Linear Classifiers II

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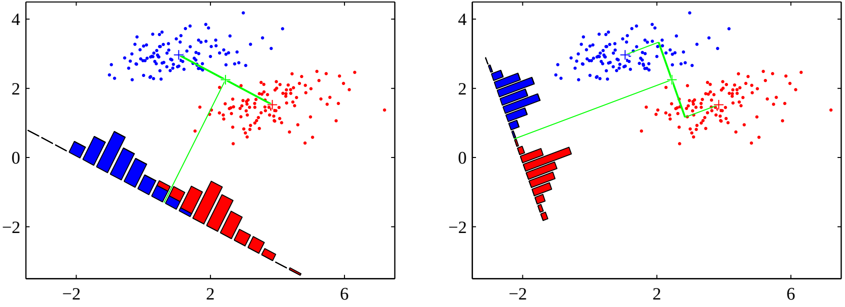
Linear Classifiers - supplement lecture

- Supplement to the lecture about learning Linear Classifiers (perceptron, . . .)
- Better etalons by applying Fischer linear discriminator analysis.
- LSQ formulation of the learning task.

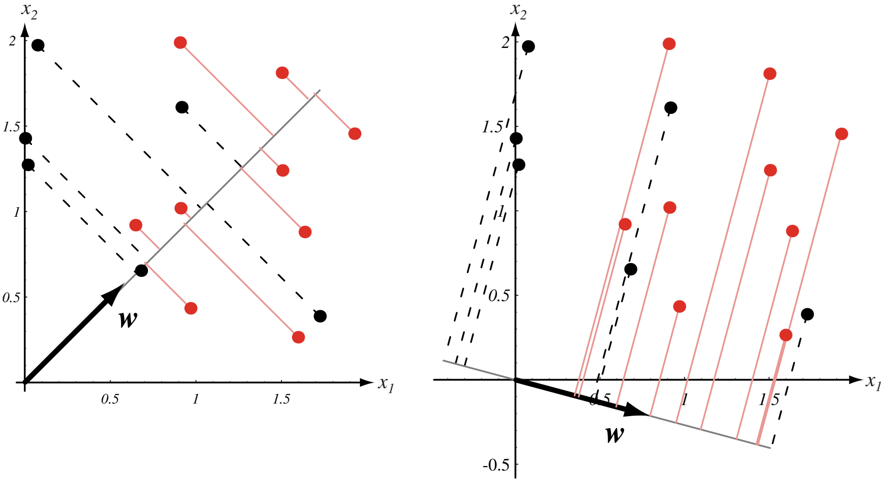
Fischer linear discriminant



- Dimensionality reduction
- Maximize distance between means, . . .
- . . . and minimize within class variance. (minimize overlap)

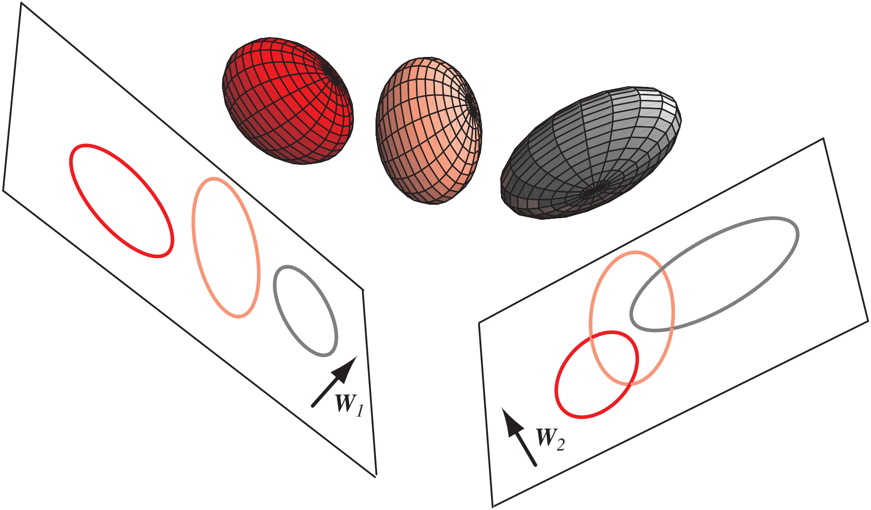
Figures from [1]

Projections to lower dimensions $y = \mathbf{w}^\top \mathbf{x}$



Figures from [2]

Projections to lower dimensions $y = \mathbf{w}^\top \mathbf{x}$



Figures from [2]

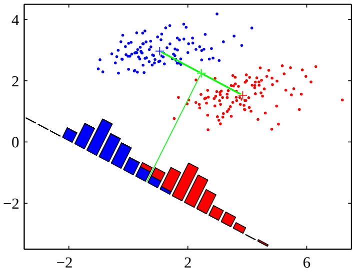
Finding the best projection

$$y = \mathbf{w}^\top \mathbf{x}$$

thresholding $y \geq -w_0$ C_1, otherwise C_2

$$\mathbf{m}_i = \frac{1}{n_i} \sum_{\mathbf{x} \in C_i} \mathbf{x}$$

$$m_2 - m_1 = \mathbf{w}^\top (\mathbf{m}_2 - \mathbf{m}_1)$$



Within class scatter of projected samples

$$s_i^2 = \sum_{y \in C_i} (y - m_i)^2$$

$$\mathrm{S}_i = \sum_{x \in C_1} (\mathbf{x} - \mathbf{m}_i)(\mathbf{x} - \mathbf{m}_i)^\top$$

$$\mathrm{S}_W = \mathrm{S}_1 + \mathrm{S}_2$$

$$\mathrm{S}_B = (\mathbf{m}_2 - \mathbf{m}_1)(\mathbf{m}_2 - \mathbf{m}_1)^\top$$

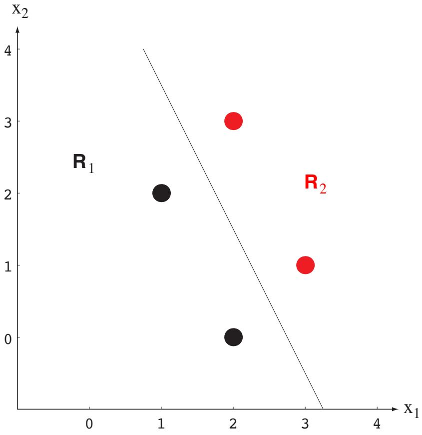
Fischer criterion:

$$J(\mathbf{w}) = \frac{(m_2 - m_1)^2}{s_1^2 + s_2^2}$$

$$J(\mathbf{w}) = \frac{\mathbf{w}^\top \mathrm{S}_B \mathbf{w}}{\mathbf{w}^\top \mathrm{S}_W \mathbf{w}}$$

LSQ approach to linear classification

$$X\mathbf{w} = \mathbf{b}$$



References I

Further reading: Chapter 4 of [1], or chapter 3 and 5 of [2].

[1] Christopher M. Bishop.
Pattern Recognition and Machine Learning.
Springer Science+Bussiness Media, New York, NY, 2006.
PDF freely downloadable.

[2] Richard O. Duda, Peter E. Hart, and David G. Stork.
Pattern Classification.
John Wiley & Sons, 2nd edition, 2001.