Problem solving by search II

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Outline

- Graph search
- Heuristics (how to search faster)
- Greedy
- A*. A-star search.

Recap: Search

A tree search recap

function TREE_SEARCH(problem) **return** a solution or failure
    initialize the frontier the initial state of the problem
    loop
        if the frontier is empty **then return** failure
        else choose a node from frontier and remove from frontier
        end if
        if the node contains a goal state **then return** the solution
        end if
        Expand the node and add the resulting nodes to frontier
    end loop
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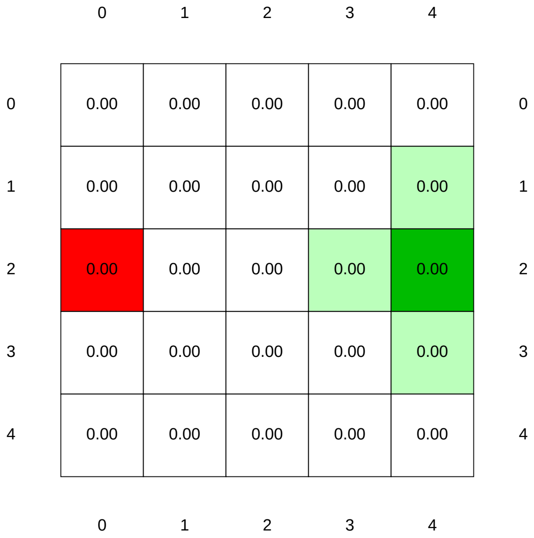
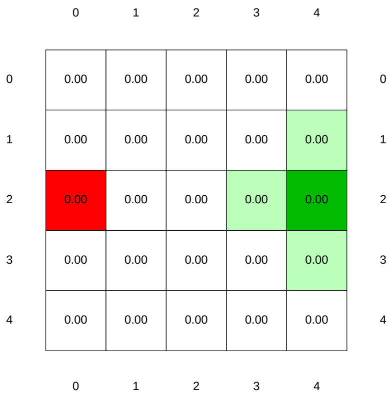
A Maze, what could possibly go wrong?



Analyze the demo run. What happened? Why did it take that long?

Tree search the maze

function TREE_SEARCH(env) **return** a solution or failure
    initialize the frontier
    while frontier **do**
        node = frontier.pop()
        if goal in node **then**
            break
        end if
        nodes = env.expand(node.state)
        Add nodes to frontier
    end while
end function



Make a `frontier` and `expand` columns on a paper and follow the algorigthm by putting and removing (scratching out) nodes from the list.

A graph search

function GRAPH_SEARCH(env) **return** a solution or failure
    init frontier by the start state
    initialize the explored set to be empty
    while frontier **do**
        node = frontier.pop()
        if goal in node **then** break
        end if
        nodes = env.expand(node.state)
        add node to explored
        for all nodes **do**
            if *node not in explored (or in frontier)* **then**
                add nodes to frontier
            end if
        end for
    end while
end function

⚠ **Do not forget:** node is not the same as state!

Think about what is node and what state. What is main difference? How are they connected? Where do they appear? What is node/state in the maze problem?

The main idea: Do not expand a state twice.

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    node ← env.observe()
    frontier ← FIFOqueue(node)
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        explored.add(node.state)                ▷ adding state not node!
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Why adding/checking `state` and note `node` in `expanded` data structure?

Can I do the simple presence check for all kind of graph search algorithms?

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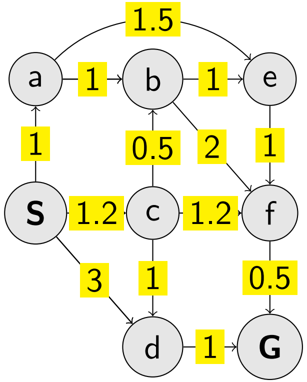
What about actual costs graph search?



When following the algorithm (animation) use the paper list of `frontier` and `expanded`

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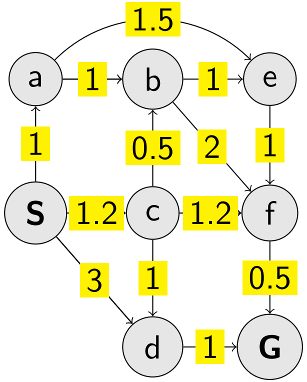

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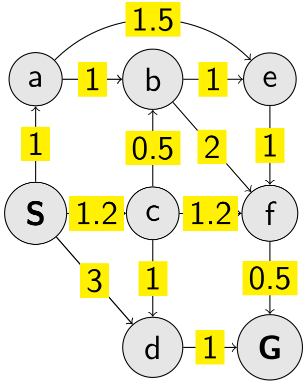

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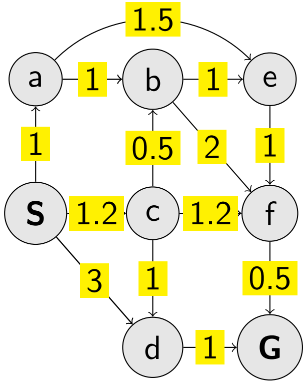

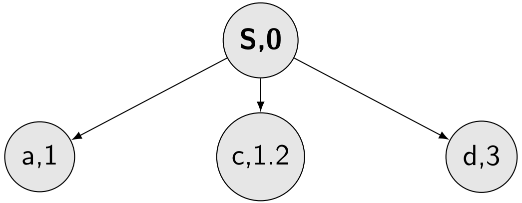

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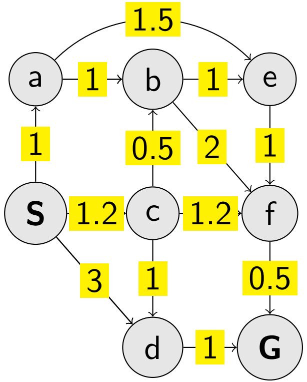

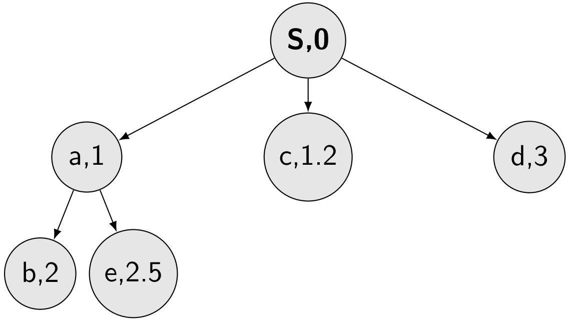

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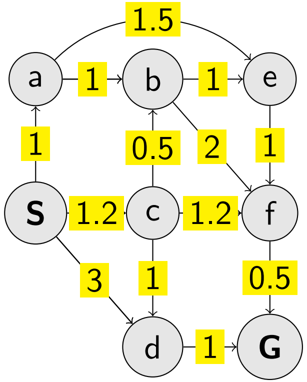

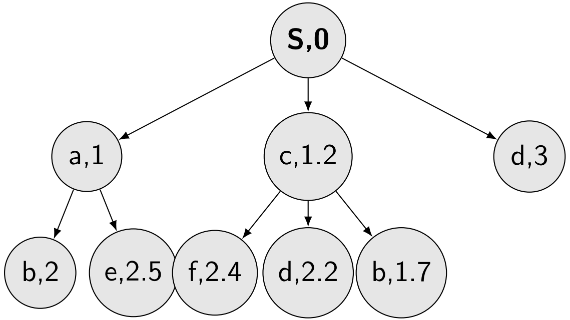

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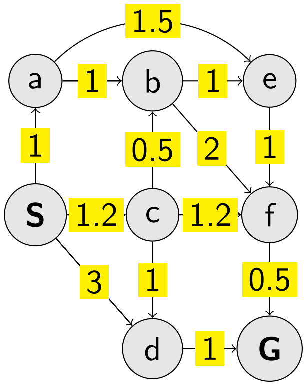

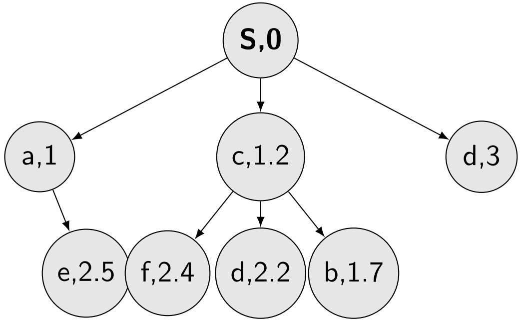

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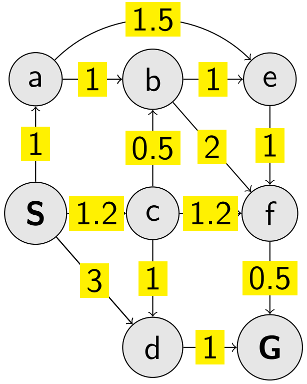

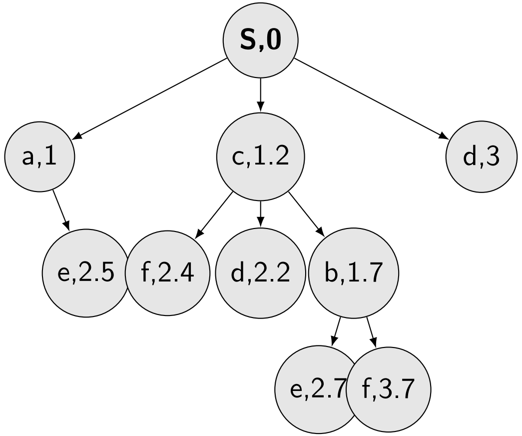

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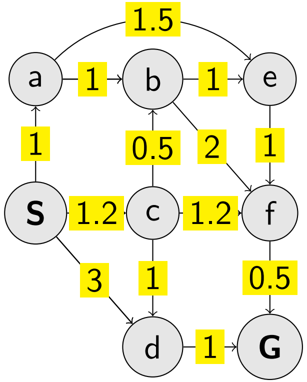

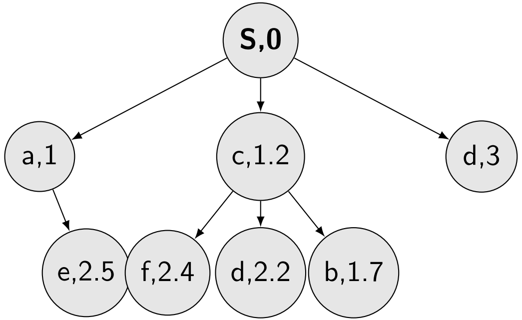

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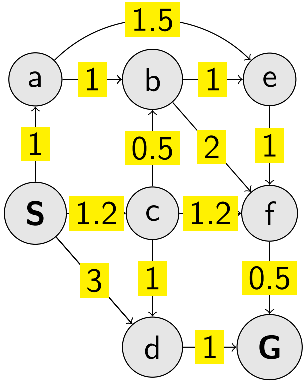

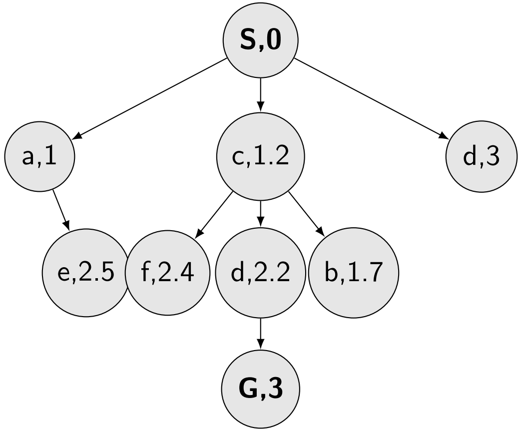

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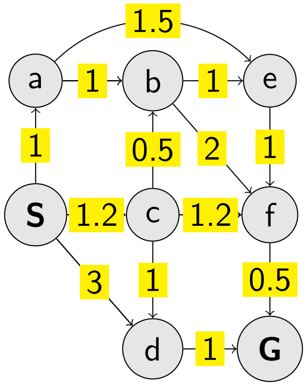

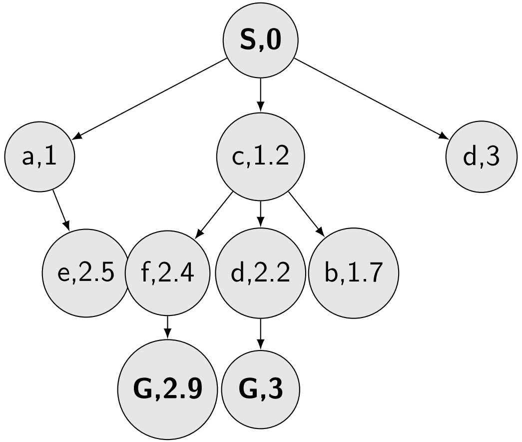

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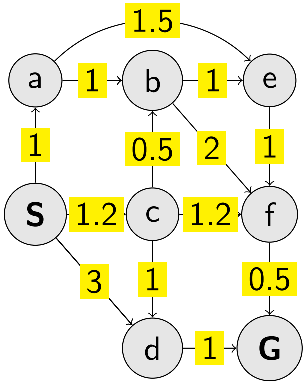

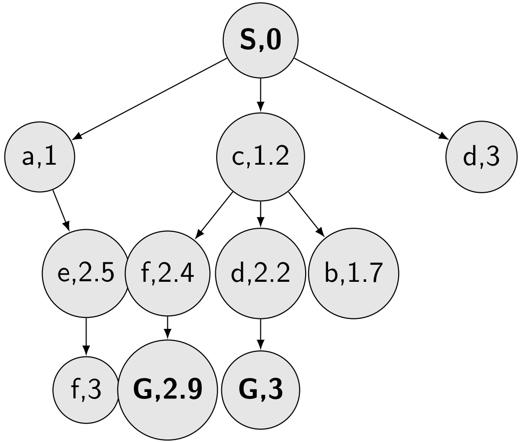

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    explored ← set()
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        end if
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        child_nodes ← env.expand(node.state)
        for all child_nodes do
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Does the algorithm always find the best (cheapest) path? Are there any requirements for the path optimality function?

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The UCS graph search

```
function UCS_GRAPH_SEARCH(env) return a solution or failure
    node ← env.observe()
    frontier ← priority_queue(node)                  ▷ path_cost for ordering
    explored ← set()
    while frontier not empty do
        node ← frontier.pop()
        if node contains Goal then return node       ▷ check here!
        end if
        explored.add(node.state)
        child_nodes ← env.expand(node.state)
        for all child_nodes do
            if child_node.state not in explored or in frontier then
                fronter.insert(child_node)
            else if child_node.state in frontier with higher cost then
                replace that with the child_node
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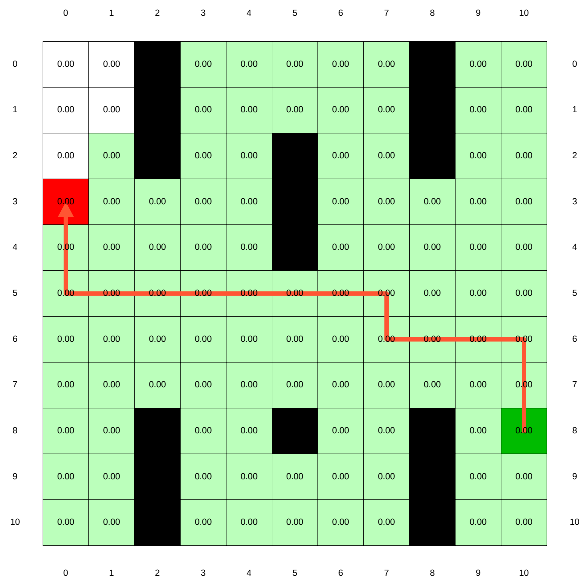
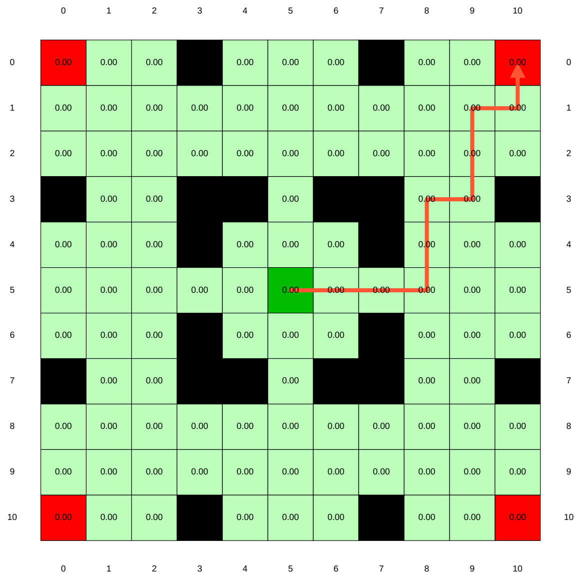
Few examples of search strategies so far



Run the demos.

What is wrong with UCS and other strategies?



Run the demo.

Node selection, take argmin $f(n)$

- DFS: $f(n) = -n.depth$
- BFS: $f(n) = n.depth$
- UCS: $f(n) = n.path\_cost$

The good: frontier as a priority queue The bad: All the $f(n)$ correspond to the cost from n to the start - only backward cost.

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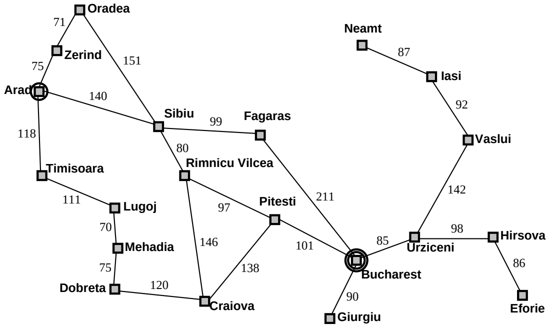
Heuristics

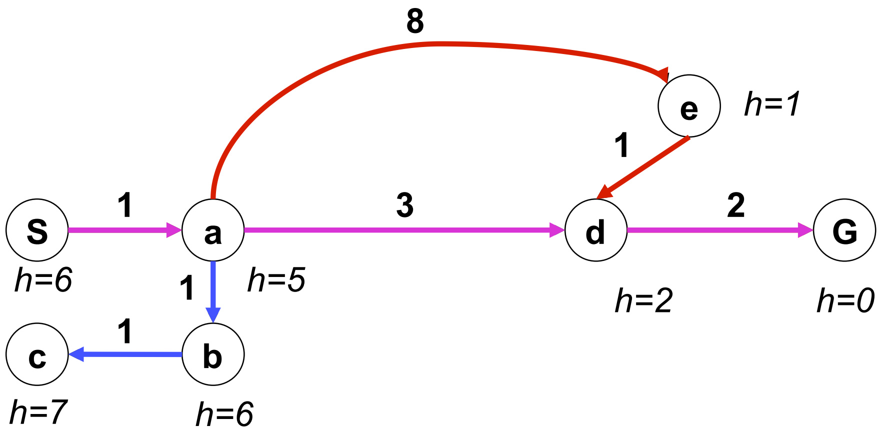
- A function that estimates how close a state to the goal.
- Designed for a particular problem.
- Examples:
- We will use $h(n)$ - heuristic value of node n

Example of heuristics



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Greedy, take the node argmin $h(n)$

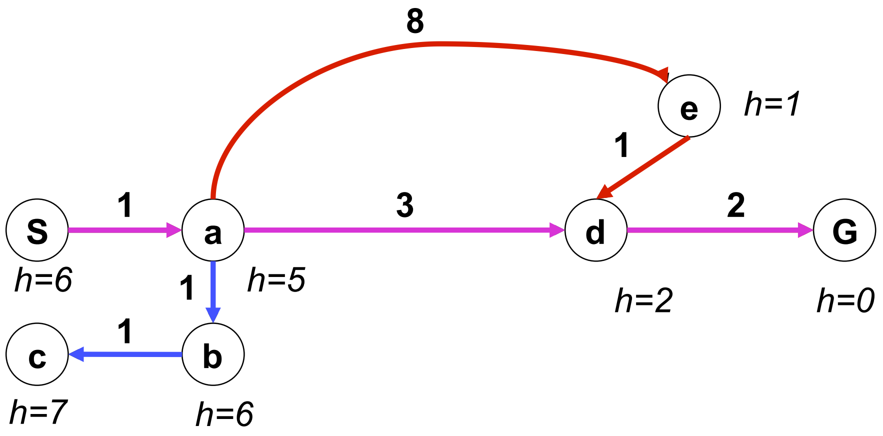


[1]

What is wrong (and nice) with the Greedy?

[1] Graph example: Ted Grenager

Greedy, take the node argmin $h(n)$



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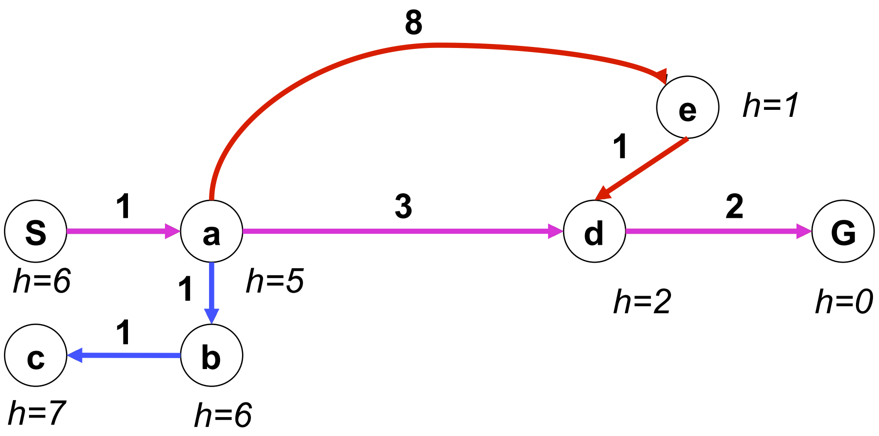
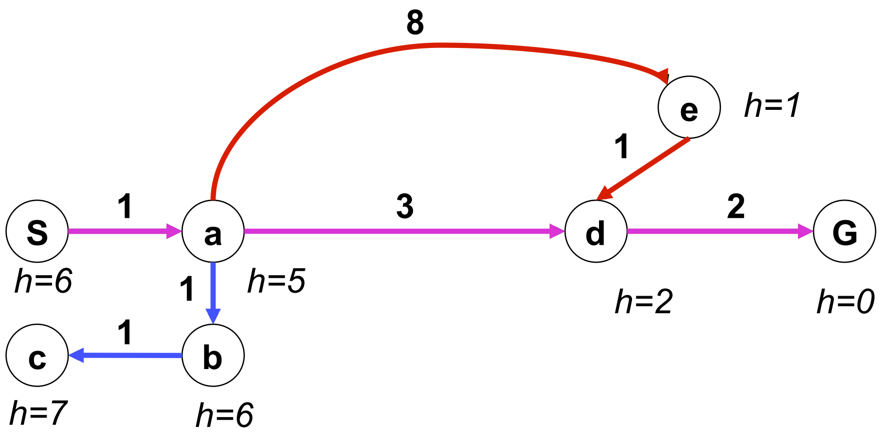
A* combines UCS and Greedy



UCS orders by backward (path) cost $g(n)$
Greedy uses heuristics (goal proximity) $h(n)$

A* orders nodes by: $f(n) = g(n) + h(n)$

A* combines UCS and Greedy



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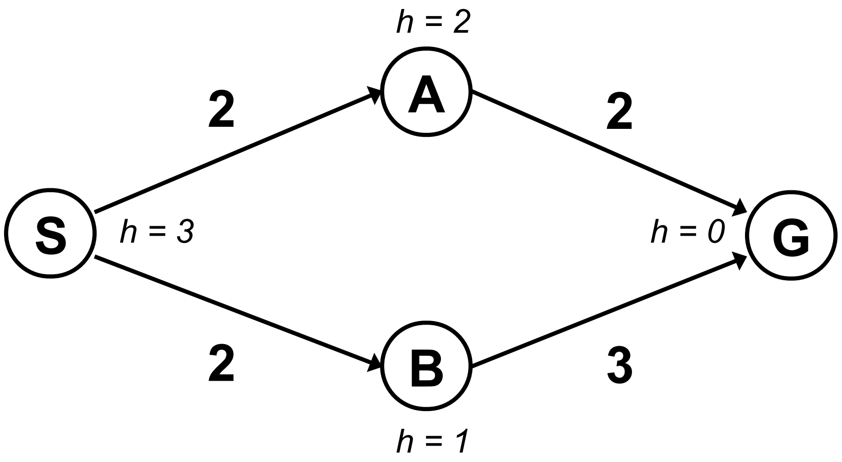
When to stop A*



[2]

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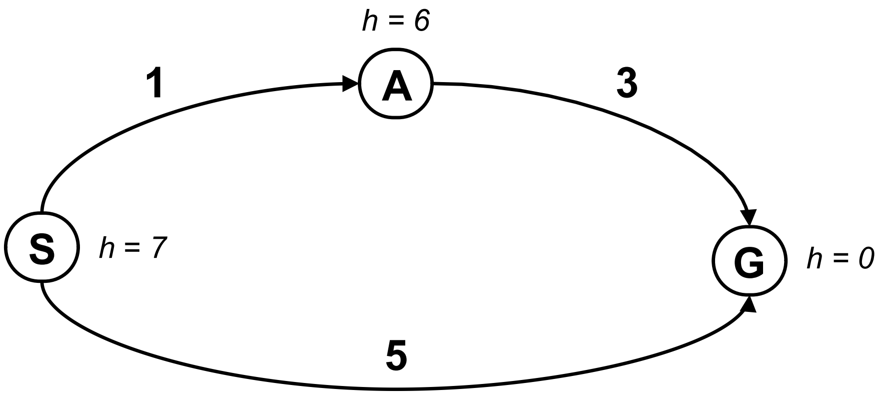
[2]Graph example: Dan Klein and Pieter Abbeel

Is A* optimal?



[3]

What is the problem?

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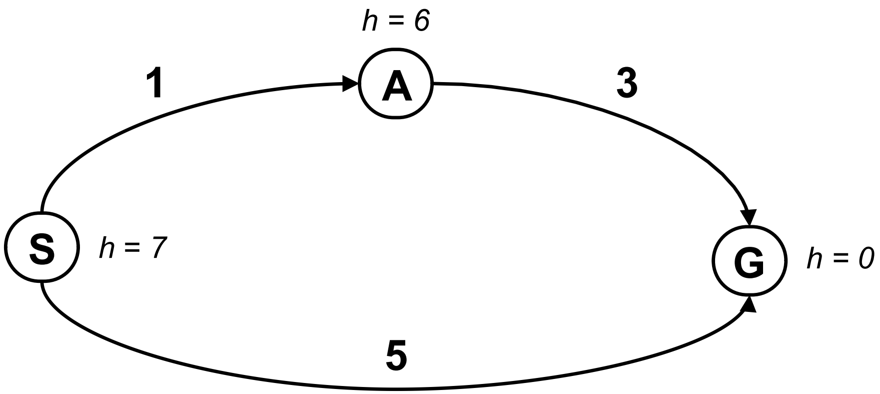
Is A* optimal?



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Admissible heuristics

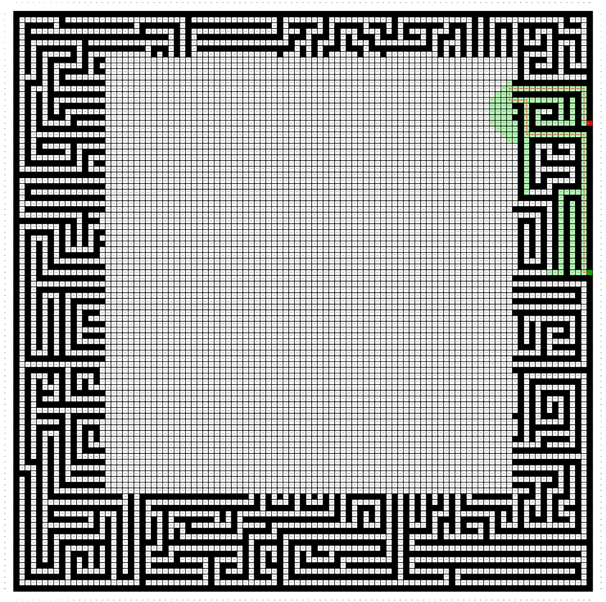
A heuristic function h is admissible if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost of going from n to the nearest goal.

Optimality of A^* tree search

Properties - does heuristic matter?



A* graph search

```
function GRAPH_SEARCH(env)
    frontier.insert(startnode)
    explored = set()
    while frontier do
        node = frontier.pop()
        if goal in node then break
        end if
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        for all nodes do
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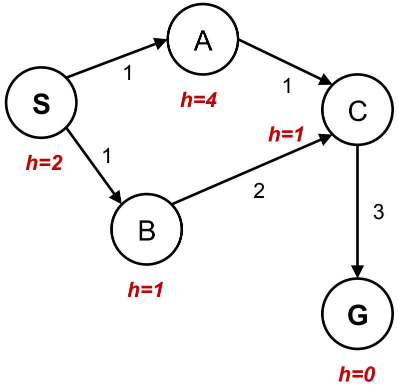

Graph example: Dan Klein and Pieter Abbeel

What went wrong?

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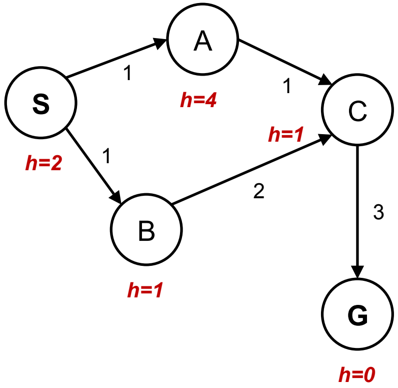

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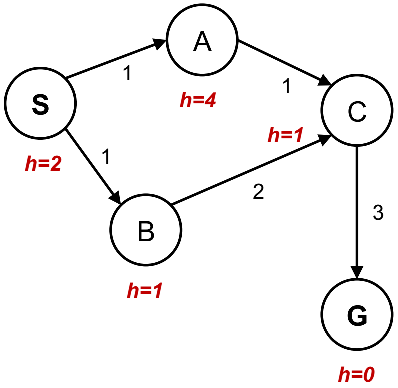
Consistent heuristics

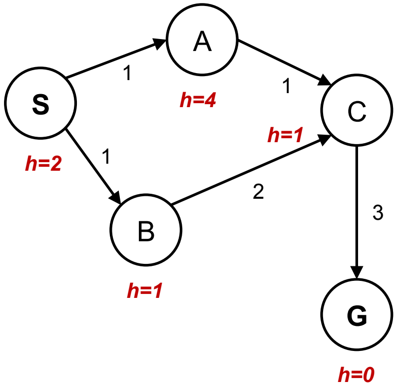


Admissible h:
$h(A) \leq$ true cost $A \to G$

Consistent h:
$h(A) - h(C) \leq$ true cost $A \to C$

f along a path never decreases!

Consistent heuristics



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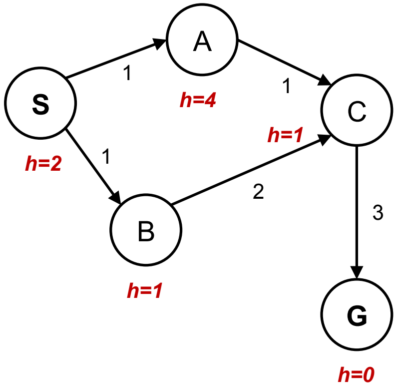

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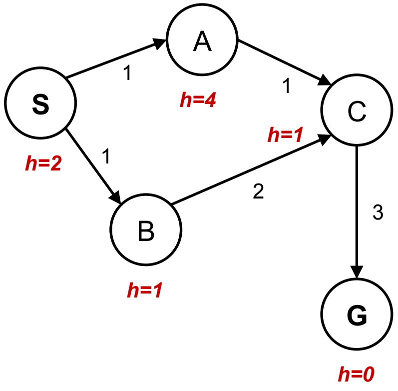

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Optimality of A*

- admissible h for tree search
- consistent h for graph search
- What about UCS?
- Are all consistent heuristics also admissible?
  $h(A) - h(C) \leq \text{cost}(A \to C)$

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How to find a heuristics?

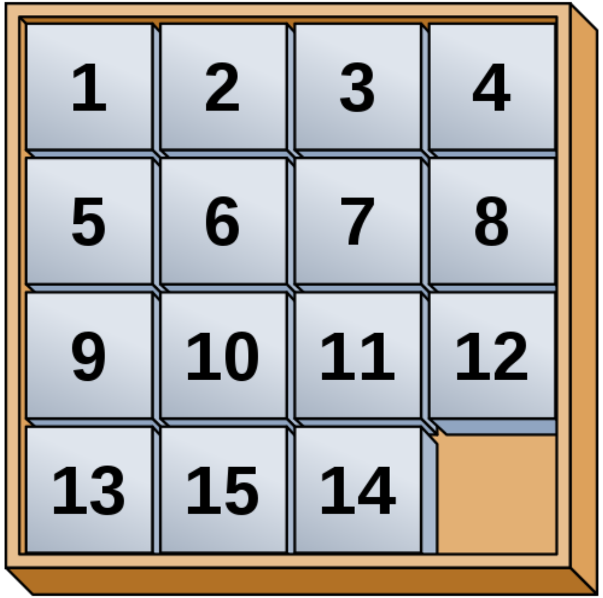

Image from: `https://commons.wikimedia.org/wiki/File:15-puzzle-loyd.svg`

Which heuristics is the best?