Classifiers, Learning

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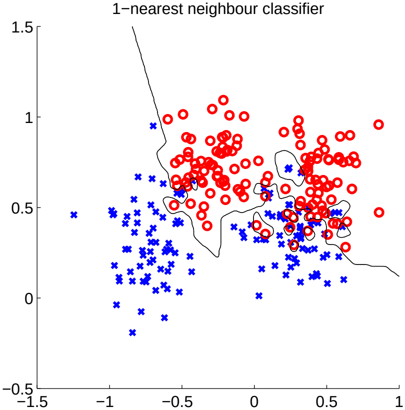
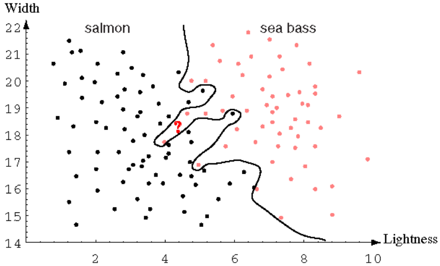
K-Nearest neighbors classification

For a query $\vec{x}$:

- Find K nearest $\vec{x}$ from the tranining (labeled) data.
- Classify to the class with the most exemplars in the set above.





$K-$ Nearest Neighbor and Bayes

Assume data:

- N points $\vec{x}$ in total.
- N_j points in s_j class. Hence, $\sum_j N_j = N$.

We want classify $\vec{x}$. We draw a sphere centered at $\vec{x}$ containing K points irrespective of class. V is the volume of this sphere. $P(s_j|\vec{x}) = ?$

$$P(s_j|\vec{x}) = \frac{P(\vec{x}|s_j)P(s_j)}{P(\vec{x})}$$

$$P(s_j) = \frac{N_j}{N}$$

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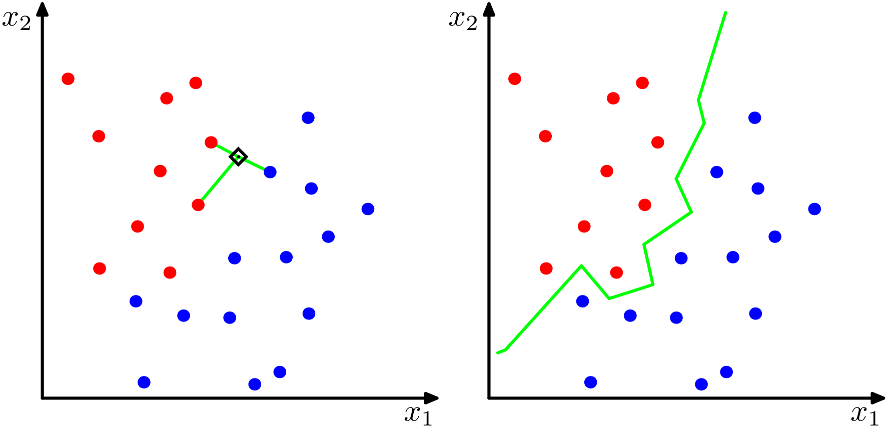
NN classification example



(a) (b) [1]

[1]Figs from [1]

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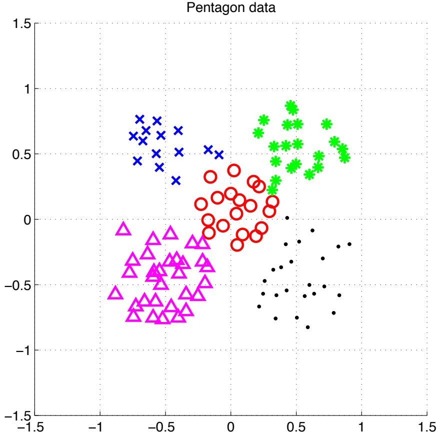

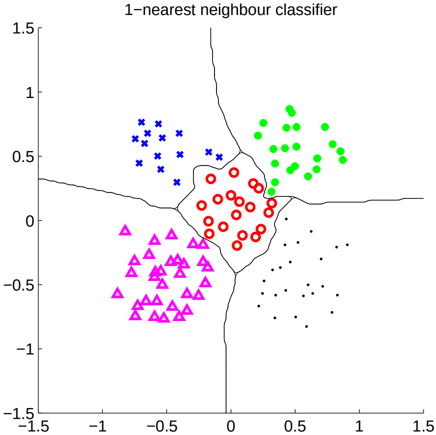

Metrics for NN classification

$D(\mathbf{a}, \mathbf{b}) \geq 0$
$D(\mathbf{a}, \mathbf{b}) = 0$ iff $\mathbf{a} = \mathbf{b}$
$D(\mathbf{a}, \mathbf{b}) = D(\mathbf{b}, \mathbf{a})$
$D(\mathbf{a}, \mathbf{b}) + D(\mathbf{b}, \mathbf{c}) \geq D(\mathbf{a}, \mathbf{c})$

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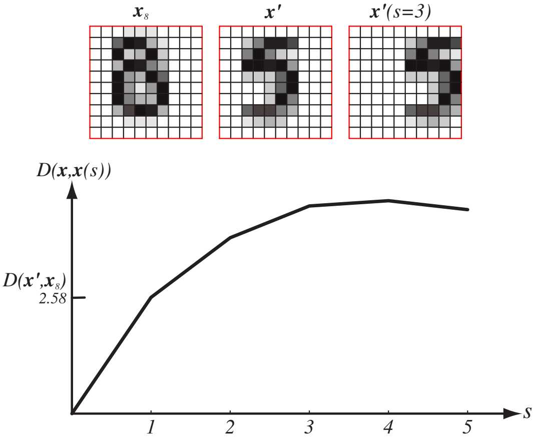
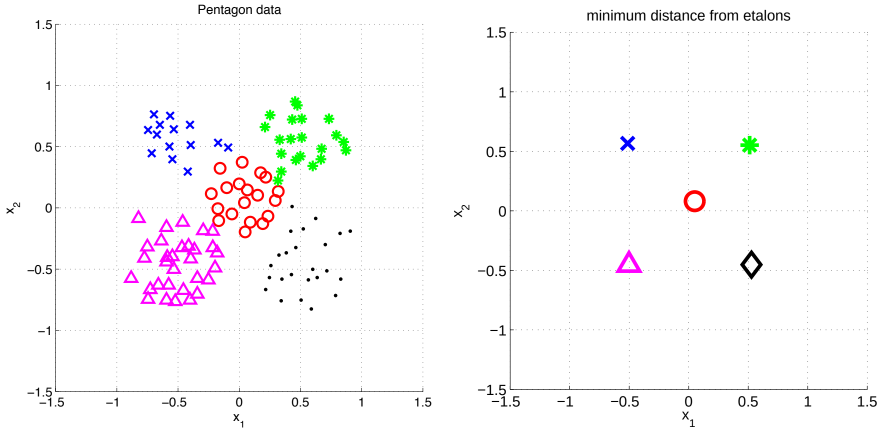


Invariance to geometrical transformations?

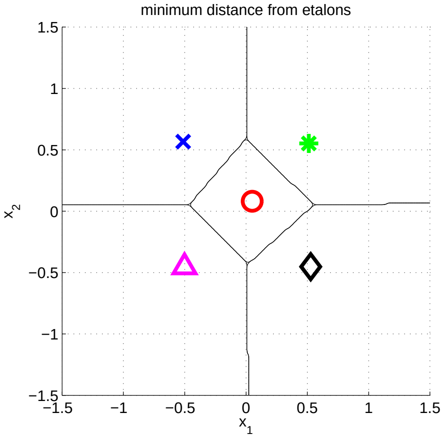
Etalon based classification



Represent $\vec{x}$ by etalon , $\vec{e}_s$ per each class $s \in S$

Separate etalons

$$s^* = \arg\min_{s \in S} \left(||\vec{x} - \vec{e}_s||^2 + o_s \right)$$

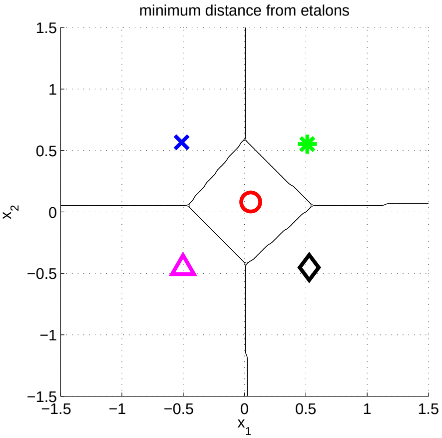


What etalons?

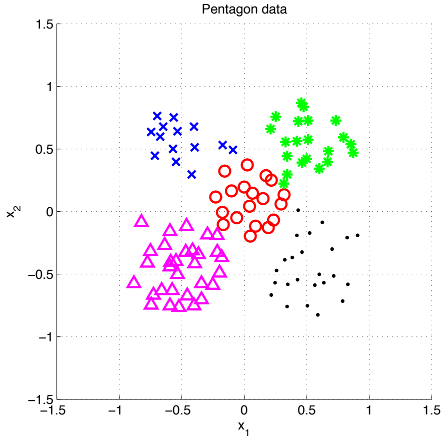
If $\mathcal{N}(\vec{x}|\vec{\mu},\Sigma)$; all classes same covariance matrices, then

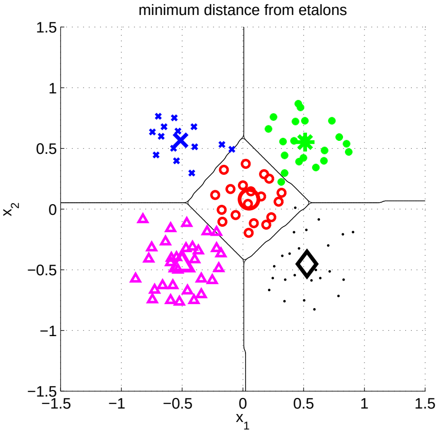
$$\vec{e}_s \stackrel{\mathrm{def}}{=} \vec{\mu}_s = \frac{1}{|\mathcal{X}^s|}\sum_{i\in\mathcal{X}^s} \vec{x}_i^s$$

and separating hyperplanes halve distances between pairs.

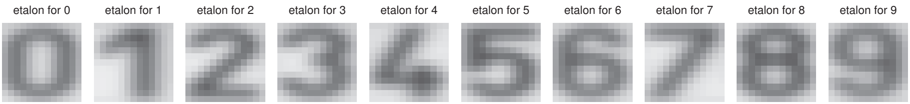


Etalon based classification, $\vec{e}_s = \vec{\mu}_s$



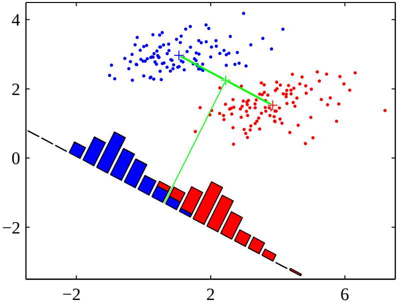


Digit recognition - etalons $\vec{e}_s = \vec{\mu}_s$



Figures from [5]

Better etalons – Fischer linear discriminant



- Dimensionality reduction
- Maximize distance between means, ...
- ...and minimize within class variance. (minimize overlap)

Figures from [1]

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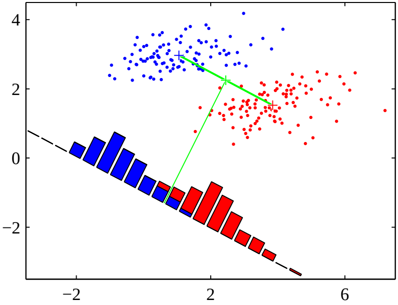

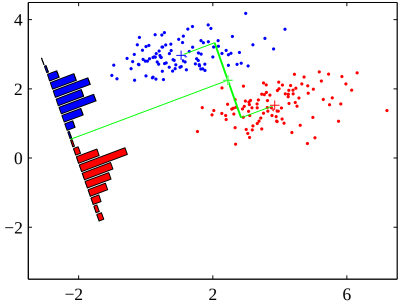

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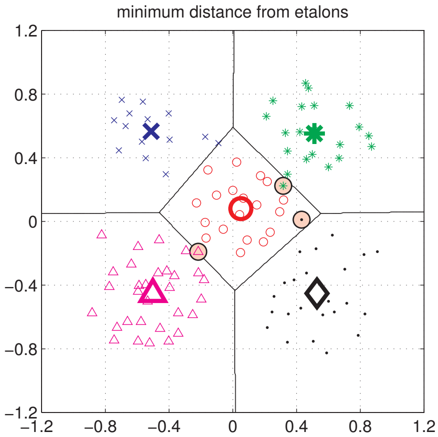

Figures from [5]

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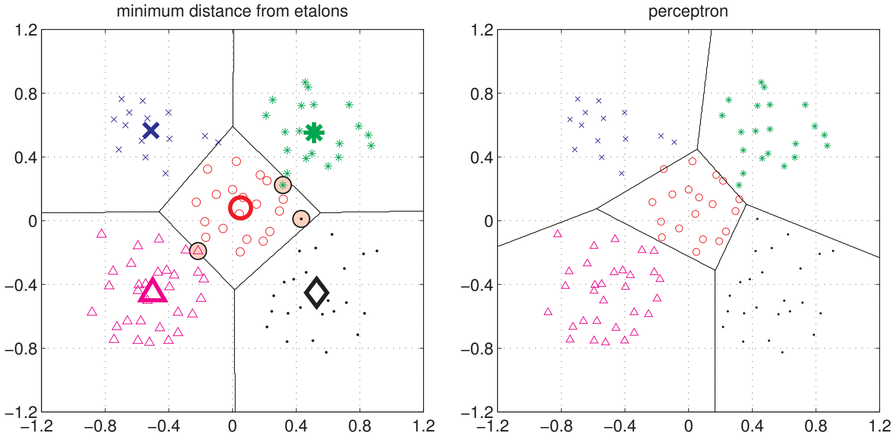

Figures from [5]

Etalon classifier – Linear classifier

$$s^* = \arg\min_{s \in S}\big(\|\vec{x} - \vec{e}_s\|^2 + o_s\big) = \arg\min_{s \in S}(\vec{x}^\top\vec{x} - 2\,\vec{e}_s^\top\vec{x} + \vec{e}_s^\top\vec{e}_s + o_s) =$$

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Linear function (plus offset)

$$g_s(\mathbf{x}) = \mathbf{w}_s^\top\mathbf{x} + w_{s0}$$

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Learning and decision

Learning stage - learning models/function/parameters from data.

Decision stage - decide about a query $\vec{x}$.

What to learn?

- Generative model : Learn $P(\vec{x}, s)$. Decide by computing $P(s|\vec{x})$.
- Discriminative model : Learn $P(s|\vec{x})$
- Discriminant function : Learn $g(\vec{x})$ which maps $\vec{x}$ directly into class labels.

Linear discriminant function - two class case

$$g(\mathbf{x}) = \mathbf{w}^\top \mathbf{x} + w_0$$

Decide s_1 if $g(\mathbf{x}) > 0$ and s_2 if $g(\mathbf{x}) < 0$

Figure from [2]

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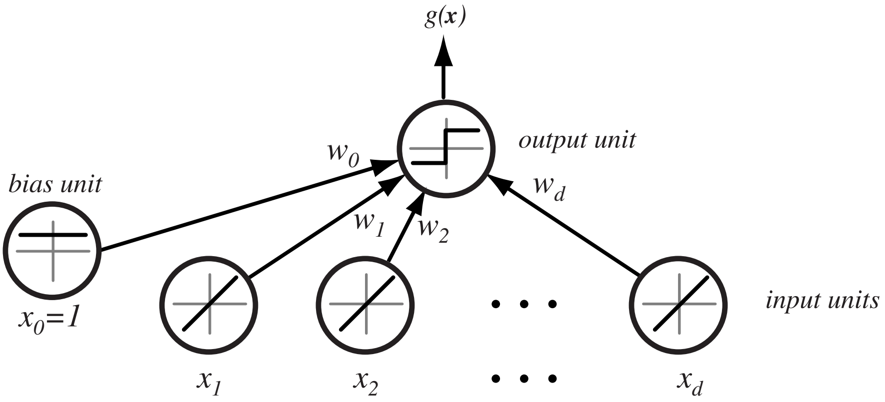


Figure from [2]

Separating hyperplane

$$\mathbf{w}^\top \mathbf{x}_1 + w_0 = \mathbf{w}^\top \mathbf{x}_2 + w_0$$

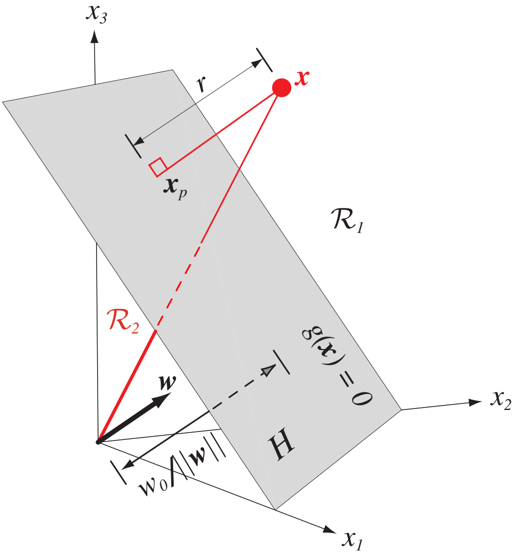
$$\mathbf{w}^\top (\mathbf{x}_1 - \mathbf{x}_2) = 0$$

$g(\mathbf{x})$ gives an algebraic measure of the distance.

$$\mathbf{x} = \mathbf{x}_p + r\frac{\mathbf{w}}{\|\mathbf{w}\|}$$

as $g(\mathbf{x}_p) = 0$,
and $g(\mathbf{x}) = \mathbf{w}^\top \mathbf{x} + w_0$, then:

$$g(\mathbf{x}) = r\|\mathbf{w}\|$$



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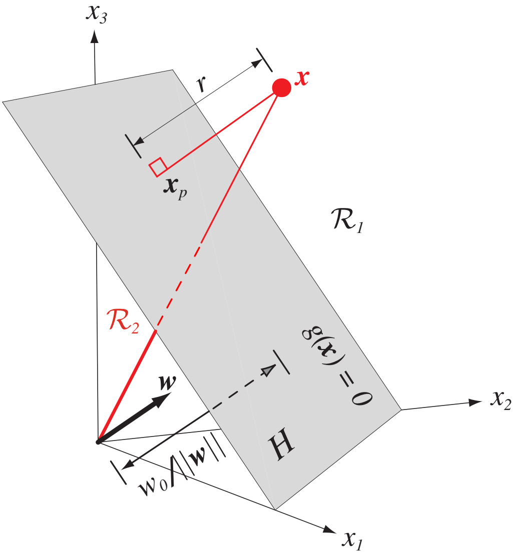

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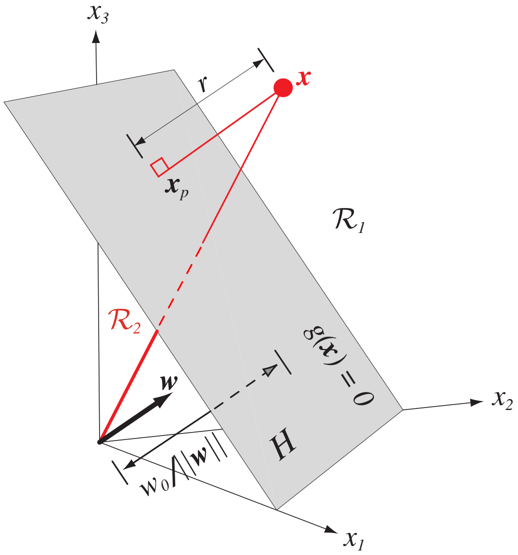

Figure from [2]

Multiclass case

each class has its own discriminant function

$$g_s(\mathbf{x}) = \mathbf{w}_s^\top \mathbf{x} + w_{s0}$$

and the classification s^* is along the max.

Two classes set-up

$|S| = 2$, i.e. two states (typically also classes)

$$g(\mathbf{x}) = \begin{cases} s = 1\,, & \text{if} \quad \mathbf{w}^\top \mathbf{x} + w_0 > 0\,, \\ s = -1\,, & \text{if} \quad \mathbf{w}^\top \mathbf{x} + w_0 < 0\,. \end{cases}$$

$$\boxed{\mathbf{x}'_j = s_j \begin{bmatrix} 1 \\ \mathbf{x}_j \end{bmatrix},\ \mathbf{w}' = \begin{bmatrix} w_0 \\ \mathbf{w} \end{bmatrix}}$$

for all $\mathbf{x}'$

$$\mathbf{w}'^\top \mathbf{x}' > 0$$

drop the dashes to avoid notation clutter.

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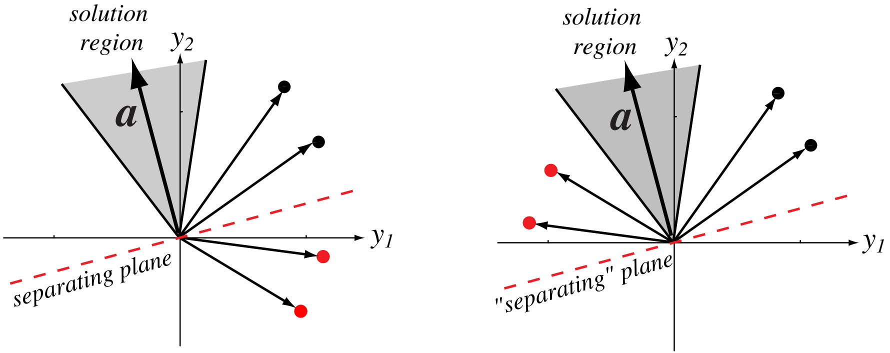
Solution (graphically)



Different notation in the book: substitute $\mathbf{a} \leftarrow \mathbf{w}$ and $y_1, y_2 \leftarrow x_1, x_2$

Figure from [2]

Learning $\mathbf{w}$, gradient descent

A criterion to be minimized $J(\mathbf{w})$

Initialize $\mathbf{w}$, threshold θ, learning rate α
$k \leftarrow 0$
repeat
$\quad k \leftarrow k + 1$
$\quad \mathbf{w} \leftarrow \mathbf{w} - \alpha(k)\nabla J(\mathbf{w})$
until $|\alpha(k)\nabla J(\mathbf{w})| < \theta$
return $\mathbf{w}$

Learning **w** - Perceptron criterion

Goal: Find a weight vector $\mathbf{w} \in \Re^{D+1}$ (original feature space dimensionality is D) such that:

$$\mathbf{w}^\top \mathbf{x}_j > 0 \qquad (\forall j \in \{1, 2, ..., m\})$$

(Perceptron) Criterion to be minimized:

$$J(\mathbf{w}) = \sum_{\mathbf{x} \in \mathcal{X}} -\mathbf{w}^\top \mathbf{x}$$

where $\mathcal{X}$ is a set of missclassified $\mathbf{x}$.

$$\nabla J(\mathbf{w}) = \sum_{\mathbf{x} \in \mathcal{X}} -\mathbf{x}$$

Learning **w** - Perceptron criterion

Goal: Find a weight vector $\mathbf{w} \in \Re^{D+1}$ (original feature space dimensionality is D) such that:

$$\mathbf{w}^\top \mathbf{x}_j > 0 \qquad (\forall j \in \{1, 2, ..., m\})$$

(Perceptron) Criterion to be minimized:

$$J(\mathbf{w}) = \sum_{\mathbf{x} \in \mathcal{X}} -\mathbf{w}^\top \mathbf{x}$$

where $\mathcal{X}$ is a set of missclassified **x**.

$$\nabla J(\mathbf{w}) = \sum_{\mathbf{x} \in \mathcal{X}} -\mathbf{x}$$

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$$\nabla J(\mathbf{w}) = \sum_{\mathbf{x} \in \mathcal{X}} -\mathbf{x}$$

(Batch) Perceptron algorithm

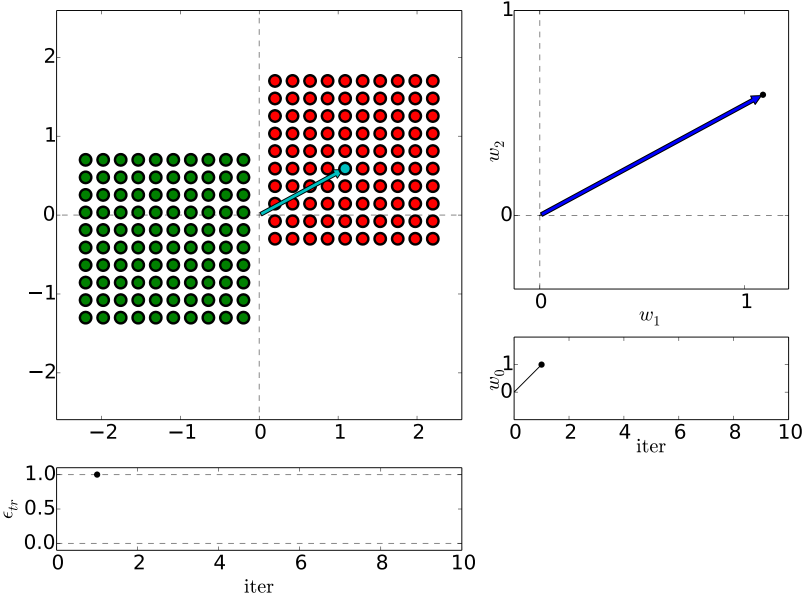
Initialize $\mathbf{w}$, threshold θ, learning rate α
$k \leftarrow 0$
repeat
  $k \leftarrow k + 1$
  $\mathbf{w} \leftarrow \mathbf{w} + \alpha(k) \sum_{\mathbf{x} \in \mathcal{X}(k)} \mathbf{x}$
until $|\alpha(k) \sum_{\mathbf{x} \in \mathcal{X}(k)} \mathbf{x}| < \theta$
return $\mathbf{w}$

Fixed-increment single-sample Perceptron

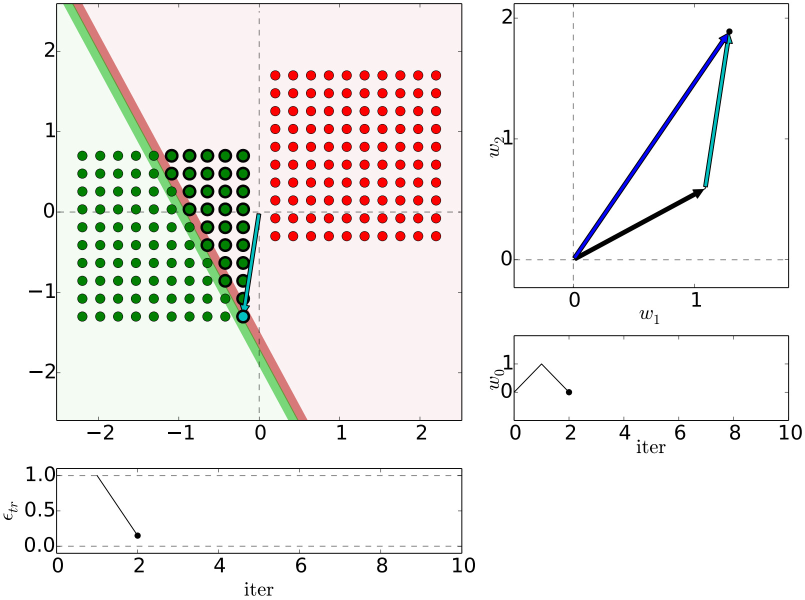
n patterns/samples, we are looping over all patterns repeatedly

Initialize $\mathbf{w}$
$k \leftarrow 0$
repeat
    $k \leftarrow (k+1) \bmod n$
    if $\mathbf{x}^k$ missclassified, **then** $\mathbf{w} \leftarrow \mathbf{w} + \mathbf{x}^k$
until all $\mathbf{x}$ correctly classified
return $\mathbf{w}$

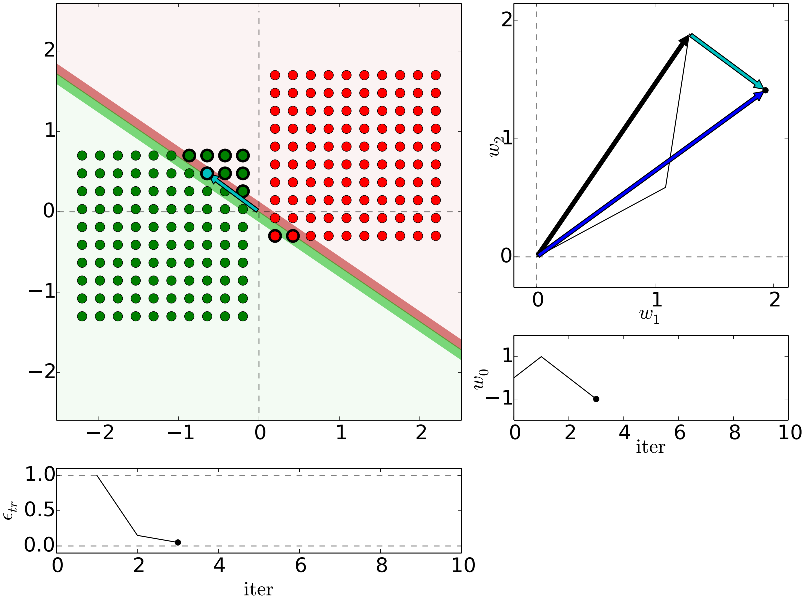
Perceptron iterations/loops



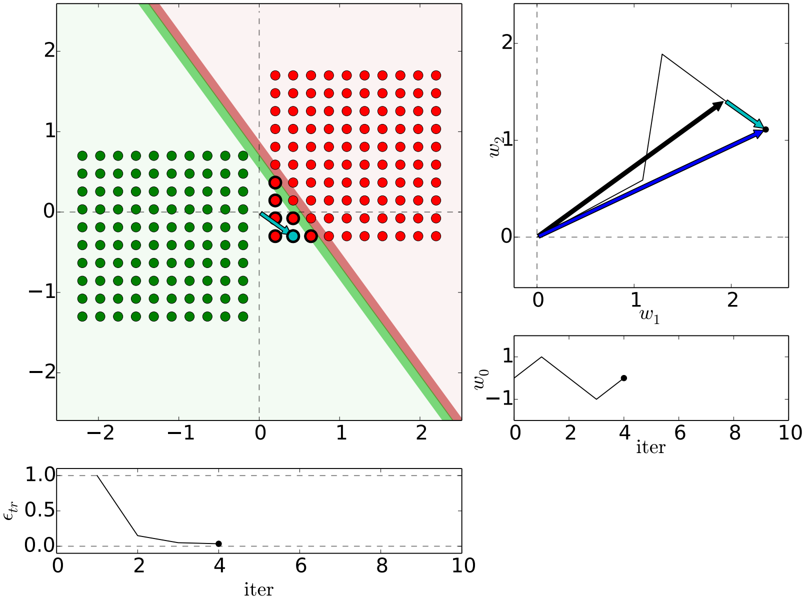
Perceptron iterations/loops



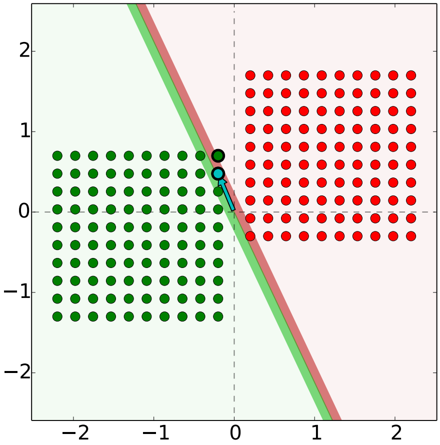
Perceptron iterations/loops

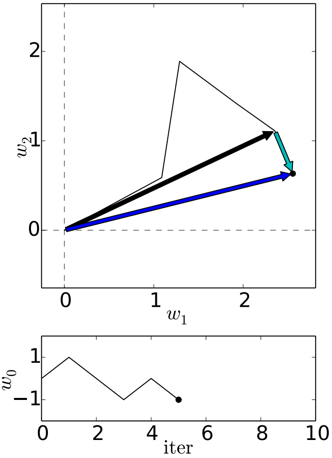


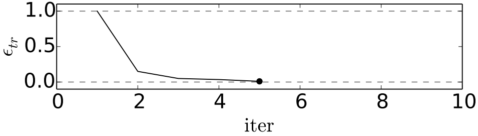
Perceptron iterations/loops



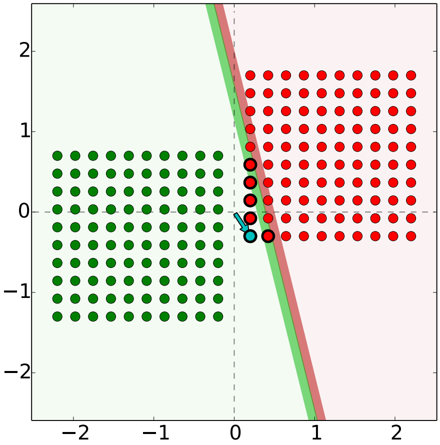
Perceptron iterations/loops

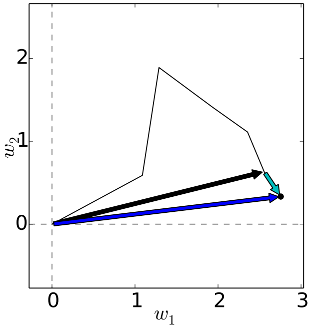


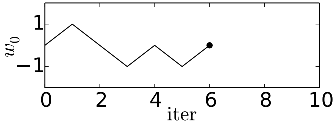


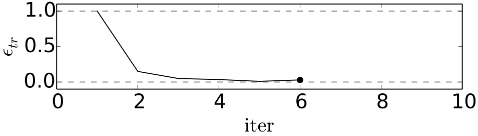


Perceptron iterations/loops

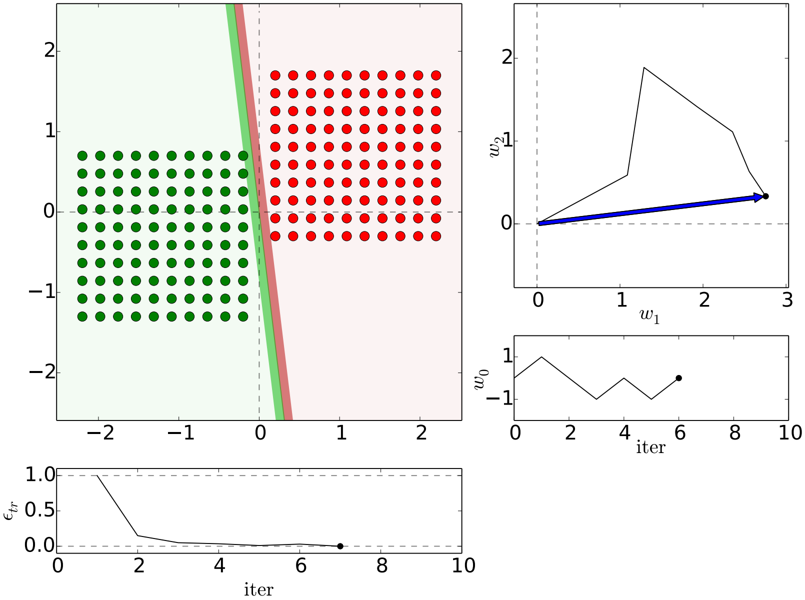




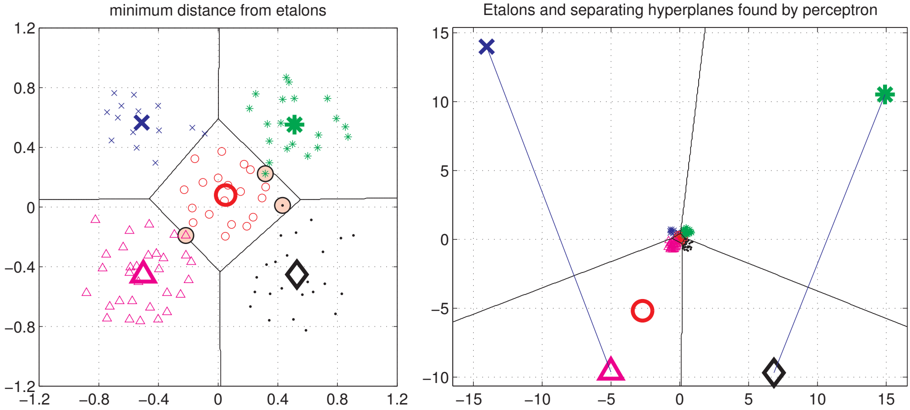




Perceptron iterations/loops

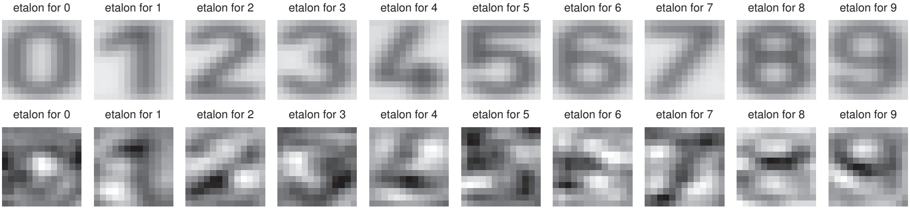


Etalons: means vs found be perceptron



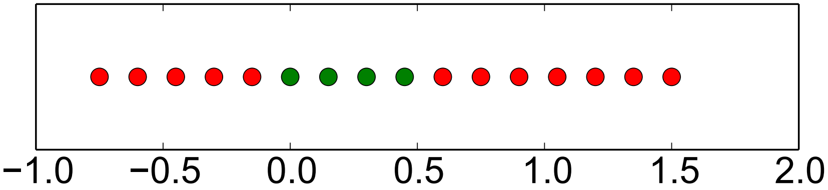
Figures from [5]

Digit recognition - etalons means vs. perceptron



Figures from [5]

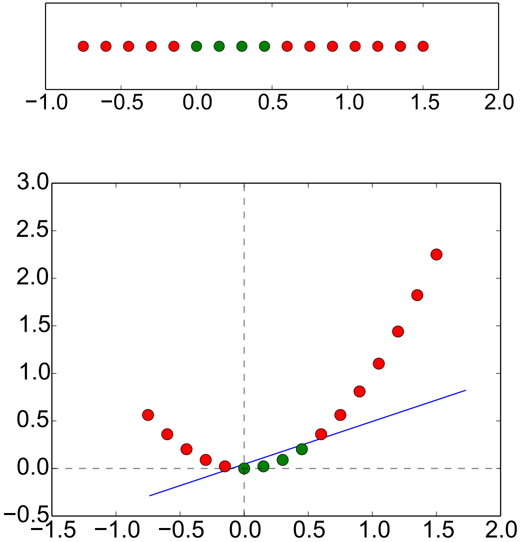
What if not lin separable?



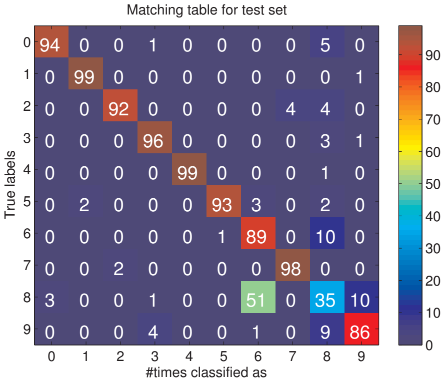
Dimension lifting

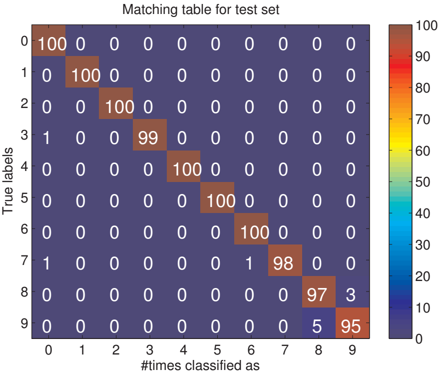
$$\mathbf{x} = [x, x^2]^\top$$

Dimension lifting, $\mathbf{x} = [x, x^2]^\top$



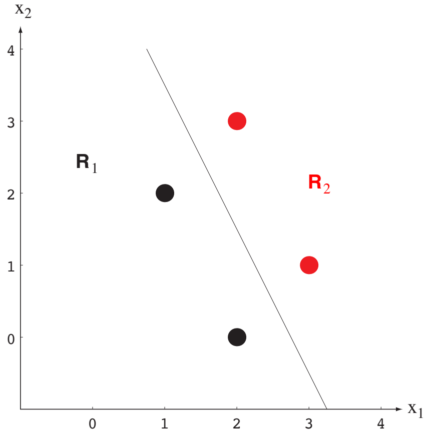
Performance comparison, parameters fixed



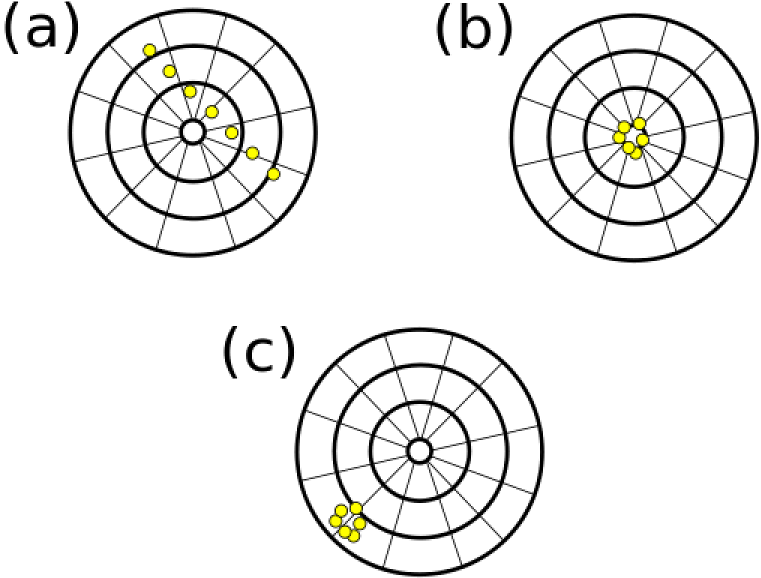


LSQ approach to linear classification

$$\mathtt{X}\mathbf{w} = \mathbf{b}$$

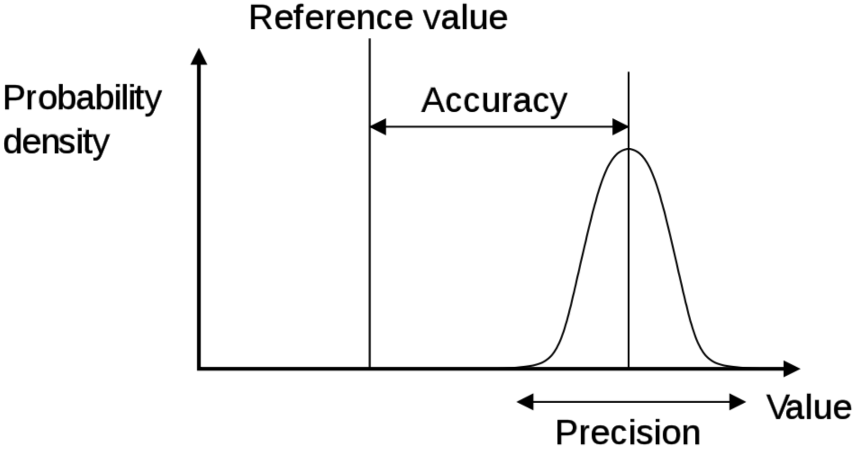


Accuracy vs precision



https://commons.wikimedia.org/wiki/File:Precision_versus_accuracy.svg

Accuracy vs precision



https://en.wikipedia.org/wiki/Accuracy_and_precision

References I

Further reading: Chapter 18 of [4], or chapter 4 of [1], or chapter 5 of [2]. Many Matlab figures created with the help of [3]. You may also play with demo functions from [5].

[1] Christopher M. Bishop.
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Springer Science+Bussiness Media, New York, NY, 2006.
PDF freely downloadable.

[2] Richard O. Duda, Peter E. Hart, and David G. Stork.
Pattern Classification.
John Wiley & Sons, 2nd edition, 2001.

[3] Votjěch Franc and Václav Hlaváč.
Statistical pattern recognition toolbox.
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References II

[4] Stuart Russell and Peter Norvig.
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Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

[5] Tomáš Svoboda, Jan Kybic, and Hlaváč Václav.
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Thomson, Toronto, Canada, 1st edition, September 2007.
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