Short introduction to motion planning and control

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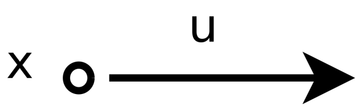

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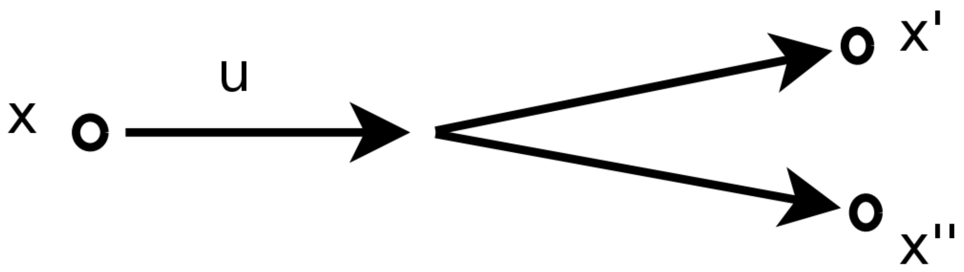

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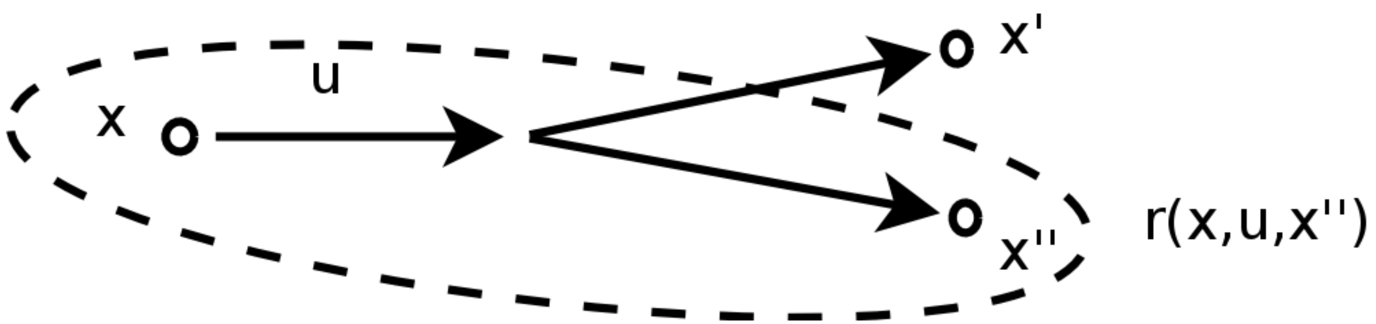

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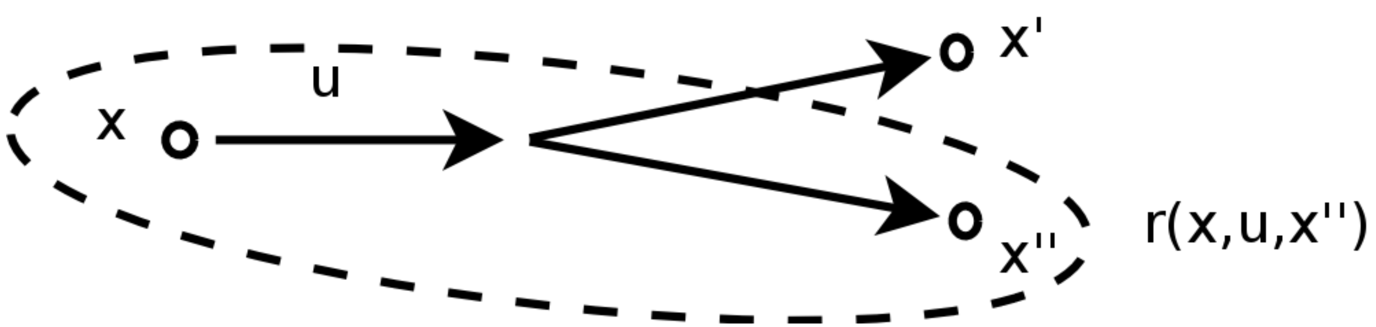
- Policy: $\pi_\theta(\mathbf{x}) : X \to U$



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- Sum of discounted rewards:

$$r(\tau) = \sum_{i=0}^{\infty} \gamma^i \cdot r(\mathbf{x}_i, \mathbf{u}_i, \mathbf{x}_{i+1})$$

Different platforms

- Many different robots with various complexity of dynamics and dimensionality of state-action space:
  - triple pendulum,
  - two-arm manipulator,
  - mobile platform with auxiliary articulated sub-tracks and lockable differential.

Different problems

- Find the shortest collision-free trajectory in configuration space from start state(s) $\mathbf{x}_s$ to goal state(s) $\mathbf{x}_g$ which respect dynamic constraints of the robot (e.g. Dijkstra, A*, RRT, PRM, LQR-trees or Guided Policy Search)

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Outline

Planning vs motion control

- Planning requires transition probability, policy is sequence of actions.
- Motion control can use but does not necessarily requires transition probability, it learns policy function.

Lecture plan:

1. Path planning via Rapidly Exploring Random Trees (RRT).
2. Reinforcement learning for robotics (with and without motion model).

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- Search in continuous high-dimensional space can get stuck in a local minimum, since you can expand infinite number of nodes in a small region.
- RRT [1] efficiently search non-convex, high-dimensional spaces by randomly building a space-filling tree.
- Initial publication [1] has over 1200 citations.

[1] LaValle, Steven M. (October 1998). "Rapidly-exploring random trees: A new tool for path planning". Technical Report (Computer Science Department, Iowa State University) (TR 98-11).

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6. Repeat from 2 until a feasible path is found.

Output: a collision-free path if exist (after infinite number of nodes expanded)

Speed-ups of Rapidly Exploring Random Tree

- Bias to goal (greediness), e.g.
  - with $P = 0.95$ choose $\mathbf{x}$ at random from X,
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[2] Karaman, Sertac; Frazzoli, Emilio, "Incremental Sampling-based Algorithms for Optimal Motion Planning", 2010

[3] J.D. Gammell, S.S. Srinivasa, T.D. Barfoot, "Informed RRT*", IROS, 2014

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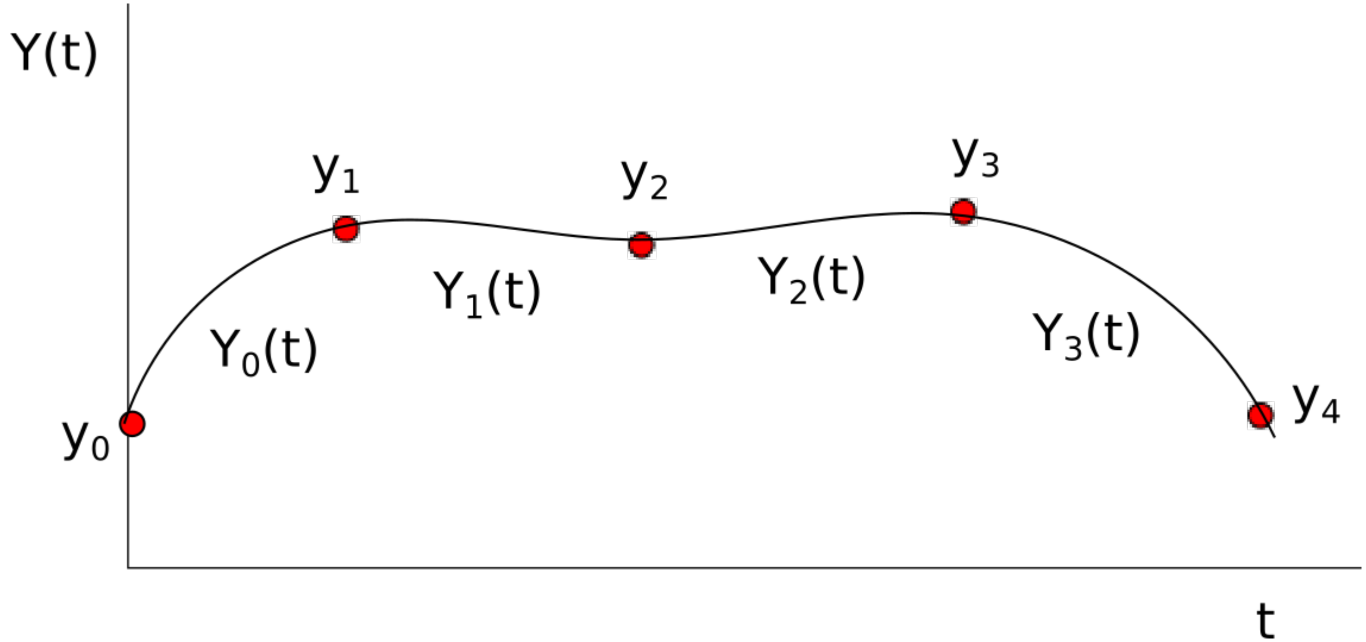
- If accurate dynamic motion model is known ($\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$ relations are determined), then RRT searches for trajectory in lifted state-space $[\mathbf{x},\ \dot{\mathbf{x}},\ \ddot{\mathbf{x}}]^\top$.

Smooth path interpolation



- If only kinematic model is available, discrete path is searched (i.e. N points $\mathbf{y}_0 \in X, \ldots \mathbf{y}_{N-1} \in X$ in configuration space).
- Trajectory is defined as a smooth interpolation of path
- Example of smooth interpolation is pair-wise cubic function Y, i.e sequence of $N-1$ cubic functions $Y_i(t) : [0, 1] \rightarrow X$ such that:
  - $Y_i(t)$ connects $\mathbf{y}_i$ with $\mathbf{y}_{i+1}$.
  - Y has continuous first and second derivatives.

Planning summary

1. Advantage: It has statistical guarantees of the optimality.
2. Drawback 1: Whenever results of action differs from the model (i.e. everytime), you need to replan the whole trajectory.
3. Drawback 2: Accurate motion model is required.

For some problems, reinforcement learning trades-off optimality (1) for (2)+(3).

What if motion model is unknown

- Can you design a motion control algorithm without motion model?

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- Given these trajectories, change the policy to increase expected sum of rewards

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  - either solve **primal task** e.g. by following gradient ∇J to maximize $J(\theta)$ directly.
  - primal is often solved in the optimal control community (e.g. LQR),
  - or solve **dual task** to find state-action function $Q(\mathbf{x}, \mathbf{u}) : X \times U \rightarrow \mathbb{R}$ which tells expected sum rewards when choosing action $\mathbf{u}$ from state $\mathbf{x}$.
  - optimal policy $\pi^* = \arg\max_{\mathbf{u}} Q(\mathbf{x}, \mathbf{u})$
  - dual is often employed by AI community as heuristics for state-space search
  - Q-values could provide metrics for RRT [Tedrake-LQR-trees-2015]

Dual task provides alternative point-of-view (e.g. shadow prices in LP or sparse feature selection for SVM)

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- We can approximate criterion value, what about gradient?

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- Of course, but doing it from one sample is quite unstable (especially for high dimensional θ).
- Perform several small random perturbations $\Delta\theta_i$ and compute $J(\theta + \Delta\theta_i)$.
- Relation to gradient $\nabla J(\theta)$ is given by the first order Taylor polynom

$$\begin{aligned}
J(\theta + \Delta\theta_i) &= J(\theta) + \nabla J(\theta)^\top \Delta\theta_i \\
\Delta\theta_i^\top \nabla J(\theta) &= J(\theta) - J(\theta + \Delta\theta_i) \\
\underbrace{\begin{bmatrix} \Delta\theta_1^\top \\ \vdots \\ \Delta\theta_n^\top \end{bmatrix}}_{\text{matrix } \mathtt{A}} \nabla J(\theta) &= \underbrace{\begin{bmatrix} J(\theta) - J(\theta + \Delta\theta_1)) \\ \vdots \\ J(\theta) - J(\theta + \Delta\theta_n)) \end{bmatrix}}_{\text{vector } \mathbf{b}}
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Primal task - solution

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$$\nabla J(\theta) = \begin{bmatrix} \Delta\theta_1^\top \\ \vdots \\ \Delta\theta_n^\top \end{bmatrix}^+ \cdot \begin{bmatrix} J(\theta) - J(\theta + \Delta\theta_1)) \\ \vdots \\ J(\theta) - J(\theta + \Delta\theta_n)) \end{bmatrix}$$

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- Algorithm is simple:
  - Randomly initialize θ
  - Use $\pi_\theta(\mathbf{x})$ to get trajectories.
  - Compute $\nabla J(\theta)$ using pseudo-inverse.
  - Update $\theta \quad \leftarrow \quad \theta + \alpha \frac{\nabla J(\theta)}{\|\nabla J(\theta)\|}$

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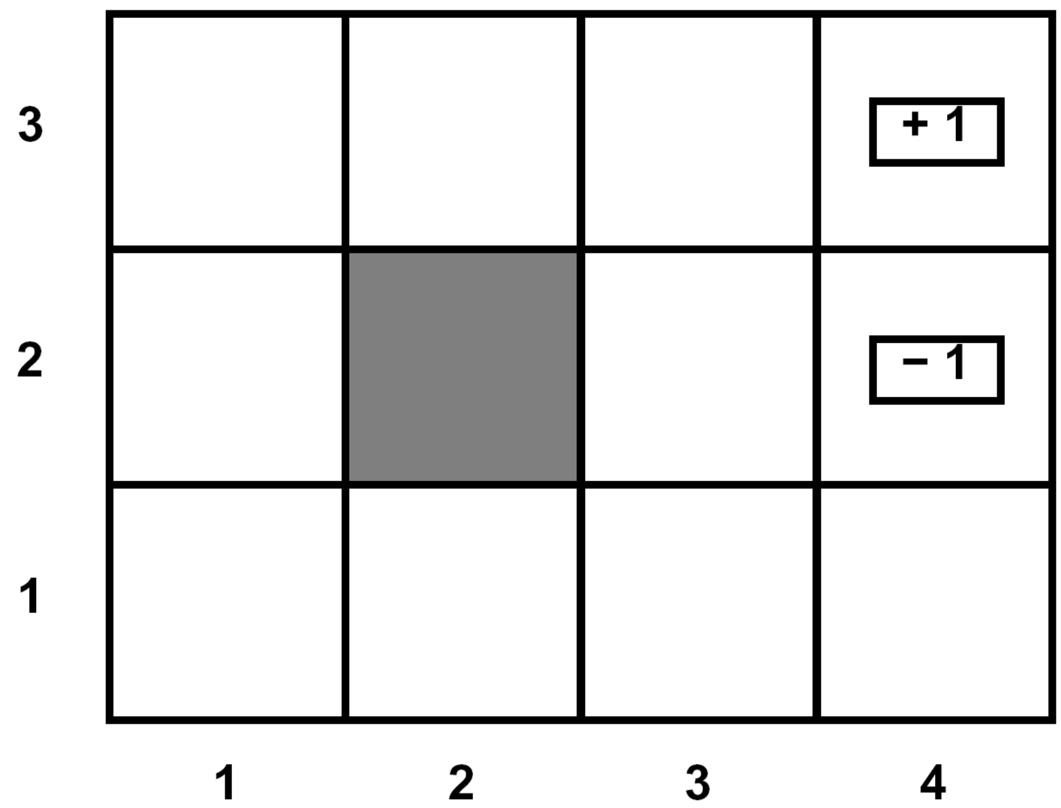
- Let us look at the grid world with stochastic transitions!



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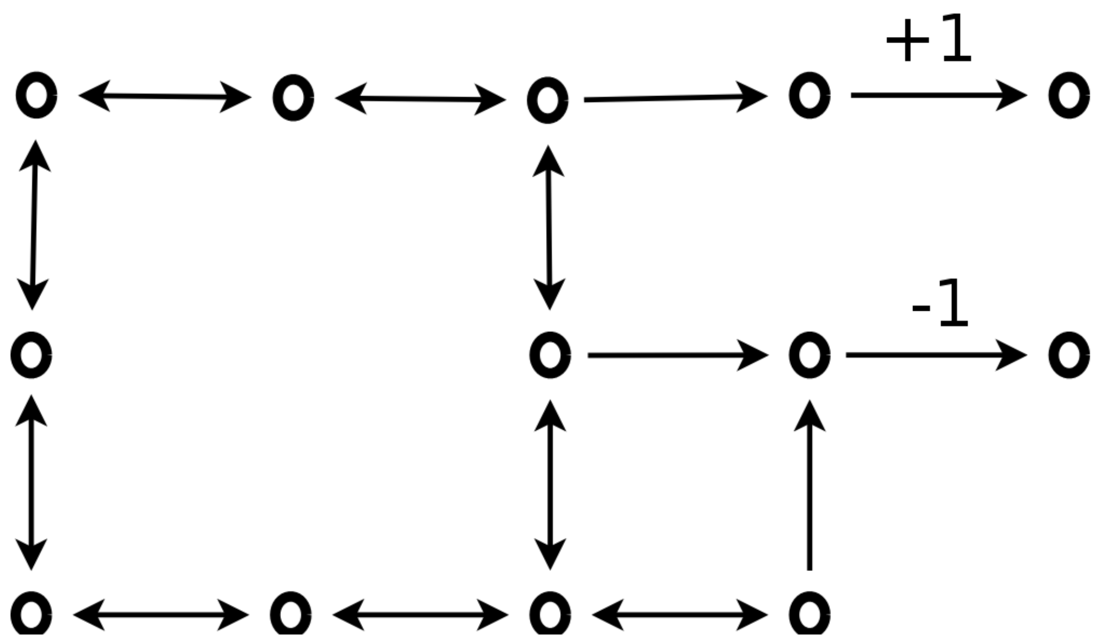

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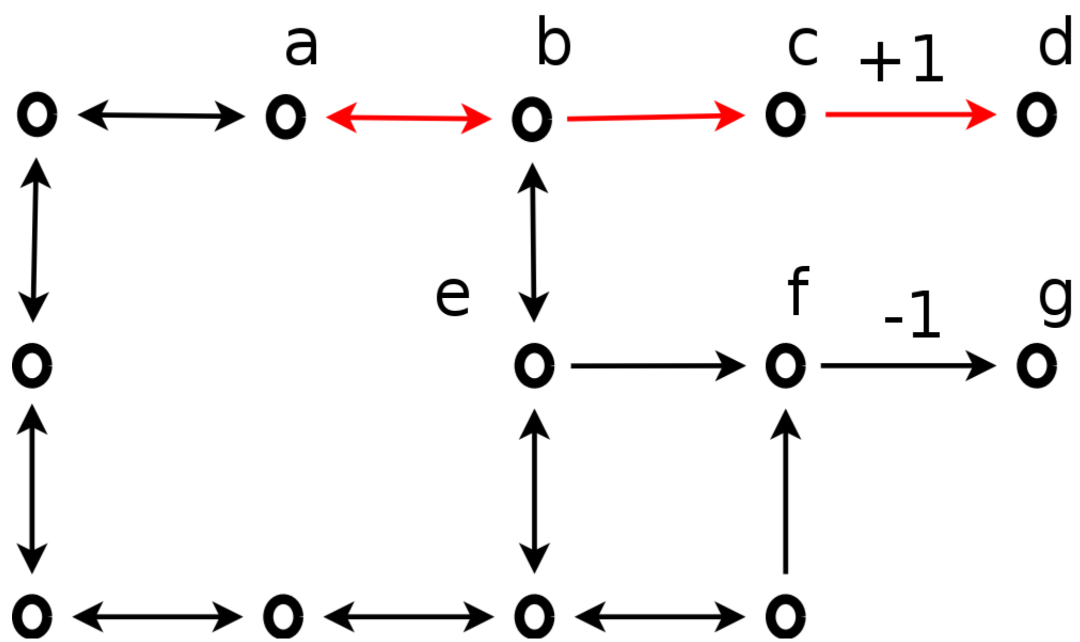

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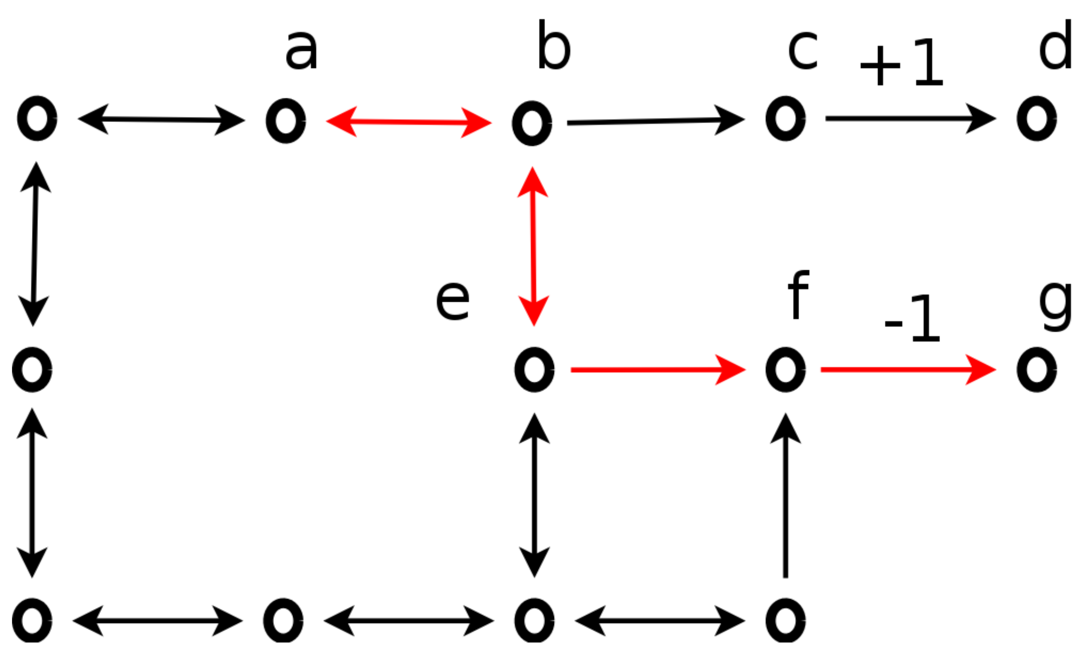

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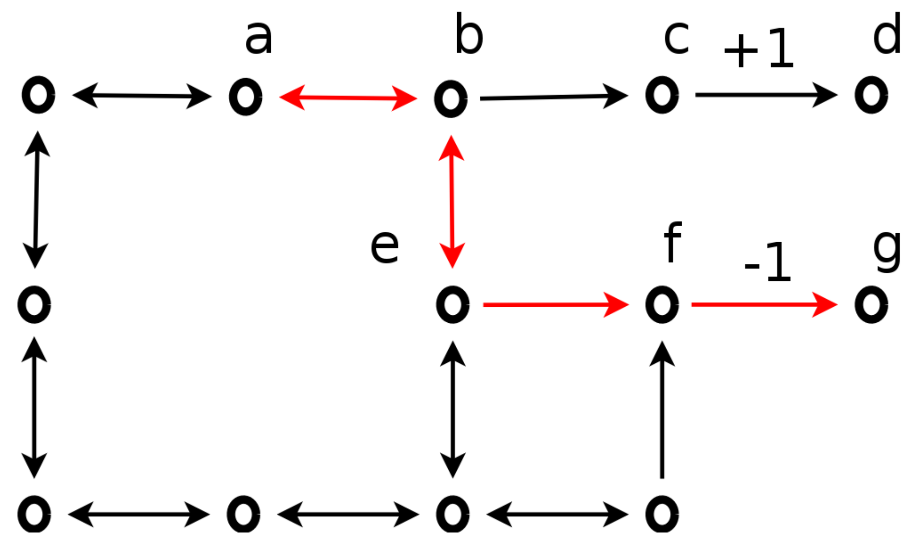
b		
c		
e	?	



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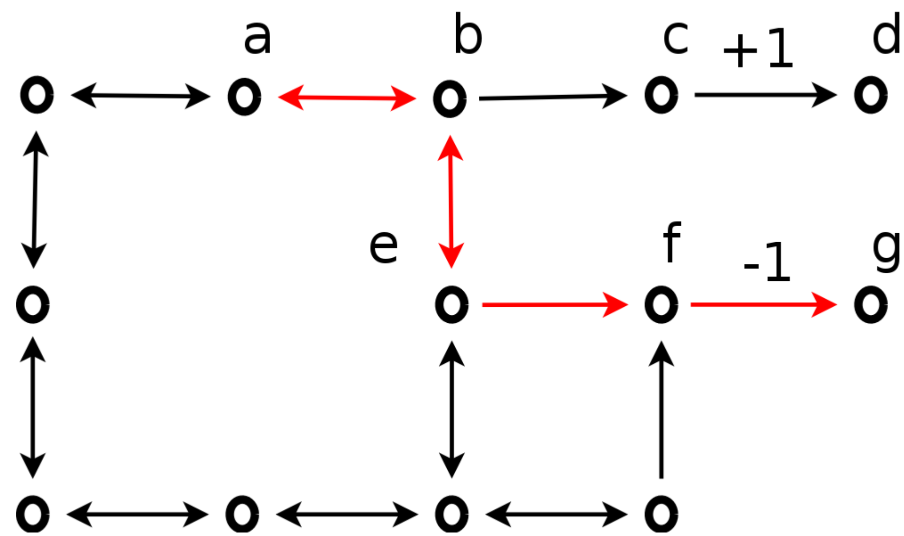

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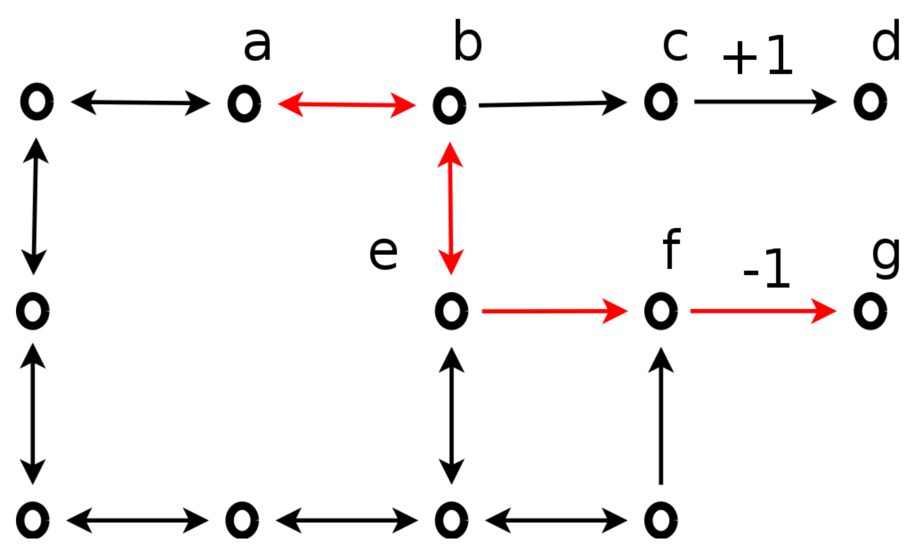
- How can we learn from recorded trajectories and corresponding rewards?
- $\tau_1 : \ (\mathrm{a}, \mathrm{R}, \mathrm{b}, \mathrm{R}, \mathrm{c}, \mathrm{R}, \mathrm{d}), \qquad r(\tau_1) = 1$
- $\tau_2 : \ (\mathrm{a}, \mathrm{R}, \mathrm{b}, \mathrm{D}, \mathrm{e}, \mathrm{R}, \mathrm{f}, \mathrm{R}, \mathrm{g}), \quad r(\tau_2) = -1$
- What is wrong? Why I learned nothing about policy for a?

Q	R - right	D - down
a	**0**	
b	1	-1
c	1	
e	-1	



Dual task - naive learning example

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- I know that I can behave better from b, can I use it?

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b	1	-1
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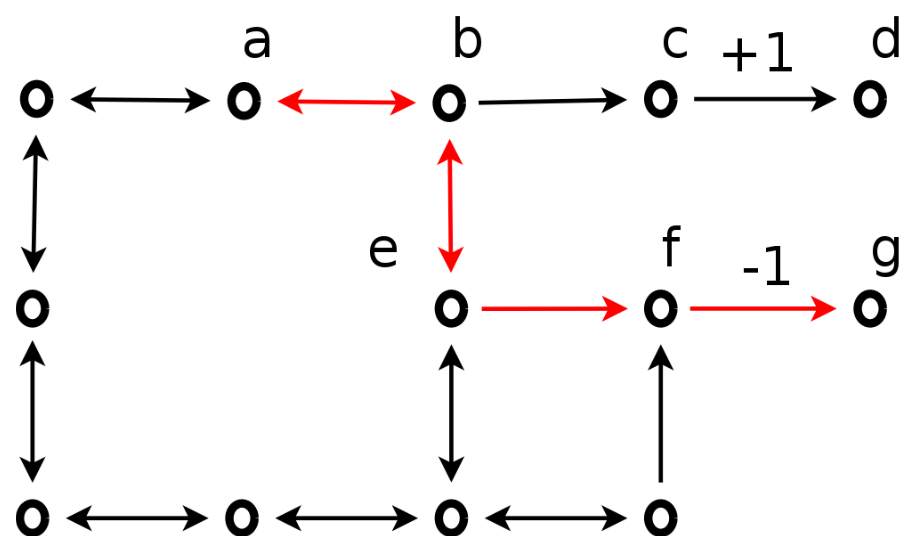

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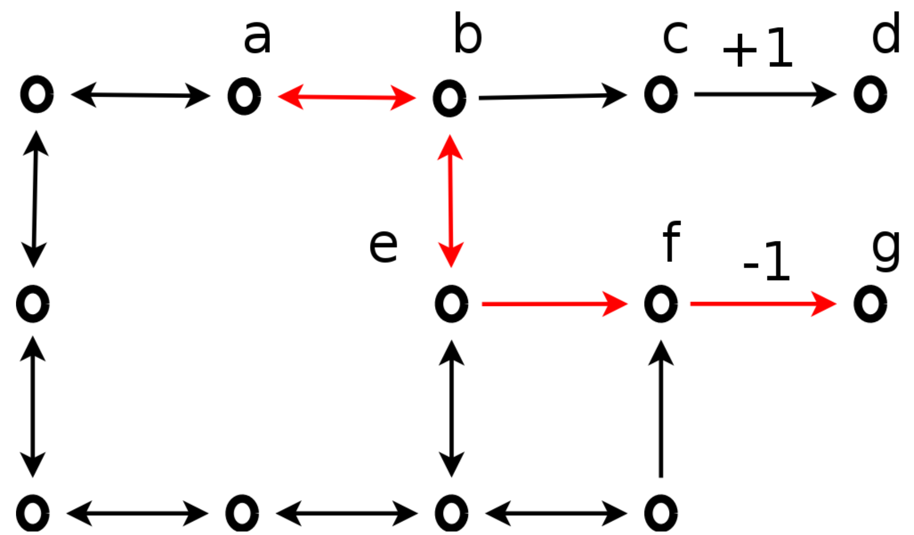
- I know that I can behave better from b, can I use it?
- Recursively: $Q(\mathrm{a}, \mathrm{R}) = \mathrm{average}(\mathrm{reward\_for\_a} + \mathrm{best\_rewards\_from\_b})$

Q	R - right	D - down
a	**1**	
b	1	-1
c	1	
e	-1	



recursive definition of Q

- Define $Q(\mathbf{x}, \mathbf{u})$ recursively:
  - If model is unknown

$$Q(\mathbf{x}, \mathbf{u}) = r(\mathbf{x}, \mathbf{u}, \mathbf{x}') + \gamma \max_{\mathbf{u}'} Q(\mathbf{x}', \mathbf{u}')$$

recursive definition of Q

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$$Q(\mathbf{x}, \mathbf{u}) = r(\mathbf{x}, \mathbf{u}, \mathbf{x}') + \gamma \max_{\mathbf{u}'} Q(\mathbf{x}', \mathbf{u}')$$

  - If a stochastic model is known

$$Q(\mathbf{x}, \mathbf{u}) = \sum_{\mathbf{x}'} p(\mathbf{x}'|\mathbf{u}, \mathbf{x}) \Big[r(\mathbf{x}, \mathbf{u}, \mathbf{x}') + \gamma \max_{\mathbf{u}'} Q(\mathbf{x}', \mathbf{u}') \Big]$$

(Bellman equation)

Q-learning

- Initialize $Q(\mathbf{x}, \mathbf{u}) = 0 \quad \forall_{\mathbf{x},\mathbf{u}}$

Q-learning

- Initialize $Q(\mathbf{x}, \mathbf{u}) = 0 \quad \forall_{\mathbf{x},\mathbf{u}}$
- Drive the robot and record trajectories like that:

$$(\mathbf{x}_0, \mathbf{u}_0, \mathbf{x'}_0, r_0), \quad (\mathbf{x}_1 = \mathbf{x'}_0, \mathbf{u}_1, \mathbf{x'}_1, r_1), \quad \ldots$$

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- End

Q-learning

- Initialize $Q(\mathbf{x},\mathbf{u}) = 0 \quad \forall_{\mathbf{x},\mathbf{u}}$
- Drive the robot and record sequences:

$(\mathbf{x}_0, \mathbf{u}_0, \mathbf{x'}_0, r_0), \quad (\mathbf{x}_1 = \mathbf{x'}_0, \mathbf{u}_1, \mathbf{x'}_1, r_1), \quad \dots$

——— Iterate until convergence ———

- For $\mathbf{x} \in X, \;\; \mathbf{u} \in U$

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- End

———(fixed point algorithm for system of lin. eq.)———

Q-learning

- Initialize $Q(\mathbf{x},\mathbf{u}) = 0 \quad \forall_{\mathbf{x},\mathbf{u}}$

Iterate until good policy found

- Drive the robot and record sequences:

$$(\mathbf{x}_0, \mathbf{u}_0, \mathbf{x'}_0, r_0), \quad (\mathbf{x}_1 = \mathbf{x'}_0, \mathbf{u}_1, \mathbf{x'}_1, r_1), \quad \ldots$$

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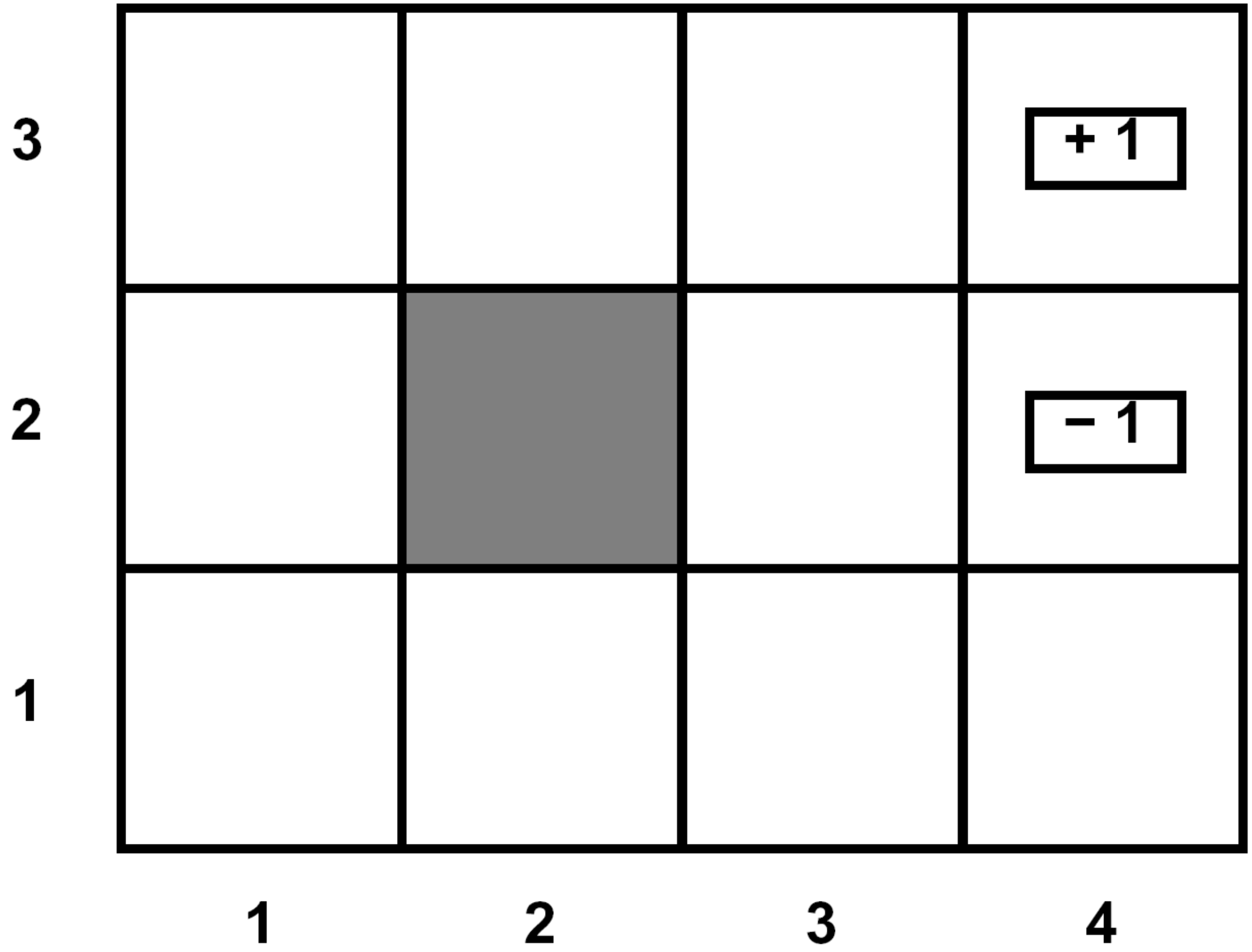
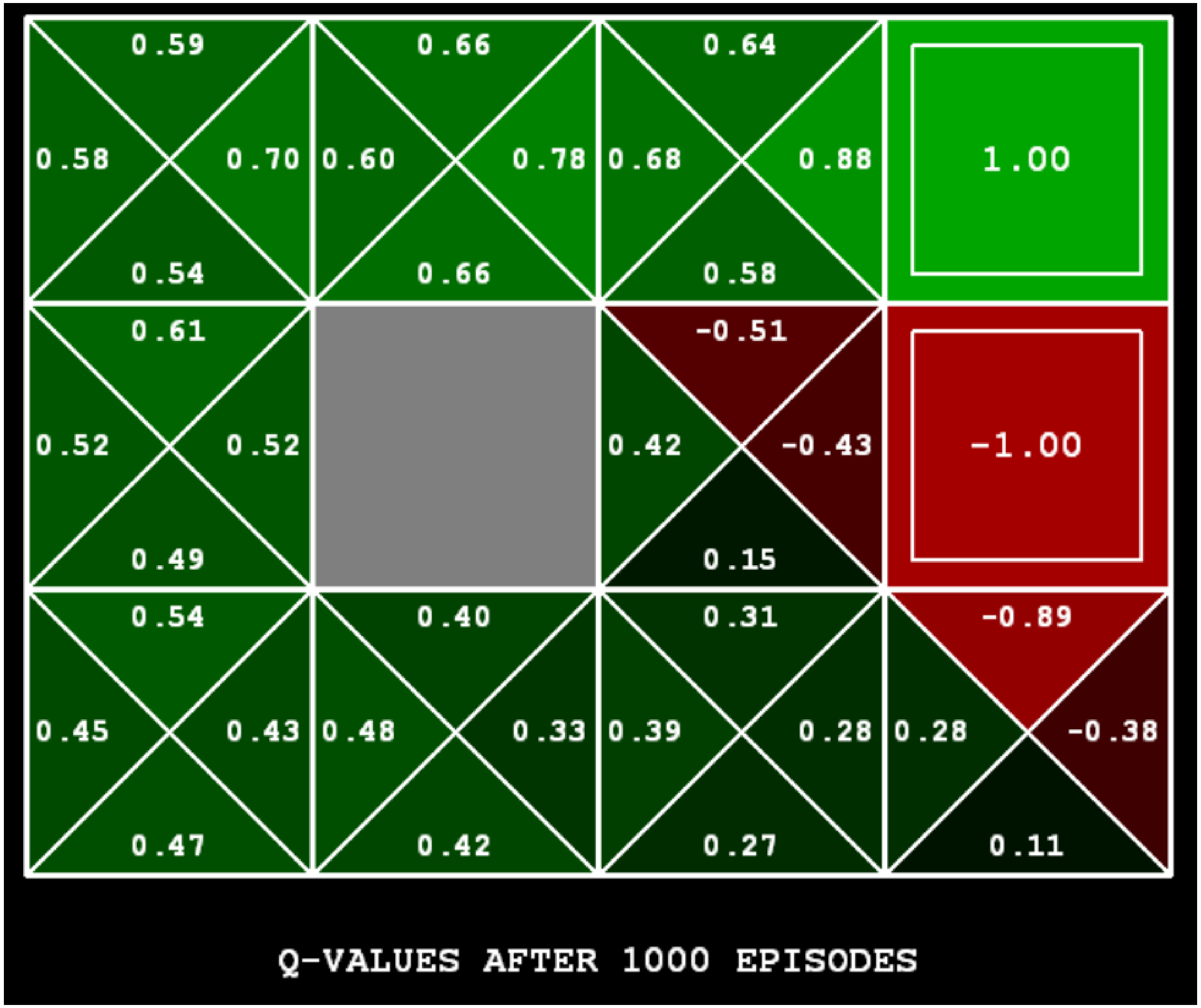
- End

State-value function example I - grid-world

- Q-learning for stochastic grid-world.



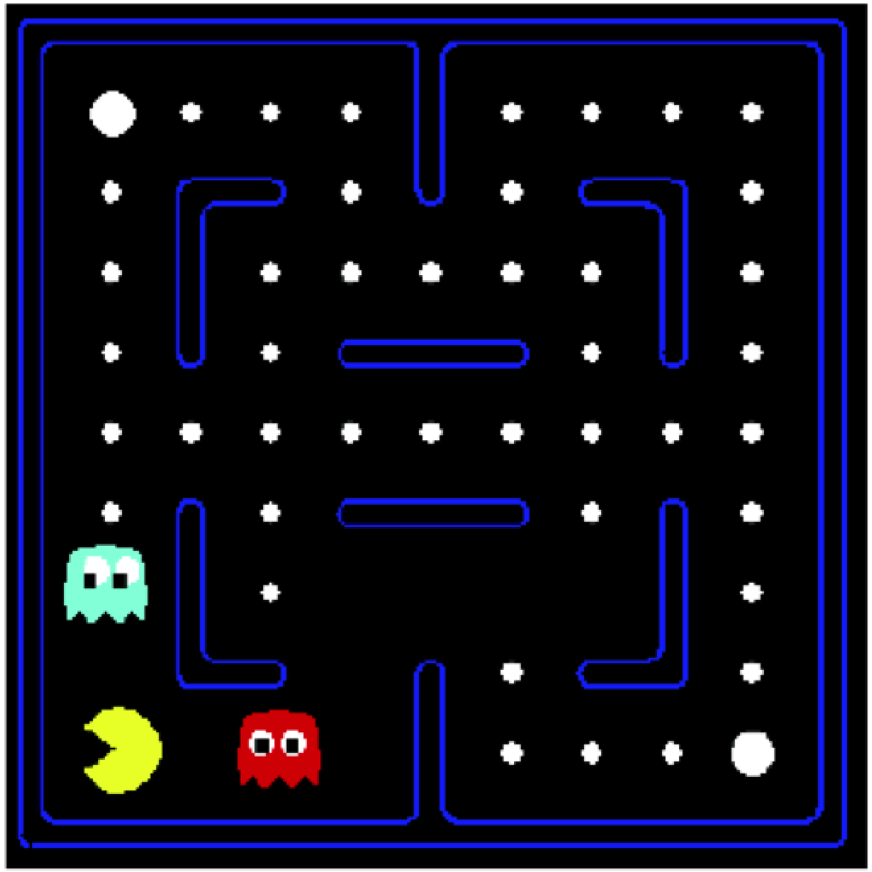
State-value function example I - grid-world



Where is the catch?

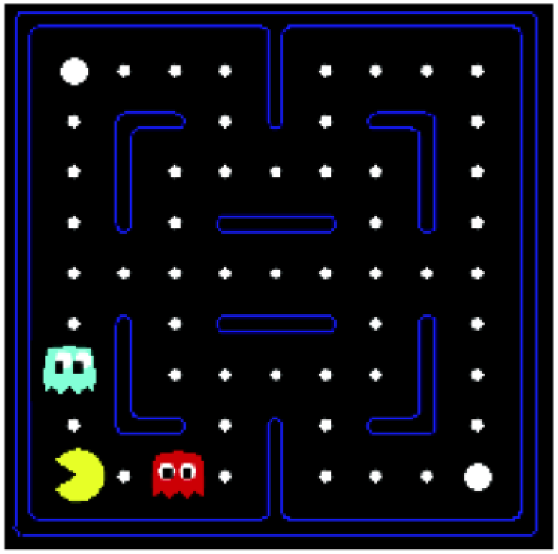
Where is the catch?

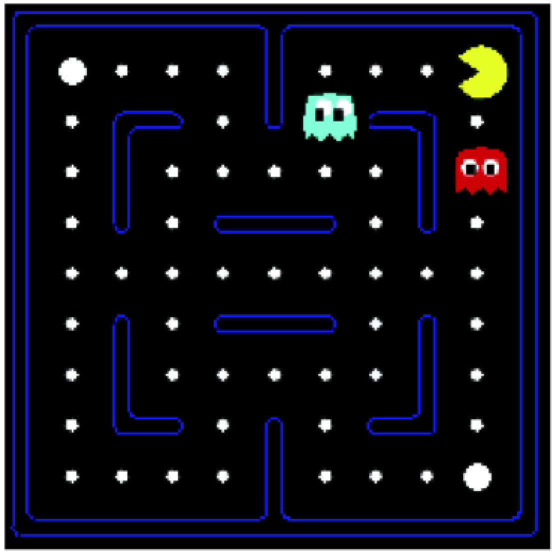
- Curse of dimensionality - considered state space for pacman.

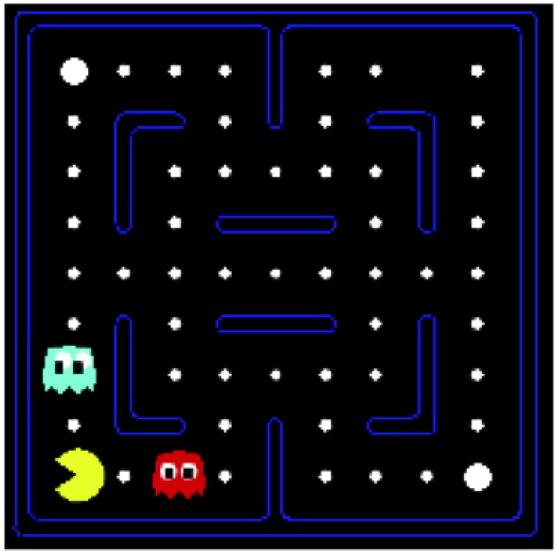


Where is the catch?

- Curse of dimensionality - are these states the same? Do we want it?



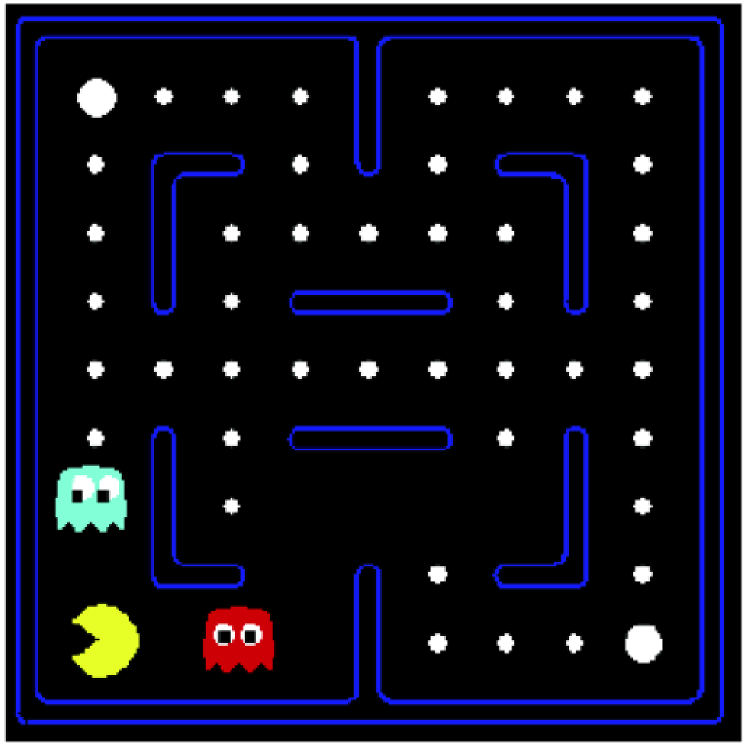




Where is the catch?

- Curse of dimensionality - we need to replace high-dimensional states $\mathbf{x}$ and control $\mathbf{u}$ by low-dimensional features $\Phi(\mathbf{x}, \mathbf{u})$.

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - ...... etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Where is the catch?

- Curse of dimensionality - Q-learning

Iterate until convergence

- For $\mathbf{x} \in X, \; \mathbf{u} \in U$

$$Q(\mathbf{x},\mathbf{u}) = \frac{1}{n} \sum_{i \in \{\mathbf{x}_i = \mathbf{x}, \mathbf{u}_i = \mathbf{u}\}} r_i + \gamma \max_{\mathbf{u}'} Q(\mathbf{x}'_i, \mathbf{u}')$$

- End

Where is the catch?

- Curse of dimensionality - approximate Q-learning

Iterate until convergence

- For all $\mathbf{x}_i,\ \mathbf{u}_i$

$$y_i = r_i + \gamma \max_{\mathbf{u}'} [\theta^\top \Phi(\mathbf{x}'_i, \mathbf{u}'))]$$

- End
- Fit Q-function to approximate mapping between $\Phi(\mathbf{x}_i, \mathbf{u}_i)$ and y_i

$$\theta \leftarrow \arg\min_{\theta} \|\theta^\top \Phi(\mathbf{x}_i, \mathbf{u}_i) - y_i\|$$

Where is the catch?

- Curse of dimensionality

Where is the catch?

- Curse of dimensionality
- Reward tuning

Where is the catch?

- Curse of dimensionality
- Reward tuning
- Exploration vs exploitation
  - ϵ-greedy exploration
  - or exploration extension $Q(\Phi(\mathbf{x},\mathbf{u})) + \frac{k}{N(\Phi)}$

Where is the catch?

- Curse of dimensionality
- Reward tuning
- Exploration vs exploitation
  - ϵ-greedy exploration
  - or exploration extension $Q(\Phi(\mathbf{x}, \mathbf{u})) + \frac{k}{N(\Phi)}$
- Safe exploration, cooperative tasks, hierarchical reinforcment learning.

Conclusions

- Primal Dual task
  - convergence issues
  - do we need to know sum of rewards?
- Do not forget features!
- What you can do?

What you can do?

- Work with us on:
  - real Search&Rescue platform
  - better IRO tasks
- TORCS - Racing **and demolishon derby** simulator competition.
  http://en.wikipedia.org/wiki/TORCS
- Starcraft competition
  http://webdocs.cs.ualberta.ca/~cdavid/starcraftaicomp/