Problem solving by search II

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Outline

- Graph search
- Heuristics (how to search faster)
- Greedy
- A^*. A-star search.

Recap: Search

A tree search recap

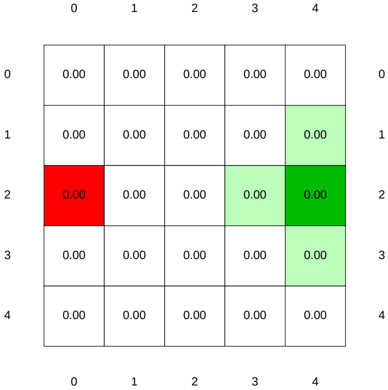
```
function TREE_SEARCH(problem) return a solution or failure
    initialize the frontier the initial state of the problem
    loop
        if the frontier is empty then return failure
        else choose a node from frontier and remove from frontier
        end if
        if the node contains a goal state then return the solution
        end if
        Expand the node and add the resulting nodes to frontier
    end loop
end function
```

A Maze, what could possibly go wrong?

	0	1	2	3	4	
0	0.00	0.00	0.00	0.00	0.00	0
1	0.00	0.00	0.00	0.00	0.00	1
2	0.00	0.00	0.00	0.00	0.00	2
3	0.00	0.00	0.00	0.00	0.00	3
4	0.00	0.00	0.00	0.00	0.00	4
	0	1	2	3	4	

Tree search the maze

function TREE_SEARCH(env) **return** a solution or failure
  initialize the frontier
  while frontier **do**
    node = frontier.pop()
    if goal in node **then**
      break
    end if
    nodes = env.expand(node.state)
    Add nodes to frontier
  end while
end function



A graph search

```
function GRAPH_SEARCH(env) return a solution or failure
    init frontier by the start state
    initialize the explored set to be empty
    while frontier do
        node = frontier.pop()
        if goal in node then break
        end if
        nodes = env.expand(node.state)
        add node to explored
        for all nodes do
            if node not in explored (or in frontier) then
                add nodes to frontier
            end if
        end for
    end while
end function
```

⚠ **Do not forget:** node is not the same as state!

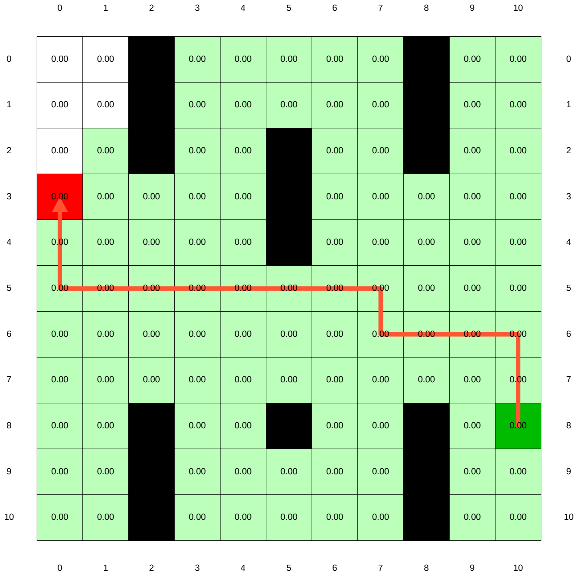
The BFS graph search

```
function BFS_GRAPH_SEARCH(env) return a solution or failure
    node ← env.observe()
    frontier ← FIFOqueue(node)
    explored ← set()
    while frontier not empty do
        node ← frontier.pop()
        explored.add(node.state)                    ▷ adding state not node!
        child_nodes ← env.expand(node.state)
        for all child_nodes do
            if child_node.state not in explored or in frontier then
                if child_node contains Goal then return child_node
                end if
                fronter.insert(child_node)
            end if
        end for
    end while
end function
```

The UCS graph search

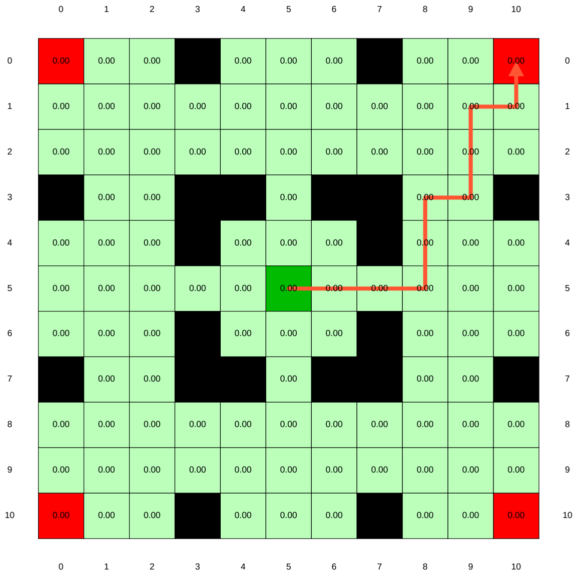
```
function UCS_GRAPH_SEARCH(env) return a solution or failure
    node ← env.observe()
    frontier ← priority_queue(node)                    ▷ path_cost for ordering
    explored ← set()
    while frontier not empty do
        node ← frontier.pop()
        if node contains Goal then return node         ▷ check here!
        end if
        explored.add(node.state)
        child_nodes ← env.expand(node.state)
        for all child_nodes do
            if child_node.state not in explored or in frontier then
                fronter.insert(child_node)
            else if child_node.state in frontier with higher cost then
                replace that with the child_node
            end if
        end for
    end while
```

Few examples of search strategies so far



Run the demos.

What is wrong with UCS and other strategies?



Run the demo.

Node selection, take argmin $f(n)$

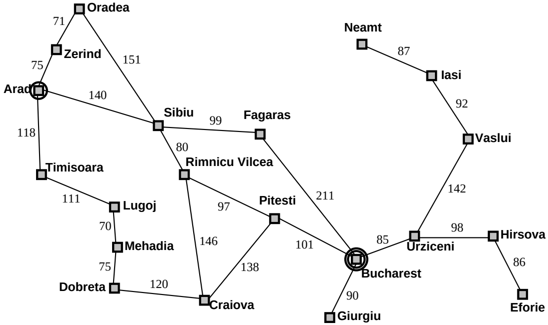
- DFS: $f(n) = -n.depth$
- BFS: $f(n) = n.depth$
- UCS: $f(n) = n.path\_cost$

The good: frontier as a priority queue The bad: All the $f(n)$ correspond to the cost from n to the start - only backward cost.

Heuristics

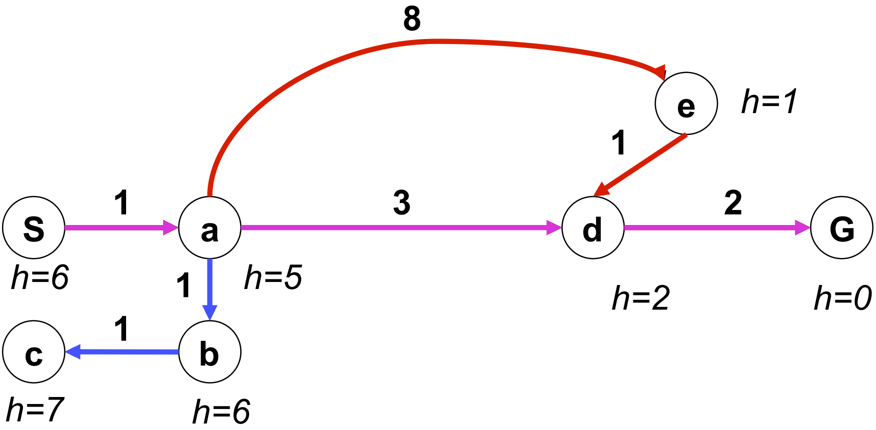
- A function that estimates how close a state to the goal.
- Designed for a particular problem.
- Examples:
- We will use $h(n)$ - heuristic value of node n

Example of hueristics



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

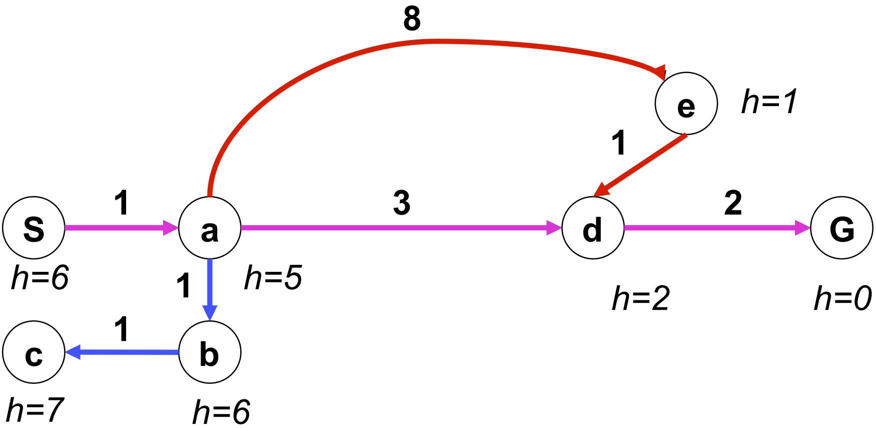
Greedy, take the node argmin $h(n)$



What is wrong (and nice) with the Greedy?

[1] Graph example: Ted Grenager

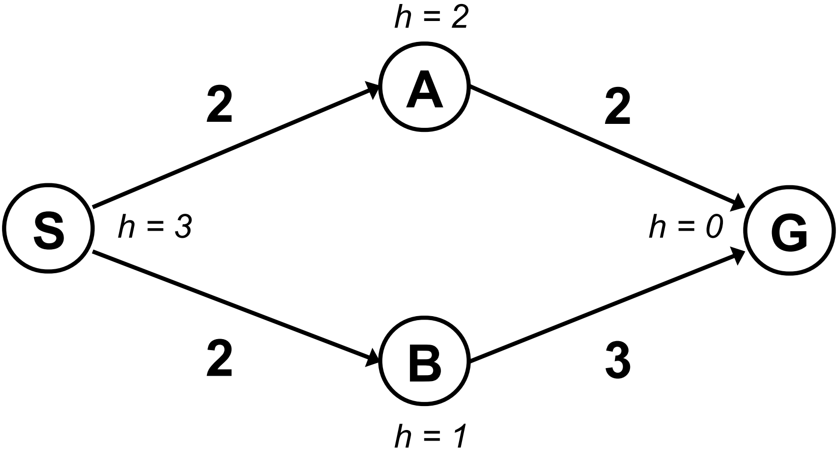
A* combines UCS and Greedy



UCS orders by backward (path) cost $g(n)$
Greedy uses heuristics (goal proximity) $h(n)$

A* orders nodes by: $f(n) = g(n) + h(n)$

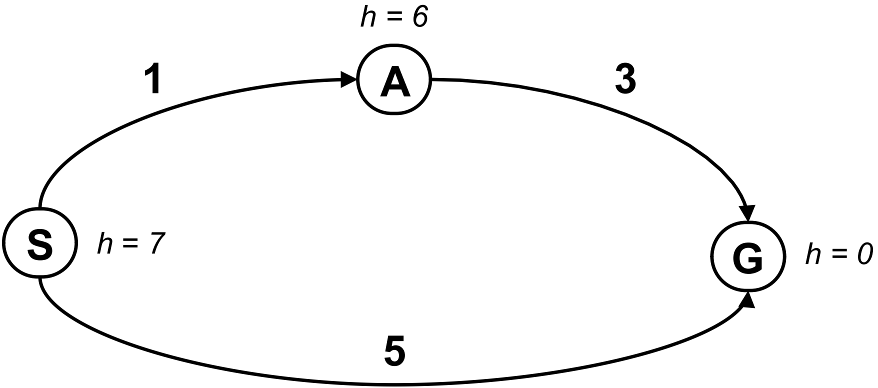
When to stop A*



[2]

[2]Graph example: Dan Klein and Pieter Abbeel

Is A* optimal?



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What is the problem?

[3]Graph example: Dan Klein and Pieter Abbeel

Admissible heuristics

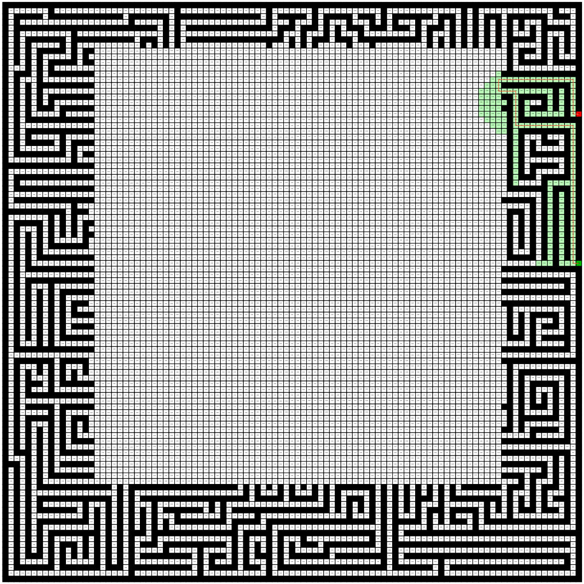
A heuristic function h is admissible if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost of going from n to the nearest goal.

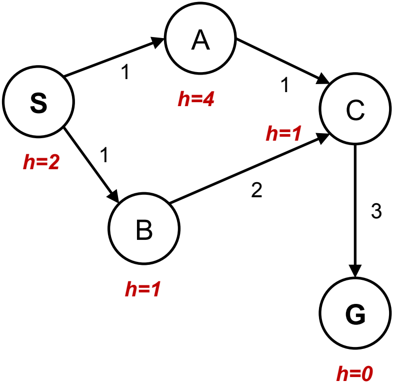
Optimality of A* tree search

Properties - does heuristic matter?



A* graph search

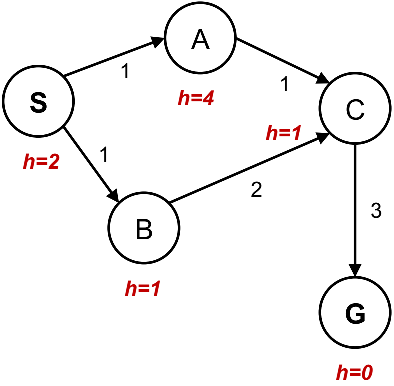
```
function GRAPH_SEARCH(env)
    frontier.insert(startnode)
    explored = set()
    while frontier do
        node = frontier.pop()
        if goal in node then break
        end if
        nodes = env.expand(node.state)
        explored.add(node)
        for all nodes do
            if node not in explored then
                frontier.insert(node)
            end if
        end for
    end while
end function
```



Graph example: Dan Klein and Pieter Abbeel

What went wrong?

Consistent heuristics



Admissible h:
$h(A) \leq$ true cost $A \rightarrow G$

Consistent h:
$h(A) - h(C) \leq$ true cost $A \rightarrow C$

f along a path never decreases!

Optimality of A*

- admissible h for tree search
- consistent h for graph search
- What about UCS?
- Are all consistent heuristics also admissible?
  $h(A) - h(C) \leq \text{cost}(A \rightarrow C)$

How to find a heuristics?

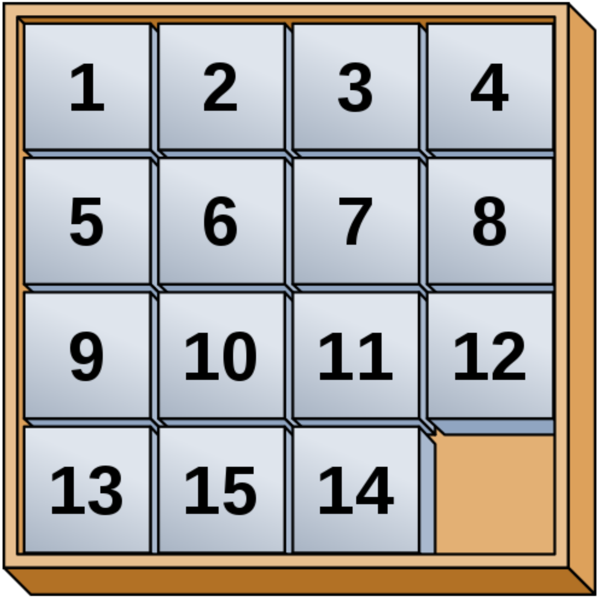


Image from: https://commons.wikimedia.org/wiki/File:15-puzzle-loyd.svg

Which heuristics is the best?