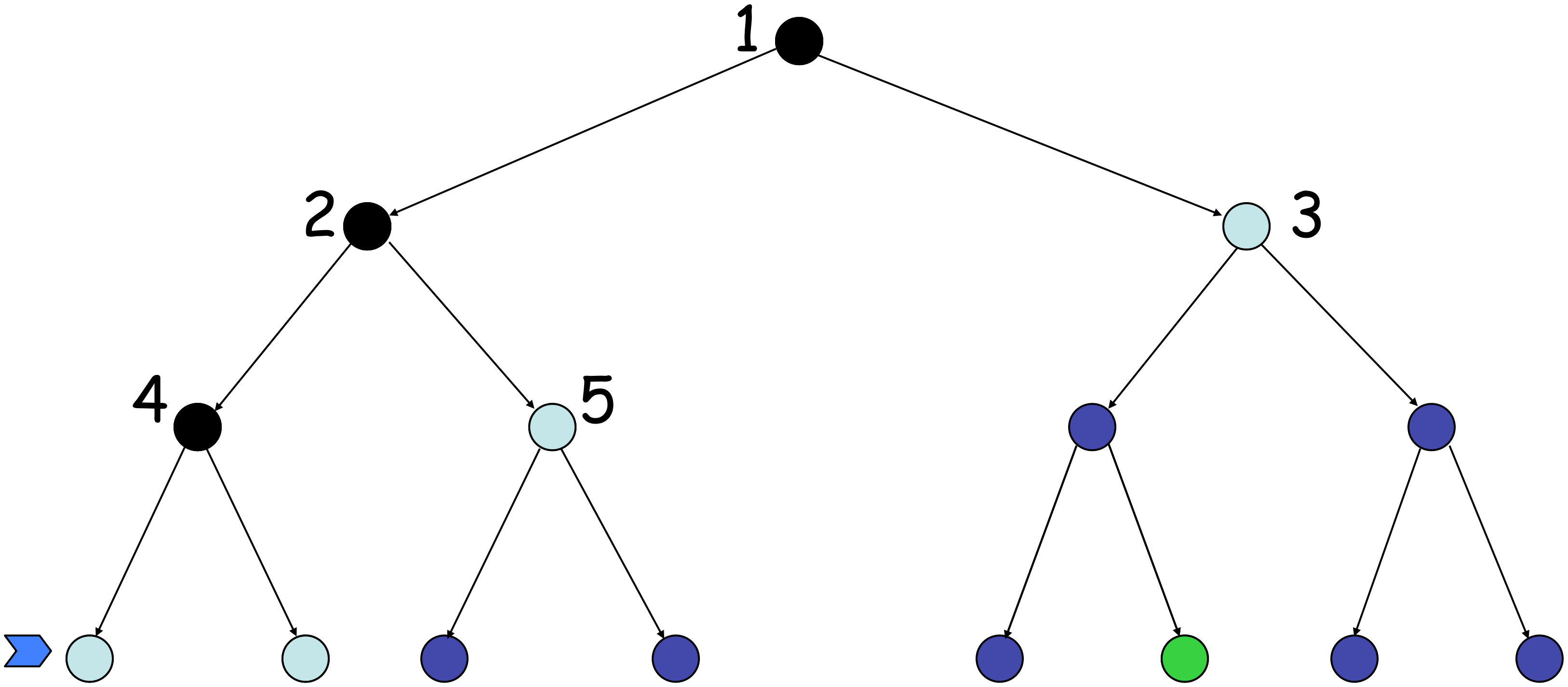

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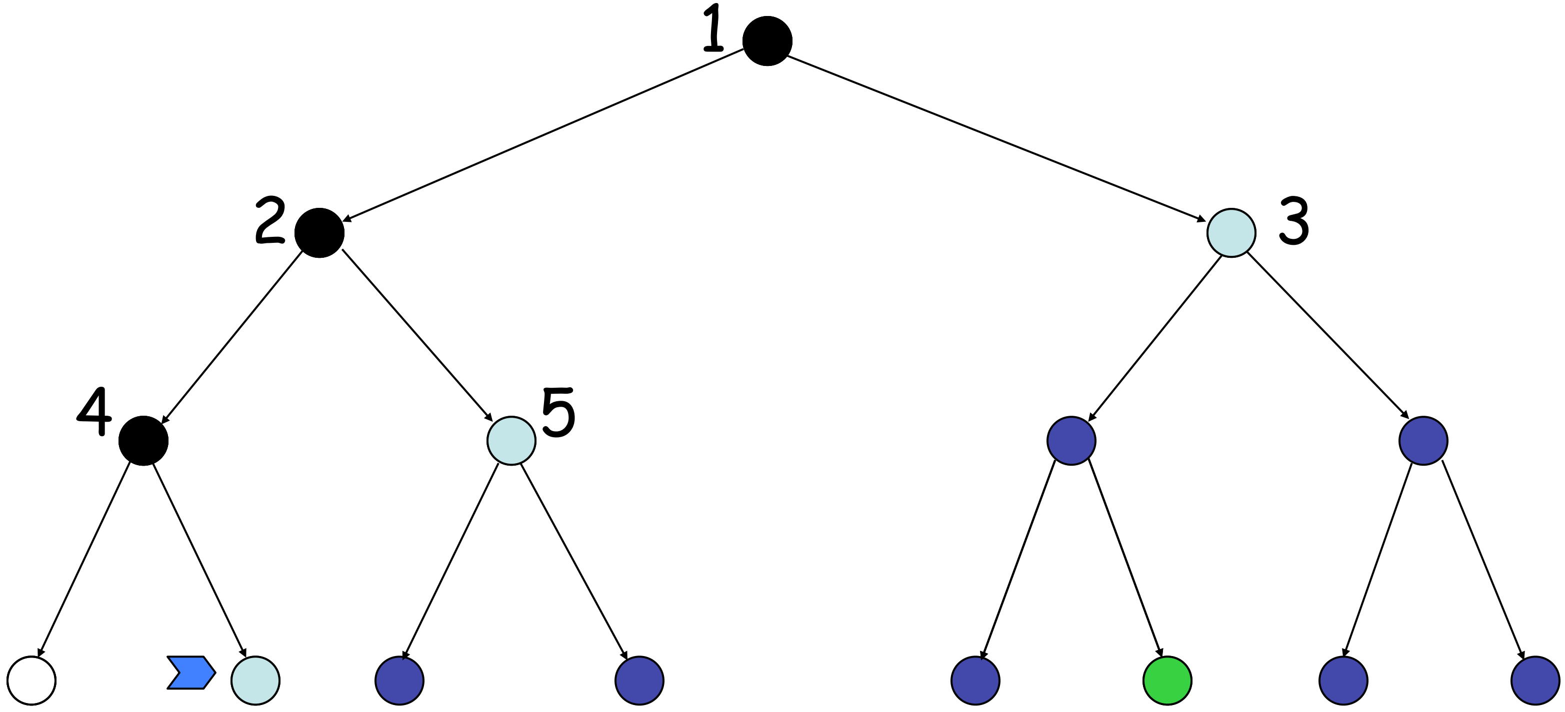

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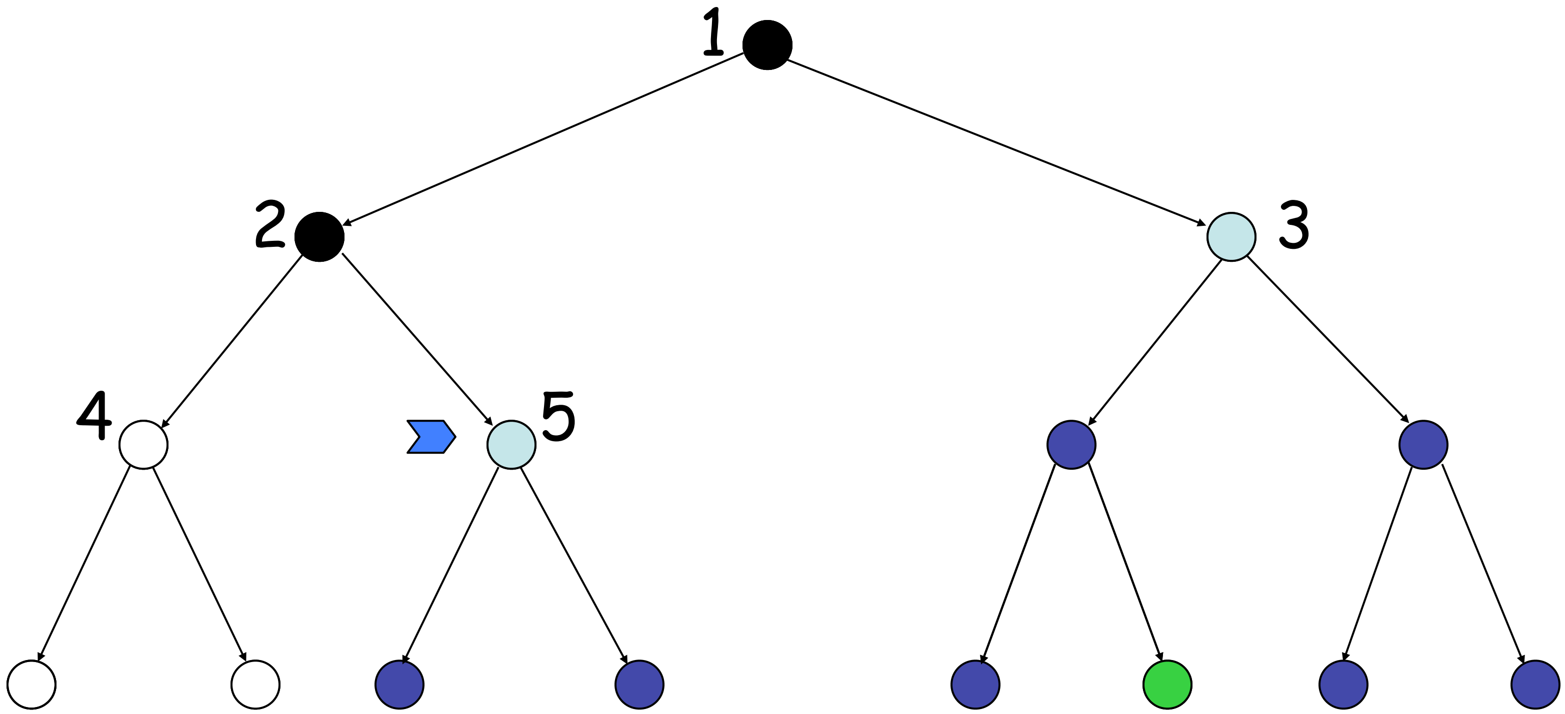

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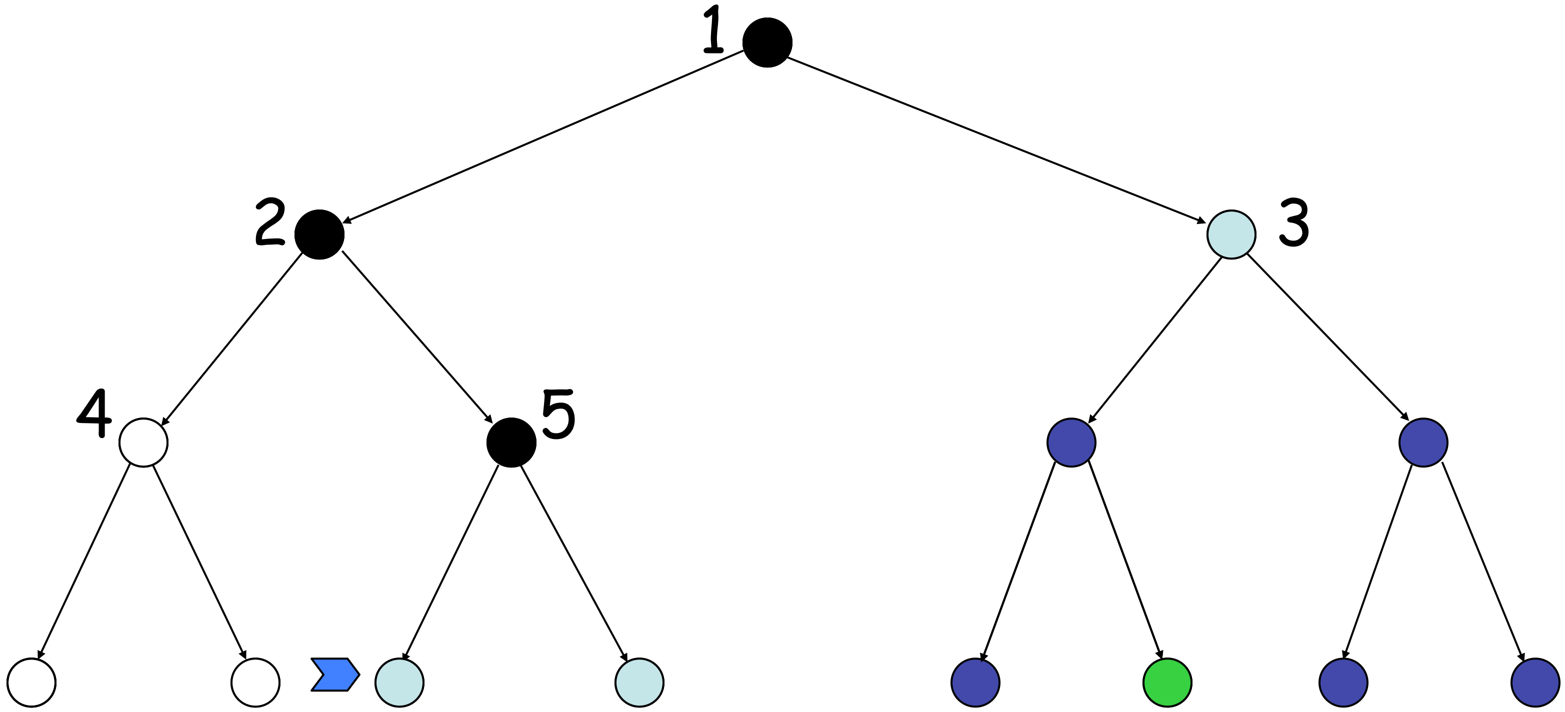

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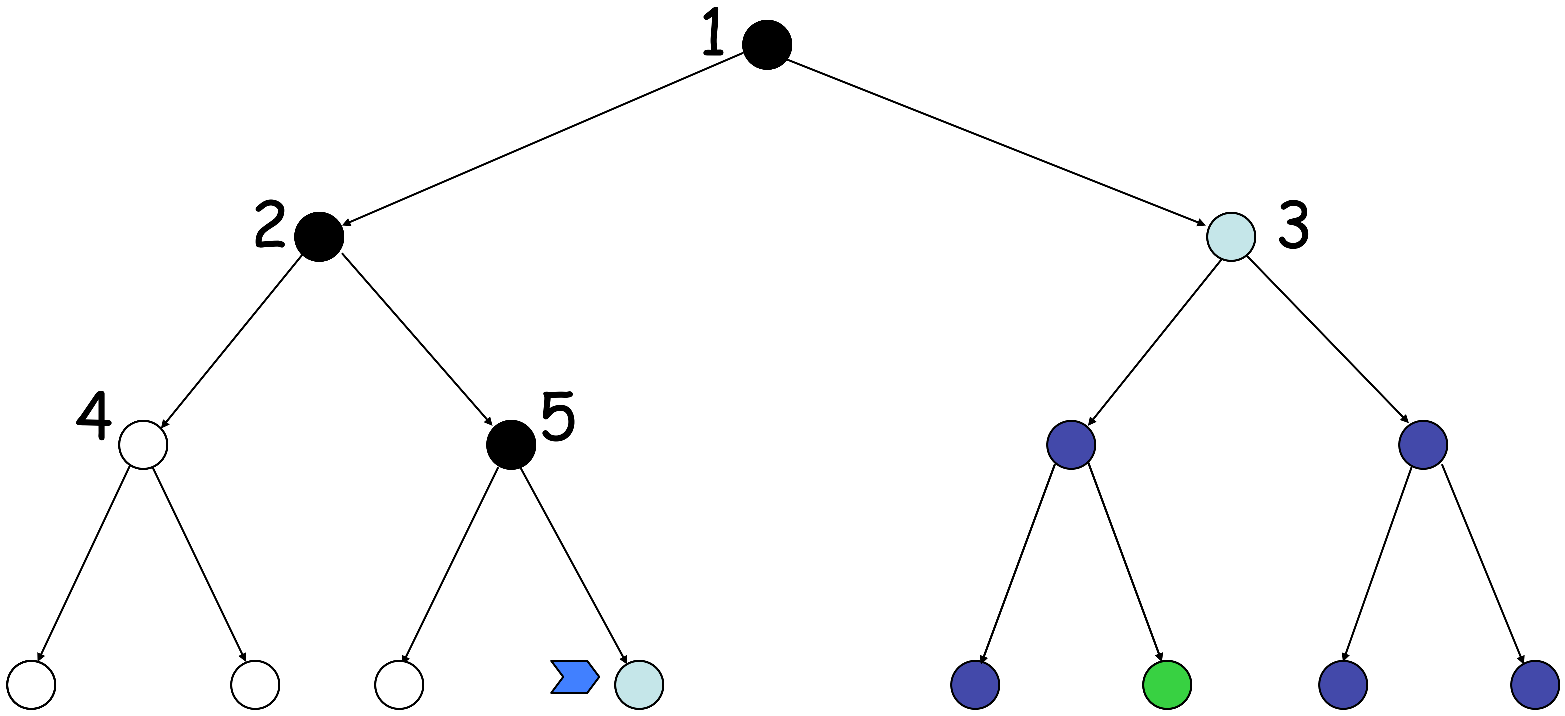

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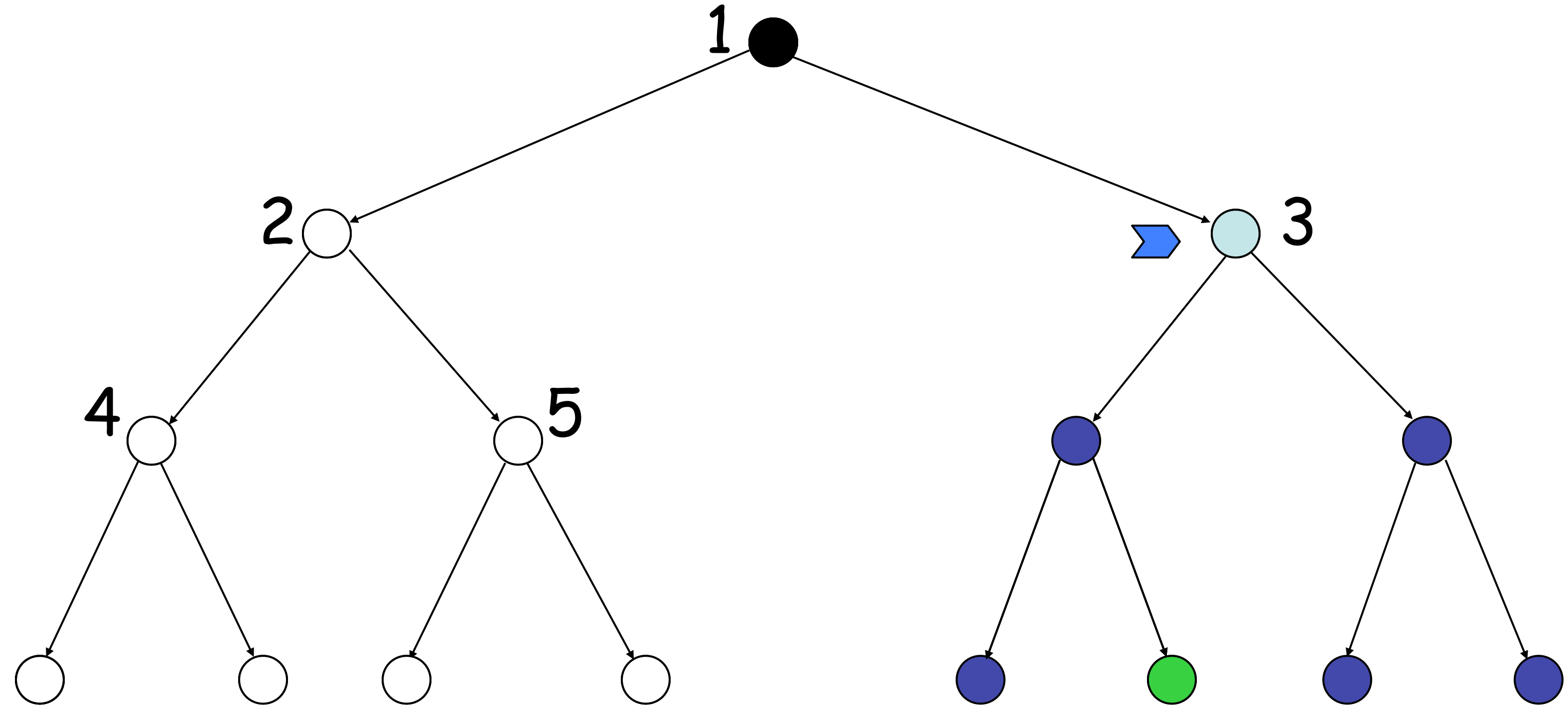

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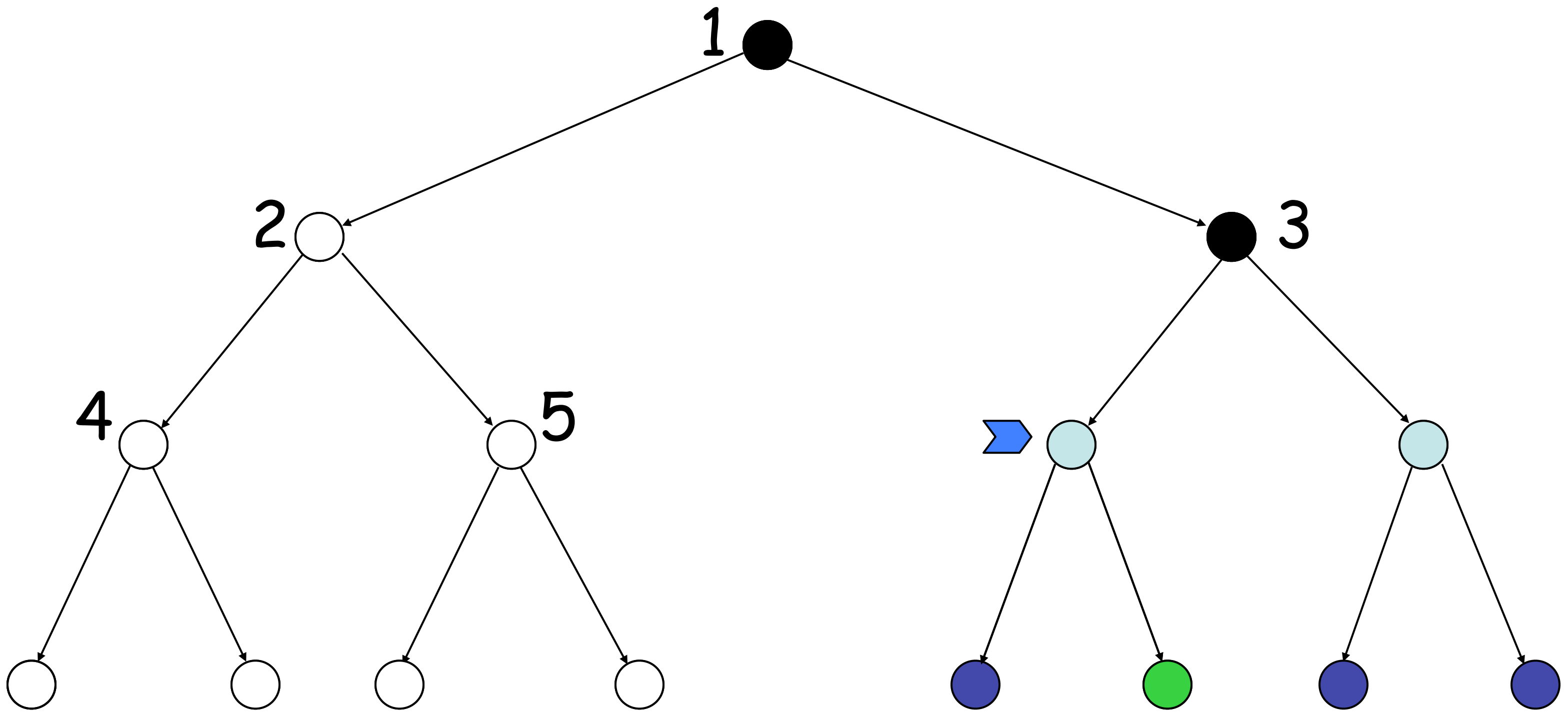

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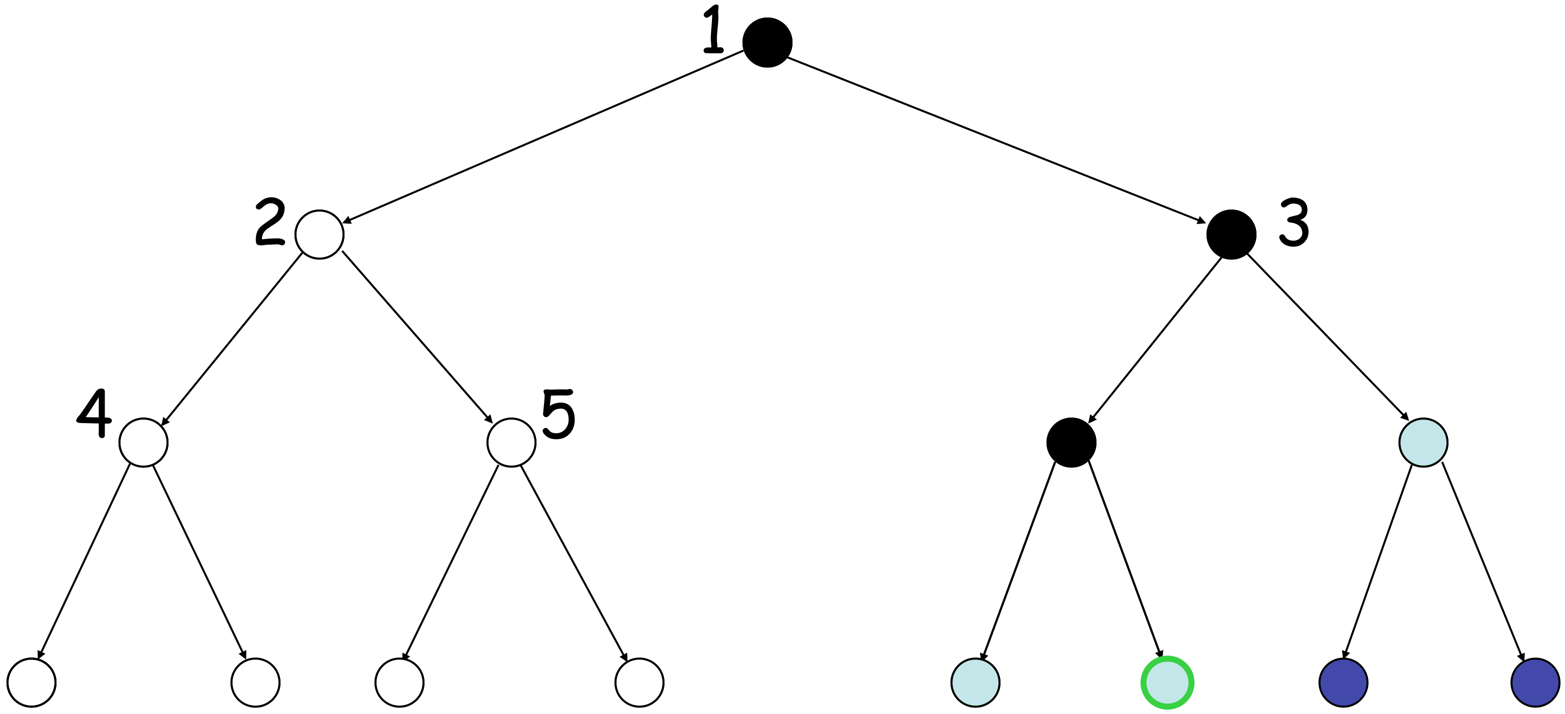

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- m: maximal depth of a leaf node
- Depth-first search is:
  - Complete?
  - Optimal?

Evaluation

Evaluation

- b: branching factor
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Evaluation

- b: branching factor
- d: depth of shallowest goal node
- m: maximal depth of a leaf node
- Depth-first search is:
  - Complete only for finite search tree
  - Not optimal

Evaluation

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- Number of nodes generated (worst case):
  $1 + b + b^2 + \ldots + b^m = O(b^m)$

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- m: maximal depth of a leaf node
- Depth-first search is:
  - Complete only for finite search tree
  - Not optimal
- Number of nodes generated (worst case): $1 + b + b^2 + \ldots + b^m = O(b^m)$
- Time complexity is $O(b^m)$

Depth-Limited Search

- Depth-first with depth cutoff k (depth at which nodes are not expanded)

- Three possible outcomes:
  - Solution
  - Failure (no solution)
  - Cutoff (no solution within cutoff)

Iterative Deepening Search

Provides the best of both breadth-first and depth-first search

Main idea: Totally horrifying !

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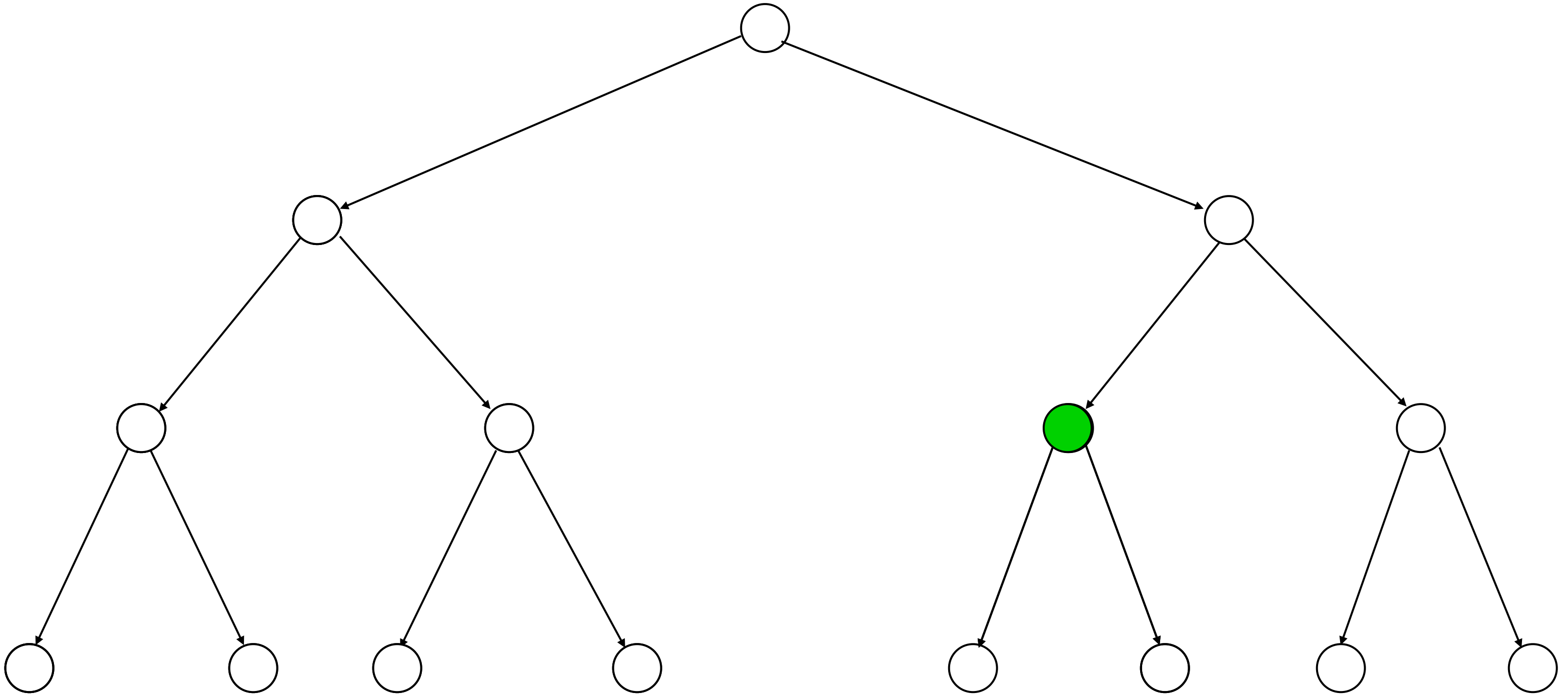
IDS

```
For k = 0, 1, 2, … do:
    Perform depth-first search with
    depth cutoff k
    (i.e., only generate nodes with depth ≤ k)
```

Iterative Deepening



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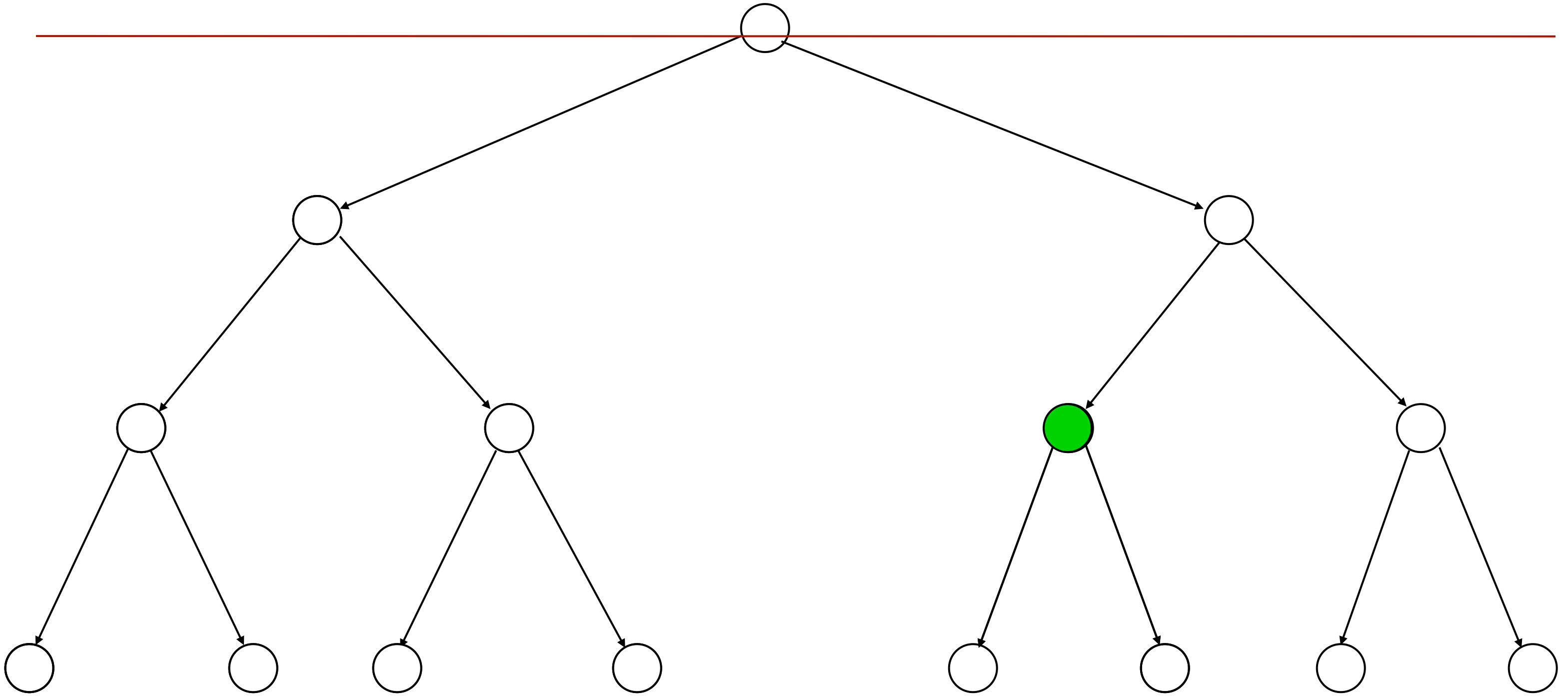

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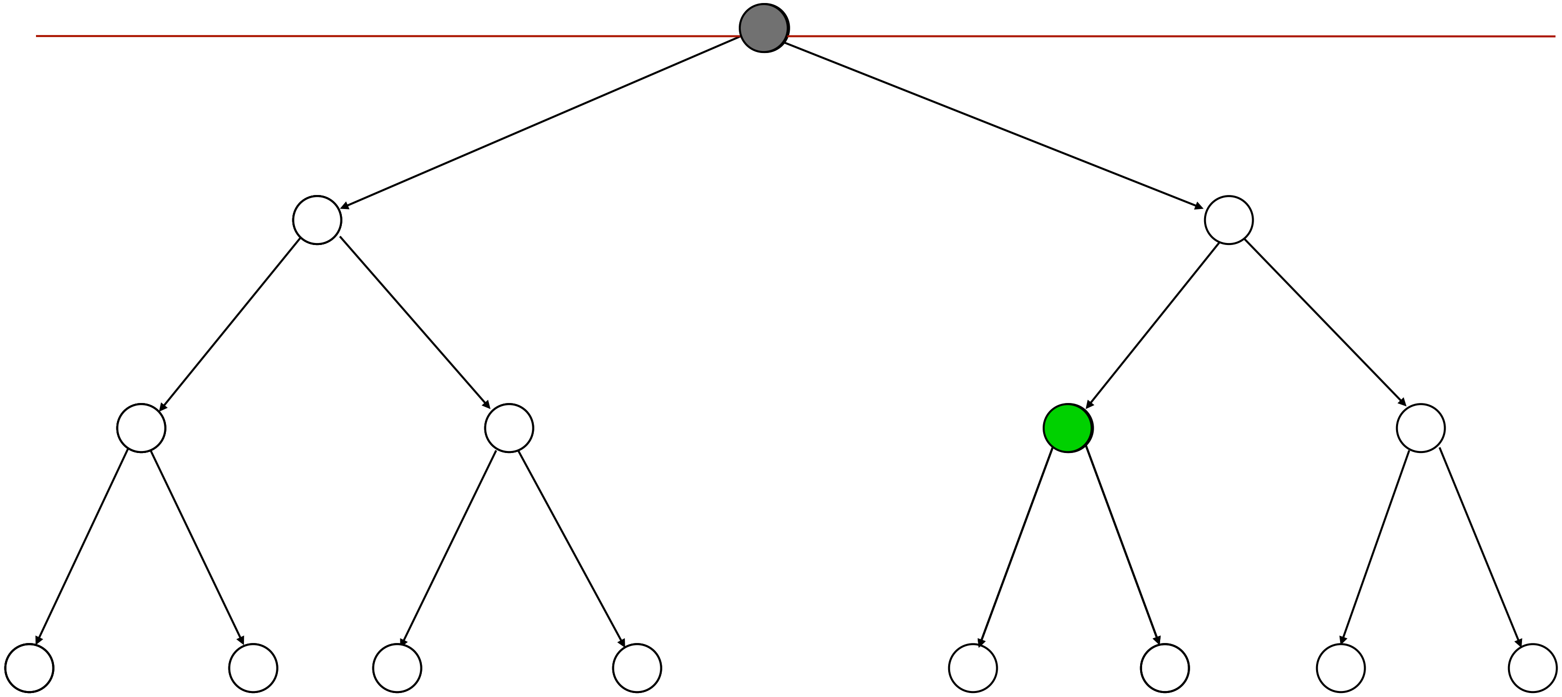

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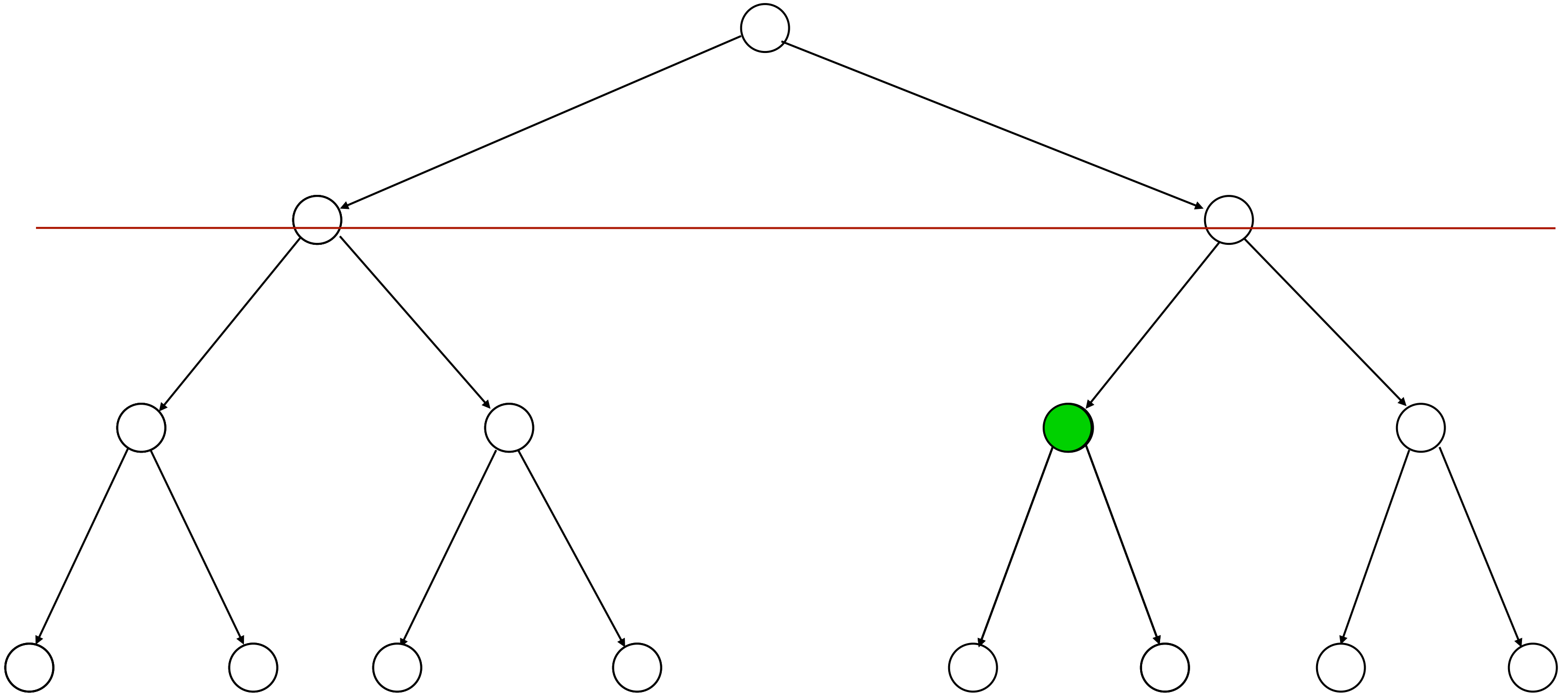
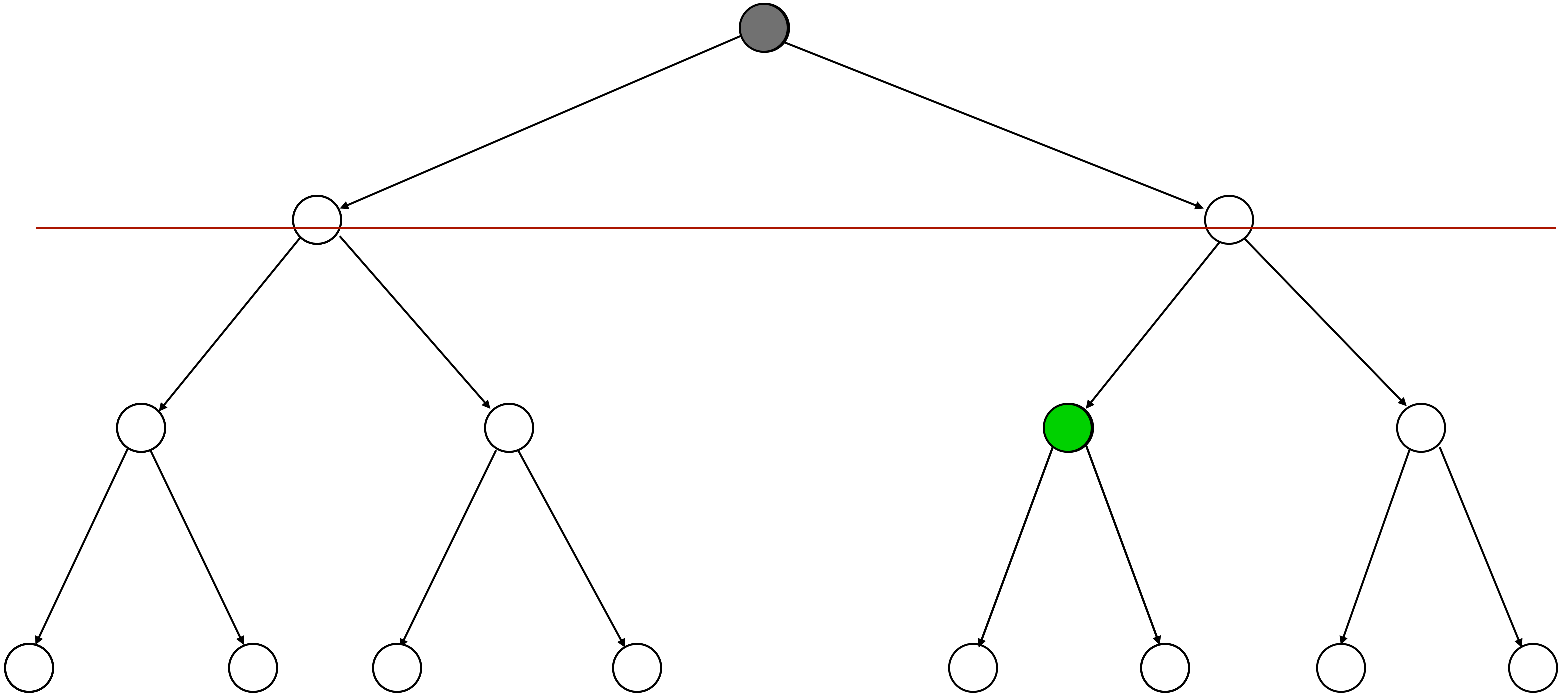


Iterative Deepening



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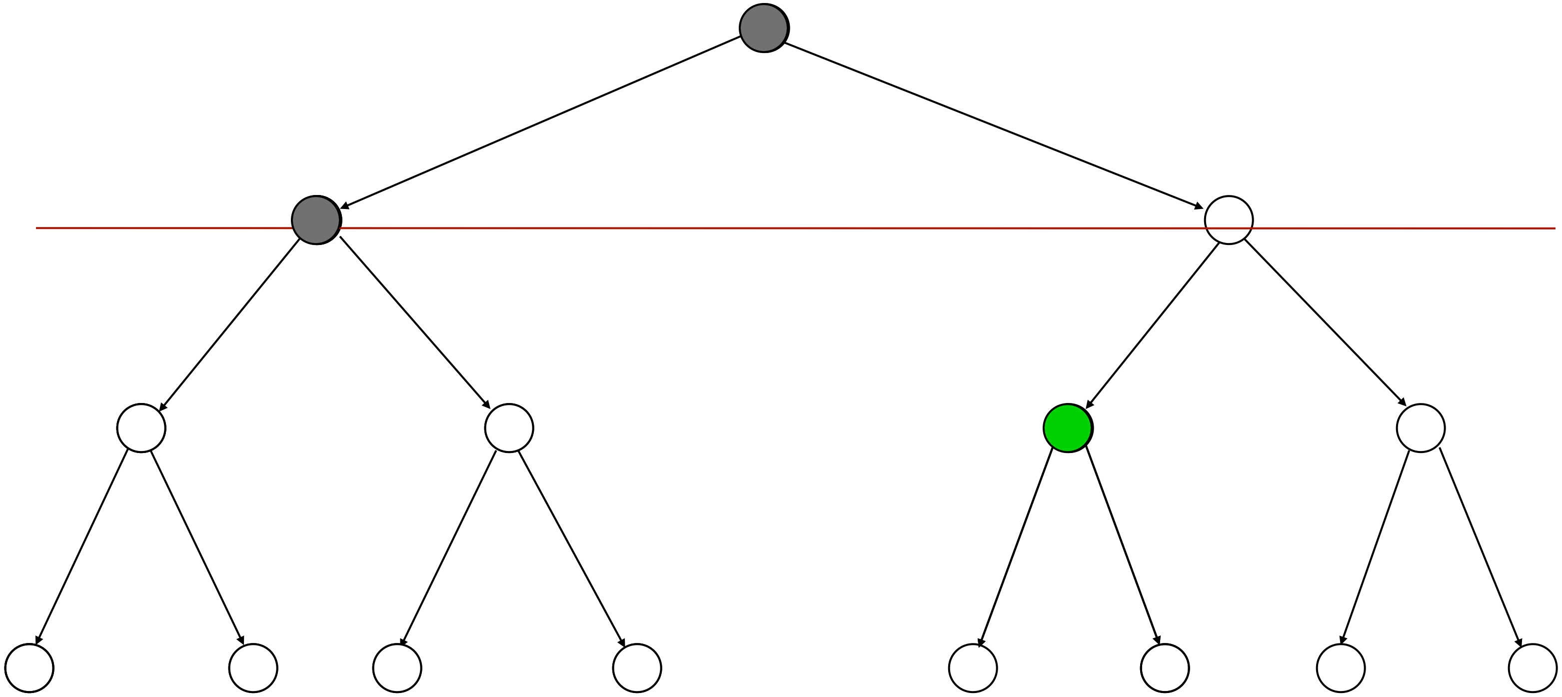

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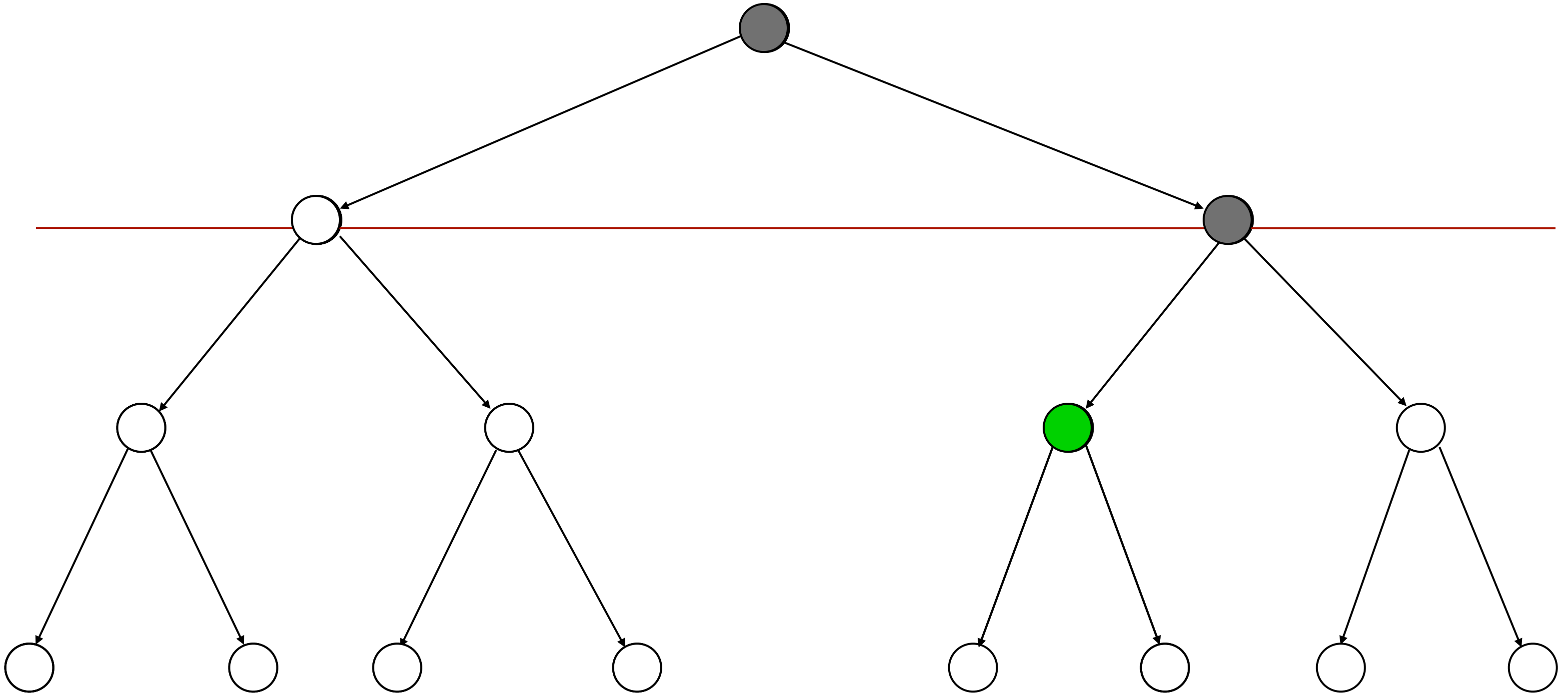

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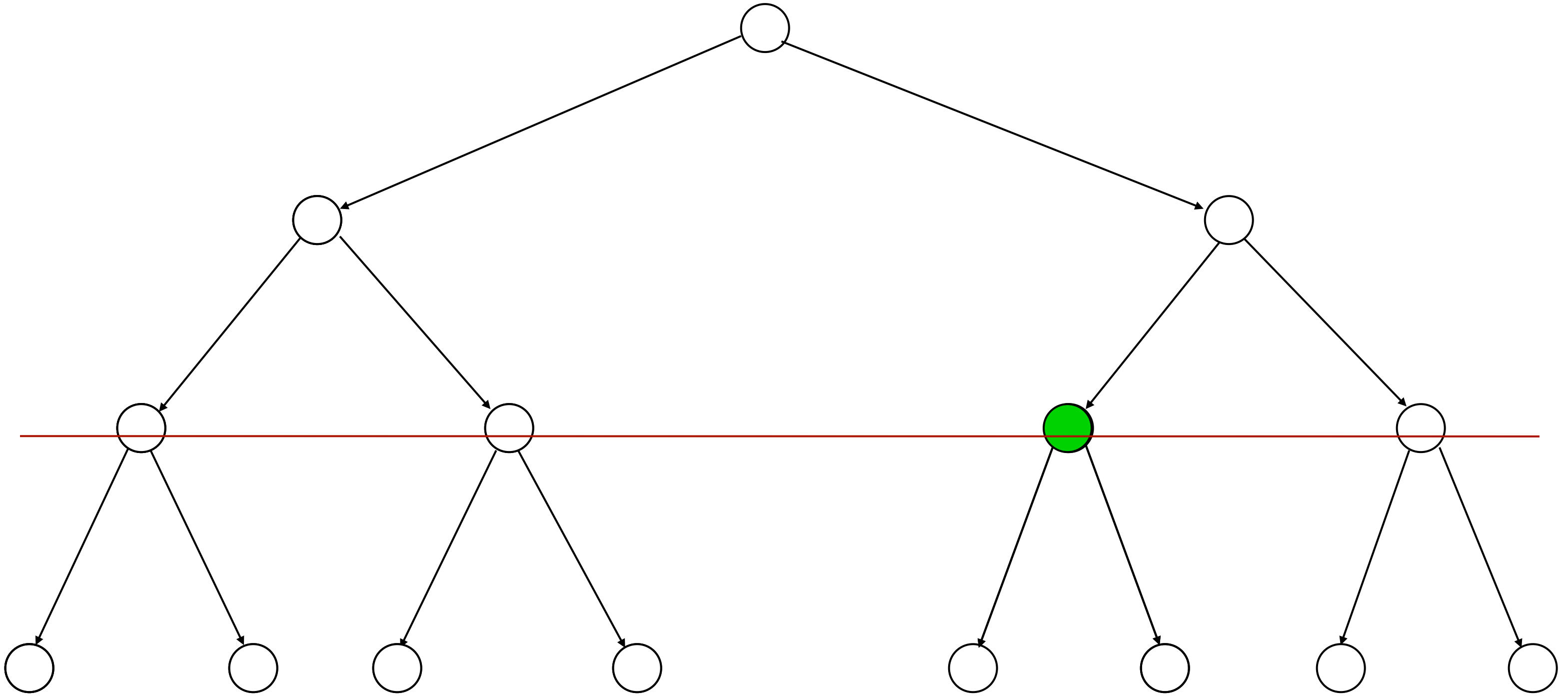

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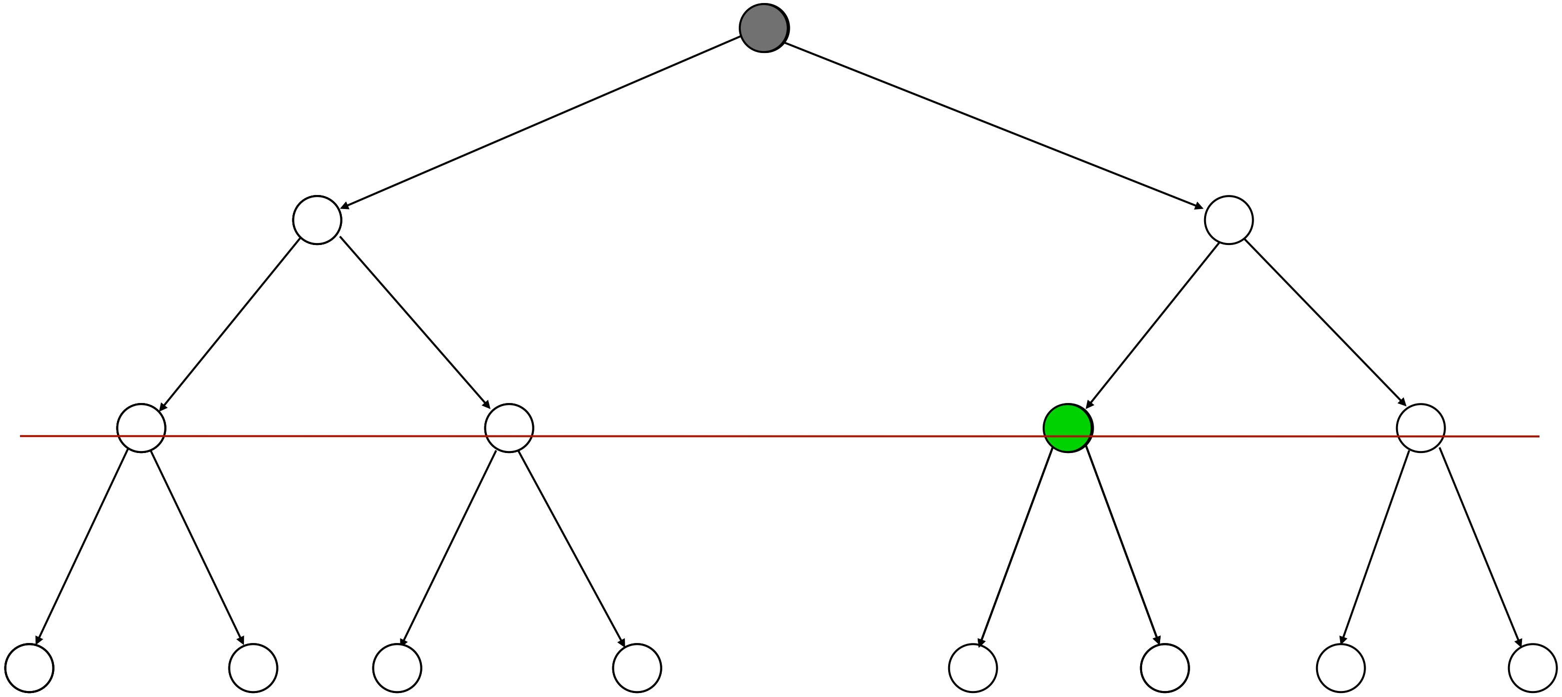
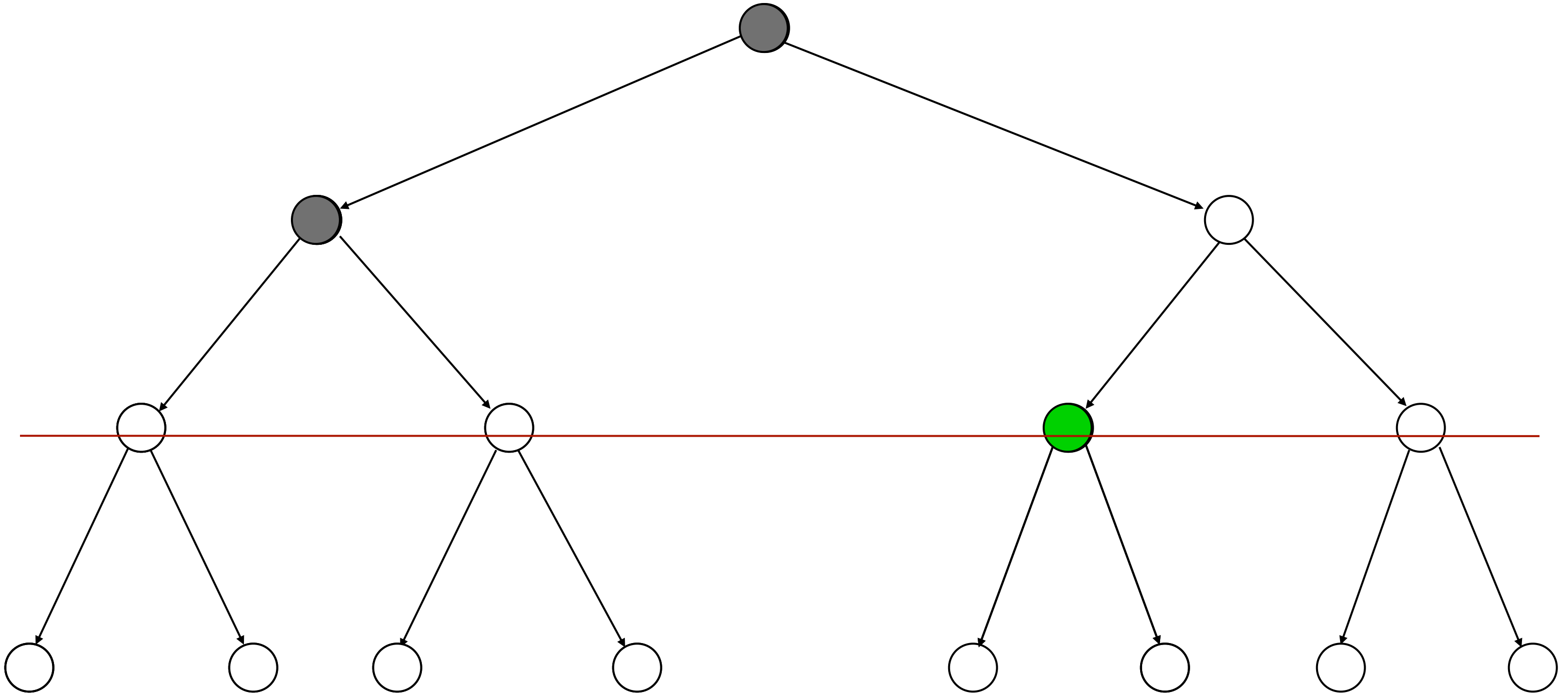


Iterative Deepening



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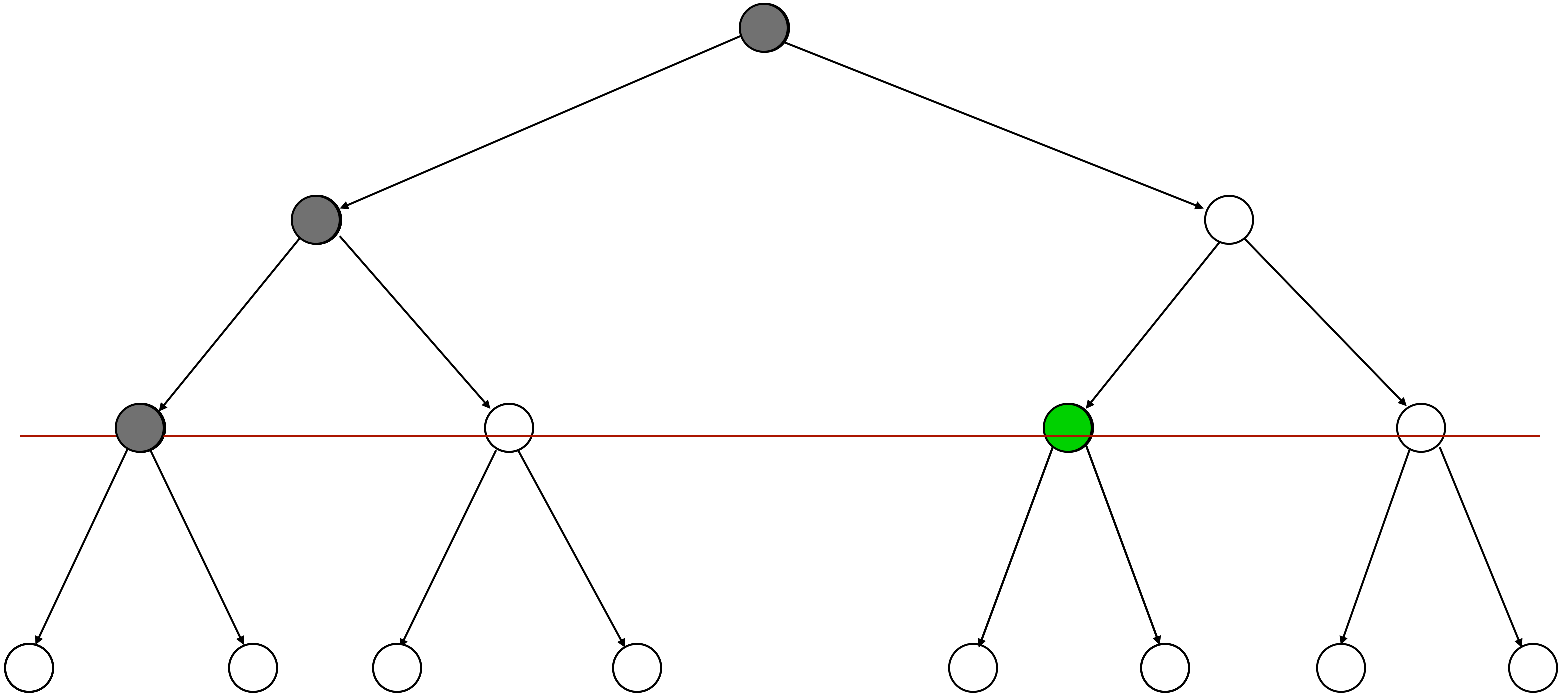

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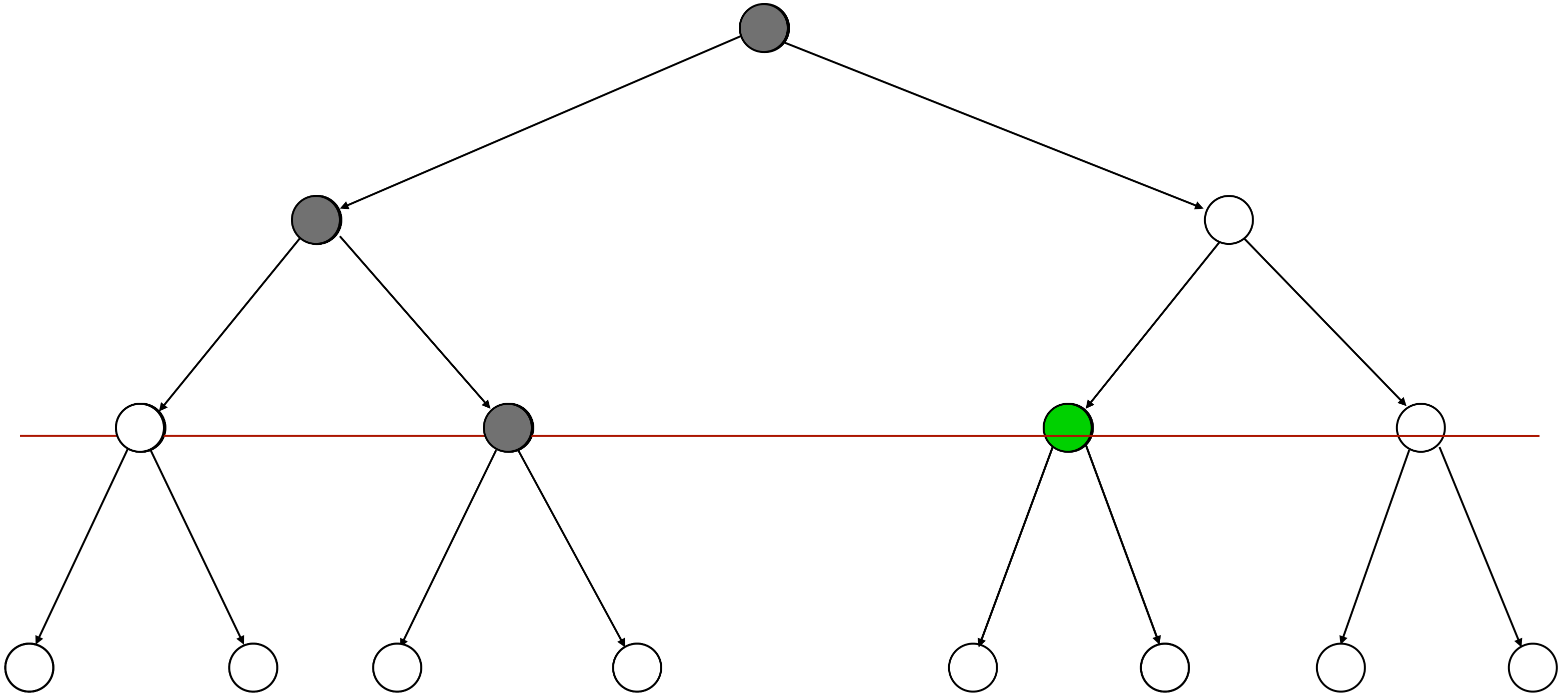

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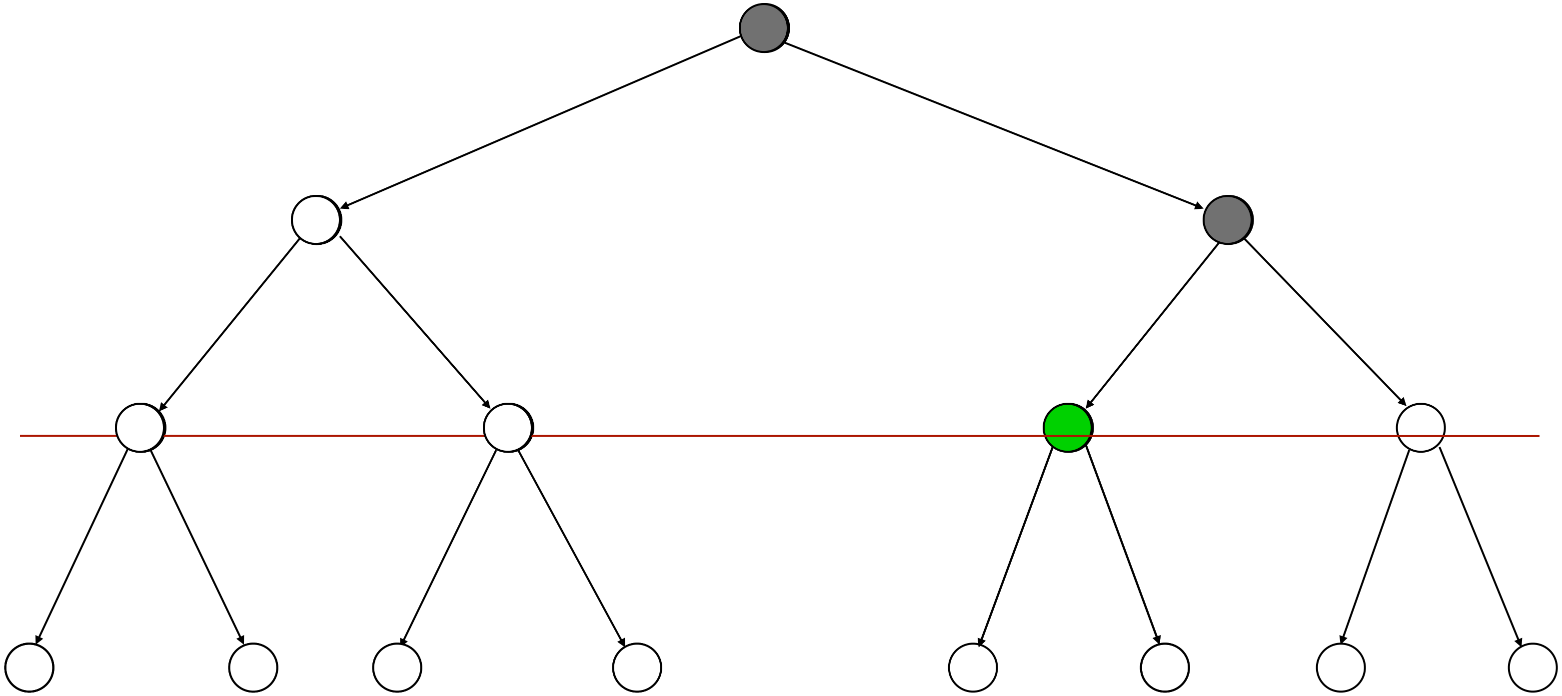
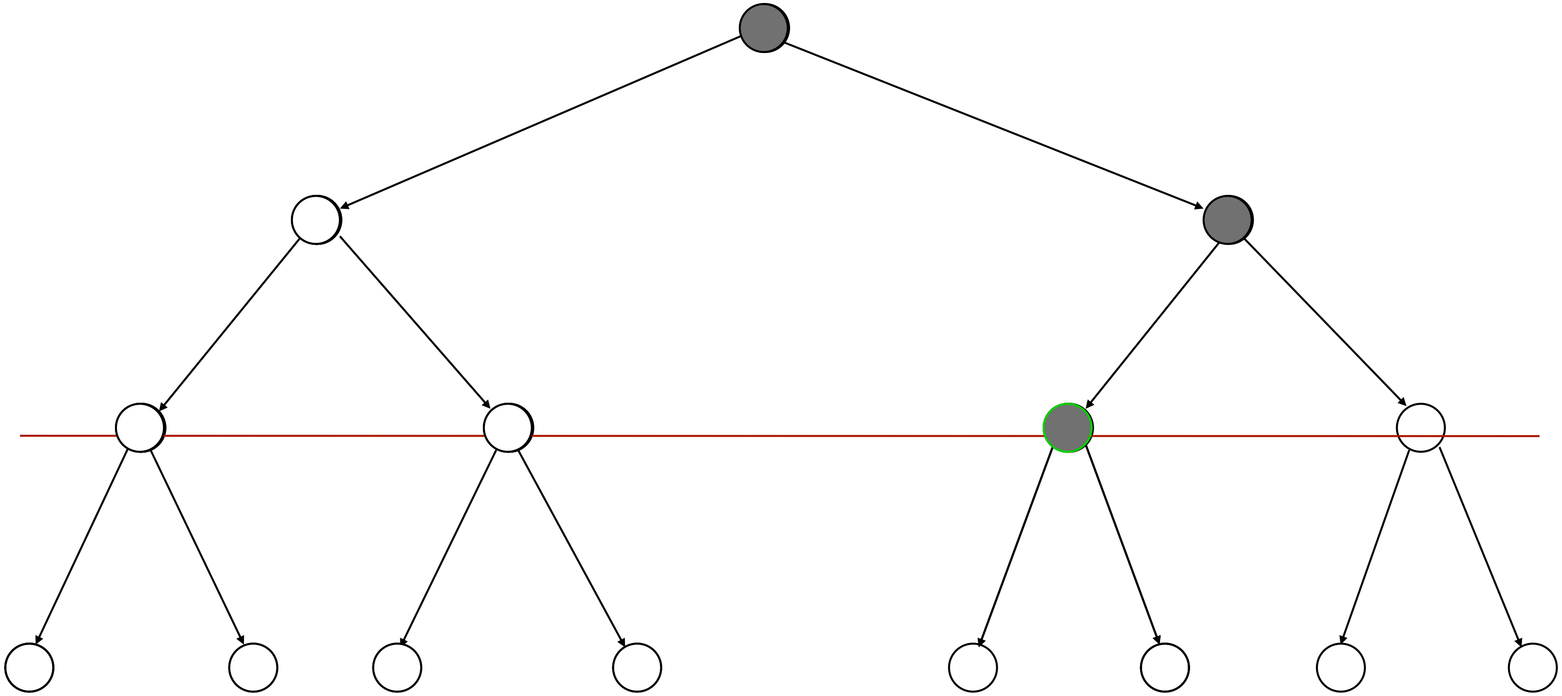


Iterative Deepening



Performance

- Iterative deepening search is:
  - Complete
  - Optimal if step cost = 1
- Time complexity is:
  $(d+1)(1) + db + (d-1)b^2 + \dots + (1)\, b^d = O(b^d)$
- Space complexity is: O(bd) or O(d)

Calculation

$$db + (d-1)b^2 + \ldots + (1)\, b^d$$

$$= b^d + 2b^{d-1} + 3b^{d-2} + \ldots + db$$

$$= (1 + 2b^{-1} + 3b^{-2} + \ldots + db^{-d}) \times b^d$$

$$\leq \left(\sum_{i=1,\ldots,\infty} ib^{(1-i)}\right) \times b^d = b^d\, (b/(b-1))^2$$

Number of Generated Nodes
(Breadth-First & Iterative Deepening)

d = 5 and b = 2

BF	ID
1	1 x 6 = 6
2	2 x 5 = 10
4	4 x 4 = 16
8	8 x 3 = 24
16	16 x 2 = 32
32	32 x 1 = 32
63	120

120/63 ~ 2

Number of Generated Nodes (Breadth-First & Iterative Deepening)

d = 5 and b = 10

BF	ID
1	6
10	50
100	400
1,000	3,000
10,000	20,000
100,000	100,000
111,111	123,456

123,456/111,111 ~ 1.111

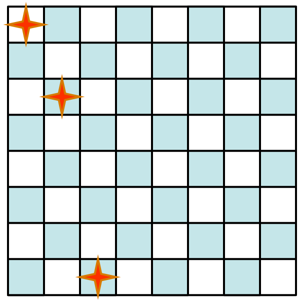
Comparison of Strategies

- Breadth-first is complete and optimal, but has high space complexity
- Depth-first is space efficient, but is neither complete, nor optimal
- Iterative deepening is complete and optimal, with the same space complexity as depth-first and almost the same time complexity as breadth-first

Revisited States

Revisited States

No

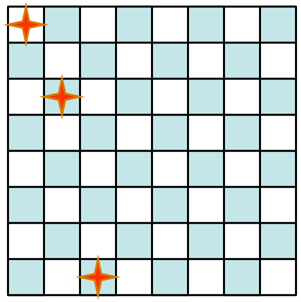


8-queens

Revisited States

No

Few



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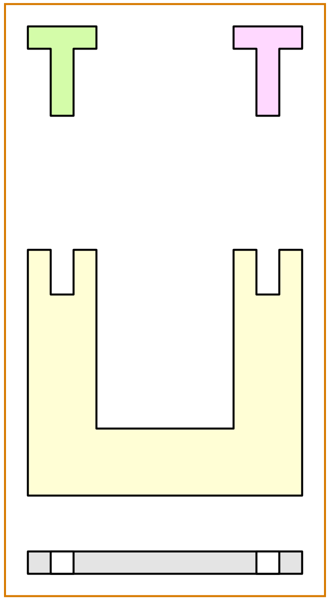


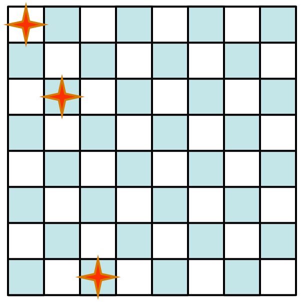
assembly planning

Revisited States

No

Few

search tree is finite



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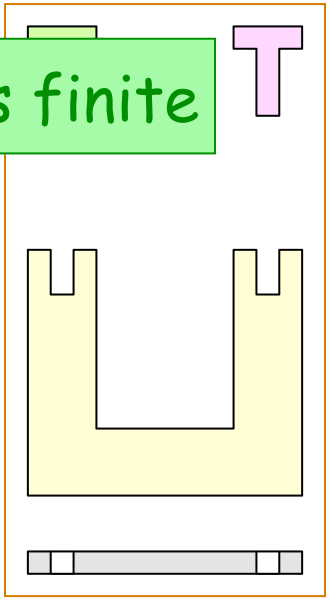


assembly planning

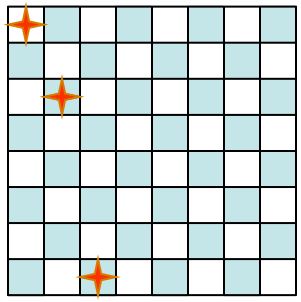
Revisited States

No

Few

Many

search tree is finite



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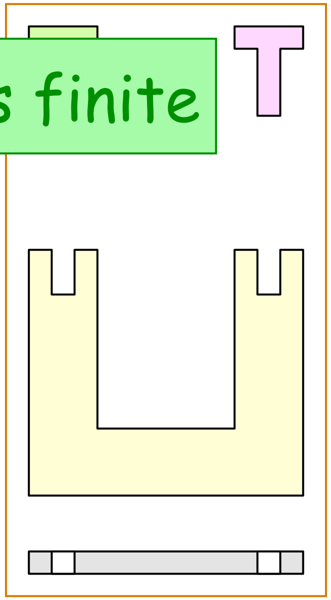
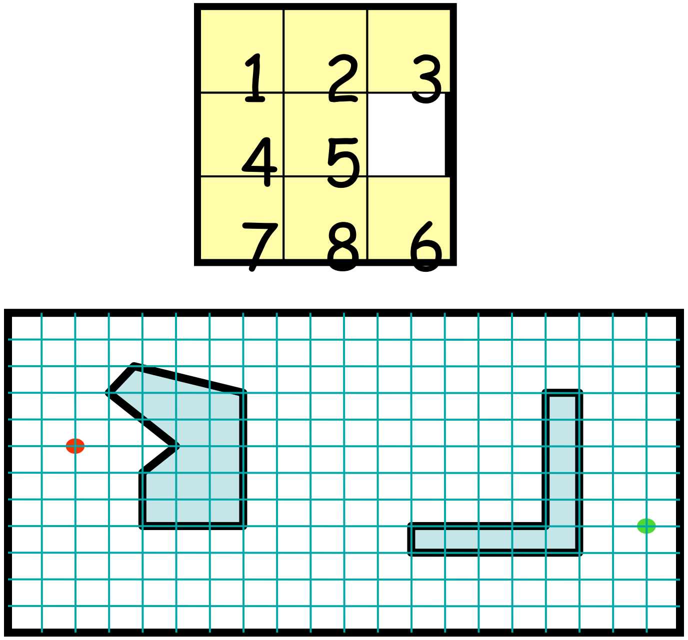


assembly planning



8-puzzle and robot navigation

Revisited States

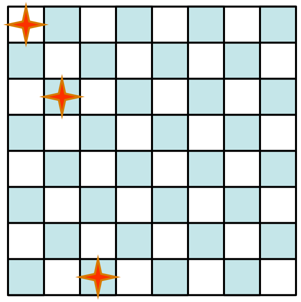
No

Few

Many

search tree is finite

search tree is infinite



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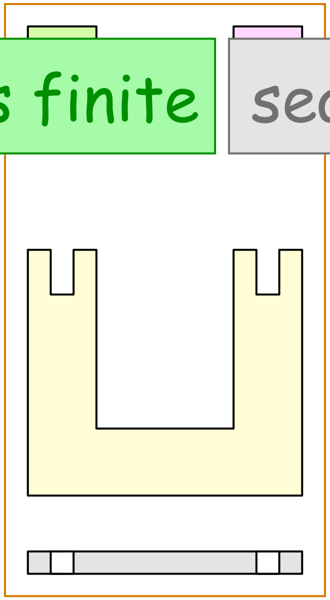
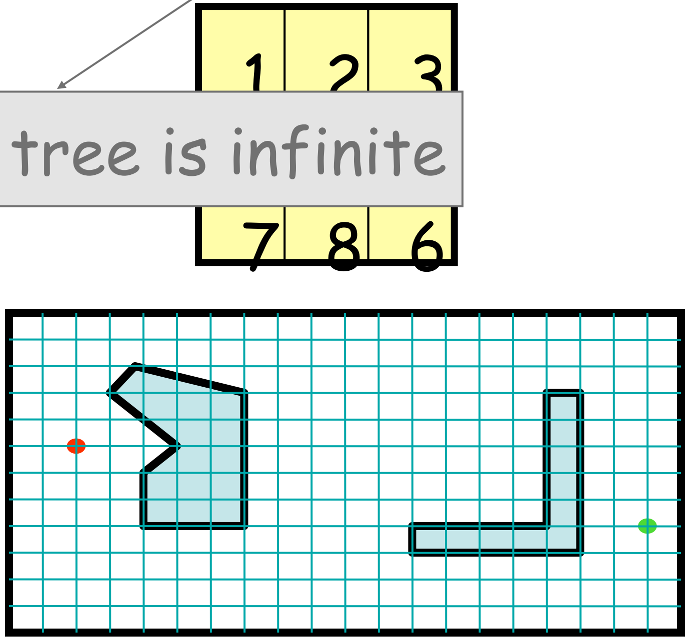


assembly planning



8-puzzle and robot navigation

Avoiding Revisited States

- Requires comparing state descriptions
- Breadth-first search:
  - Store all states associated with **generated** nodes in CLOSED LIST (CL)
  - If the state of a new node is in CL, then discard the node

Avoiding Revisited States

Avoiding Revisited States

- Depth-first search:

  Solution 1:
  - Store all generated states in CL

Avoiding Revisited States

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  Solution 2:

Avoiding Revisited States

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  Solution 2:

  - Store all states associated with nodes in current path in CL

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  → Only avoids loops

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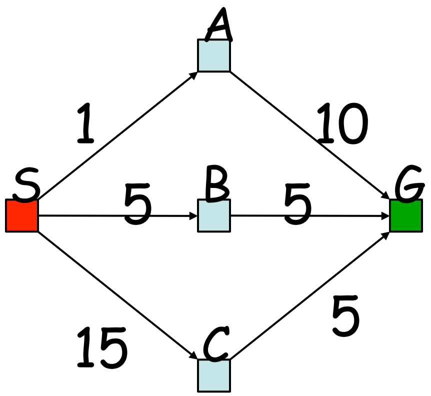
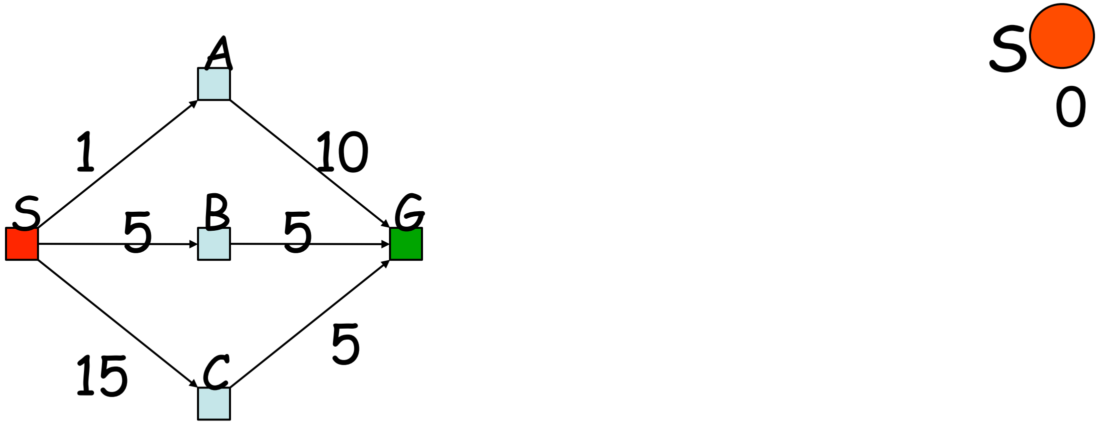
Uniform-Cost Search

Uniform-Cost Search



Uniform-Cost Search



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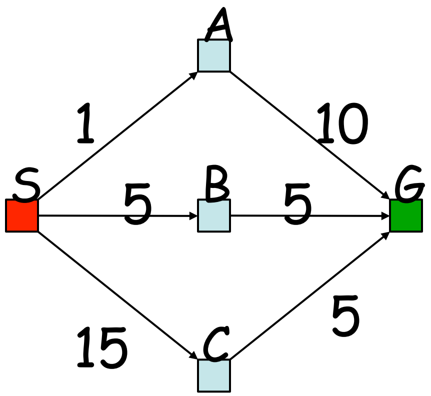

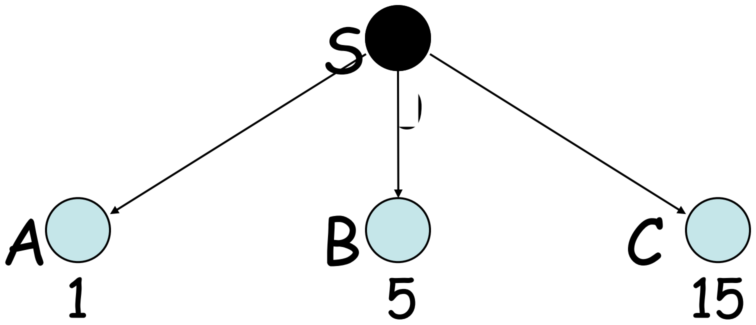

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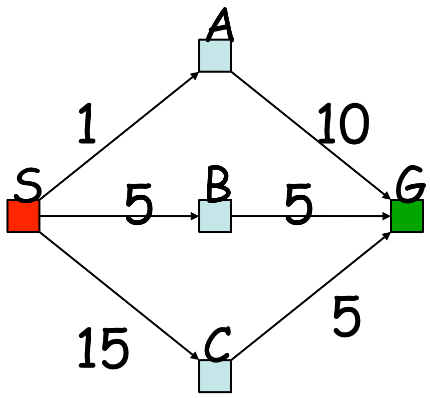

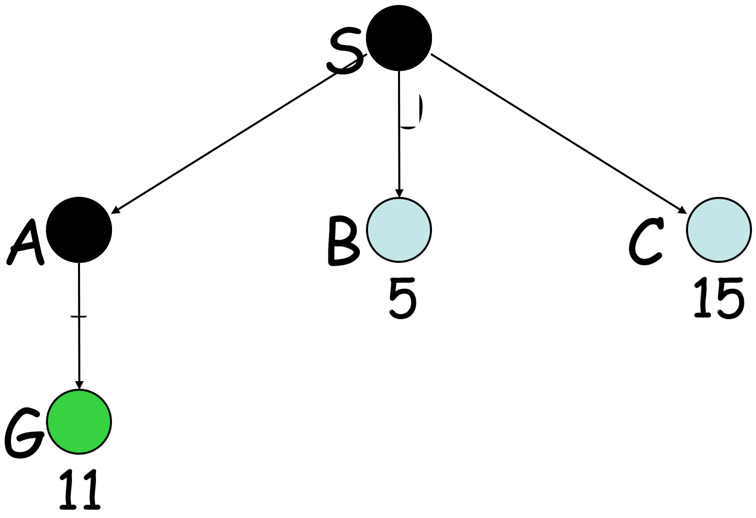

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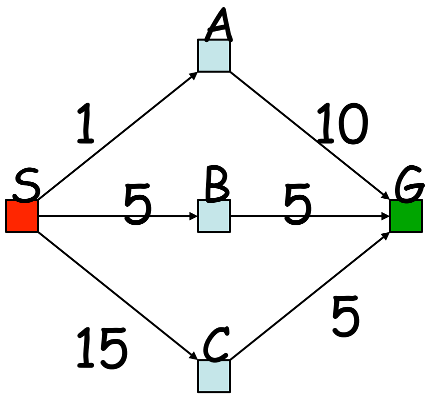
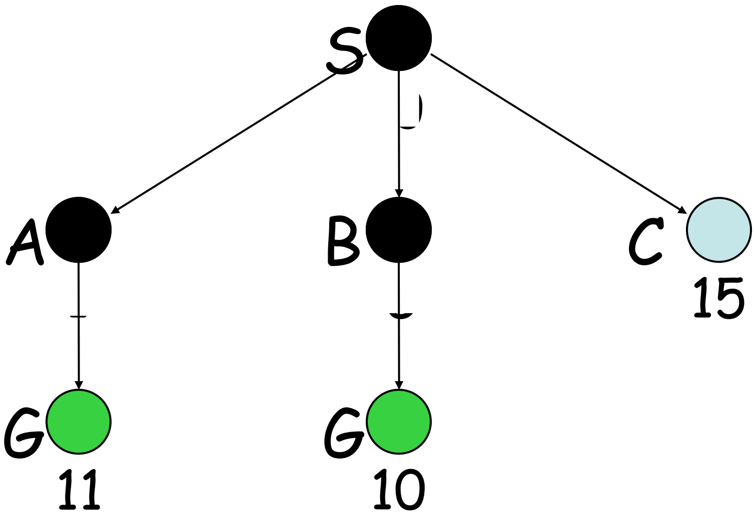
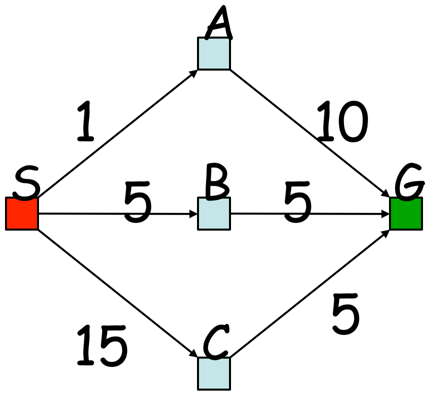
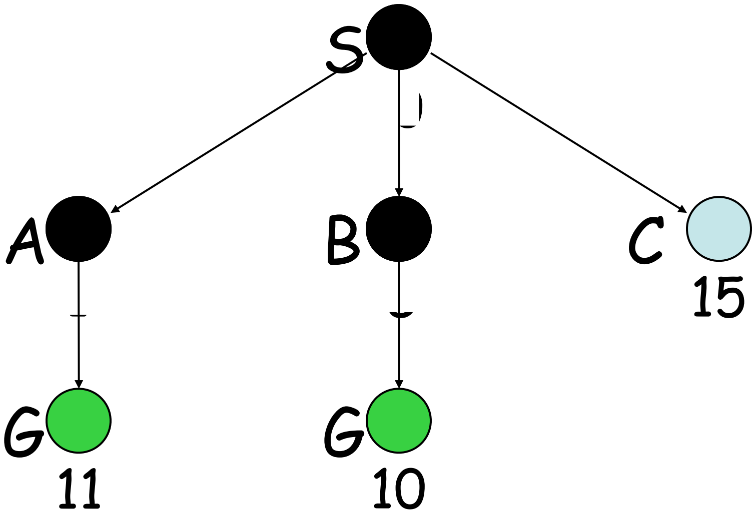




Uniform-Cost Search

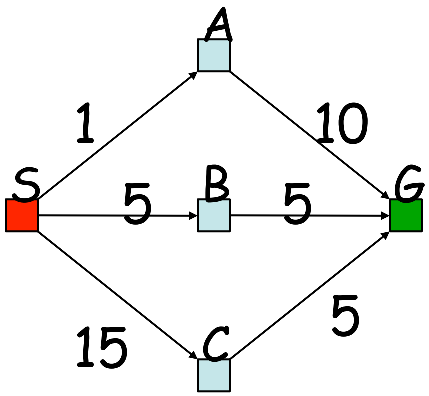
- Each arc has some cost c ≥ ε > 0

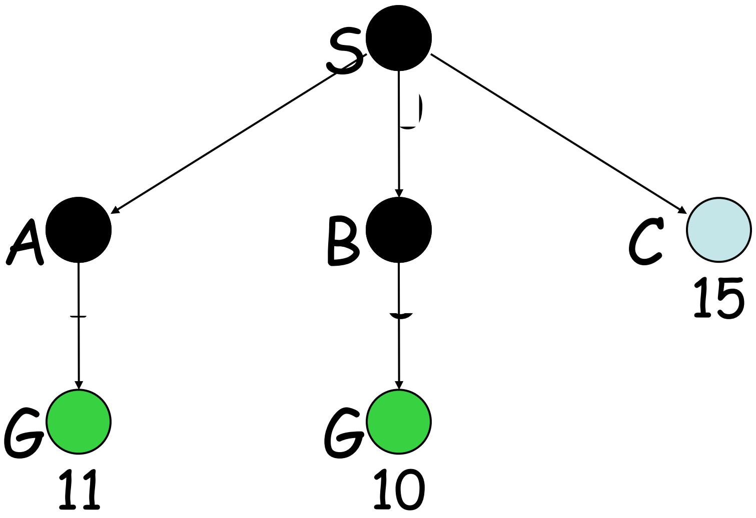




Uniform-Cost Search

- Each arc has some cost c ≥ ε > 0
- The cost of the path to each node N is

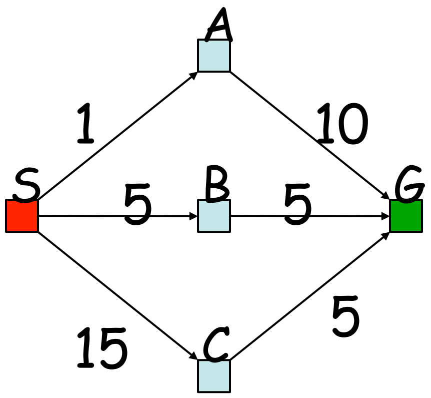


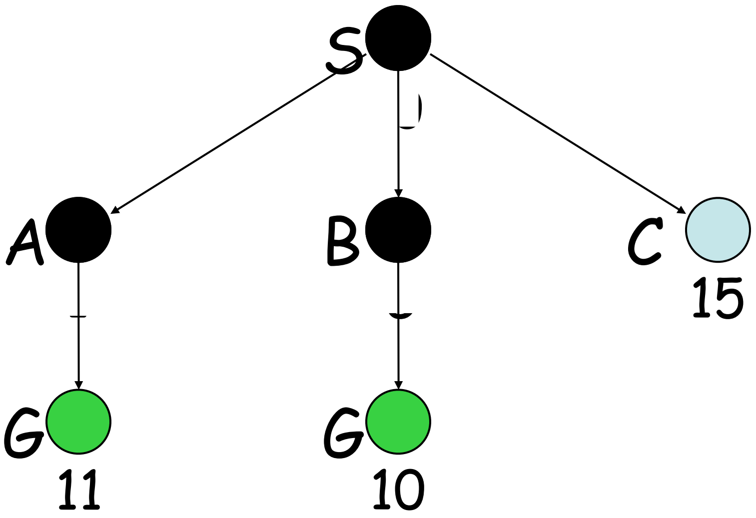


Uniform-Cost Search

- Each arc has some cost $c \geq \varepsilon > 0$
- The cost of the path to each node N is

$$g(N) = \Sigma \text{ costs of arcs}$$





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- Each arc has some cost c ≥ ε > 0
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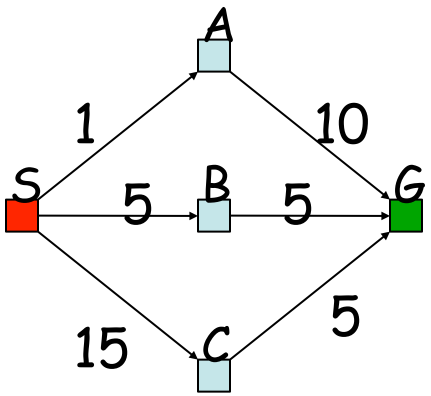
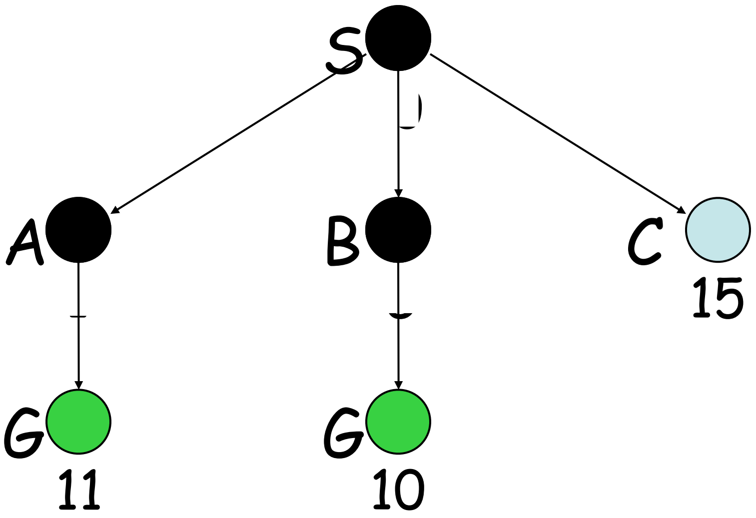
  g(N) = Σ costs of arcs
- The goal is to generate a solution path of minimal cost





Uniform-Cost Search

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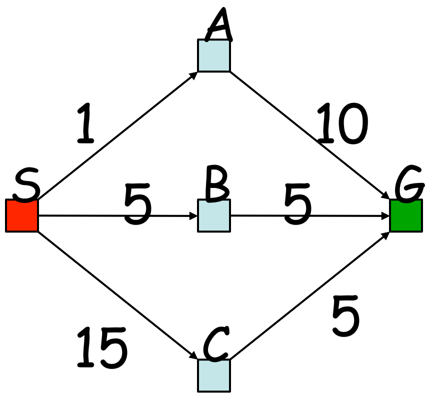
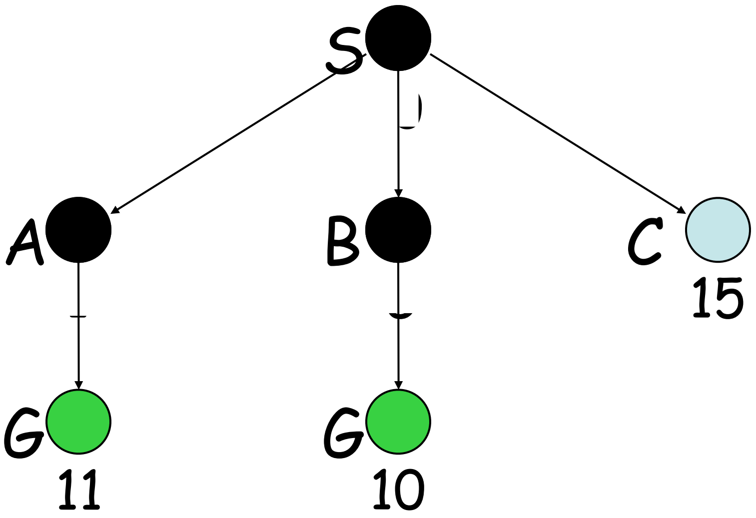
  $g(N) = \Sigma$ costs of arcs
- The goal is to generate a solution path of minimal cost
- The nodes N in the queue OL are sorted in increasing g(N)





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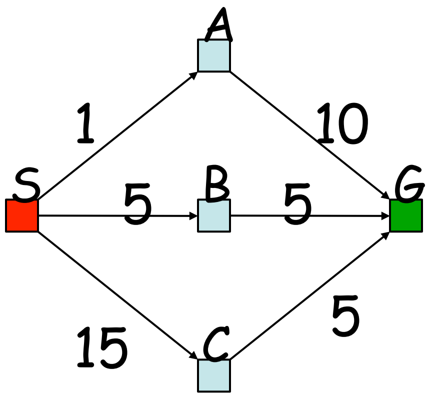
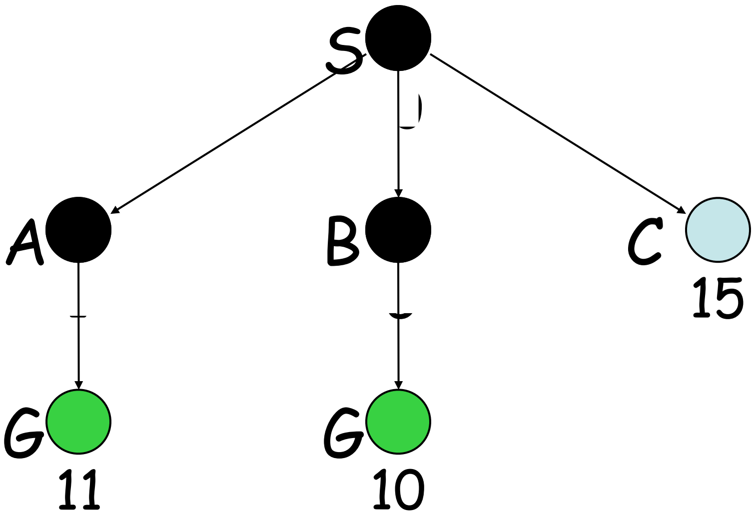




- Need to modify search algorithm

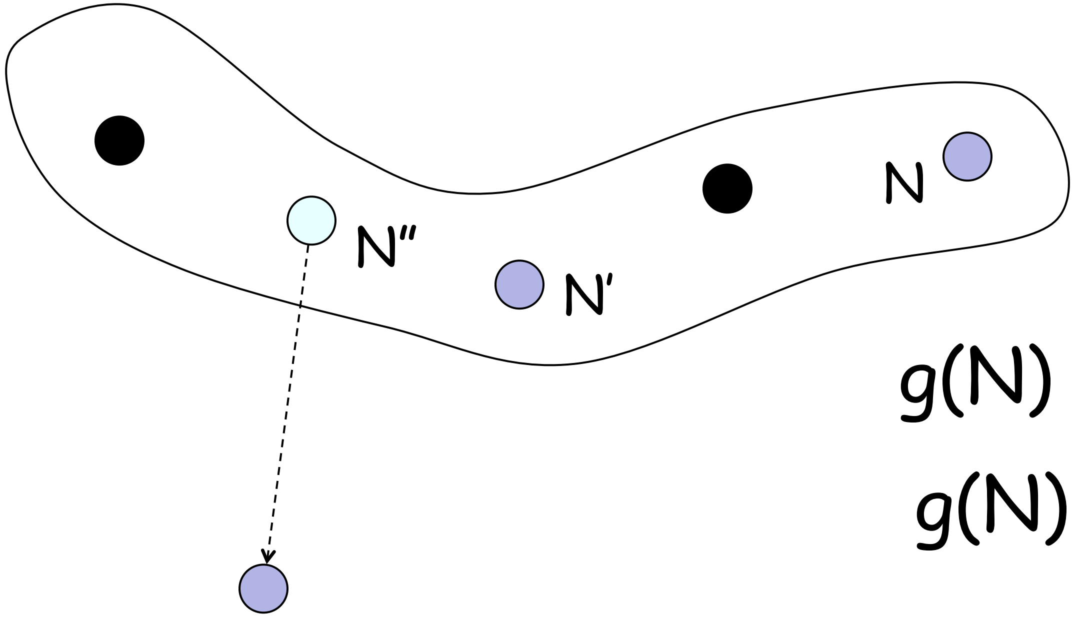
Search Algorithm #2

The goal test is applied to a node when this node is **expanded**, not when it is generated.

```
SEARCH#2
1. INSERT(initial-node,OL)
2. Repeat:
     a. If empty(OL) then return failure
     b. N ← REMOVE(OL)
     c. s ← STATE(N)
   ➤ d. If GOAL?(s) then return path or goal state
     e. For every state s’ in SUCCESSORS(s)
          i.  Create a new node N’ as a child of N
          ii. INSERT(N’,OL)
```

Avoiding Revisited States in Uniform-Cost Search

- For any state S, when the first node N such that STATE(N) = S is expanded, the path to N is the best path from the initial state to S



$$g(N) \leq g(N')$$

$$g(N) \leq g(N'')$$

Avoiding Revisited States in Uniform-Cost Search

- For any state S, when the first node N such that STATE(N) = S is expanded, the path to N is the best path from the initial state to S

- So:
  - When a node is **expanded**, store its state into CL
  - When a new node N is generated:
    - If STATE(N) is in CL, discard N
    - If there exits a node N' in OL such that STATE(N') = STATE(N), discard the node -- N or N' -- with the highest-cost path