Problem solving by search

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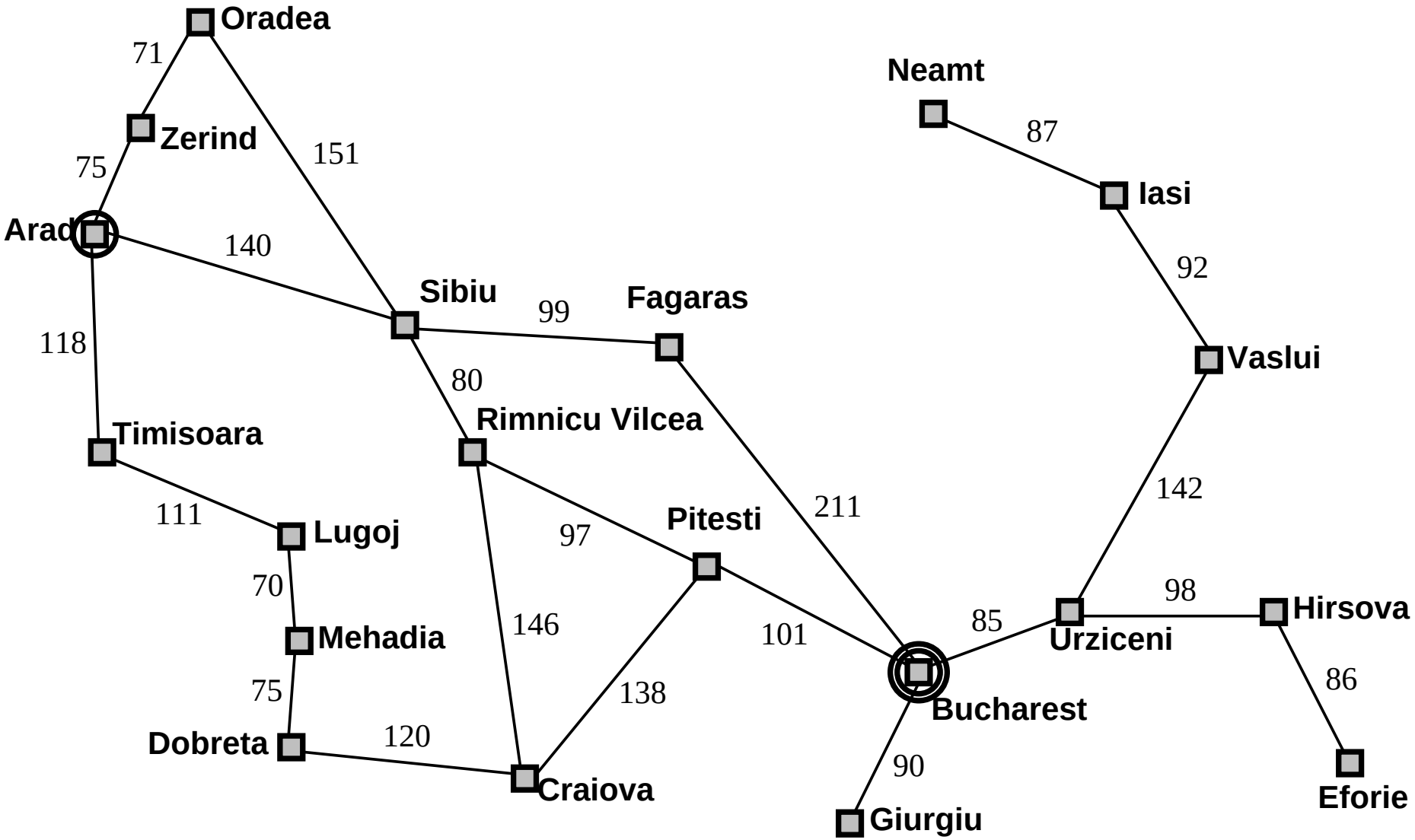
Outline

- Search problem.
- State space graphs.
- Search trees.
- Strategies, which tree branches to choose?
- Strategy/Algorithm properties?
- Programming instrastructure

Example: Romania



Example: Romania

Goal:

be in Bucharest

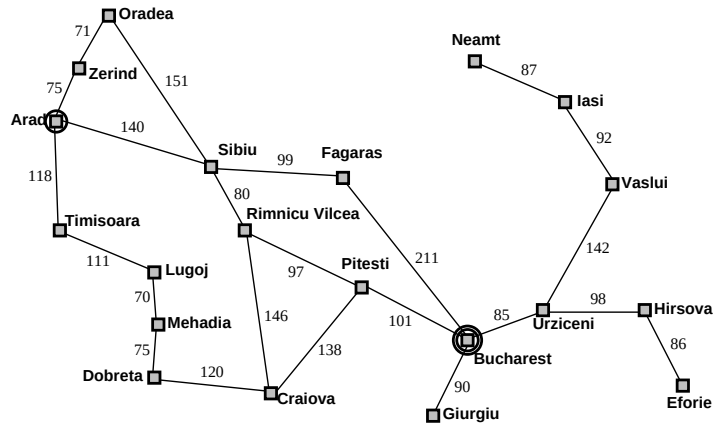
Problem formulation:

states: position in a city (cities)

actions: drive between cities

Solution:

Sequence of cities (path)



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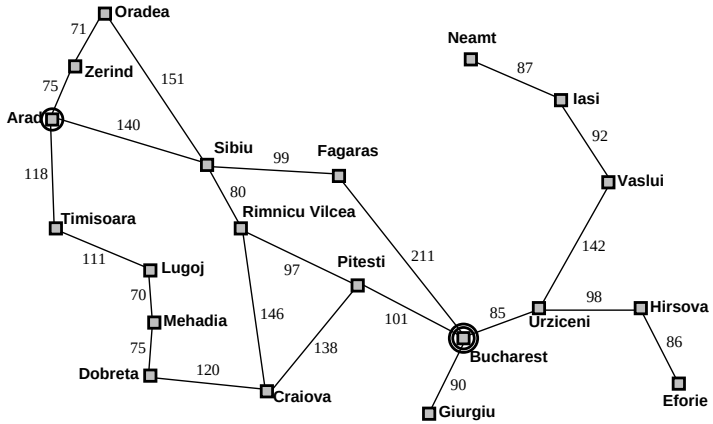

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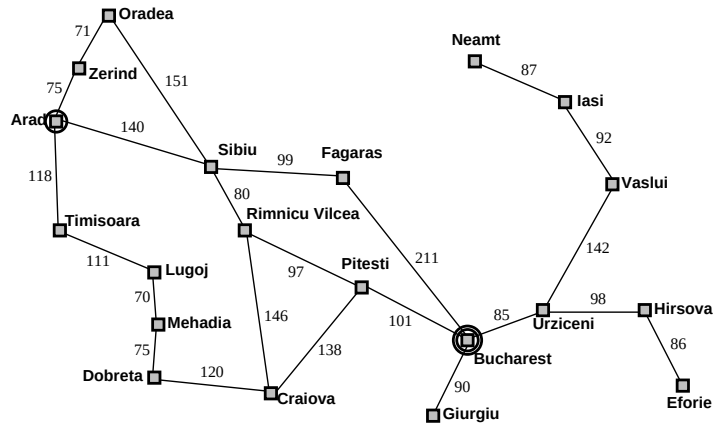

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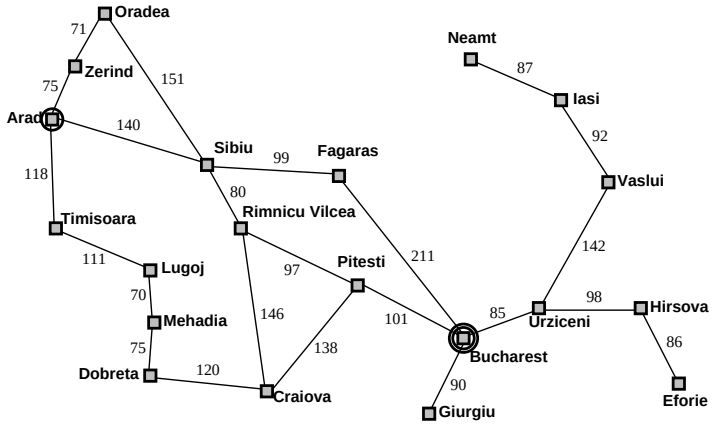

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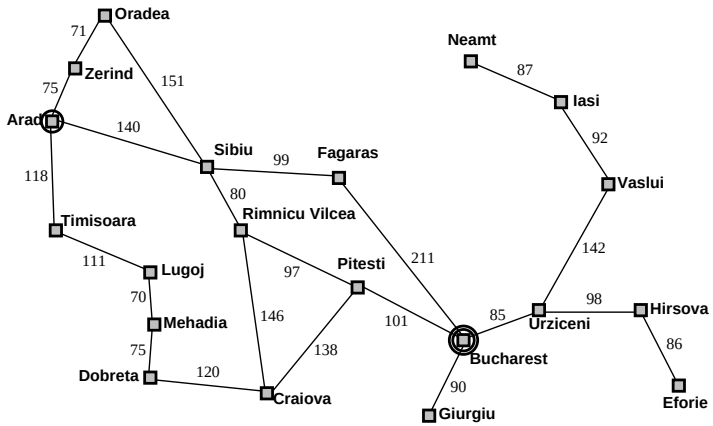
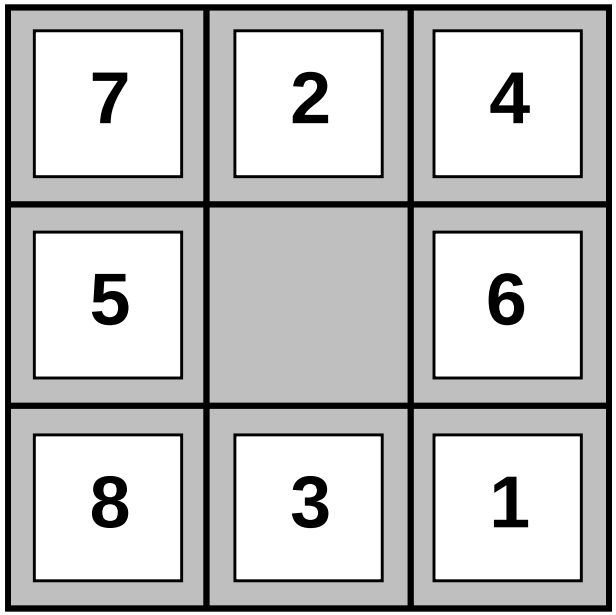
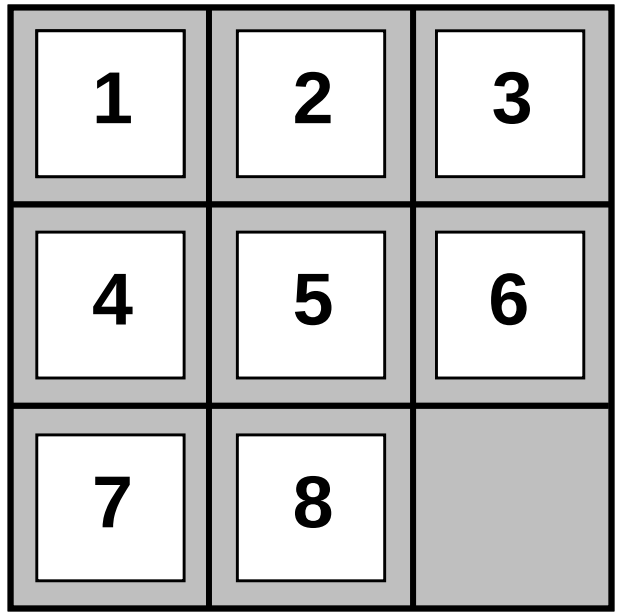


Example: The 8-puzzle



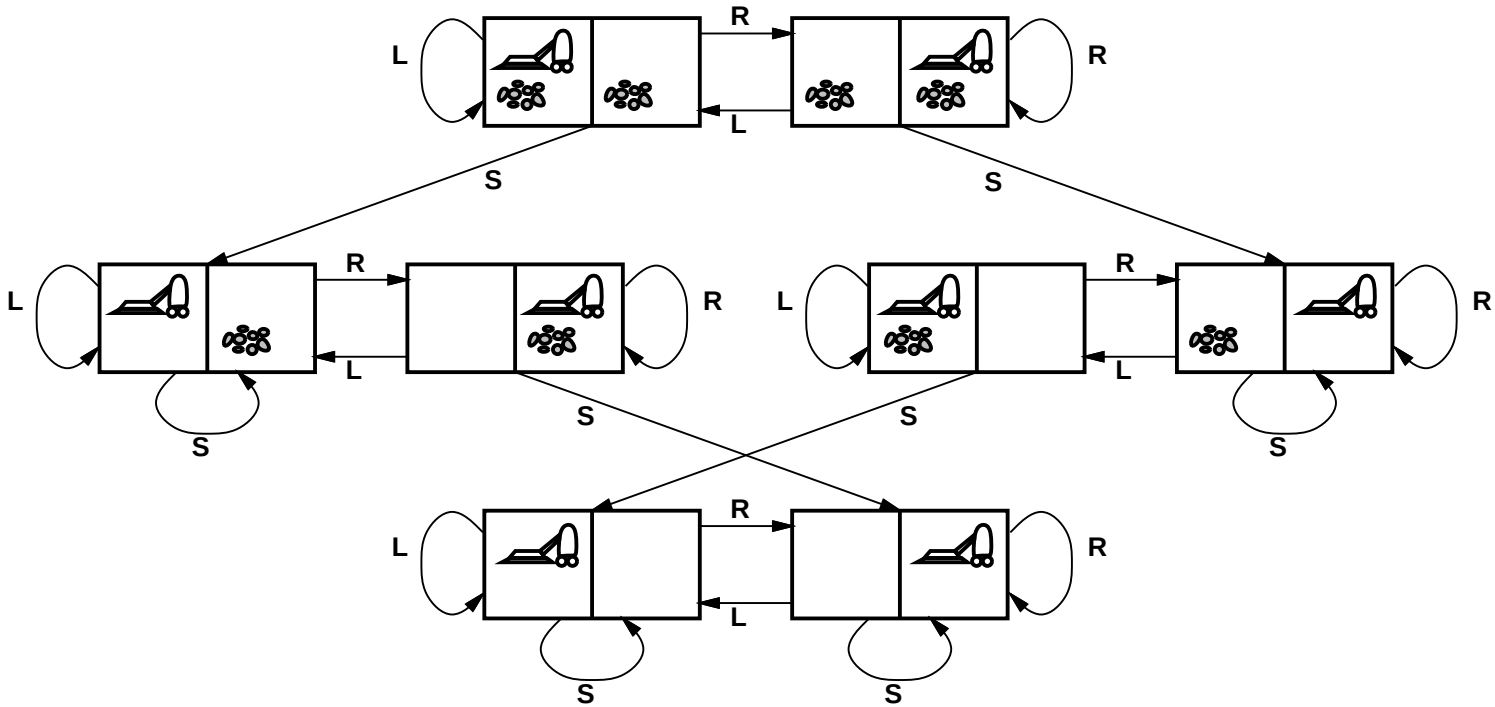
Start State



Goal State

states?
actions?
solution?
cost?

Example: Vacuum cleaner



states?

actions?

solution?

cost?

A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left . . .
- Transition model : Given state and action return state (and cost)
- Goal test : Are we done?

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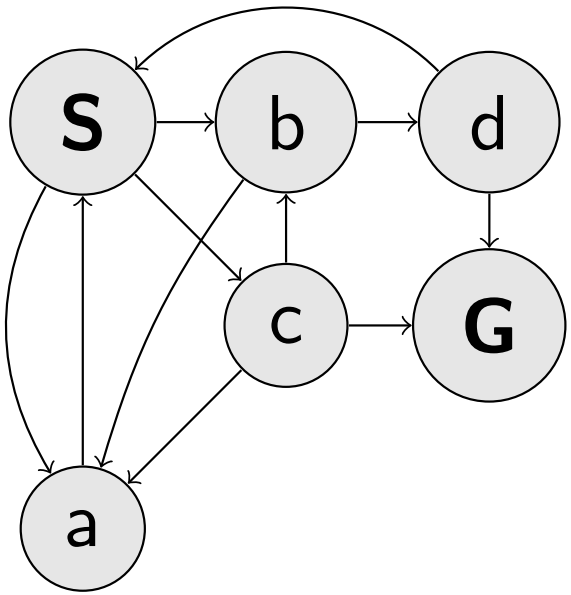
State Space Graphs

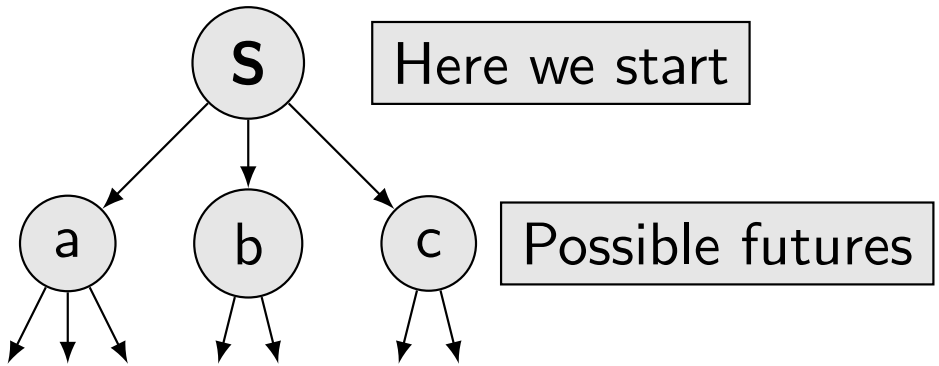
State space graph: a representation of a search problem

- Graph Nodes – states – are abstracted world configurations
- Arcs represent action results
- Goal test – a set of goal nodes

Each state occurs only *once* in a state (search) space.



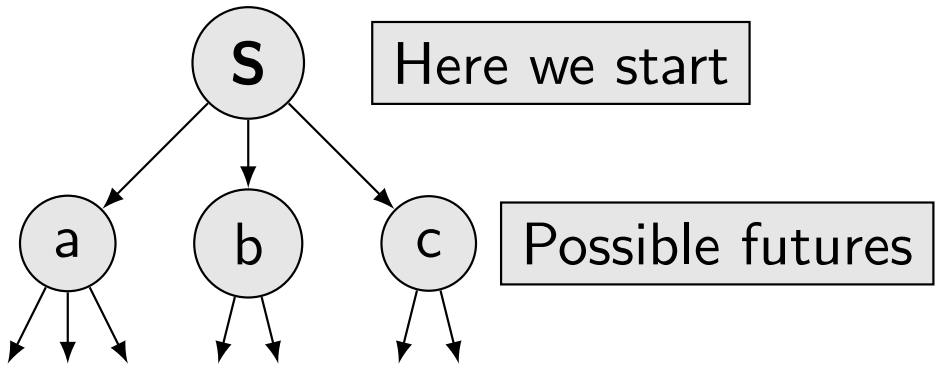
Search Trees



- A "what if" tree of plans and their outcomes
- Start node is the root
- Children are successors
- Nodes show states, but correspond to *plans* that achieve those states

What does the last item mean, actually?

Search Trees



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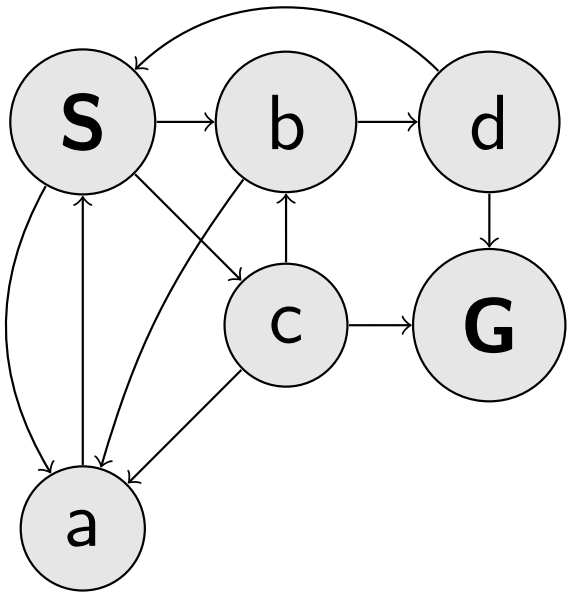

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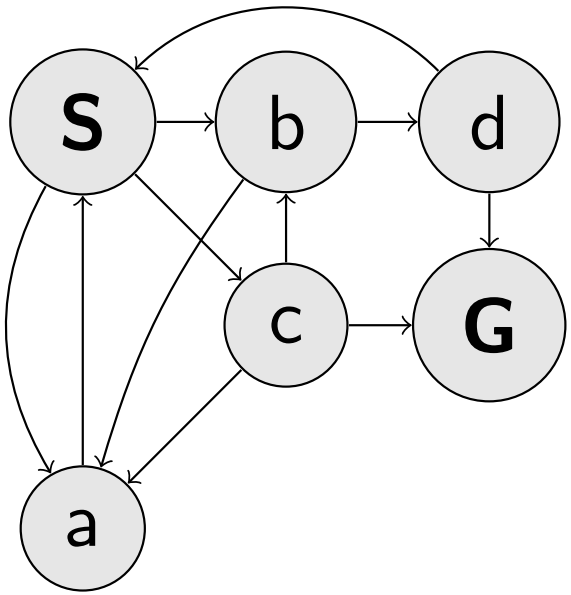

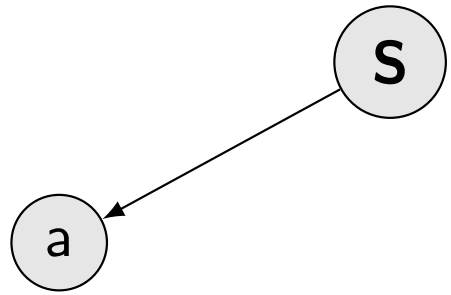

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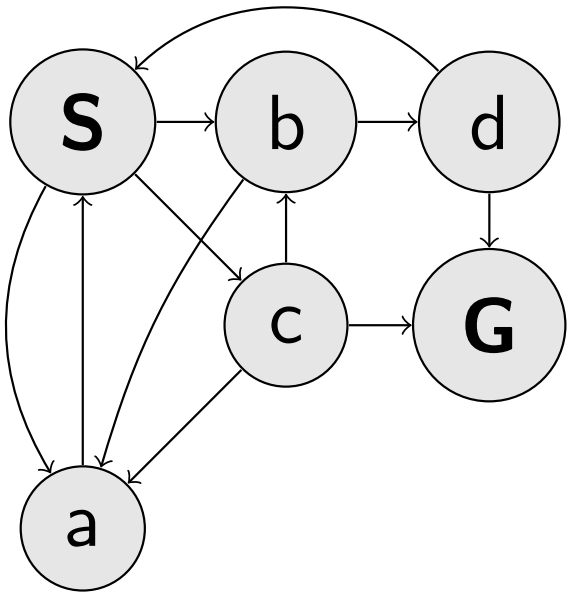

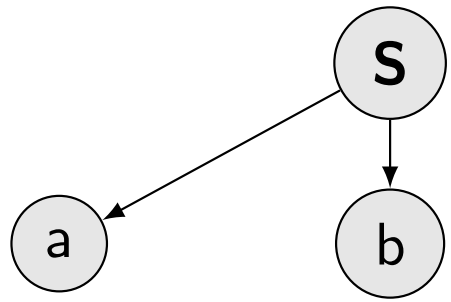

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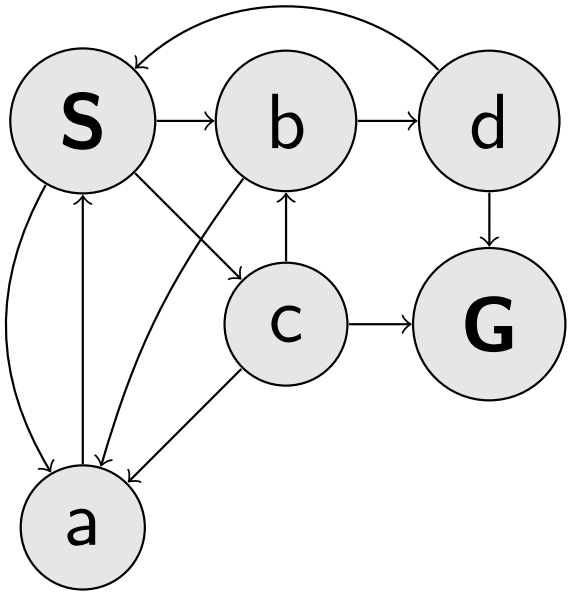

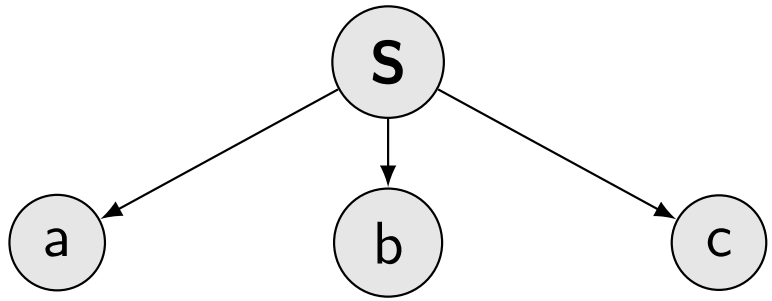

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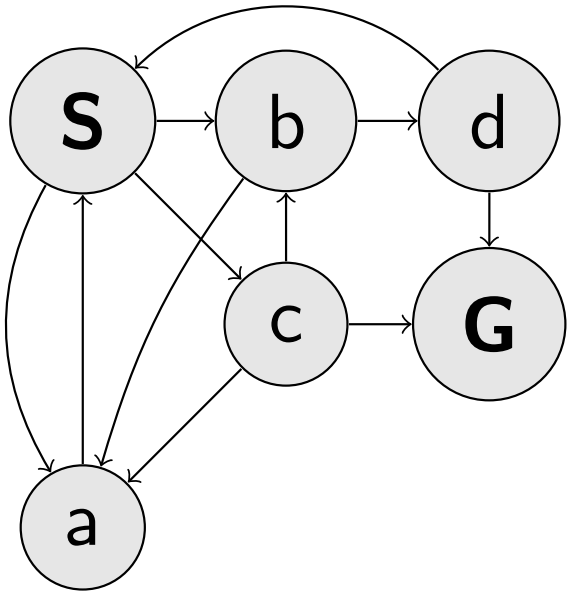

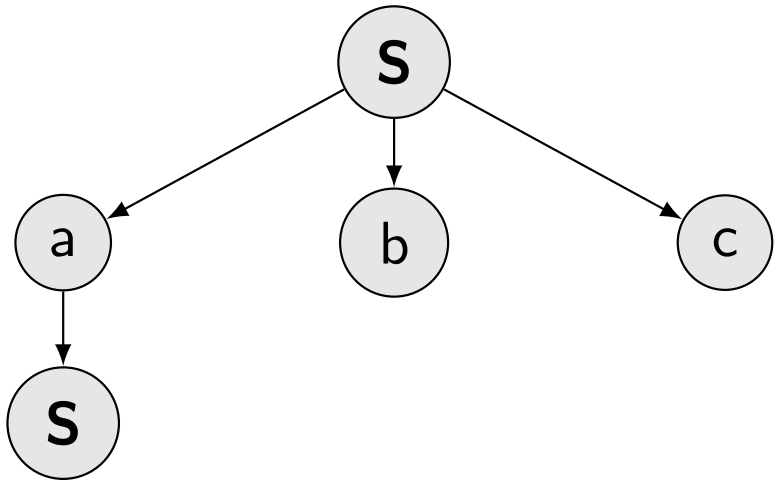

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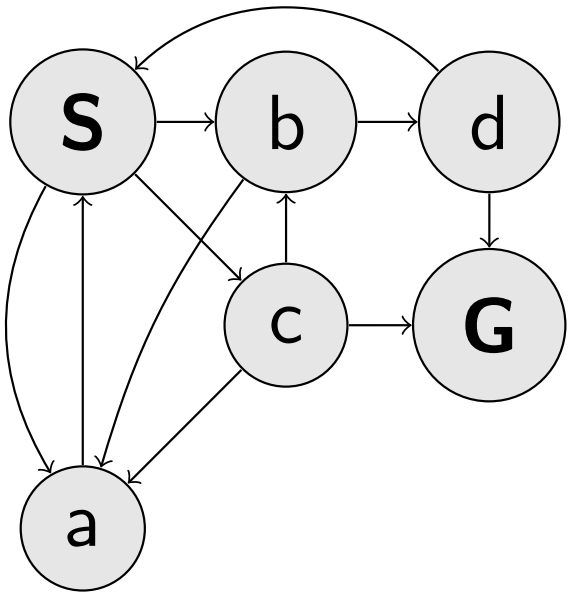

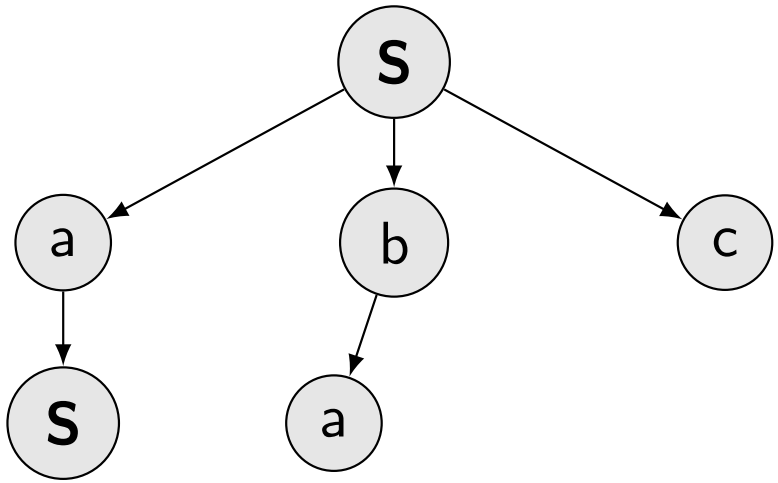

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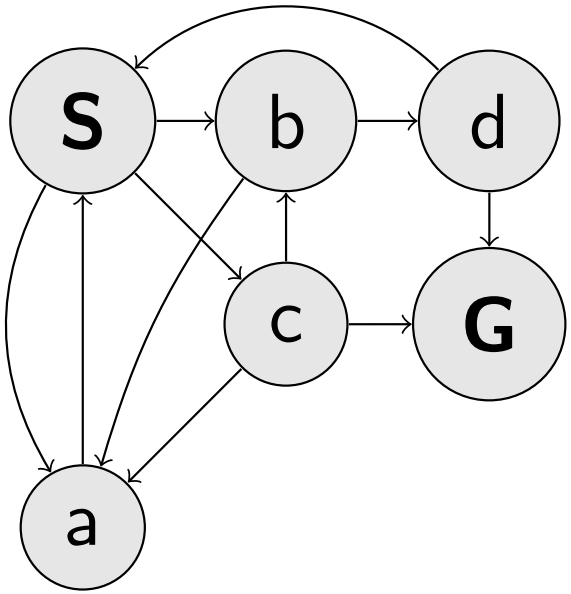

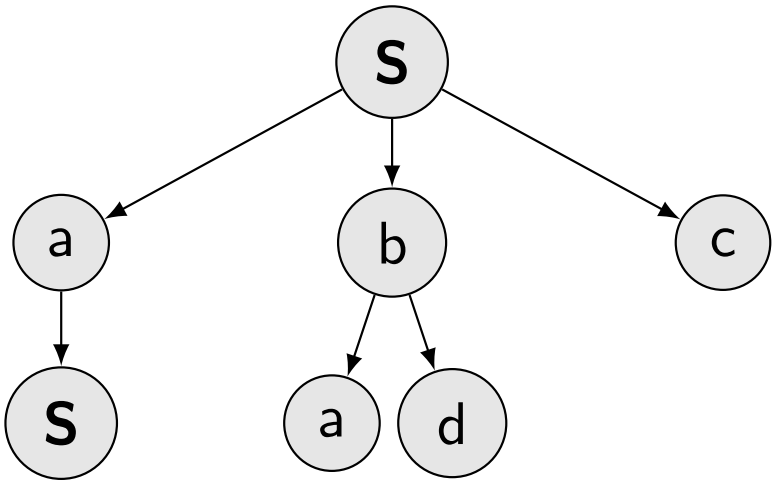

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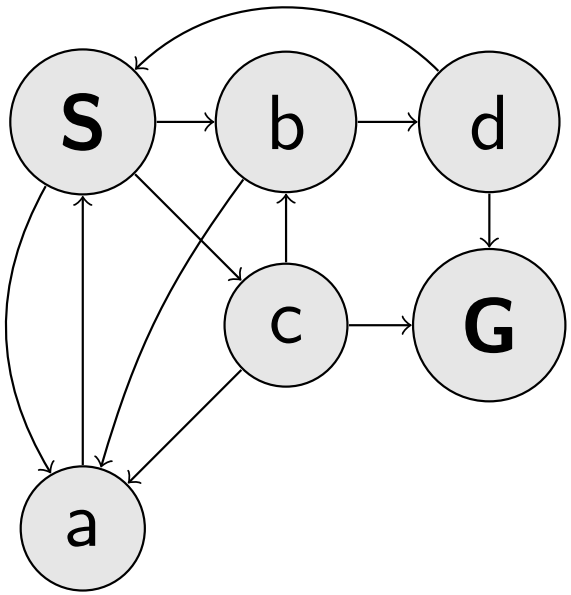

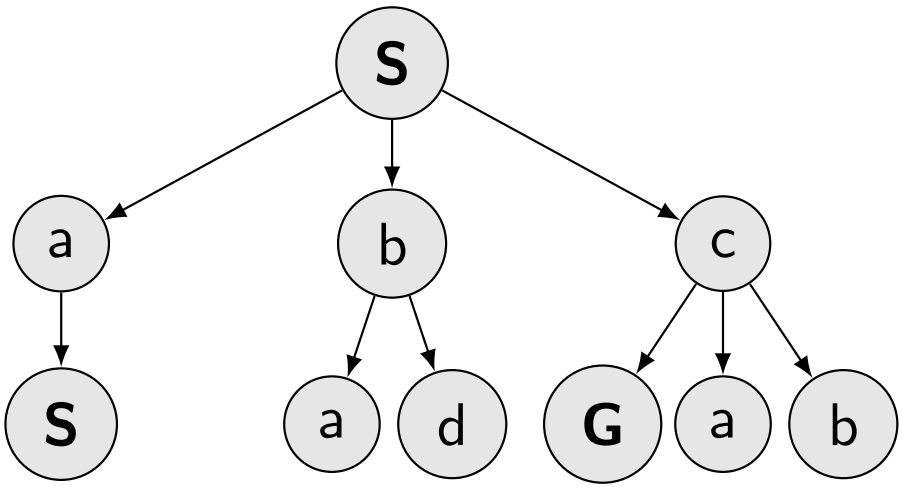

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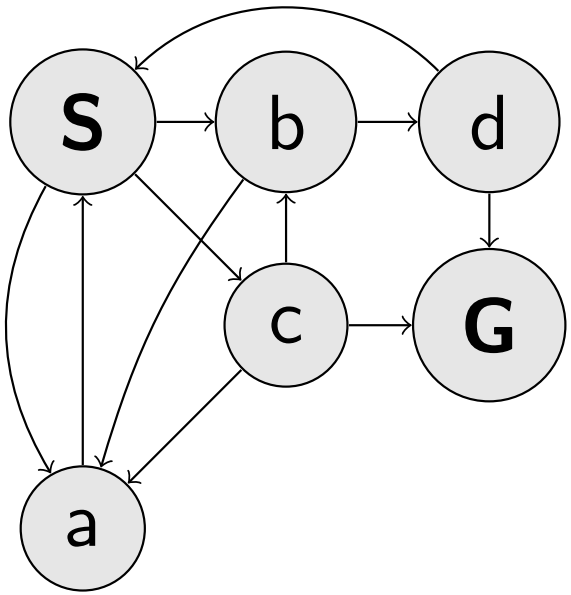

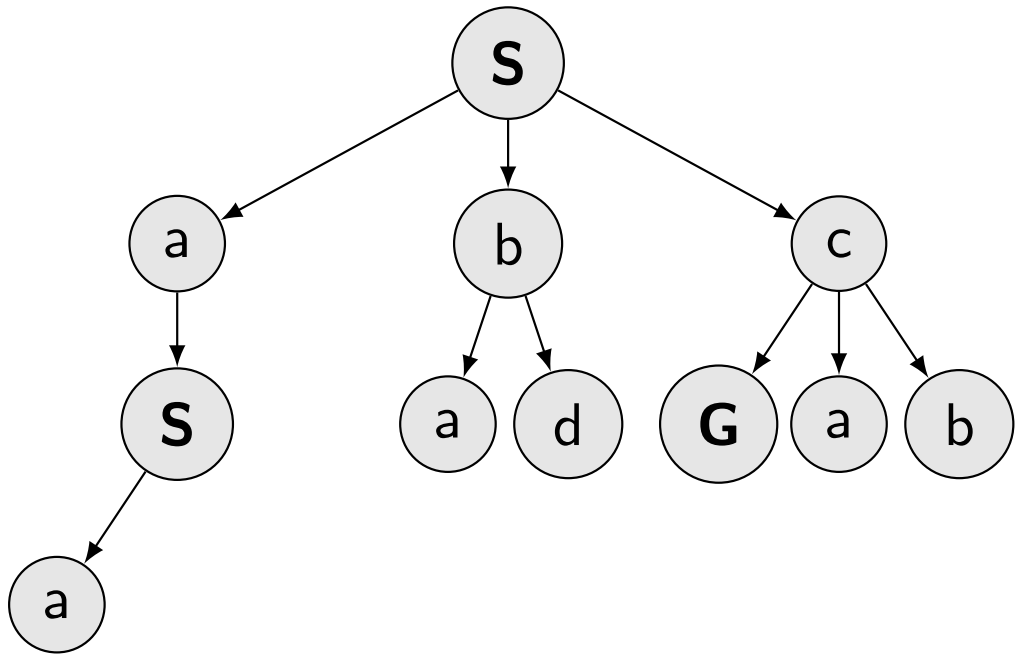

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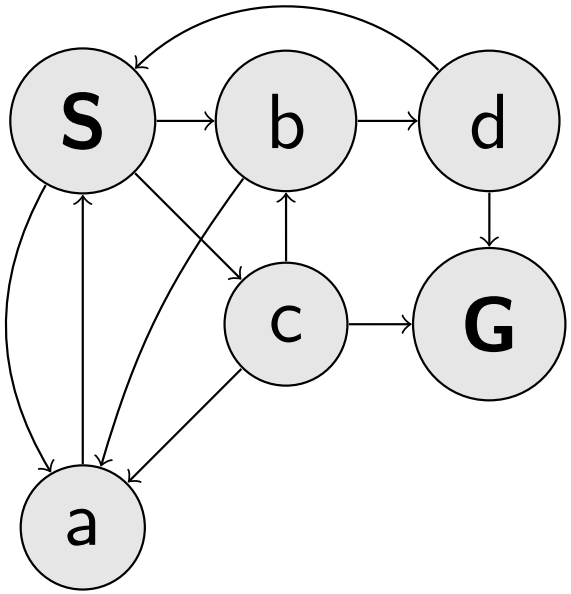

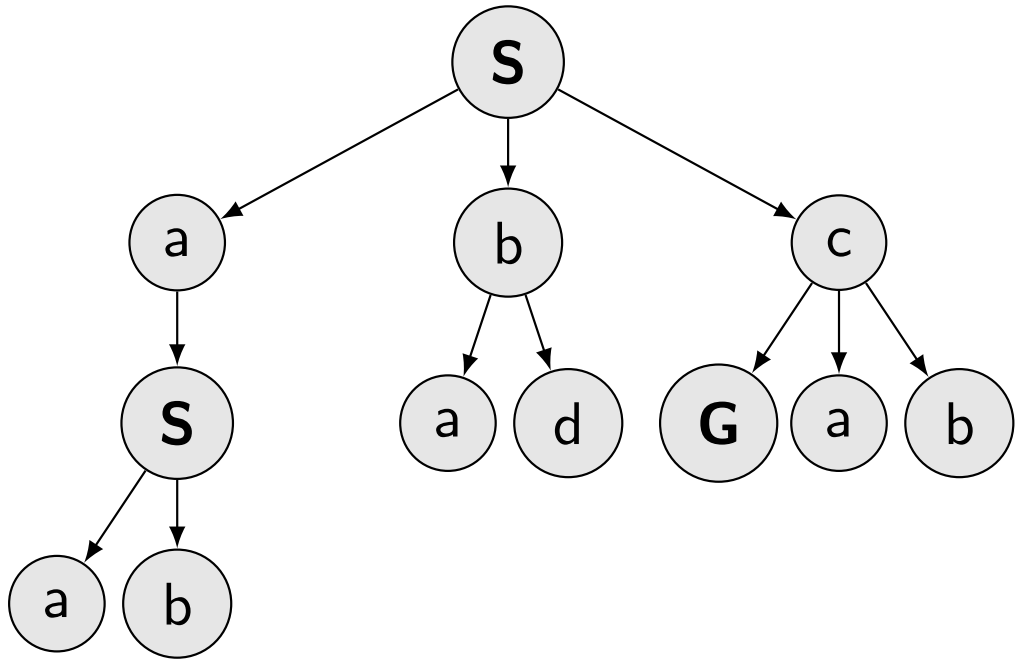

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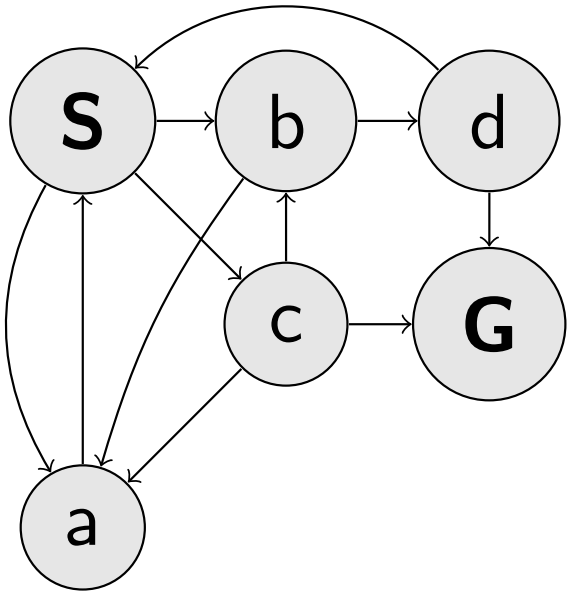

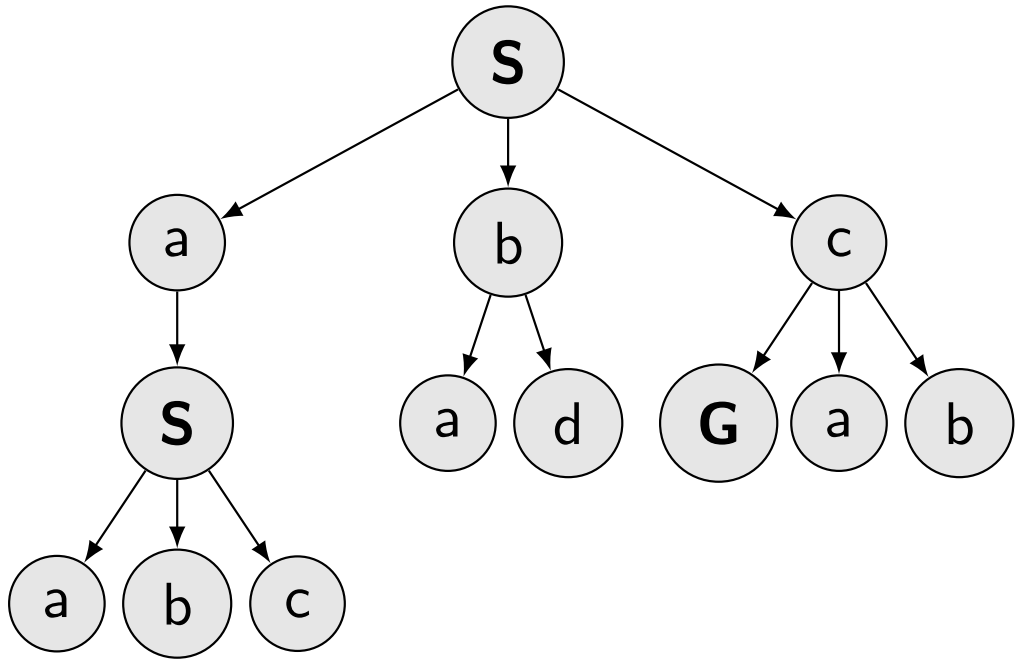

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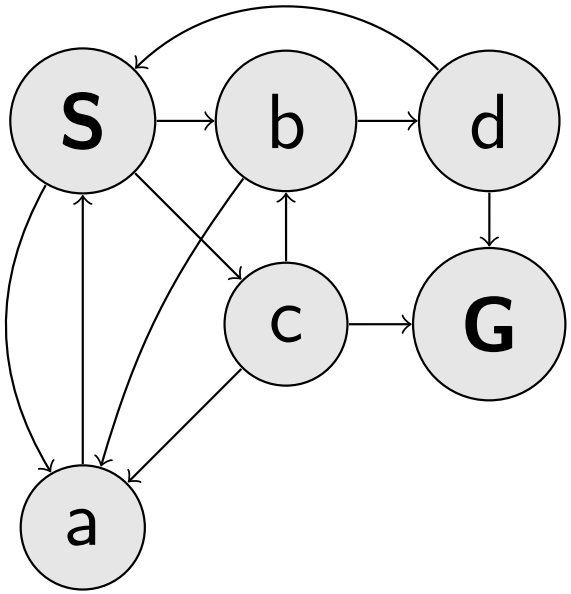

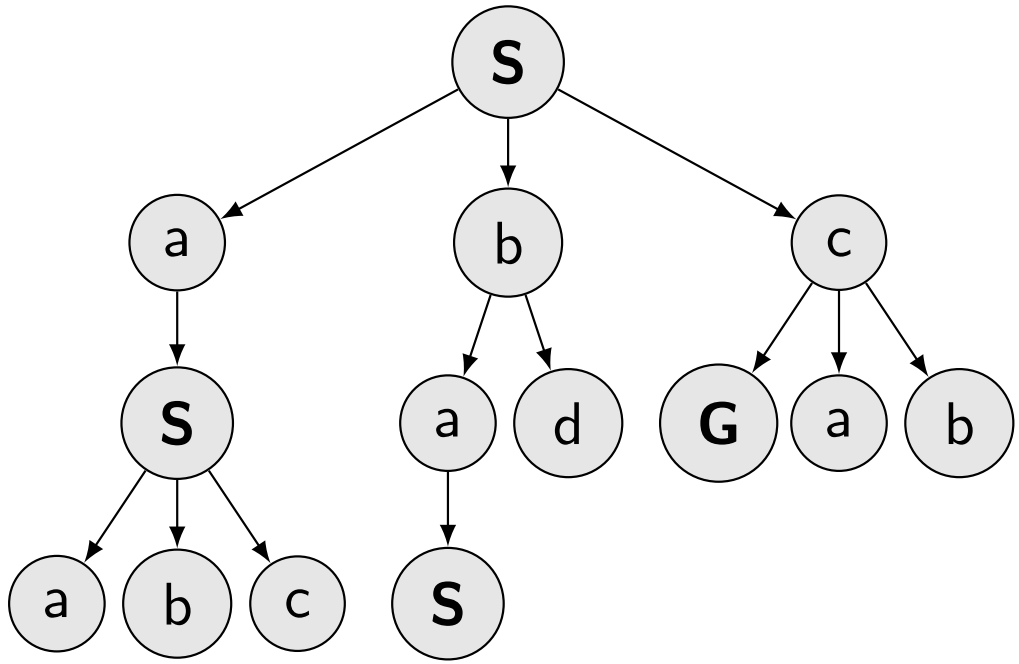

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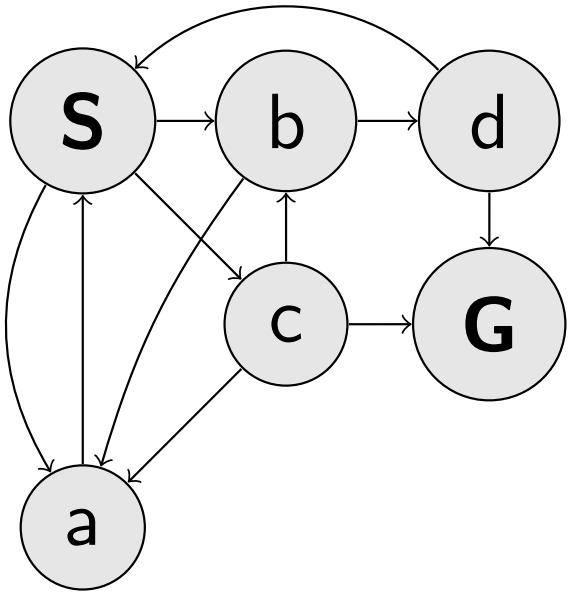

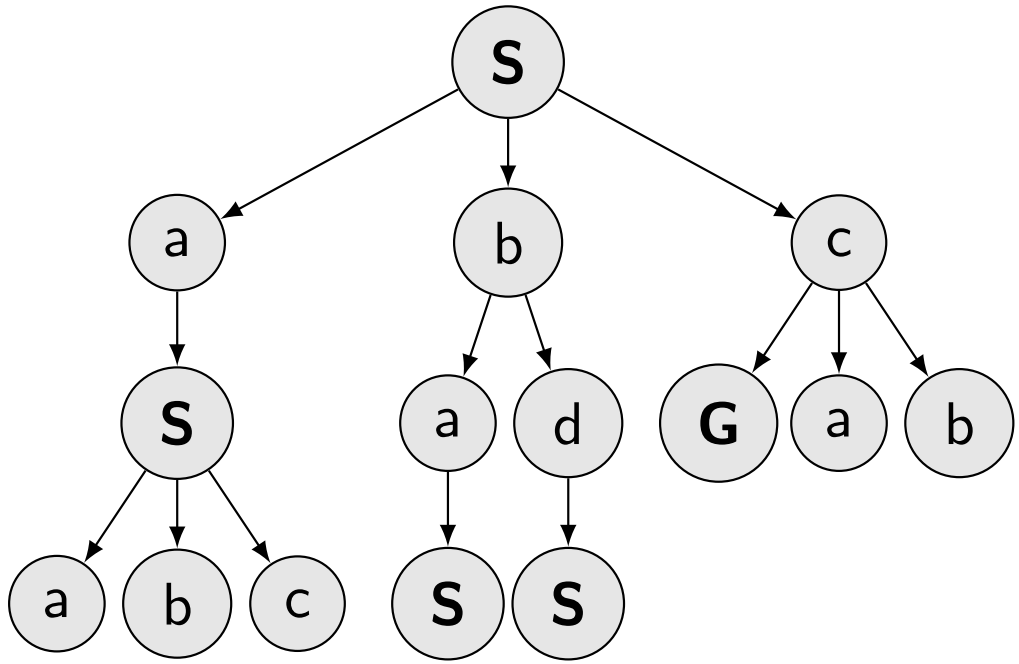

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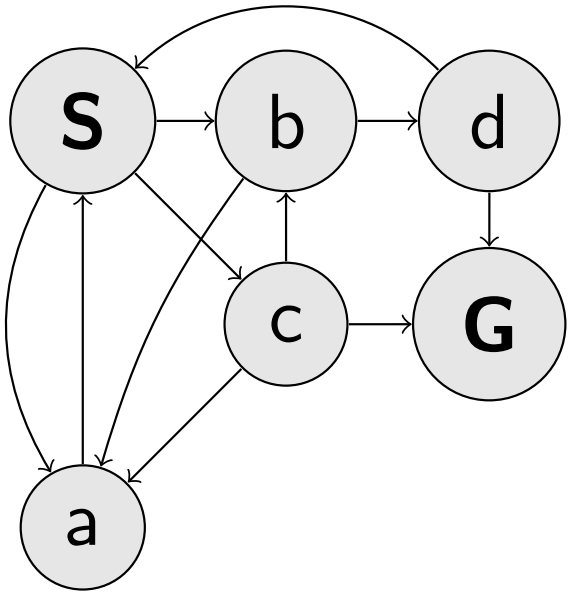

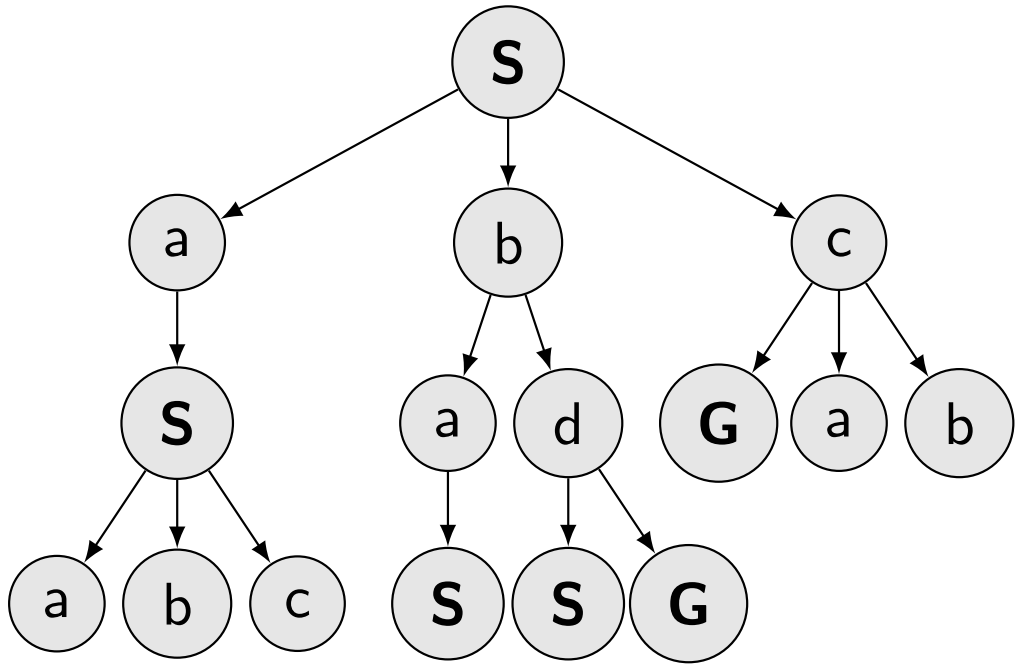

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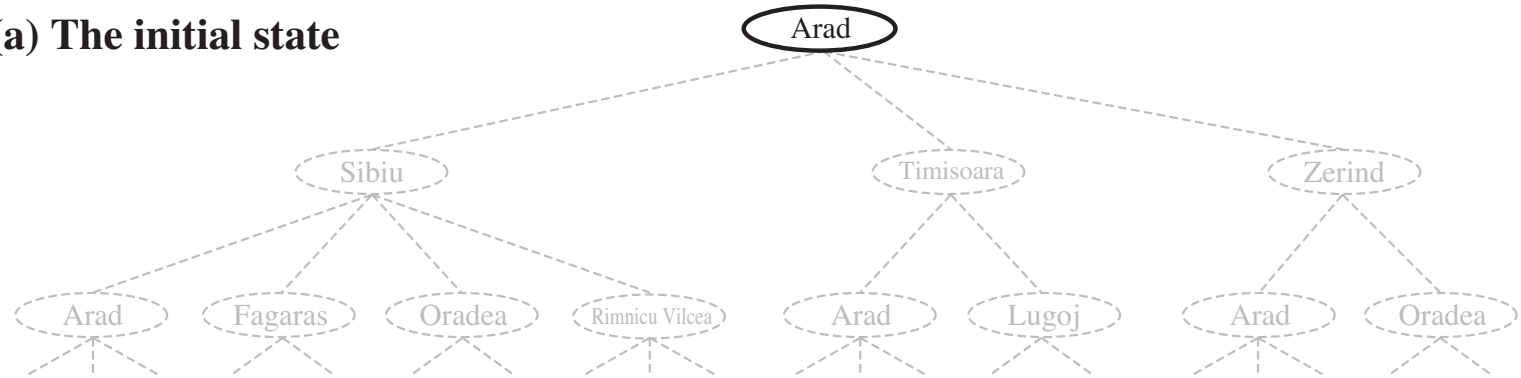
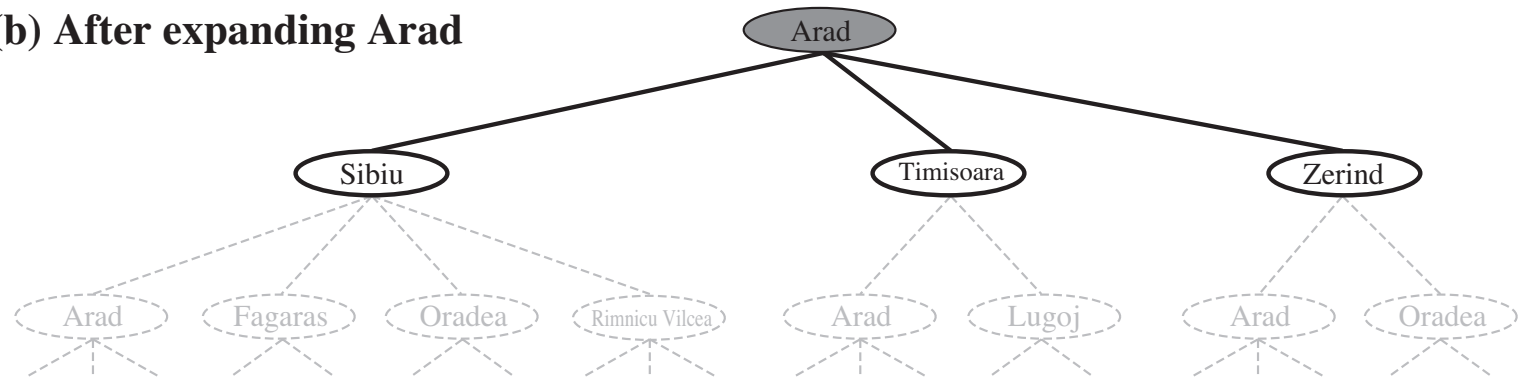
Search tree for Romania

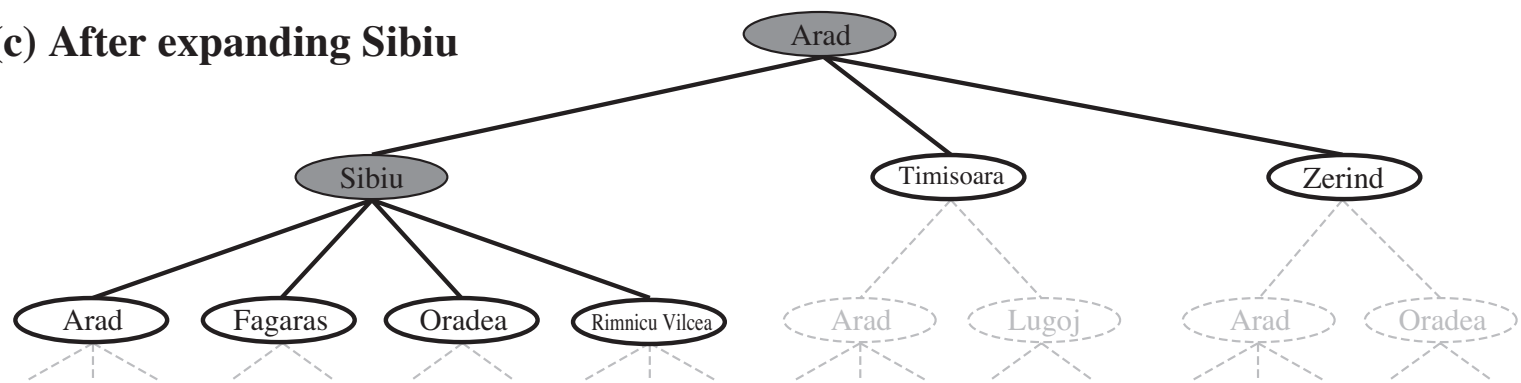
(a) The initial state



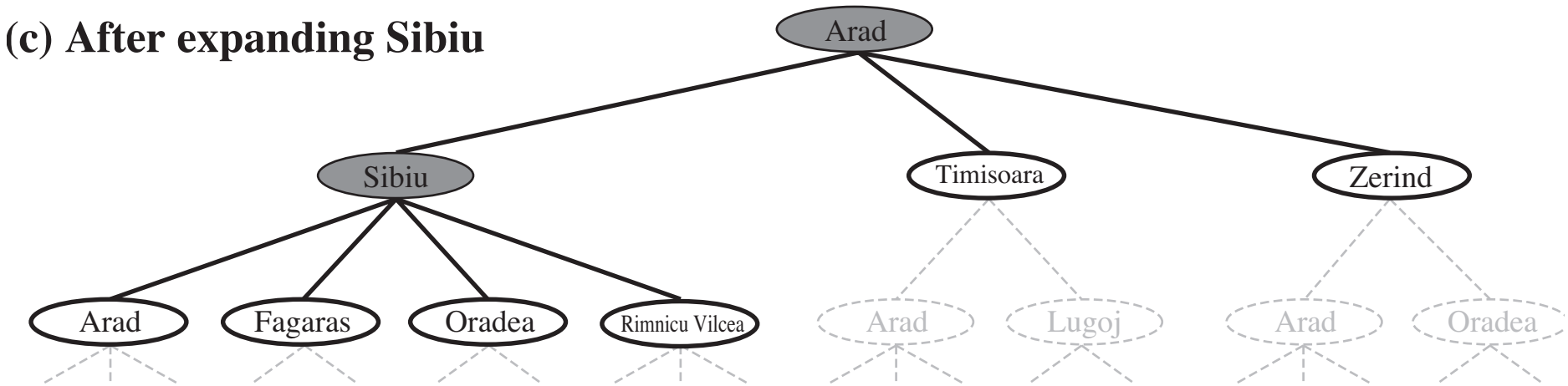
(b) After expanding Arad



(c) After expanding Sibiu



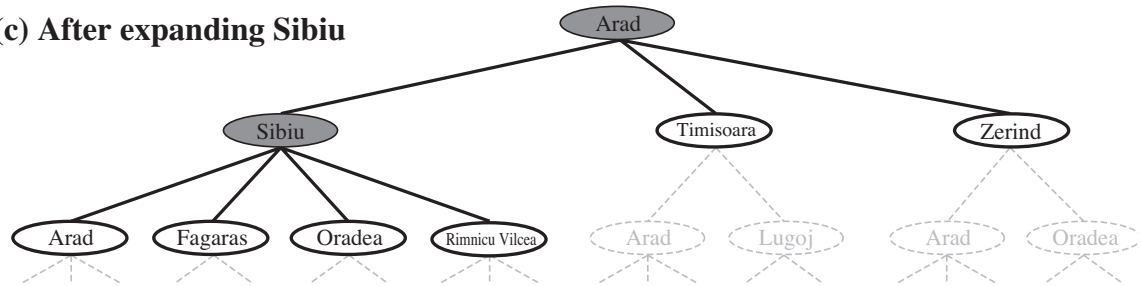
Search elements



- Expand plans - possible ways (tree nodes).
- Manage/Maintain fringe (or frontier) of plans under consideration.
- Expand new nodes *wisely(?).*

Tree search algorithm

(c) After expanding Sibiu



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initialize by using the initial state of the problem
loop
if no candidates for expansion **then return** failure
else choose a leaf node for expansion
end if
if the node contains a goal state **then return** the solution
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Expand the node and add the resulting nodes to the tree
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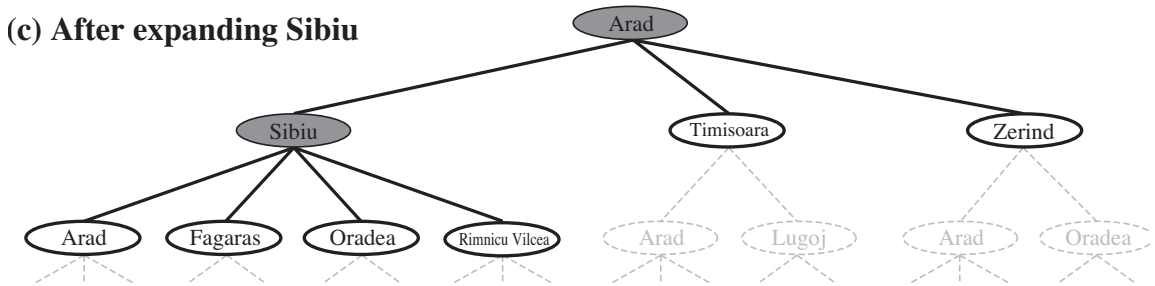
Tree search algorithm



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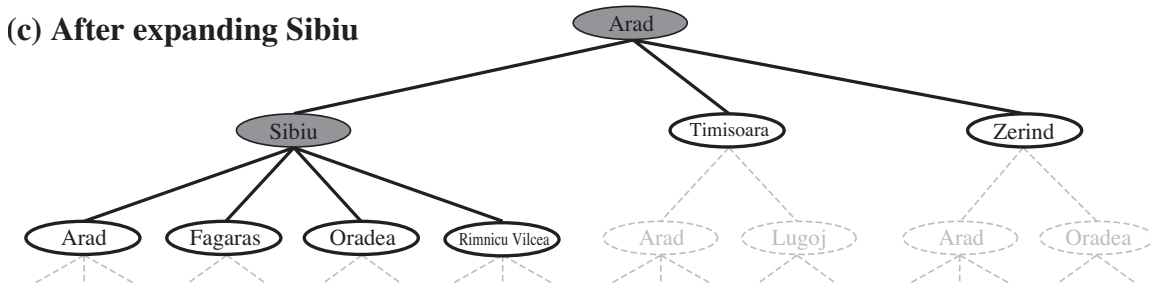

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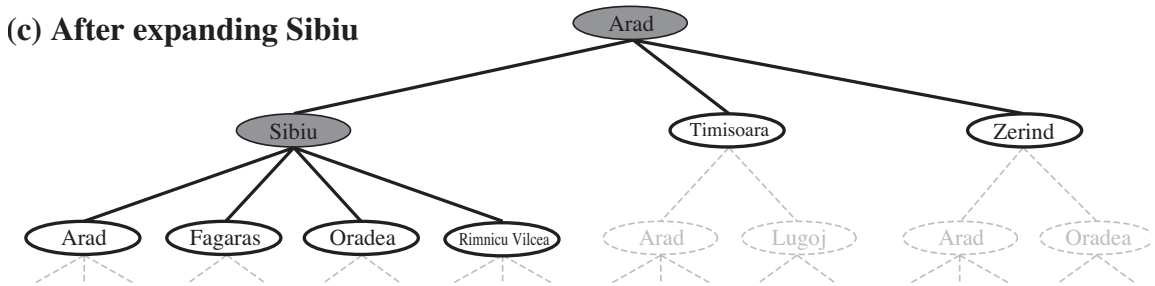

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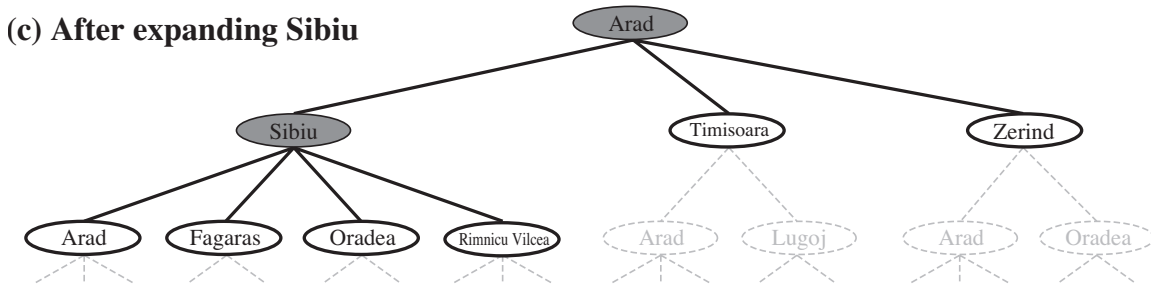

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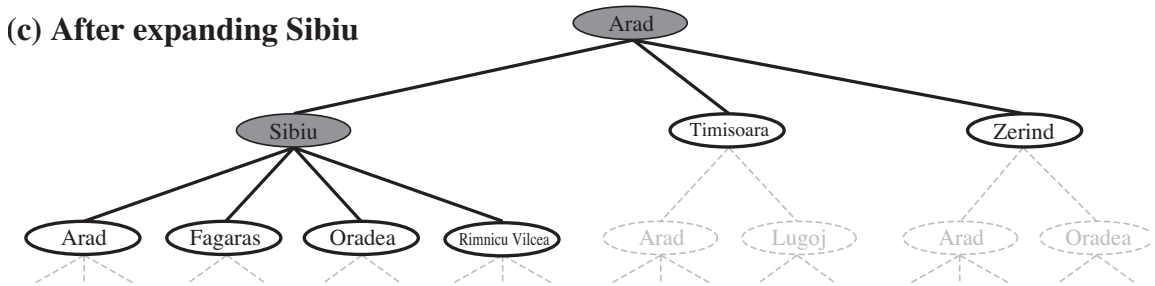

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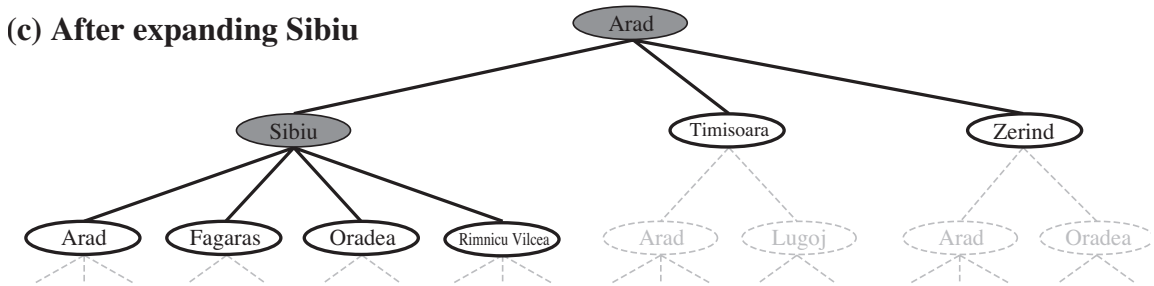

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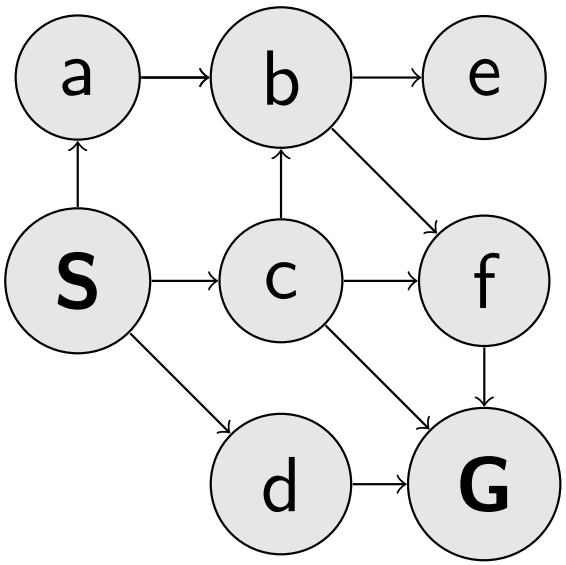

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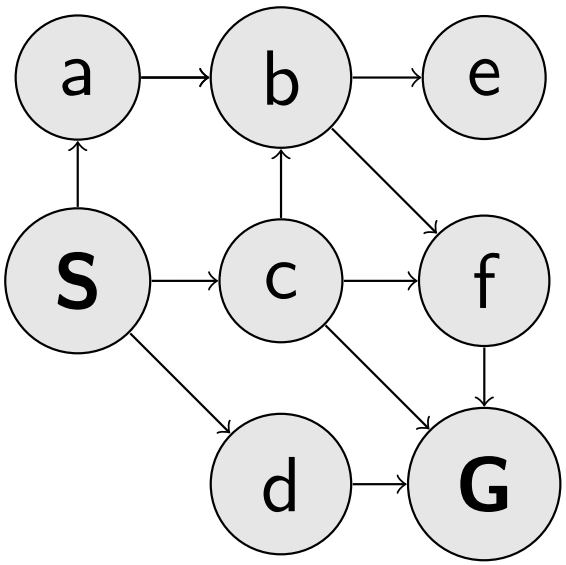

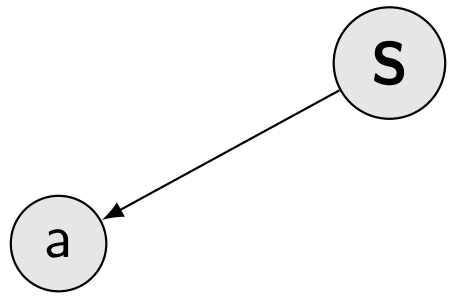

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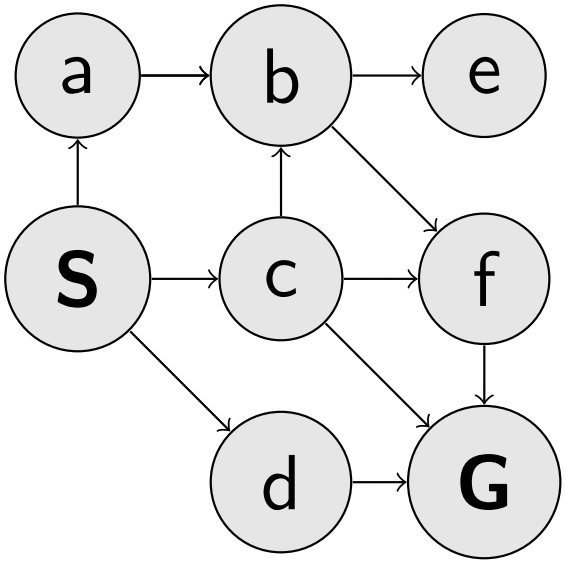

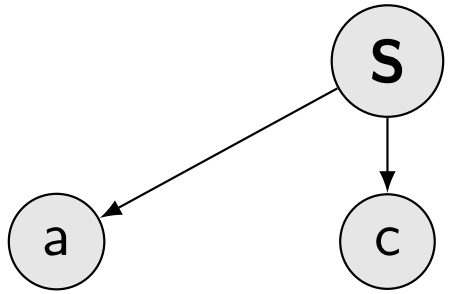

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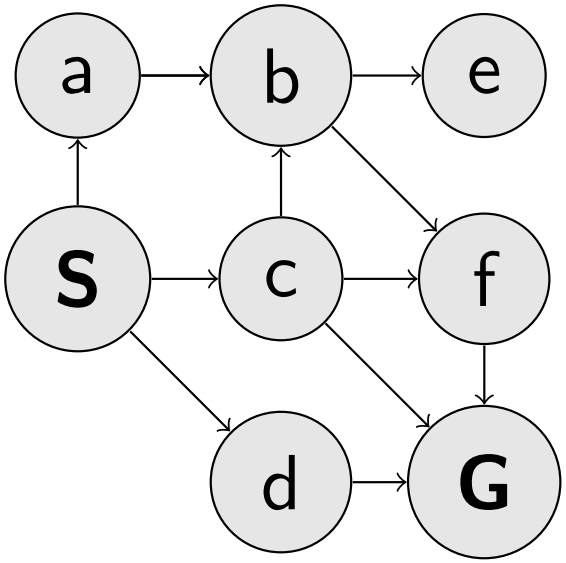

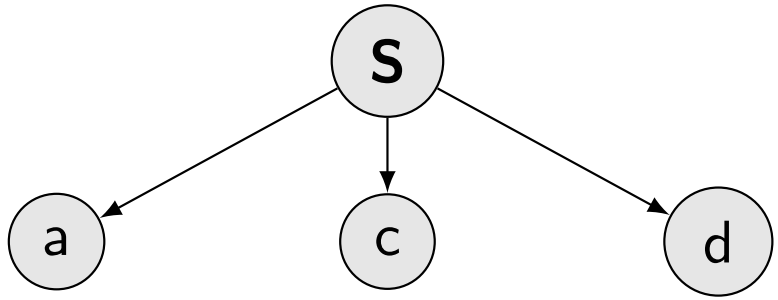

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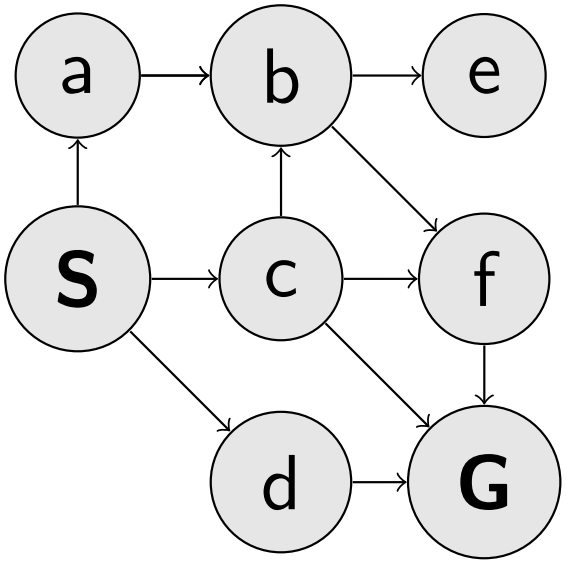

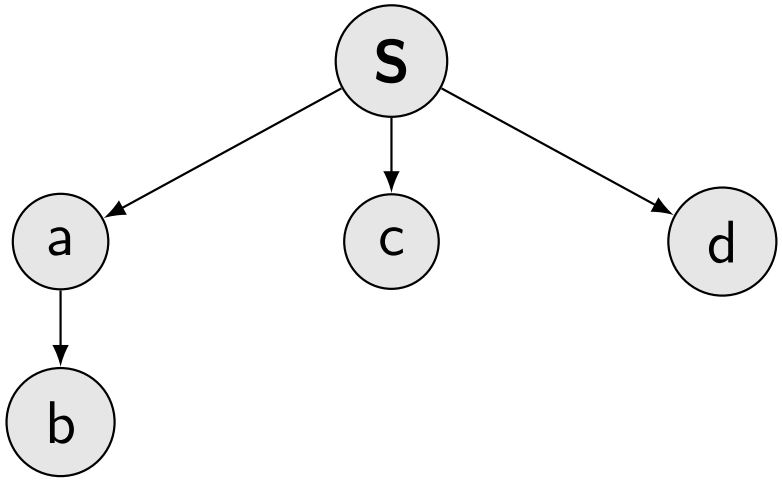

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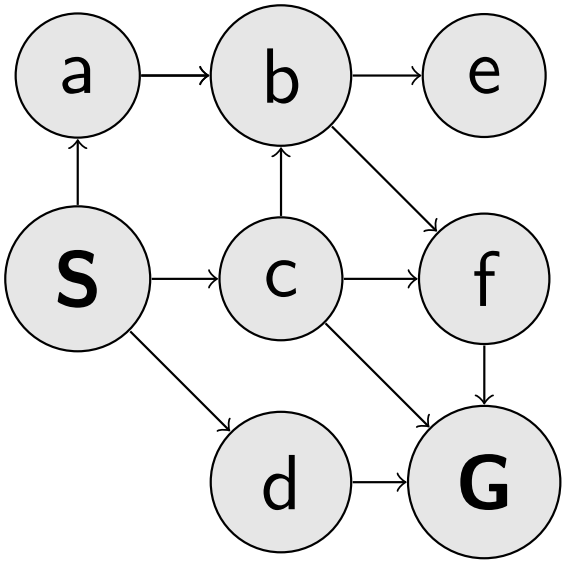

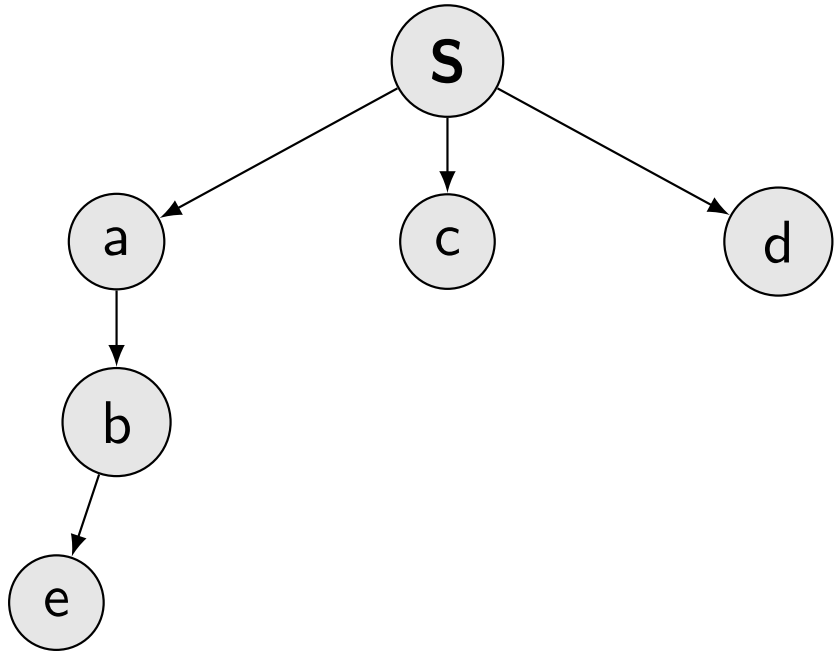

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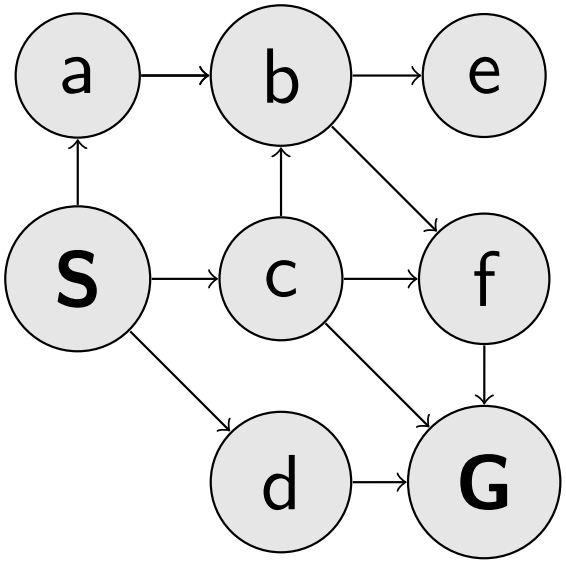

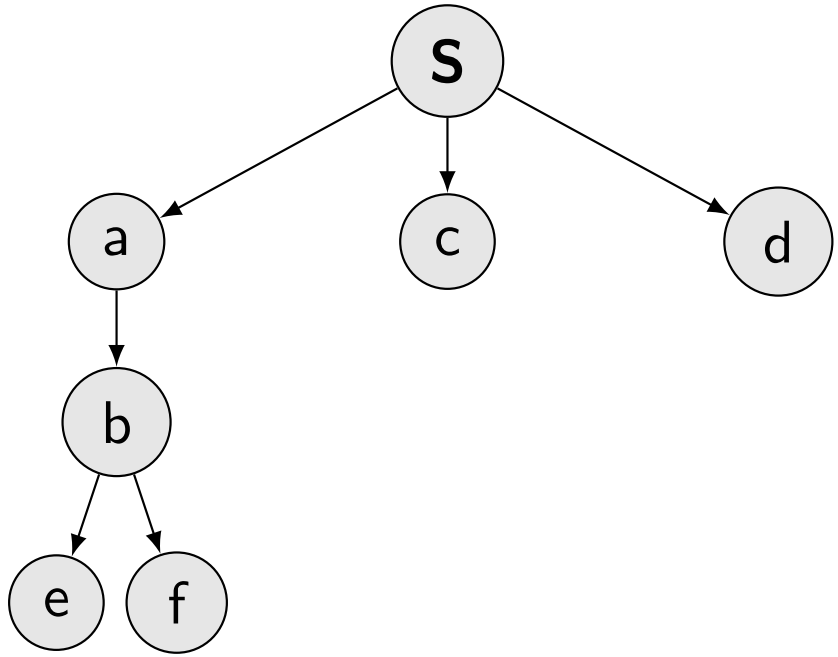

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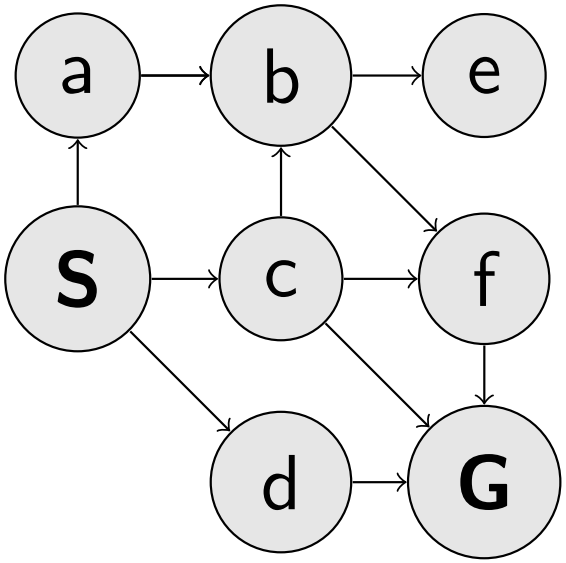

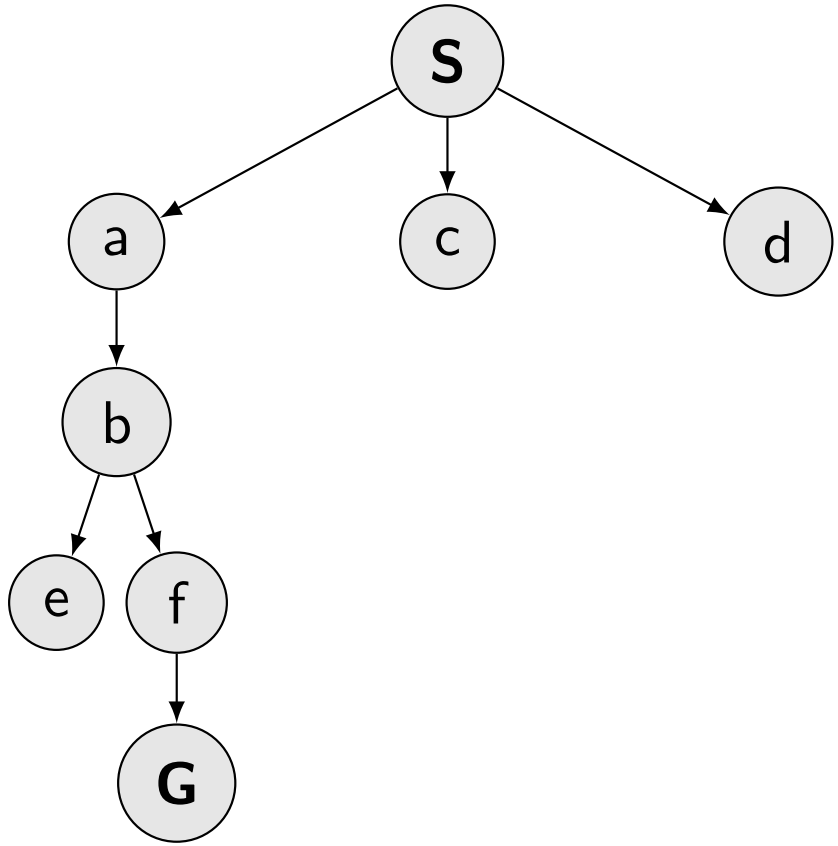

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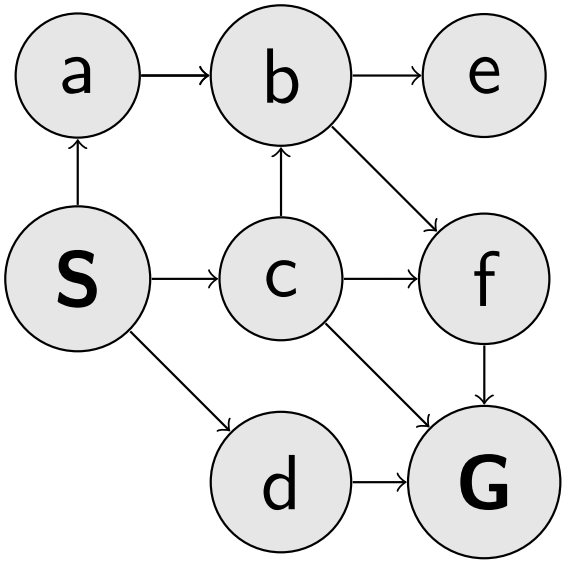

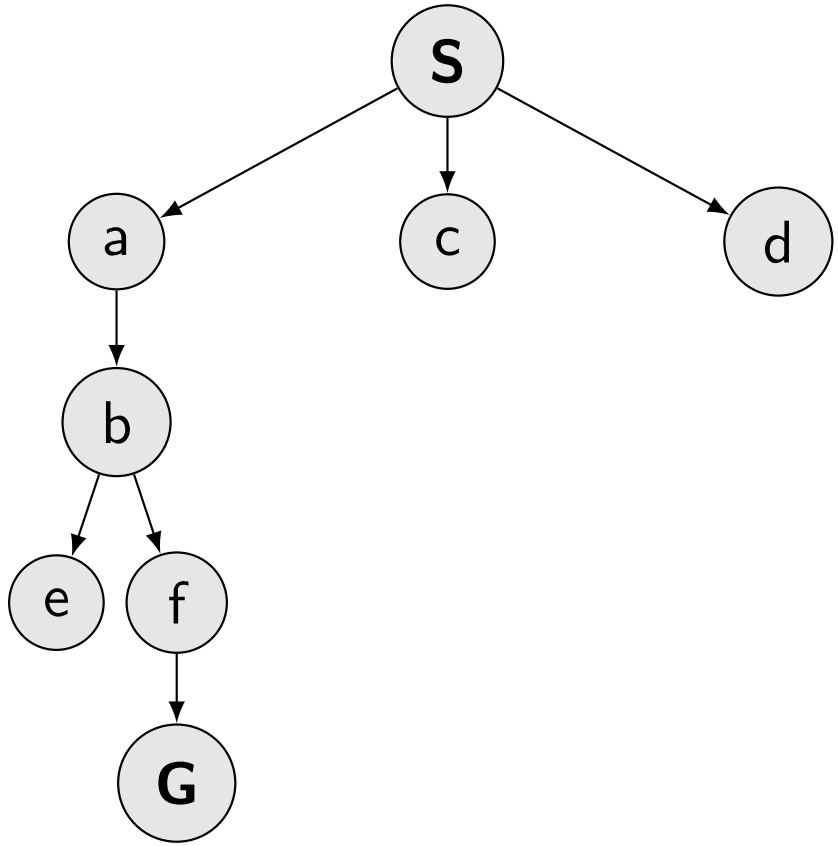

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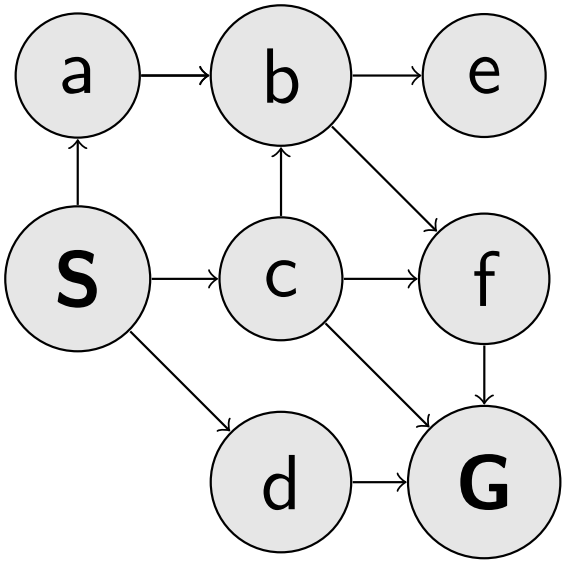

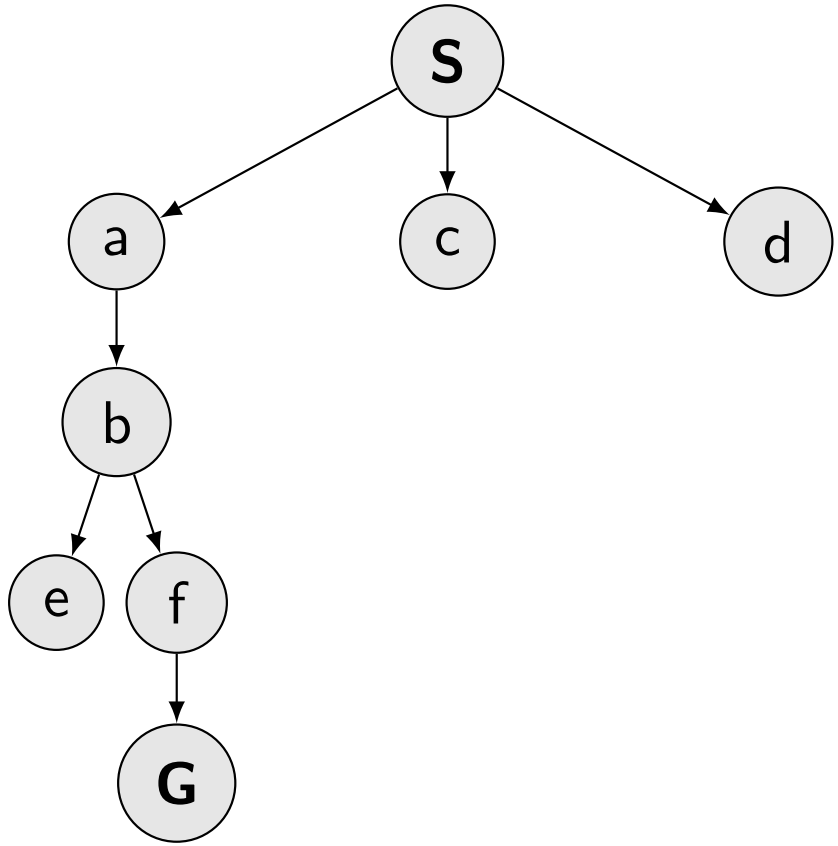

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Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
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- How many steps - an operation with a node? Time complexity?
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How many nodes in a tree? What are tree parameters?

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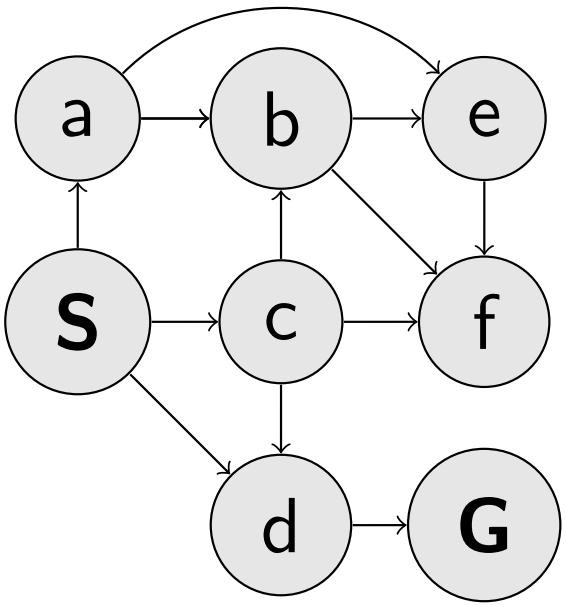
Strategies

How to traverse/build a search tree?

Depth-First Search (DFS)



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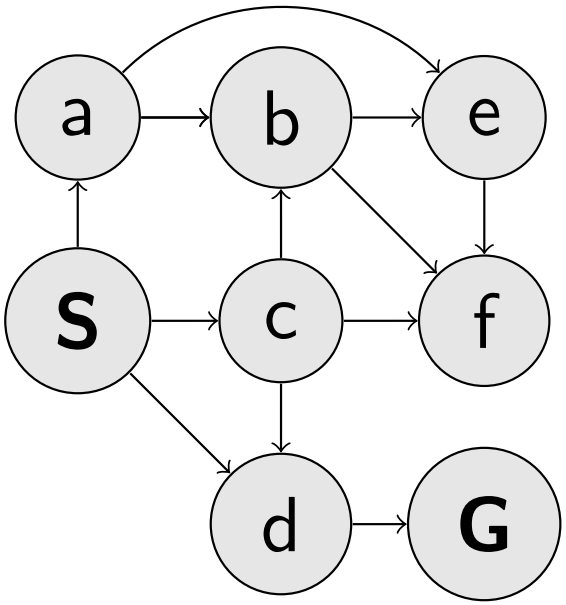

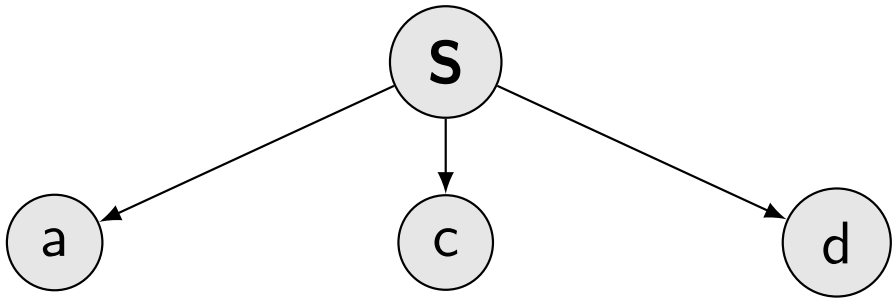

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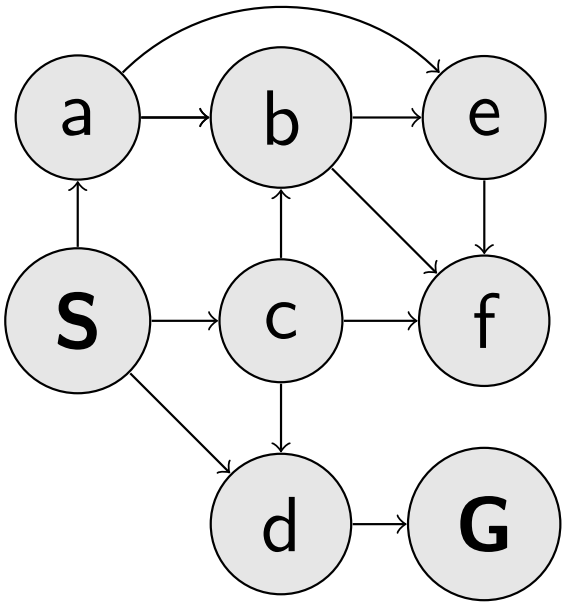

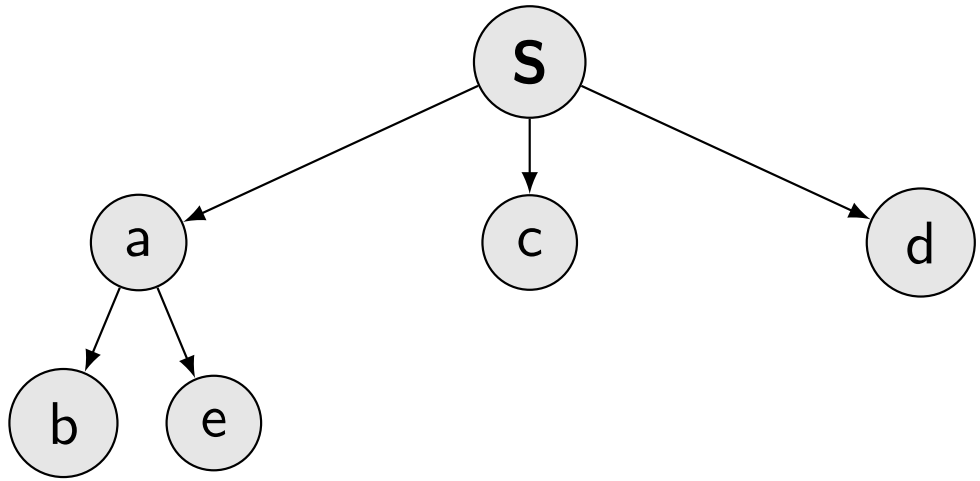

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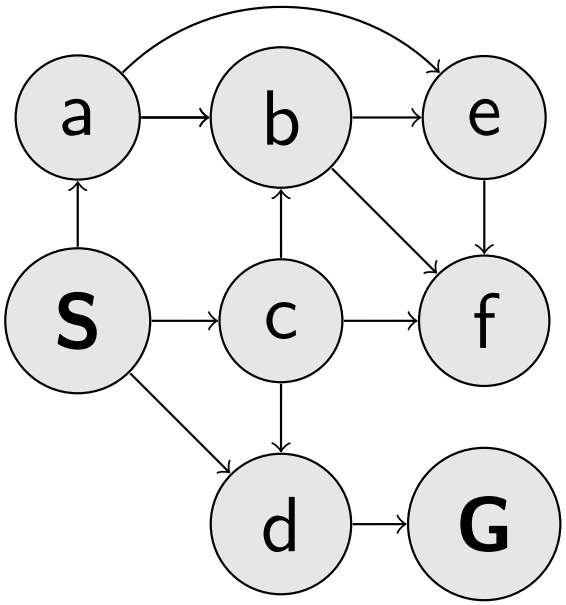

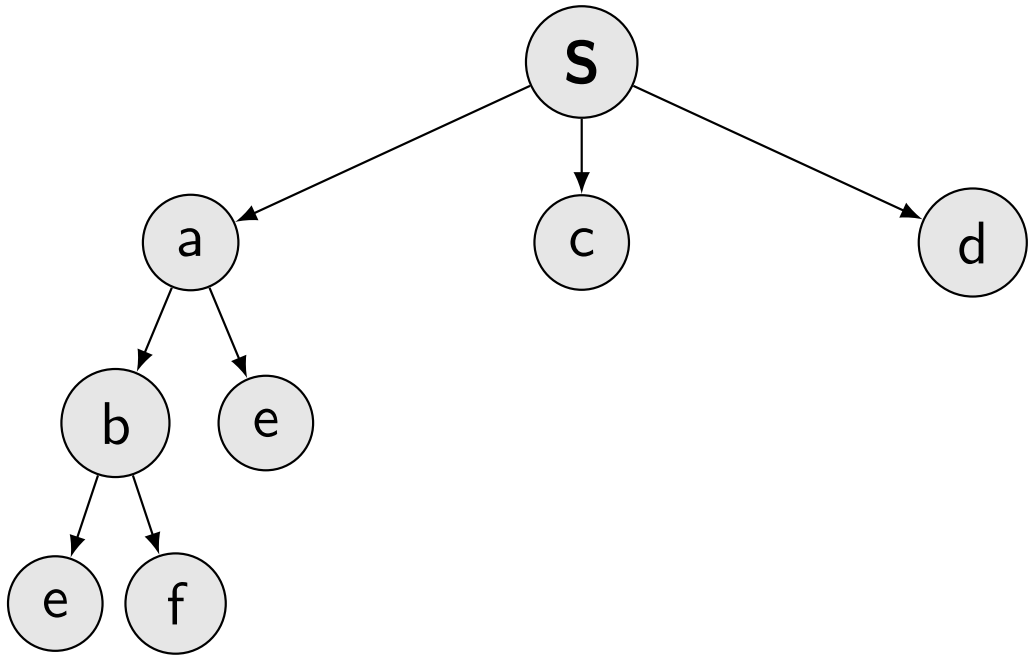

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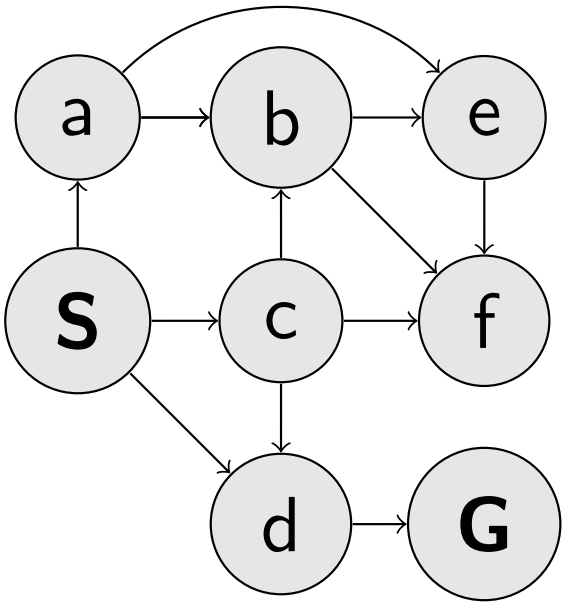

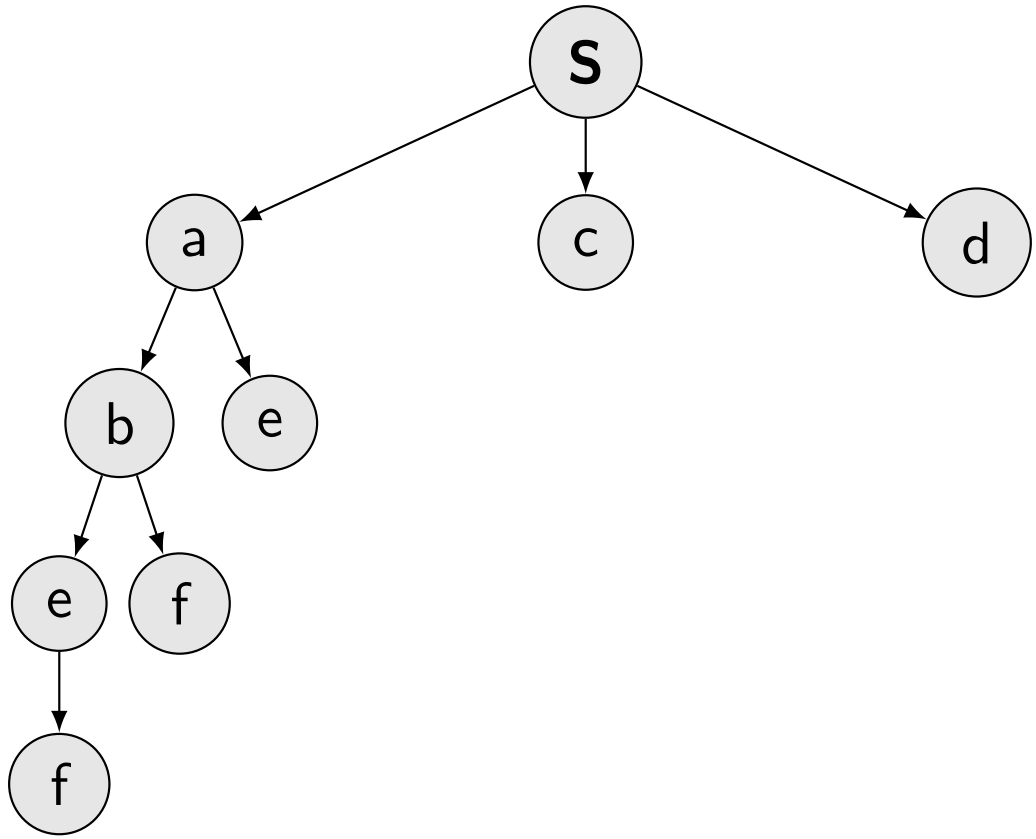

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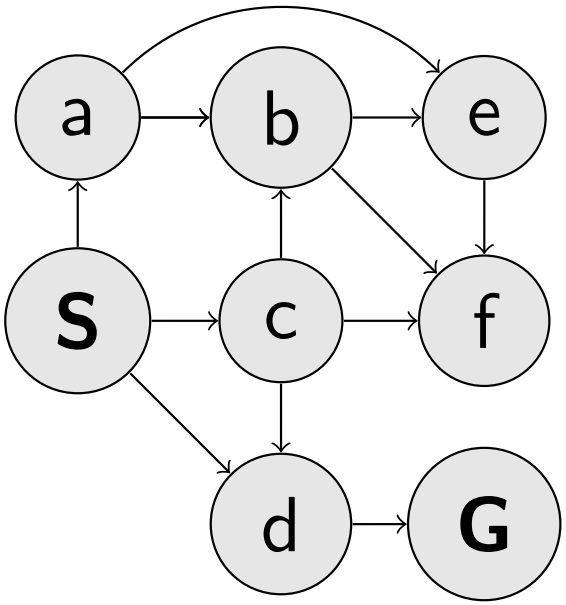

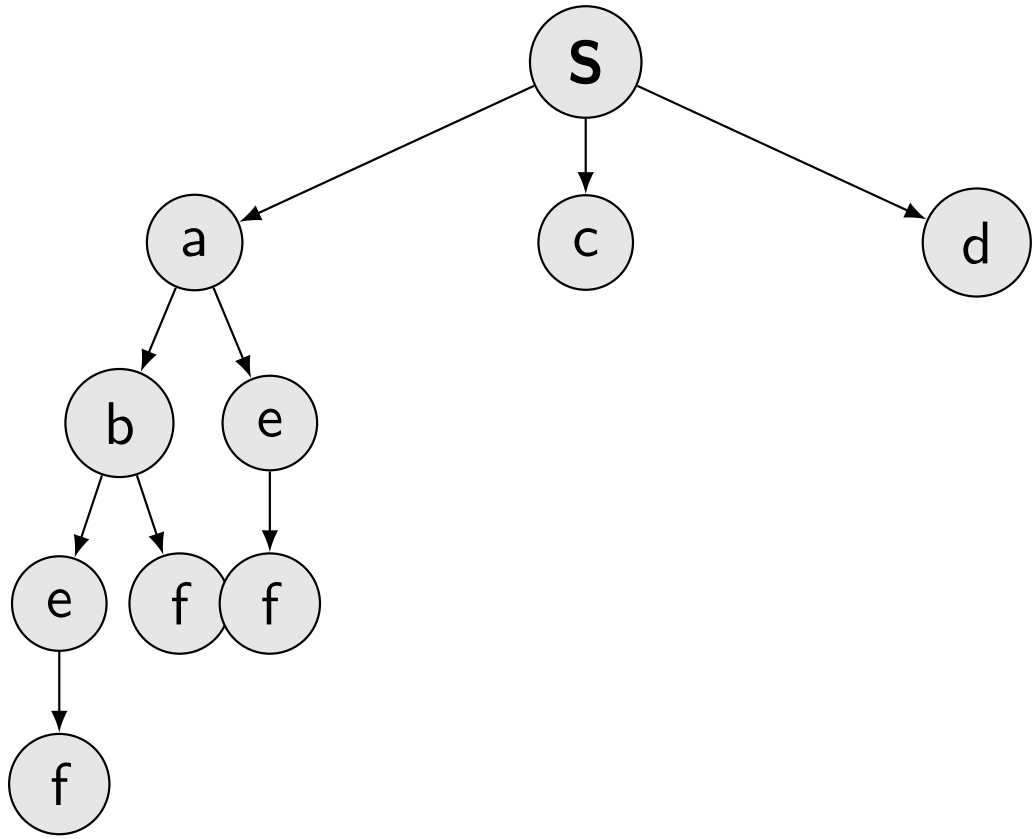

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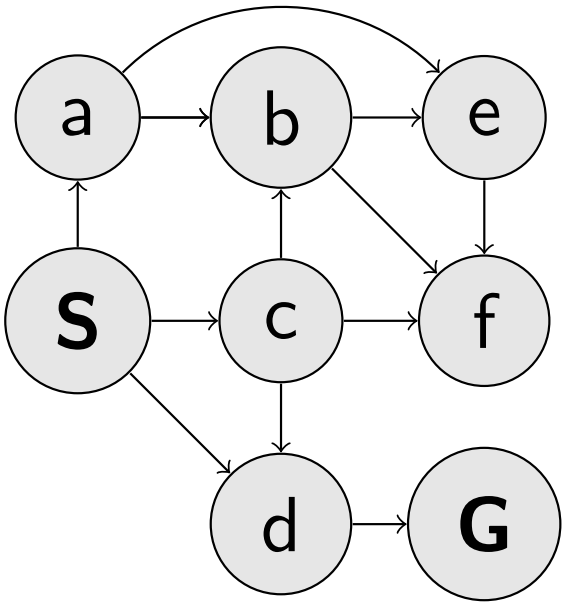

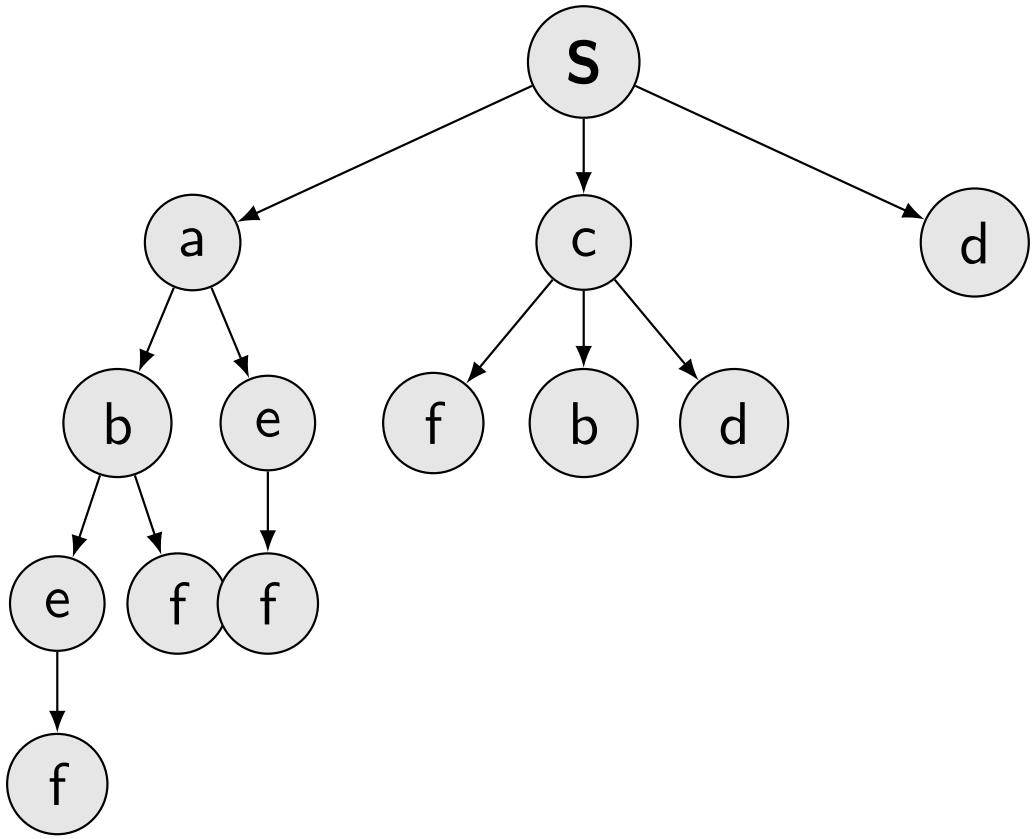

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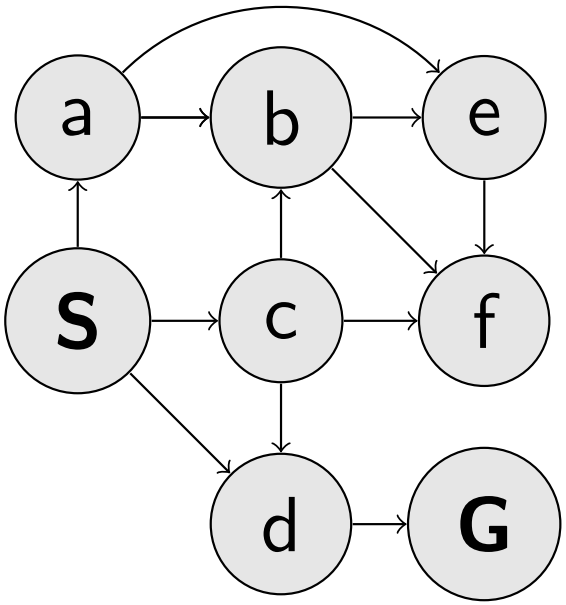

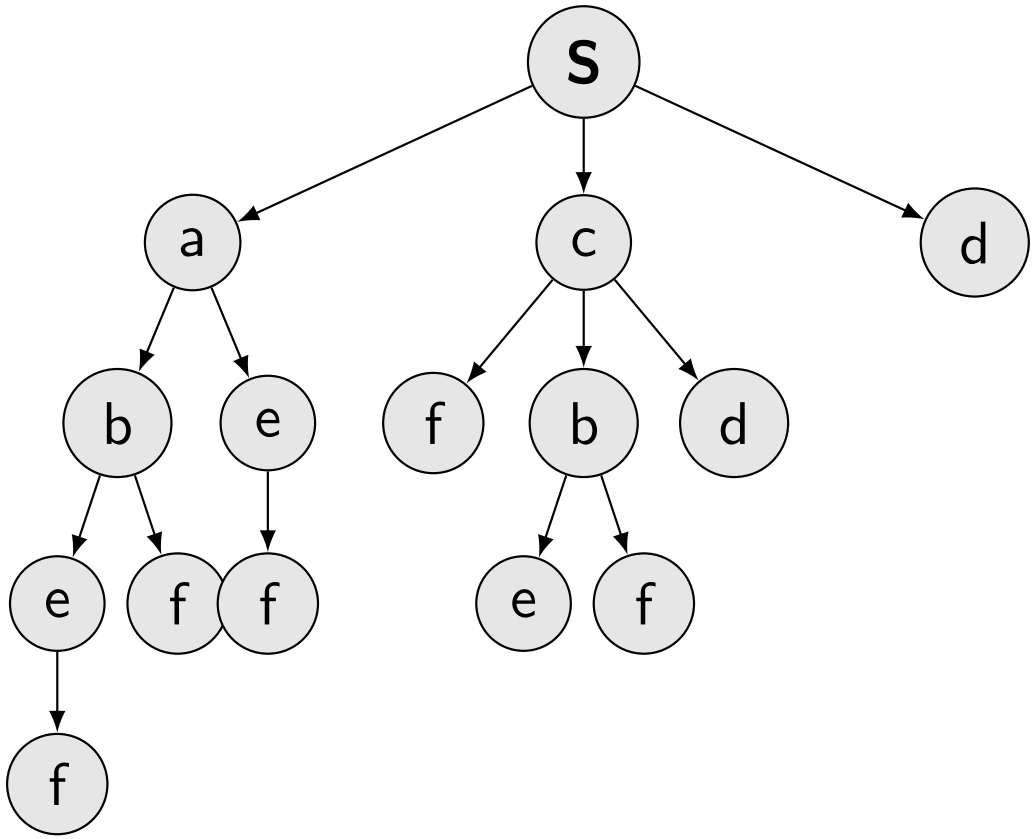

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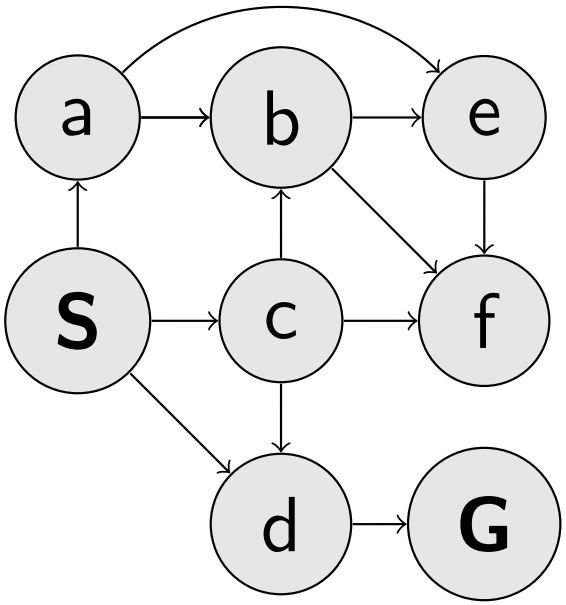

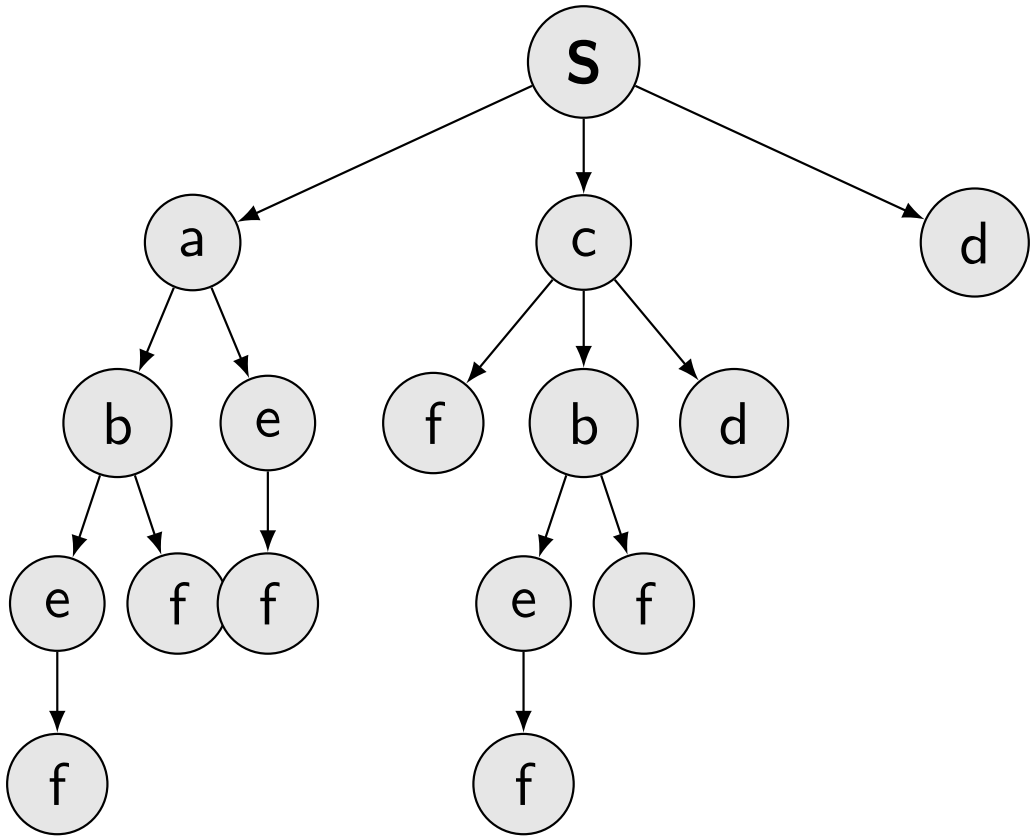

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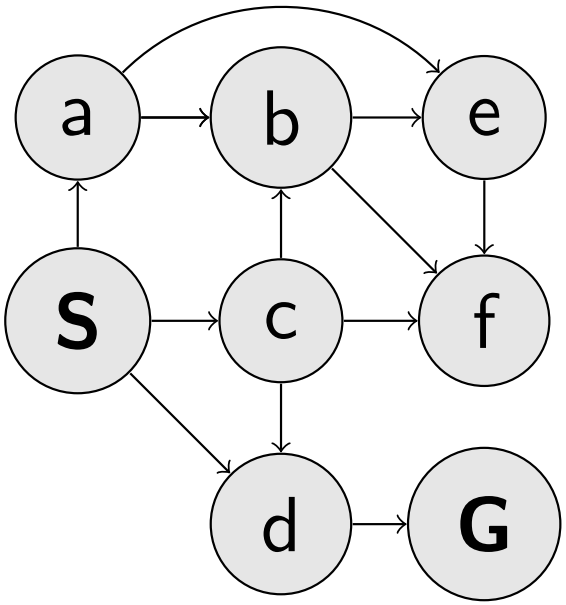

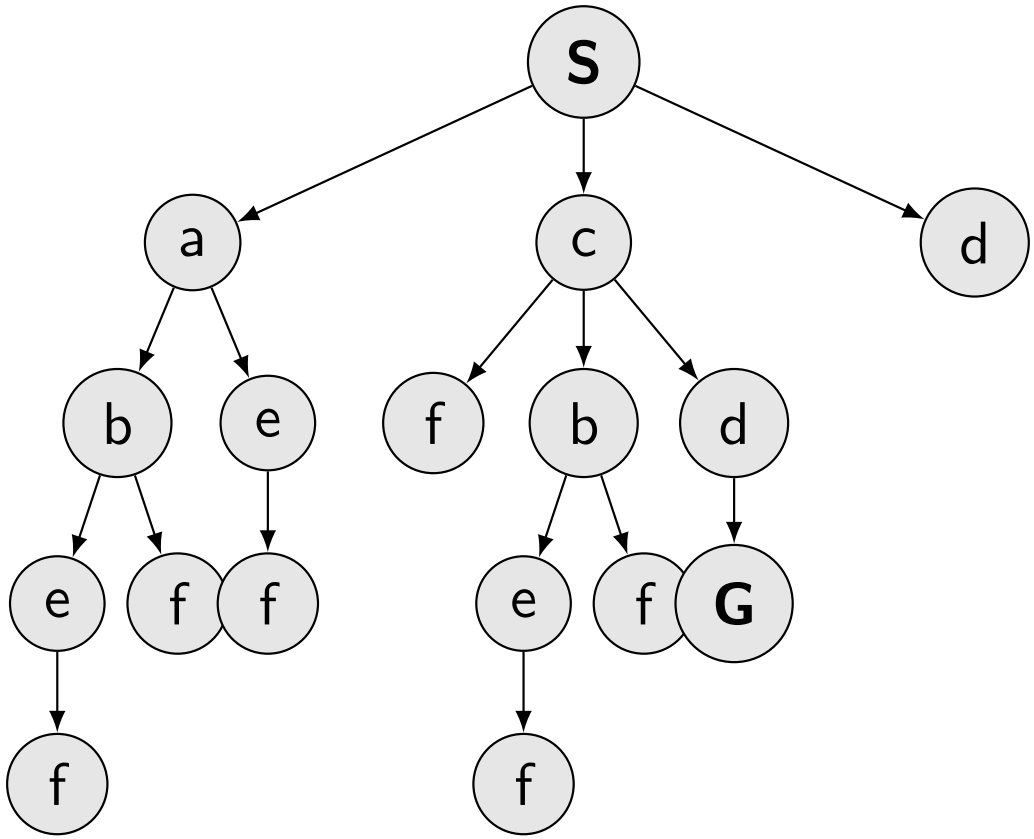

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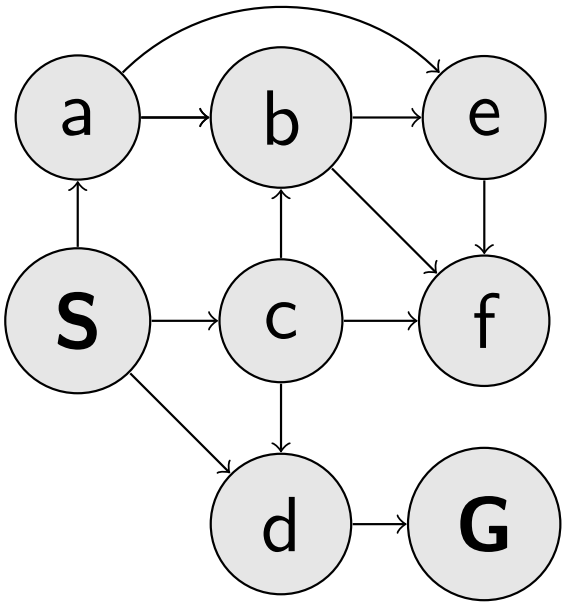

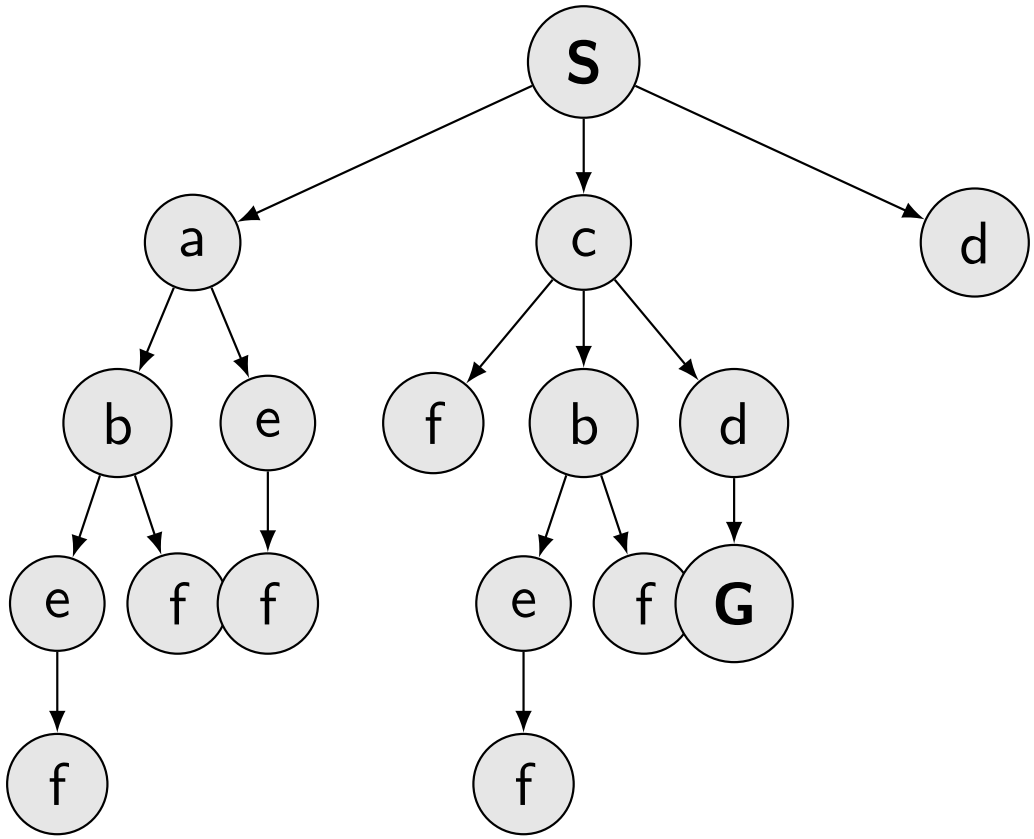

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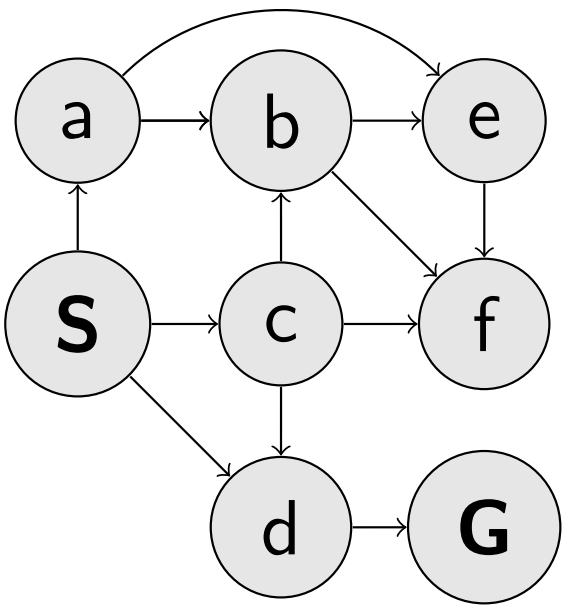

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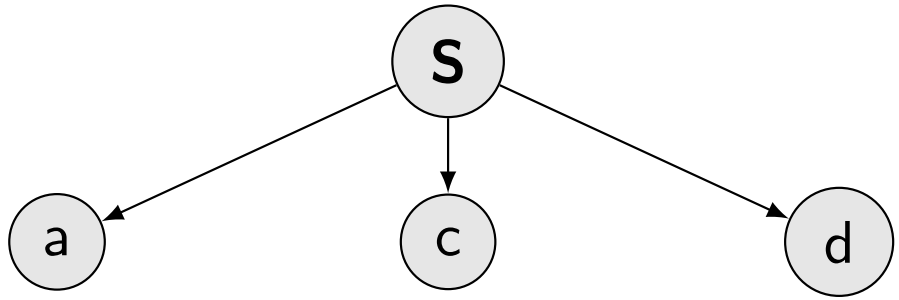

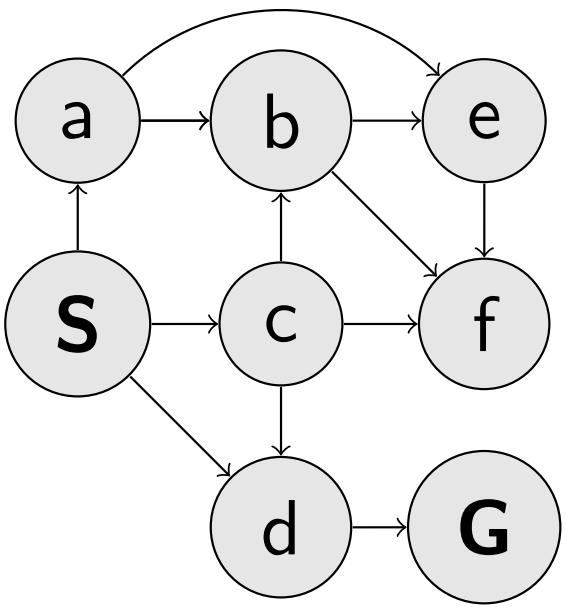

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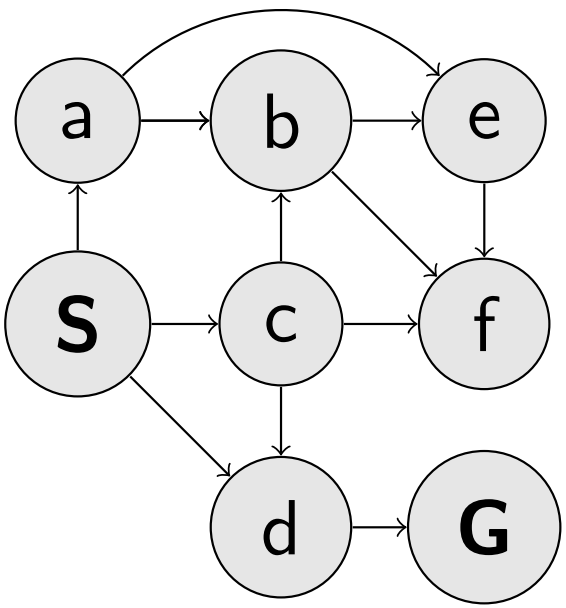

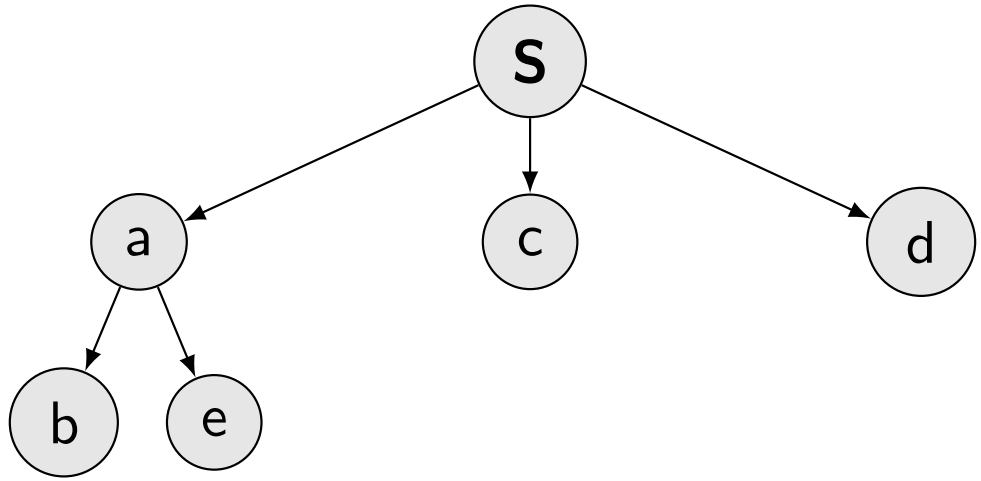

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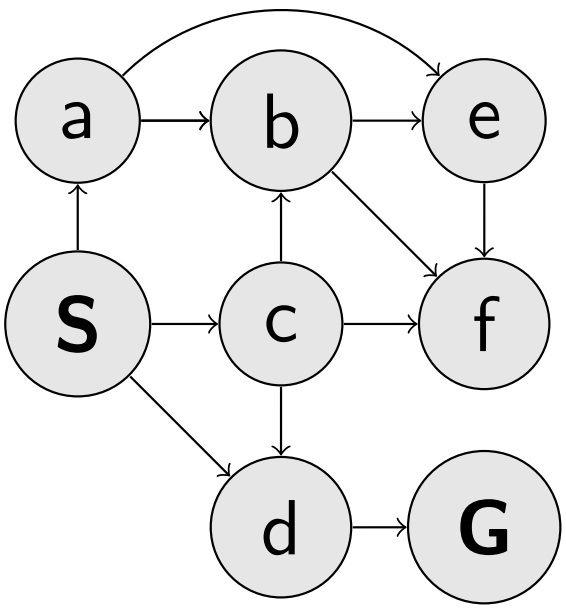

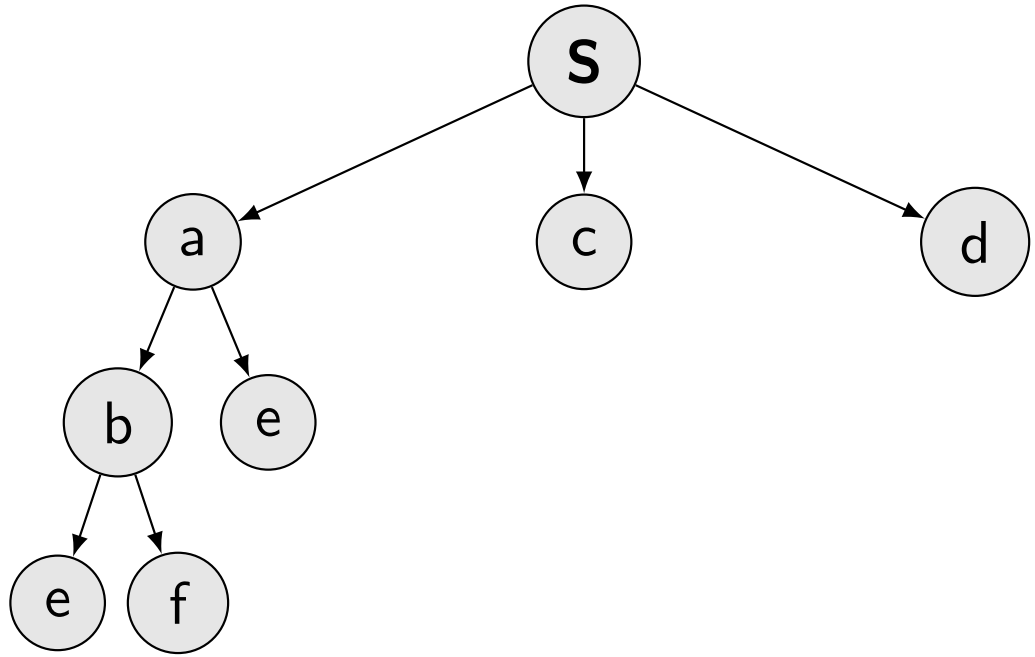

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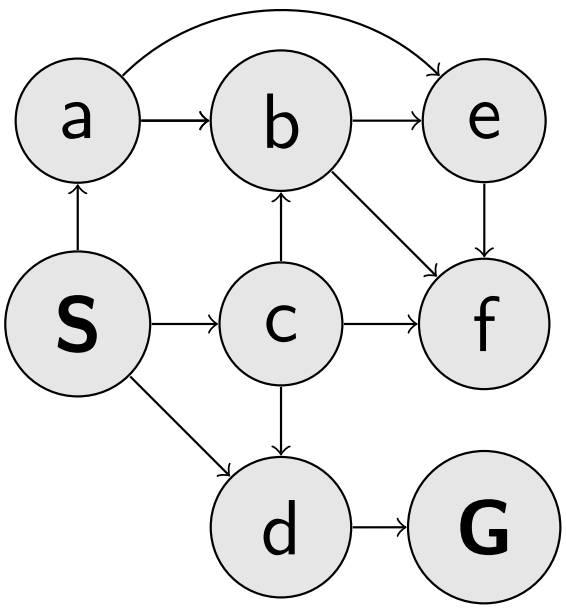

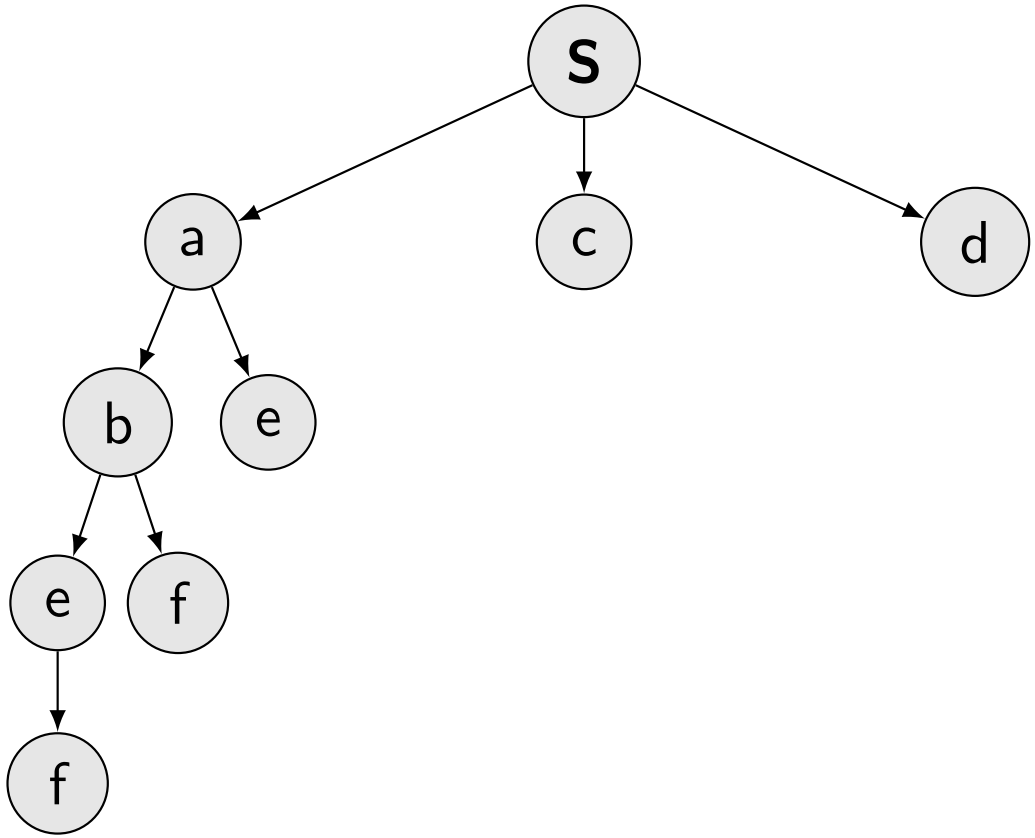

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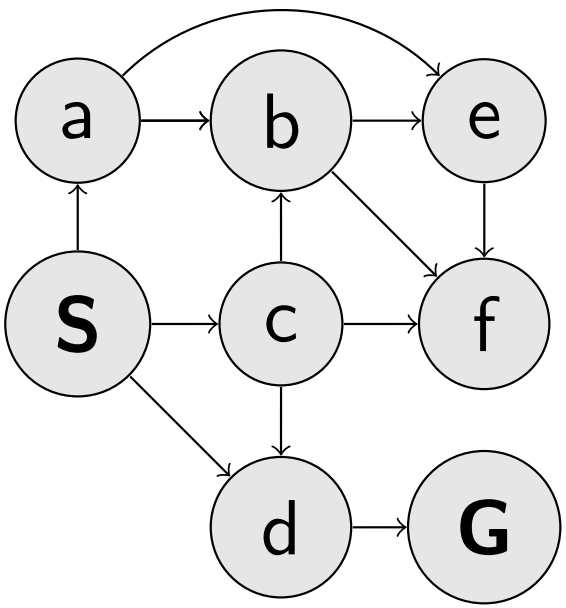

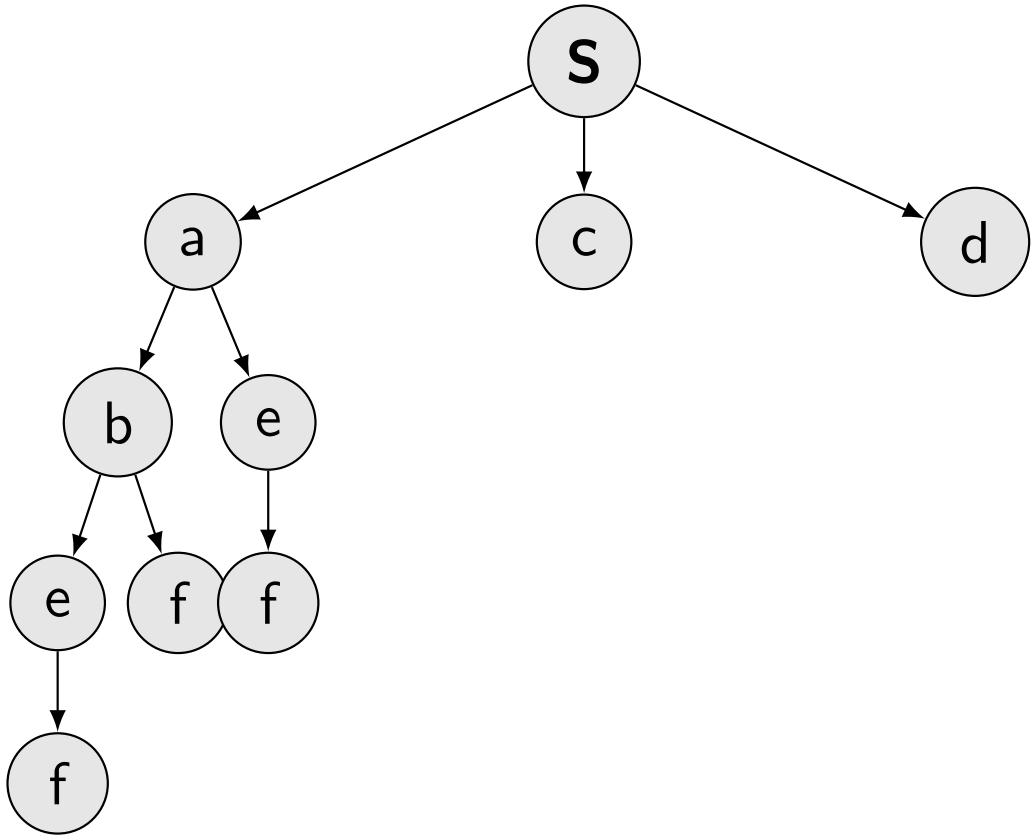

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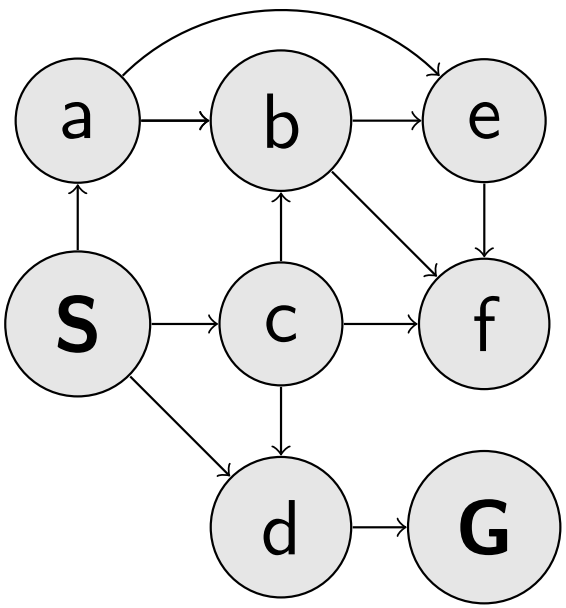

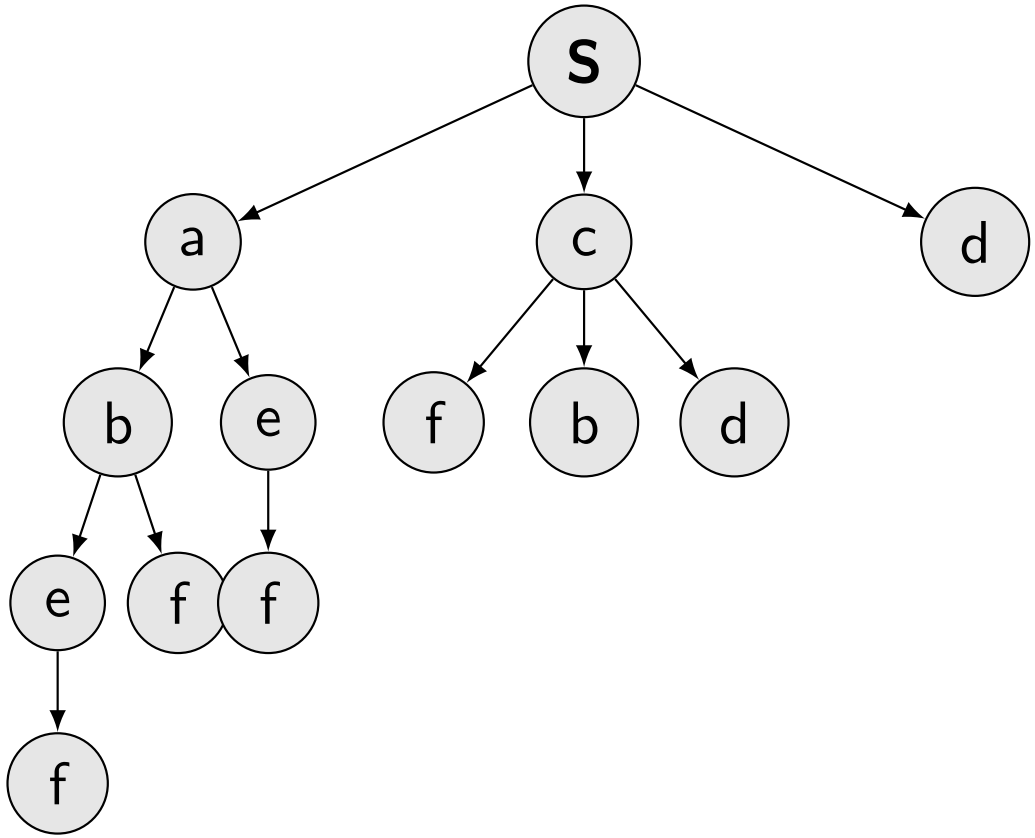

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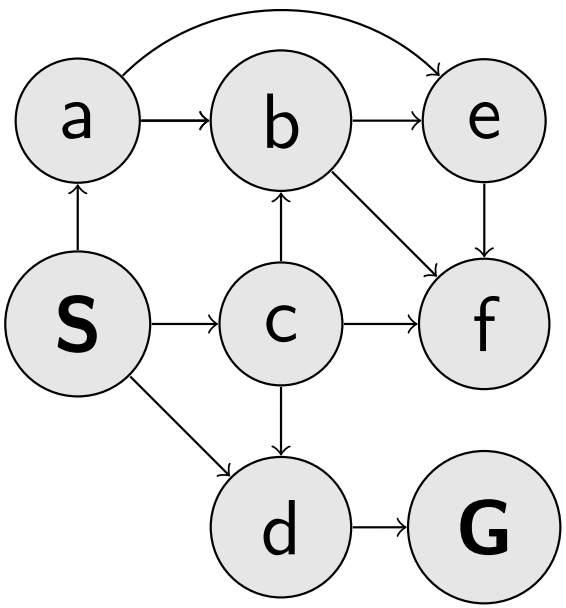

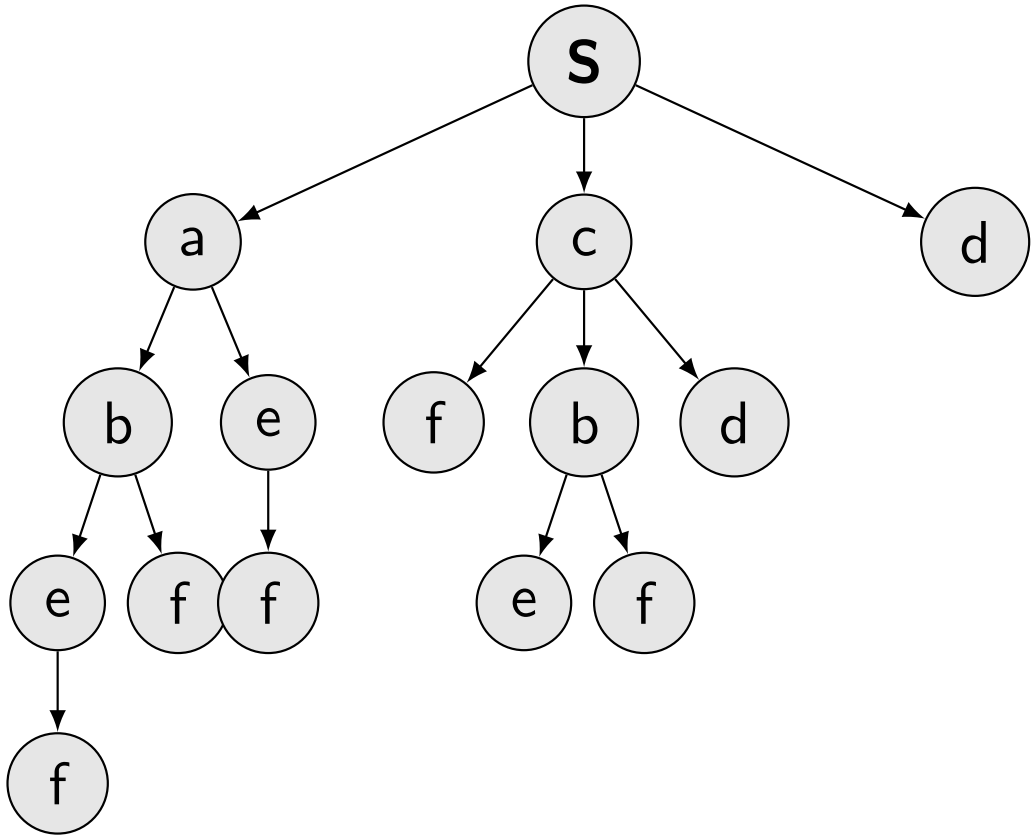

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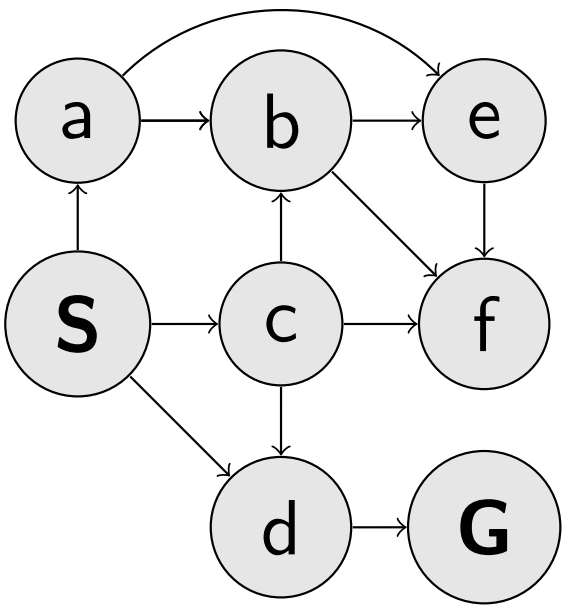

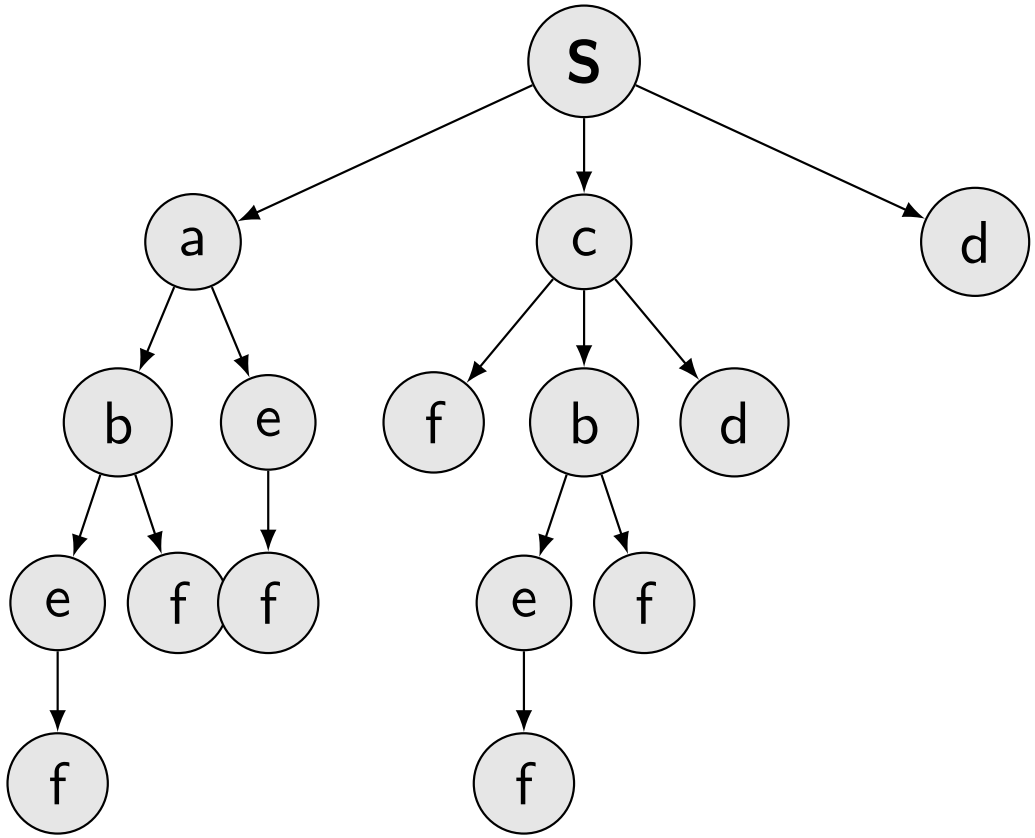

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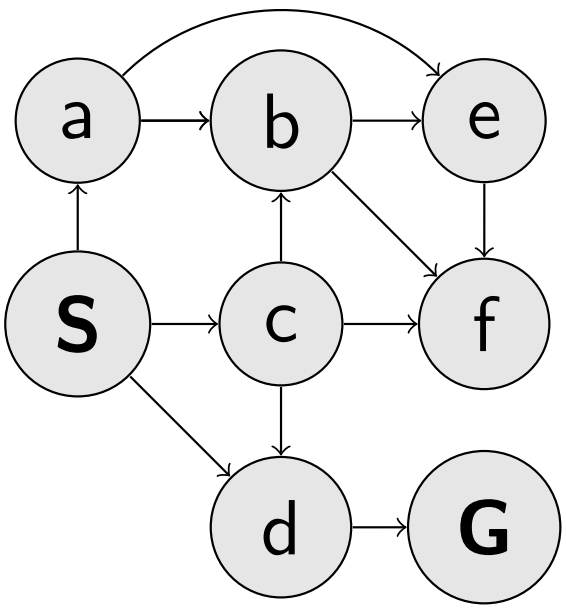
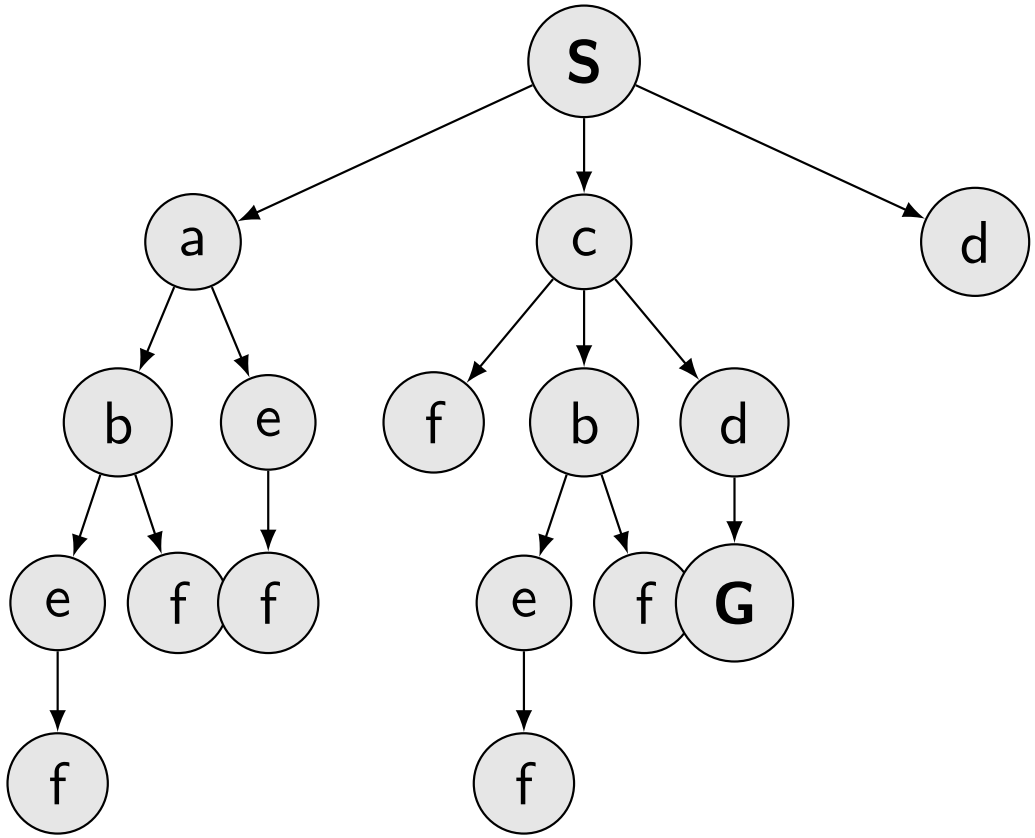
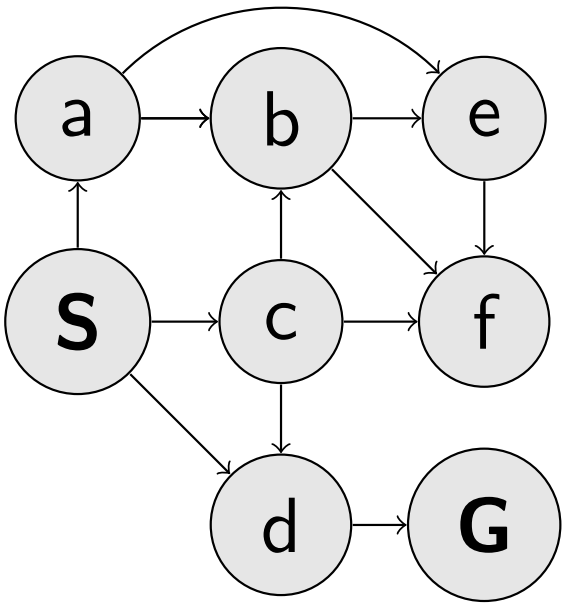
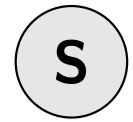




Breadth-First Search (BFS)





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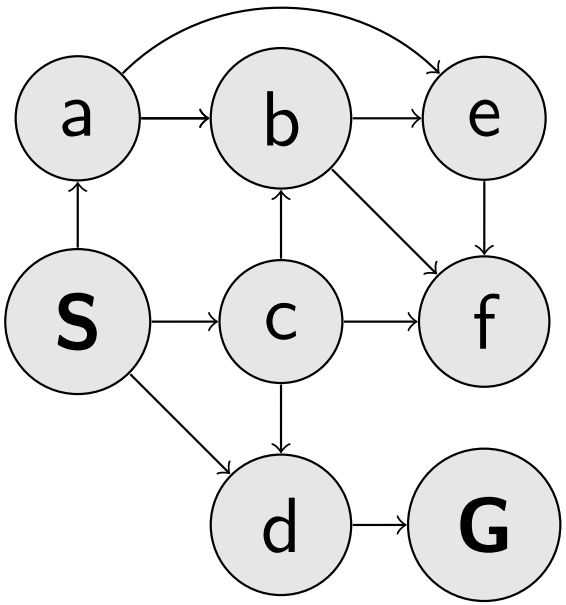

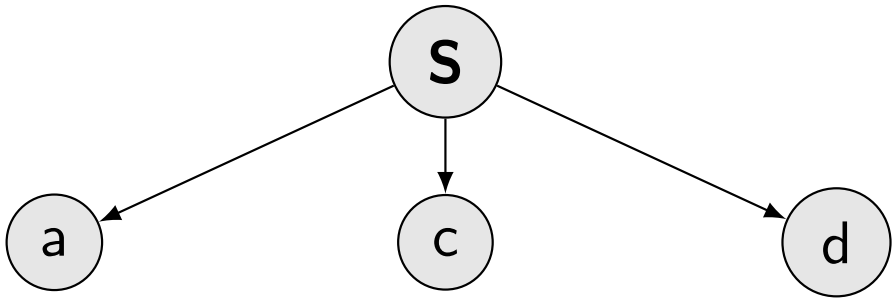

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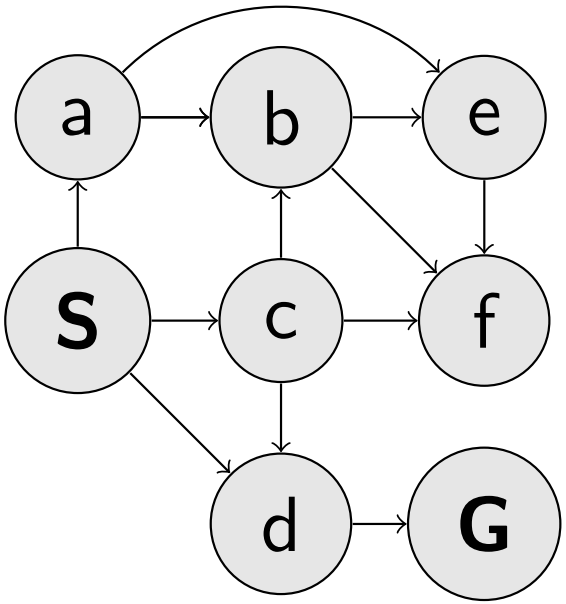

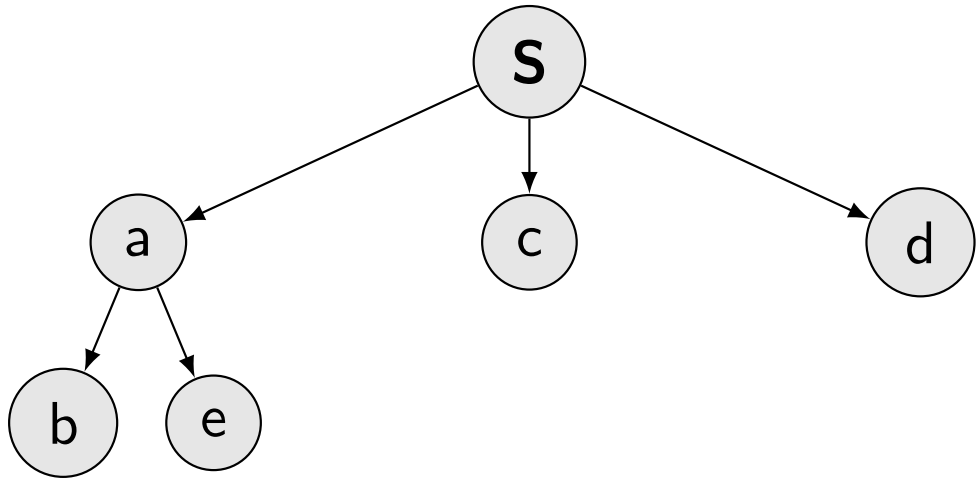

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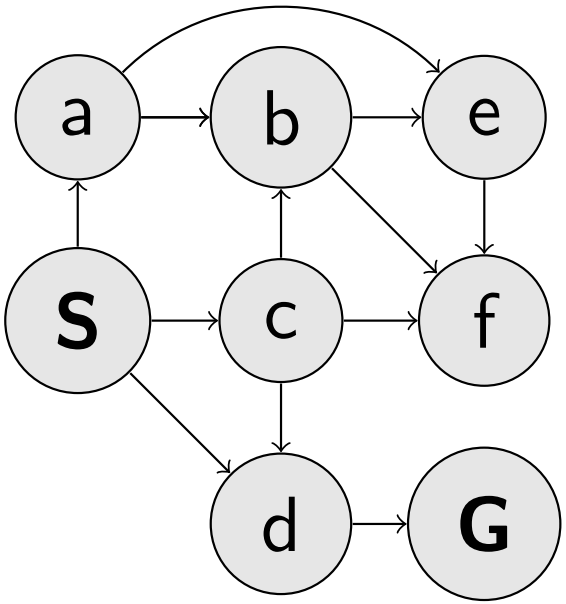

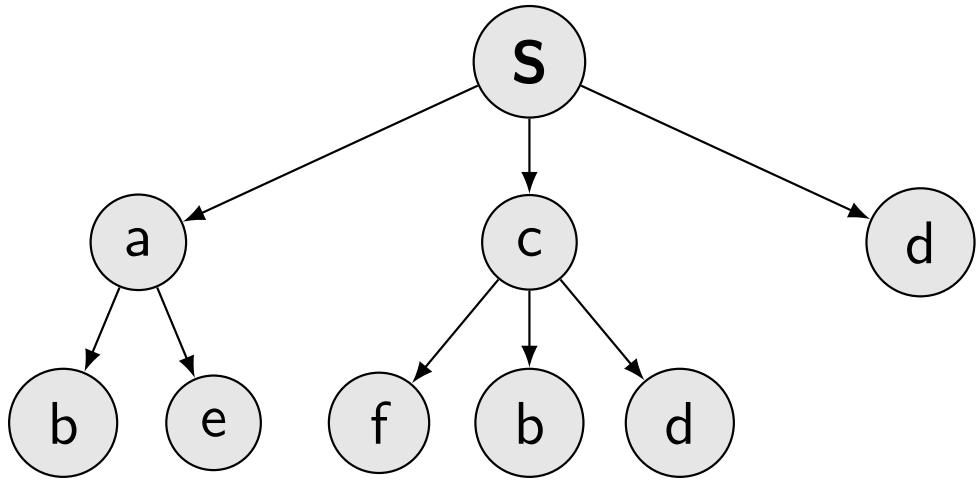

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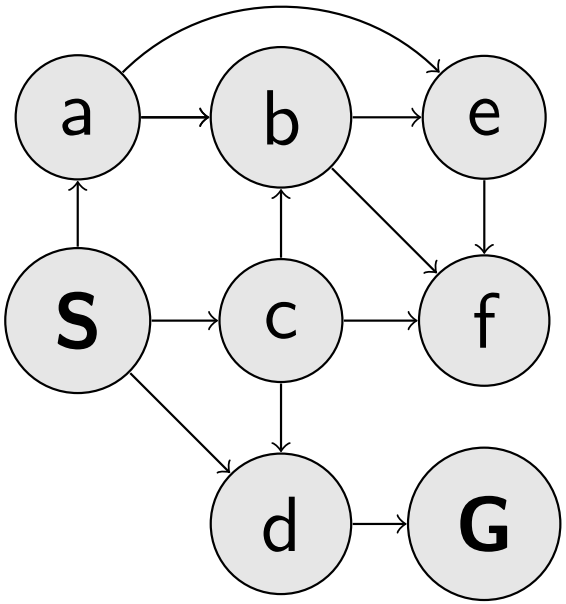

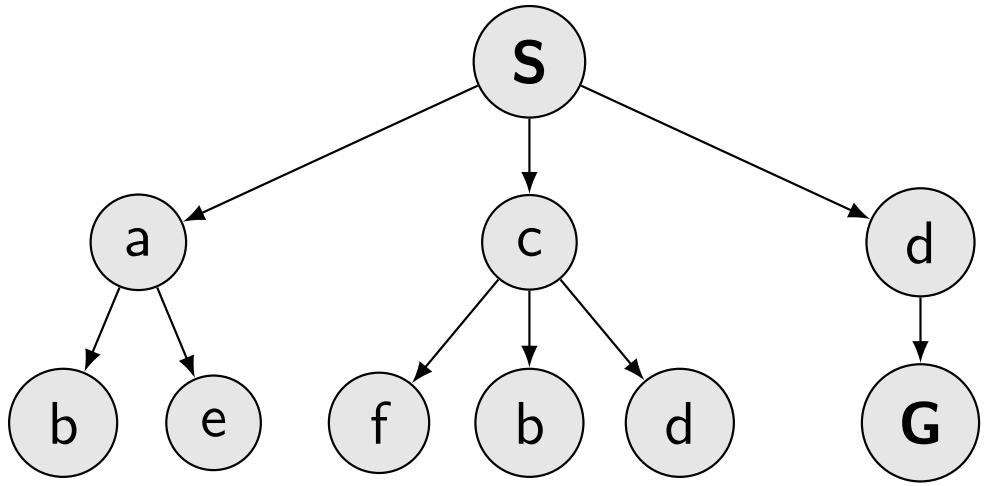

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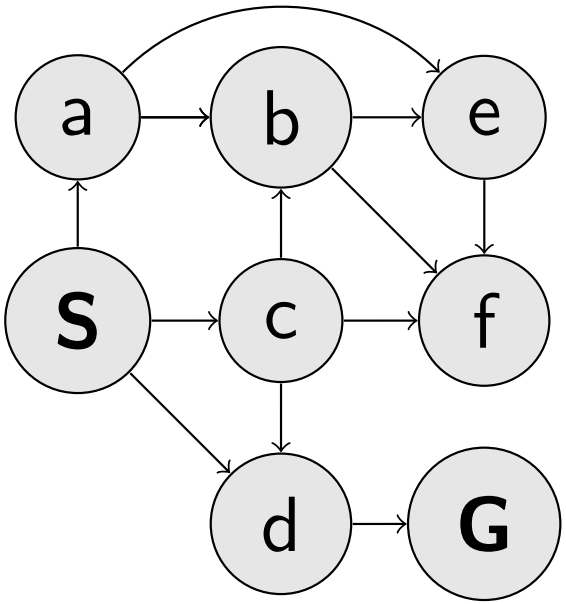

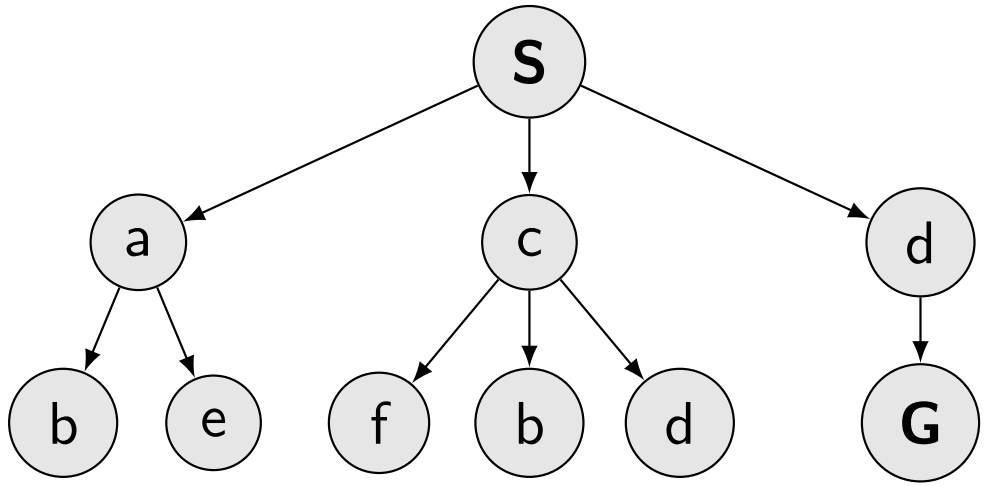

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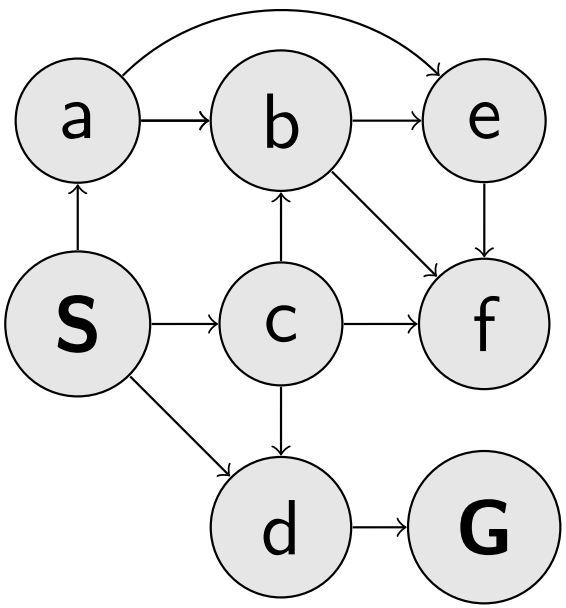

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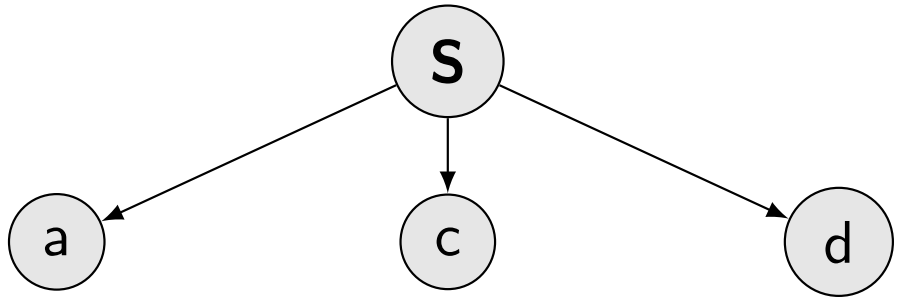

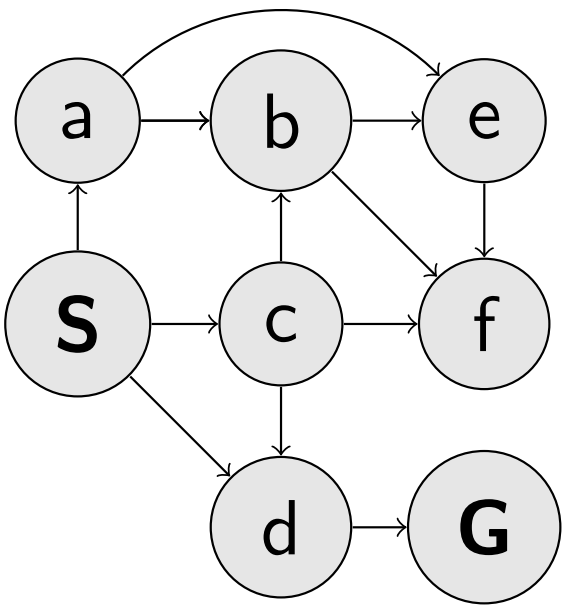

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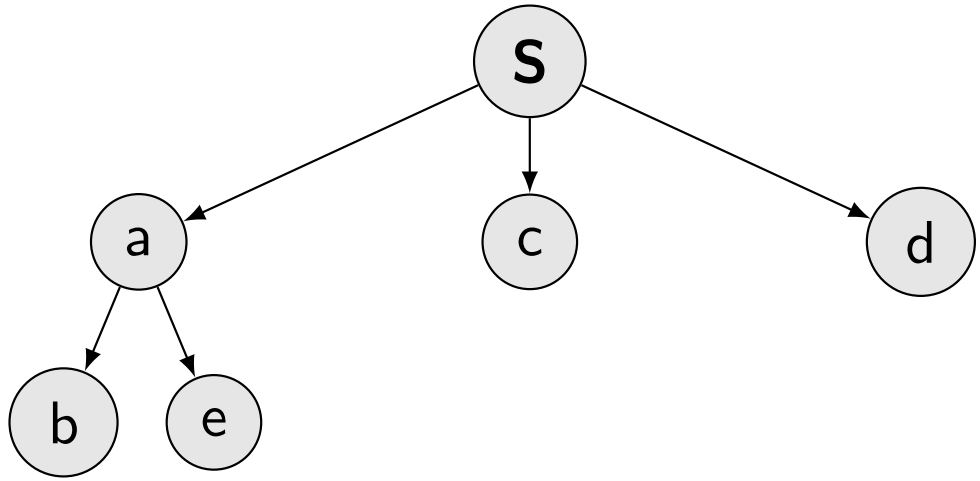

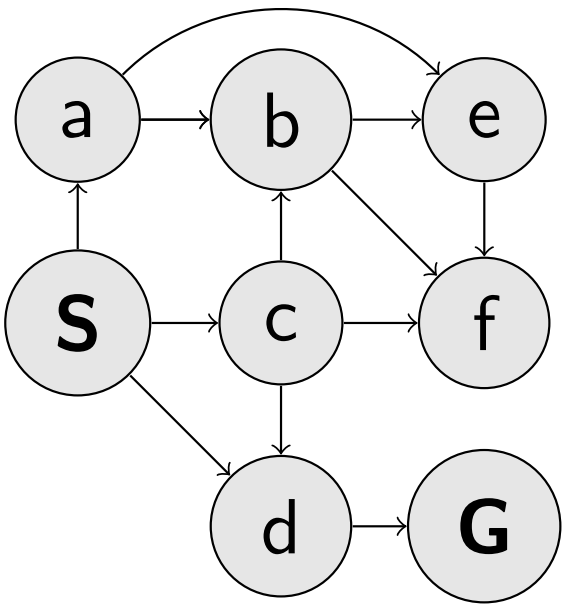

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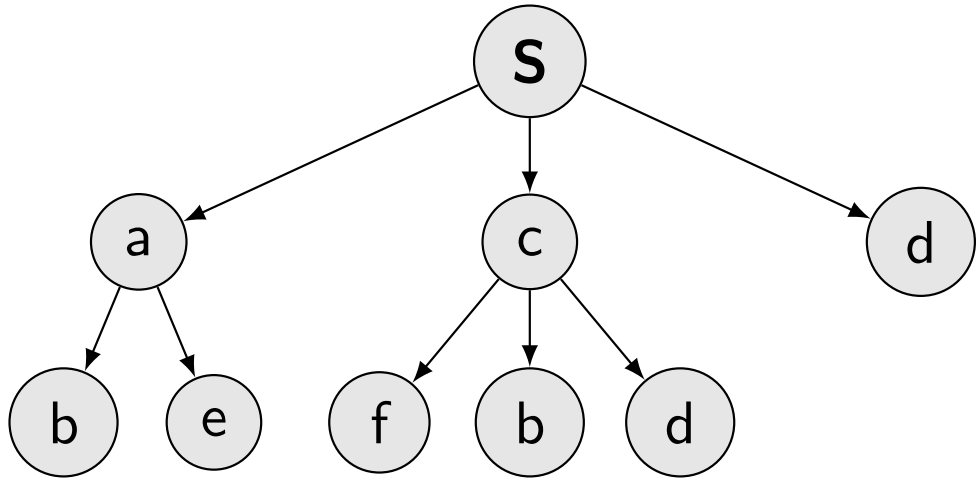

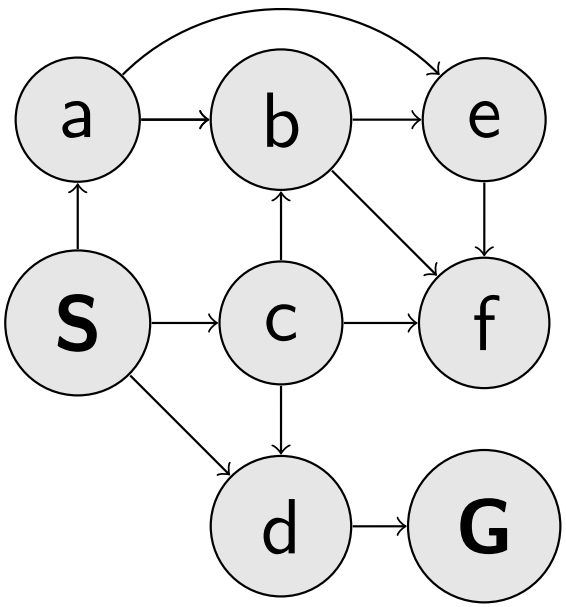

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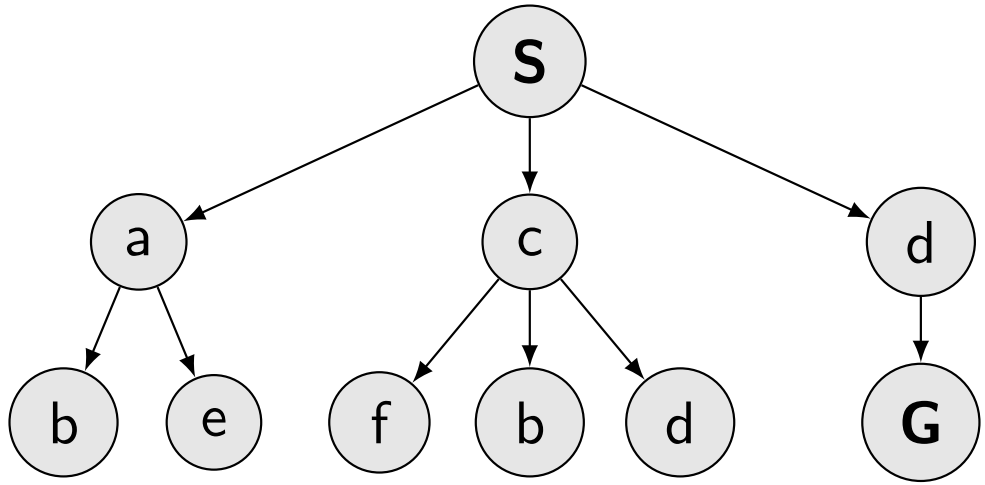

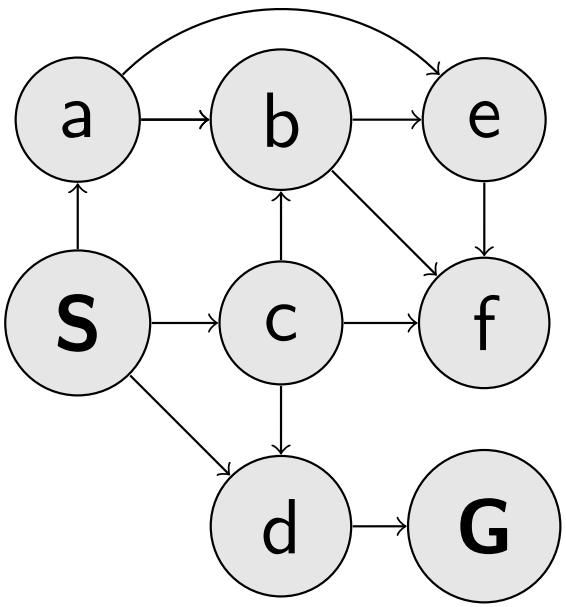

DFS vs BFS

What are (dis)advantages of the individual strategies?

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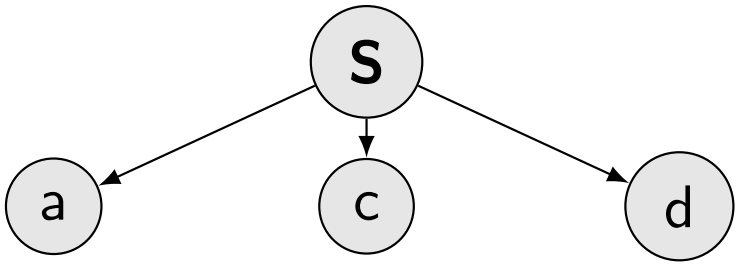

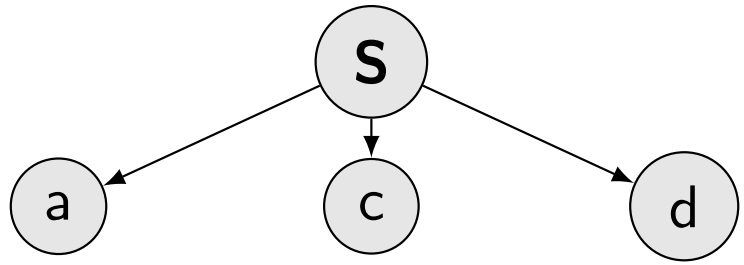

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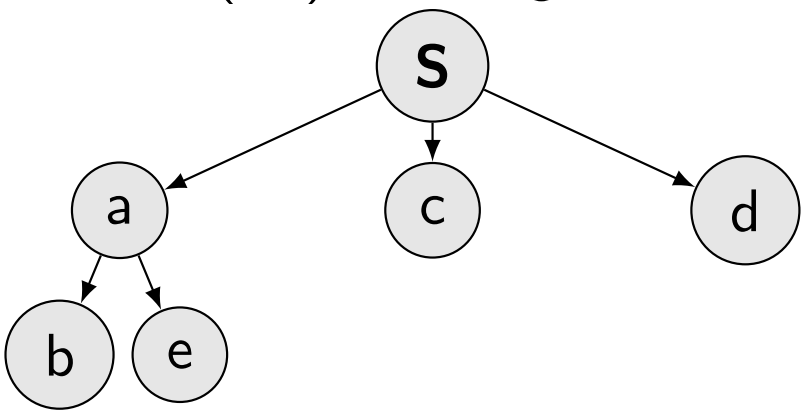

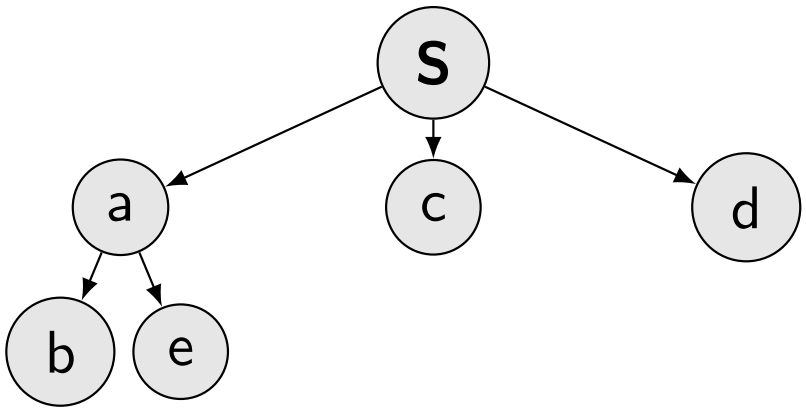

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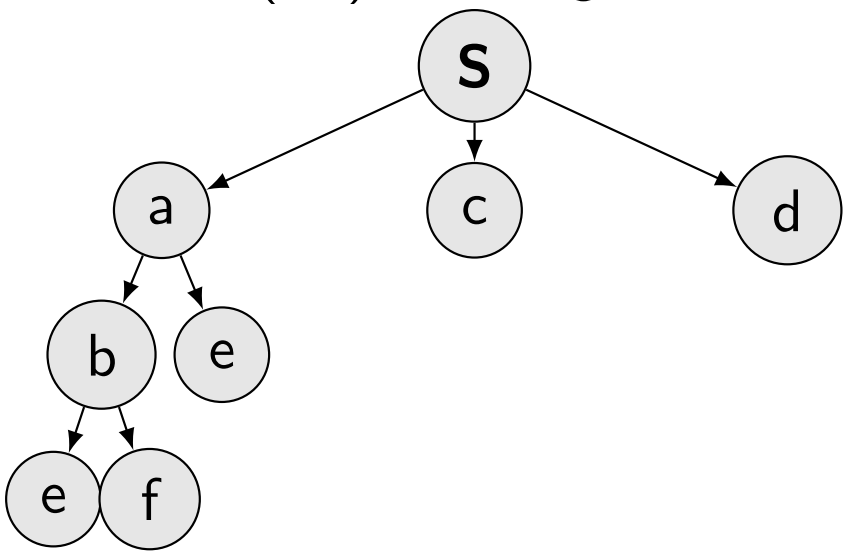

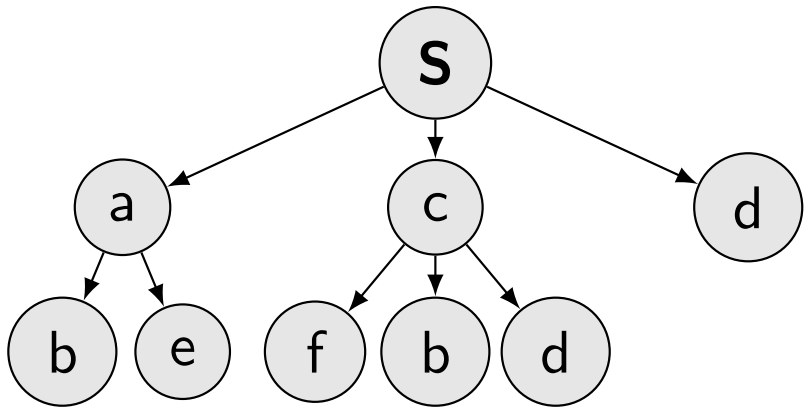

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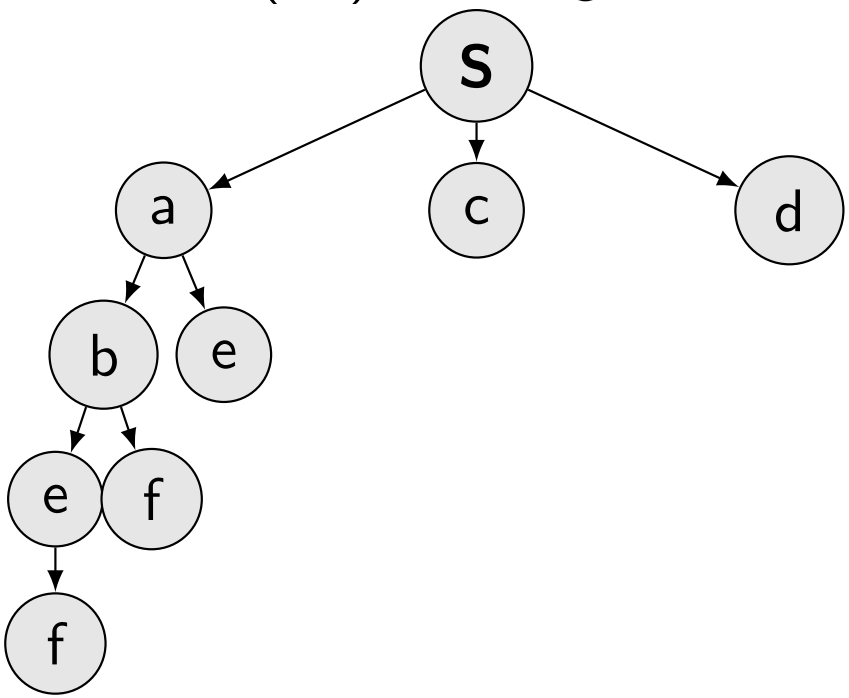

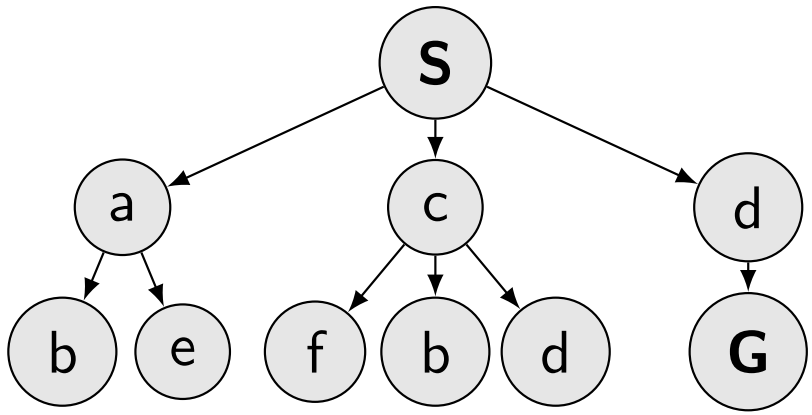

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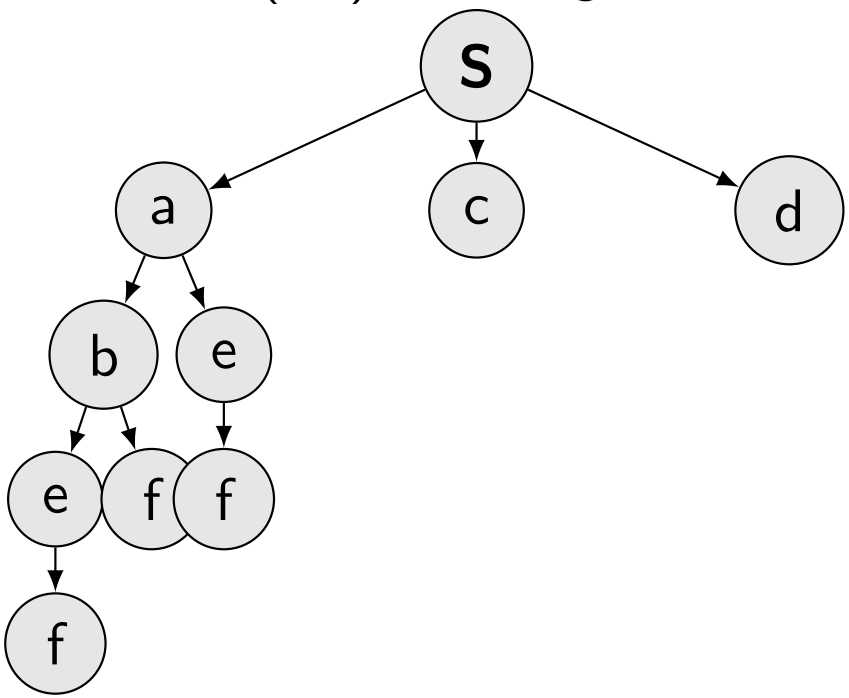

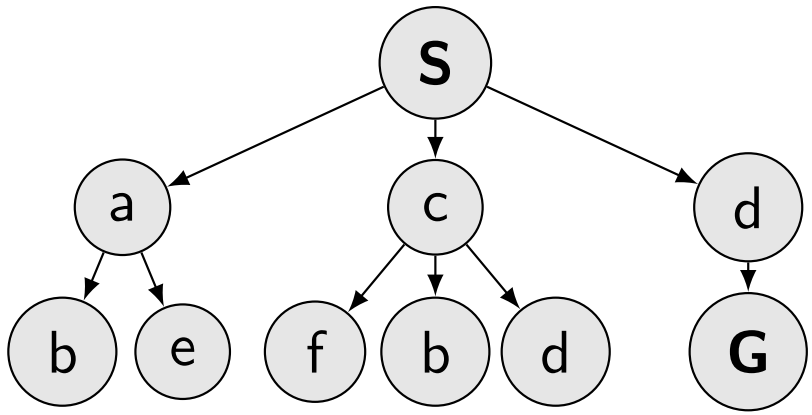

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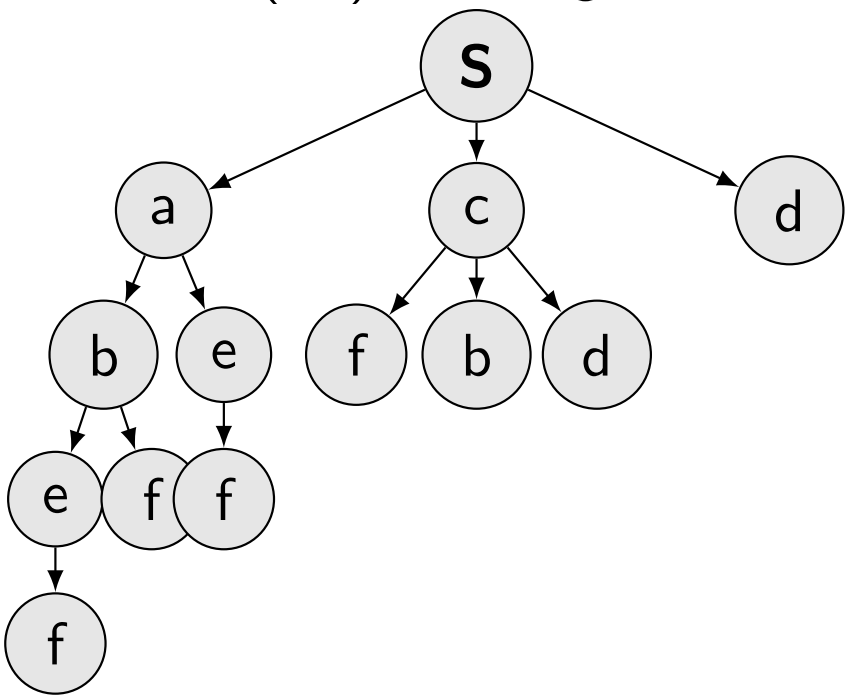

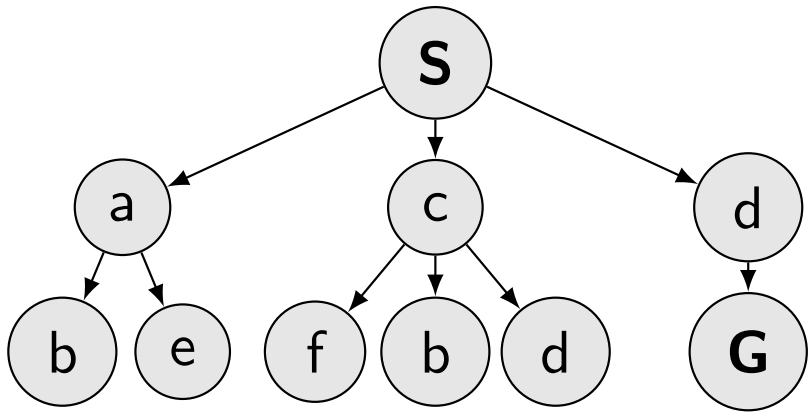

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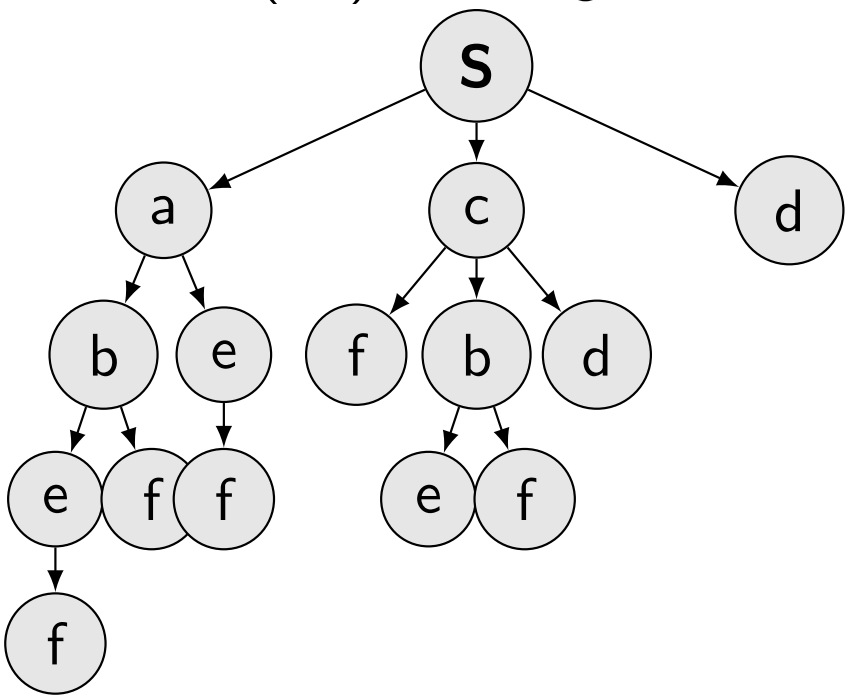

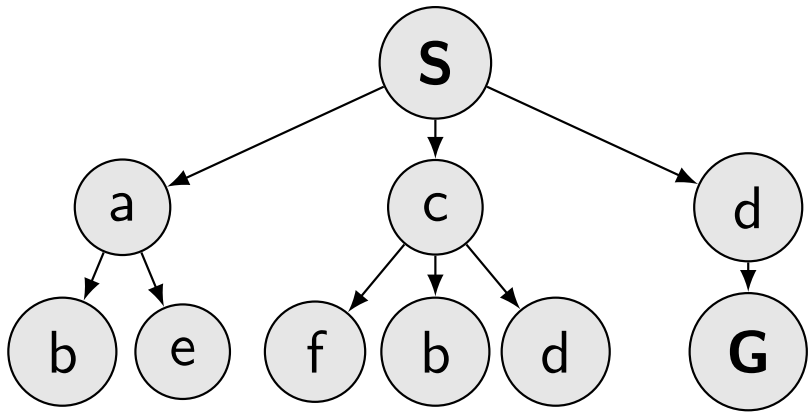

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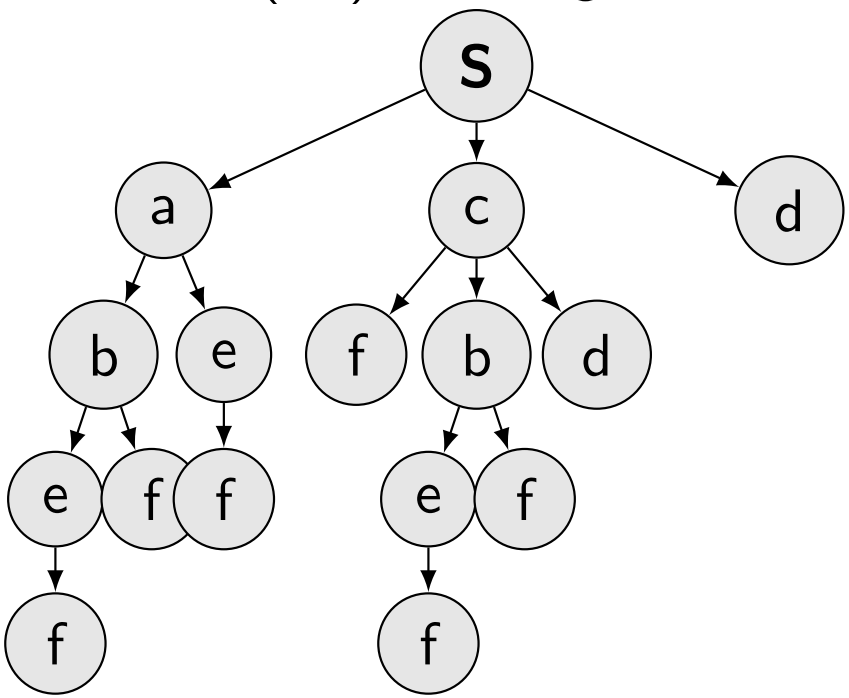

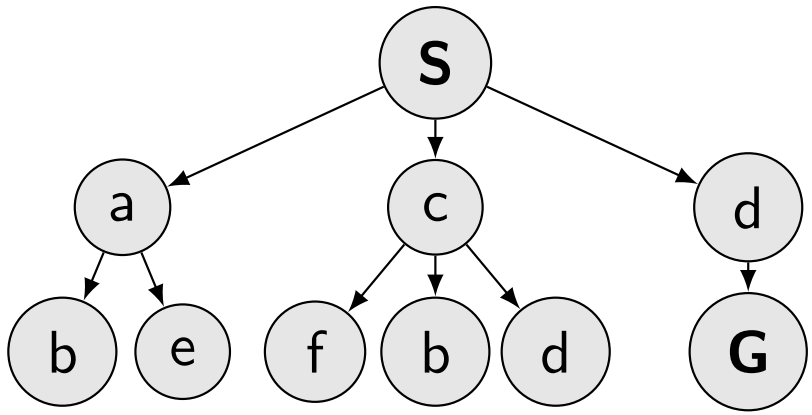

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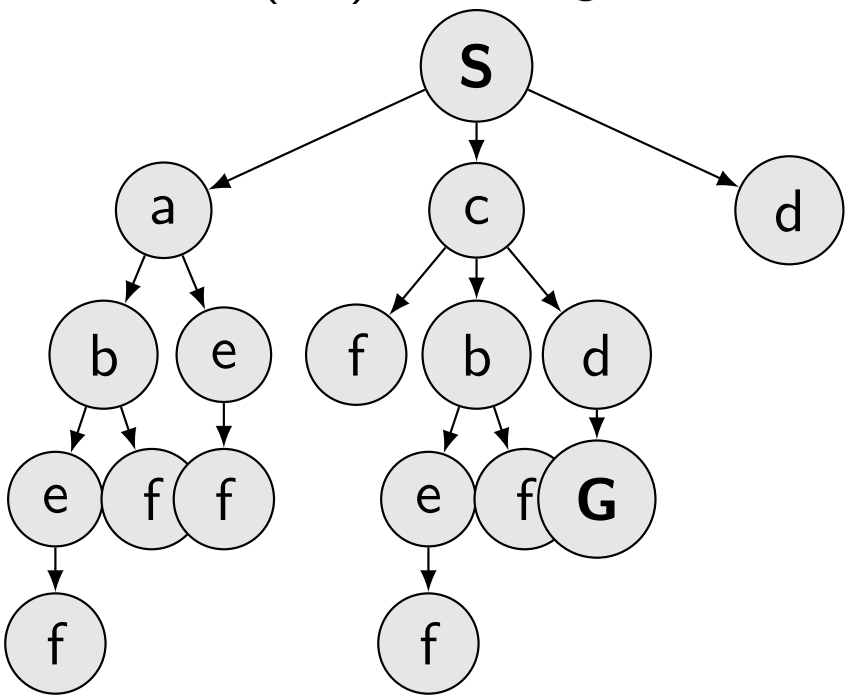

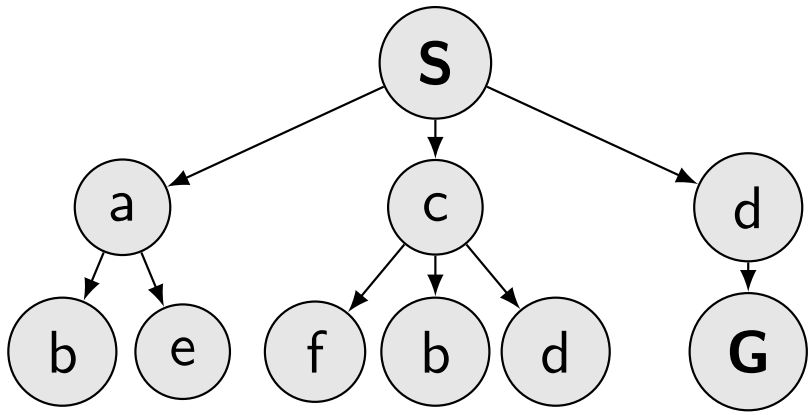

DFS with limited depth, `maxdepth=2`

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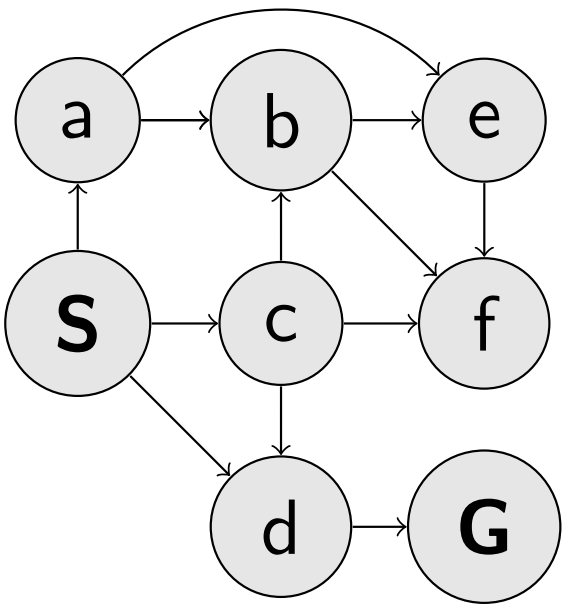

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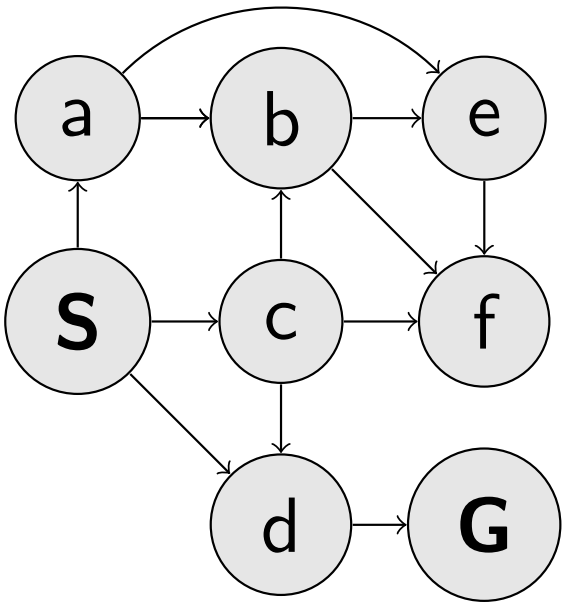

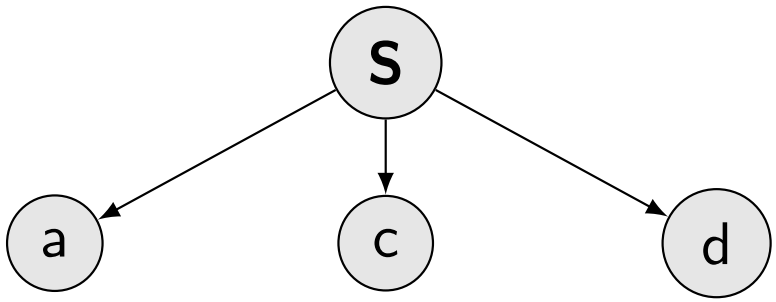

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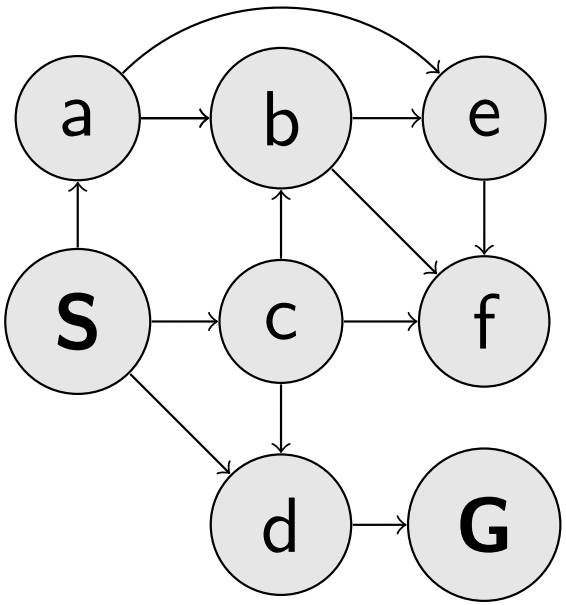

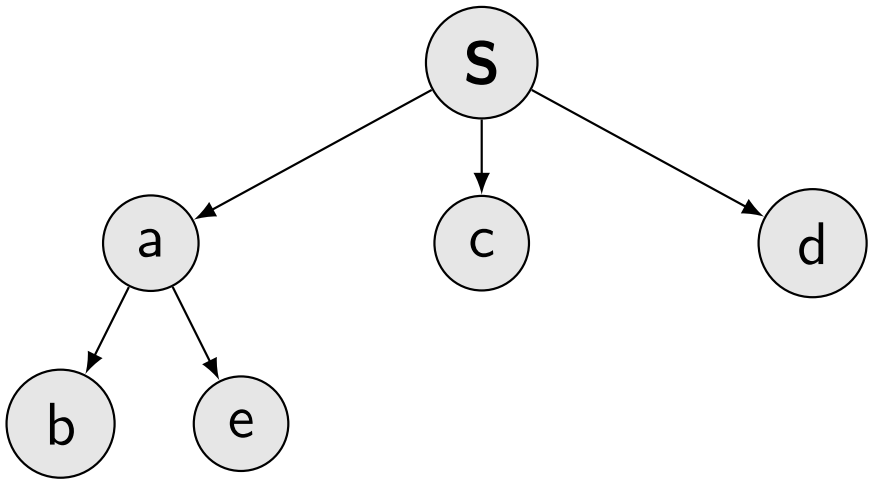

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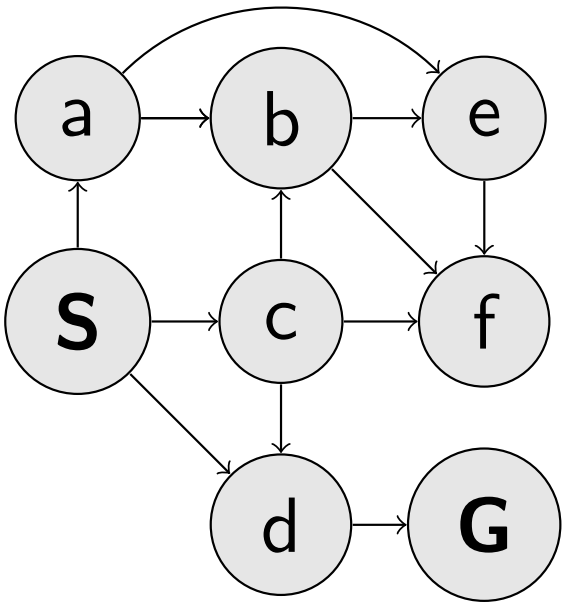

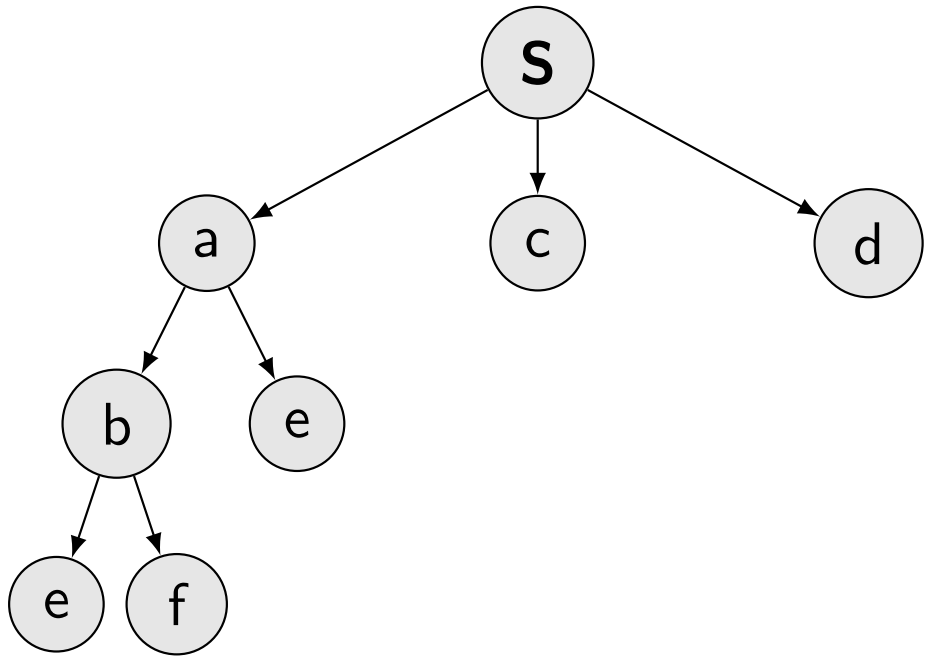

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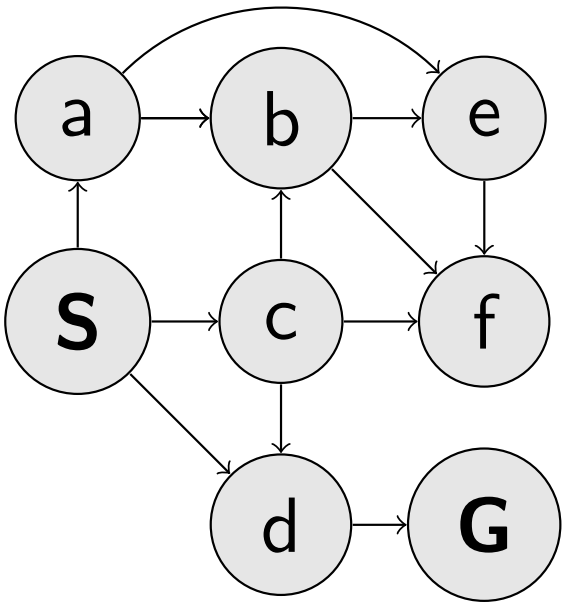

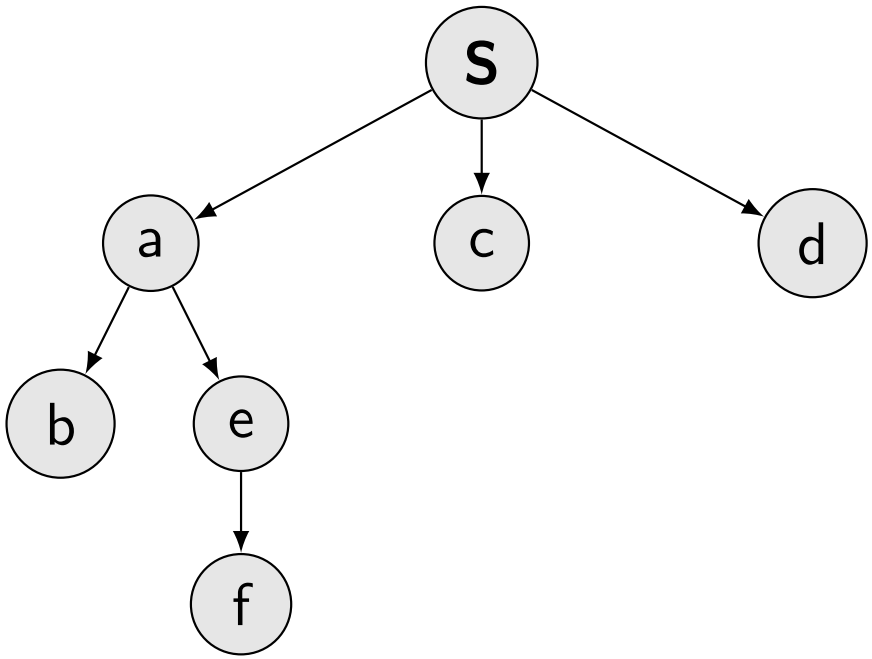

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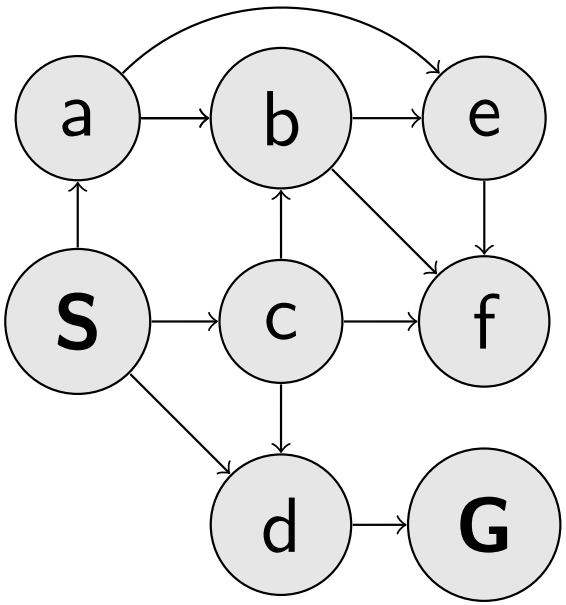

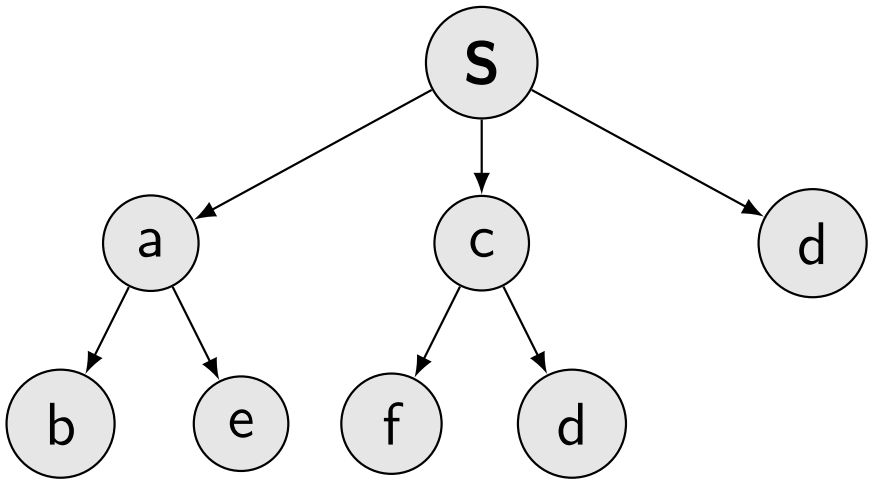

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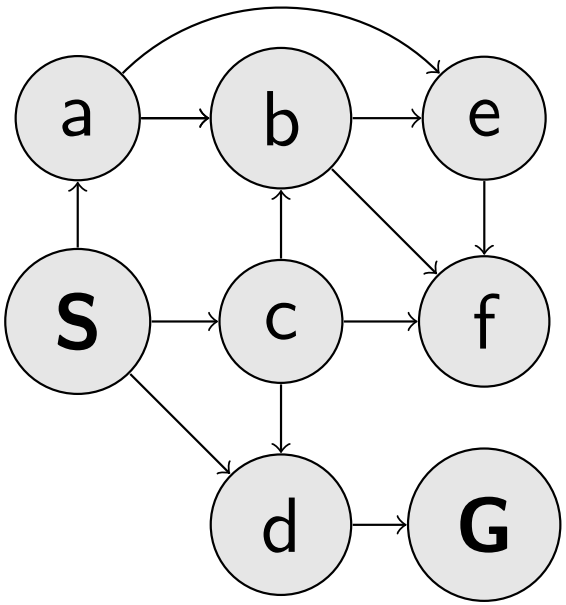

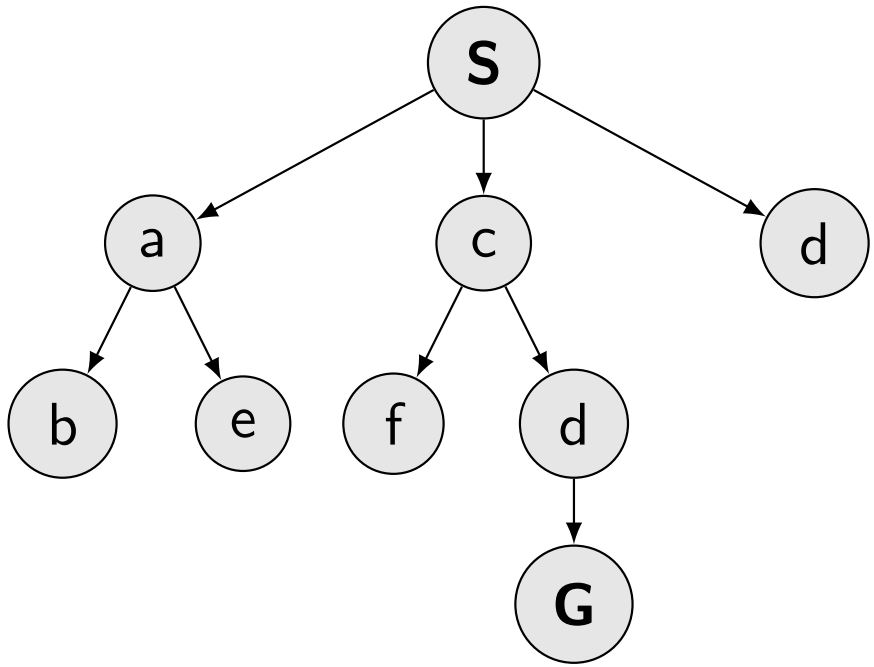

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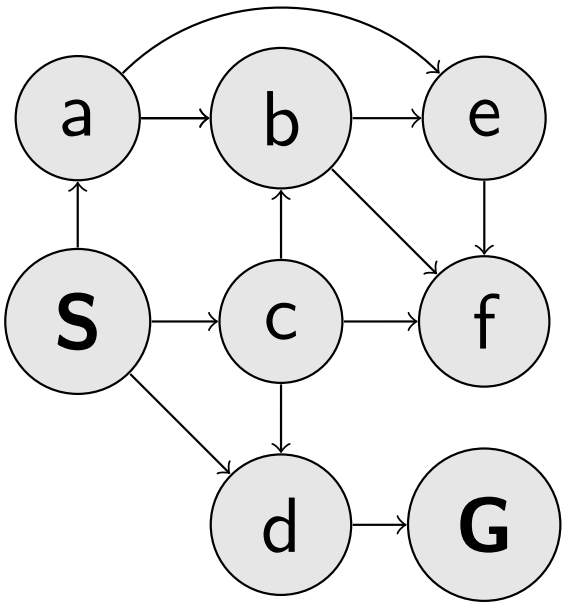

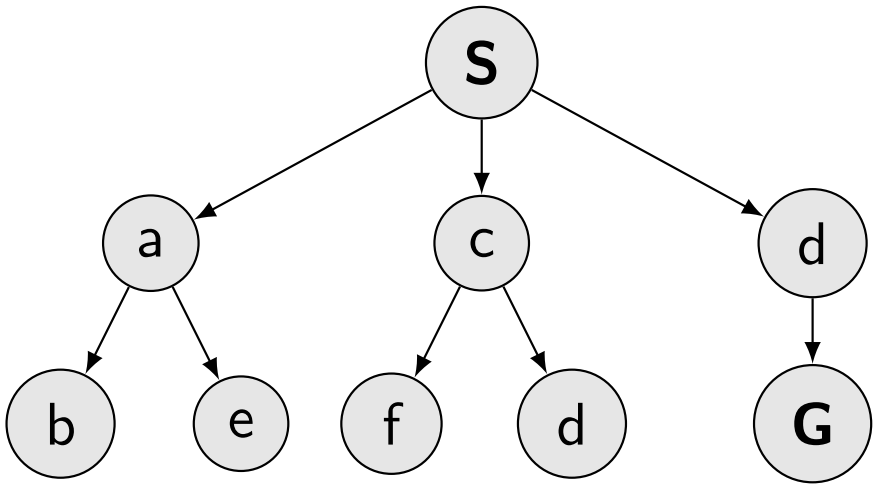

Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

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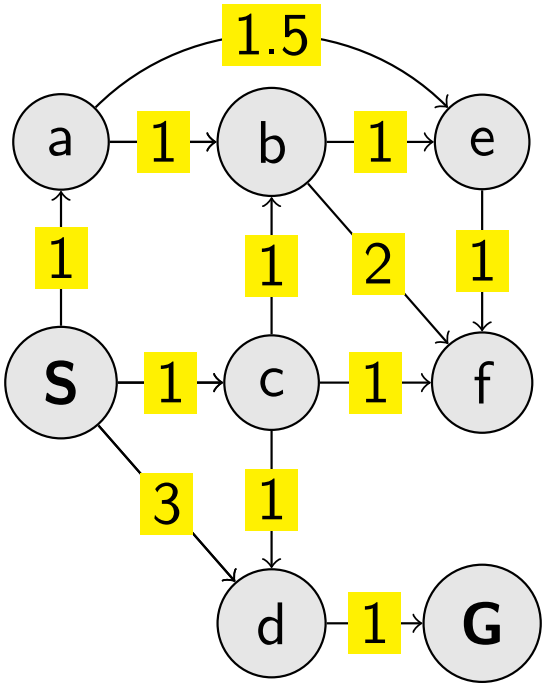
Cost sensitive search



- In BFS, DFS, node ±depth was the node-value.
- How was the depth actually computed?
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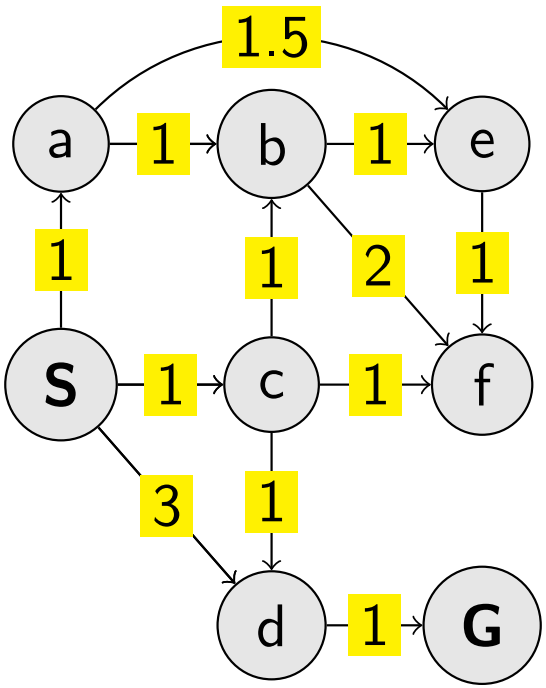

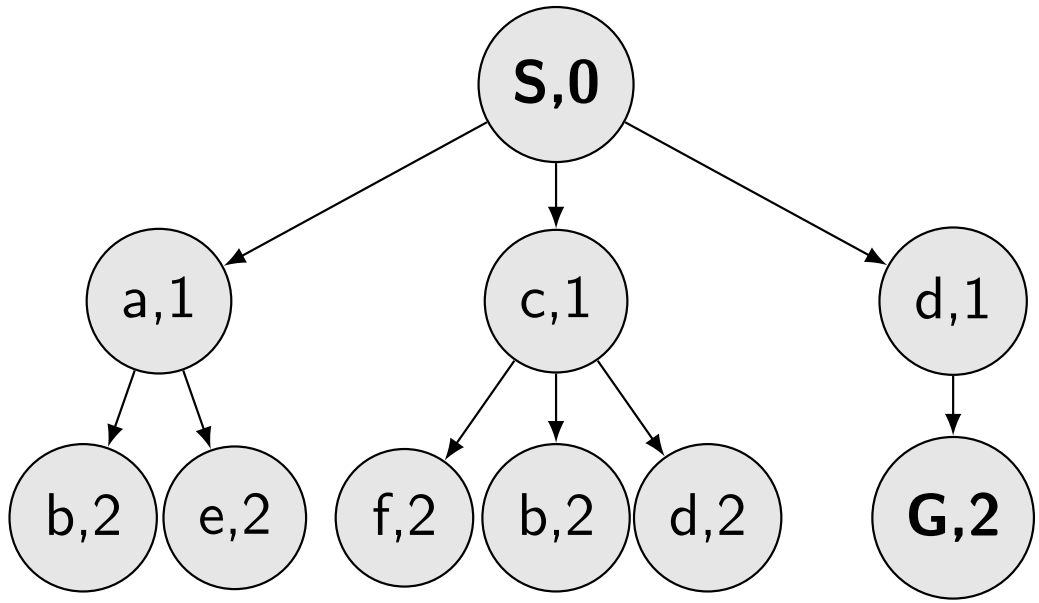

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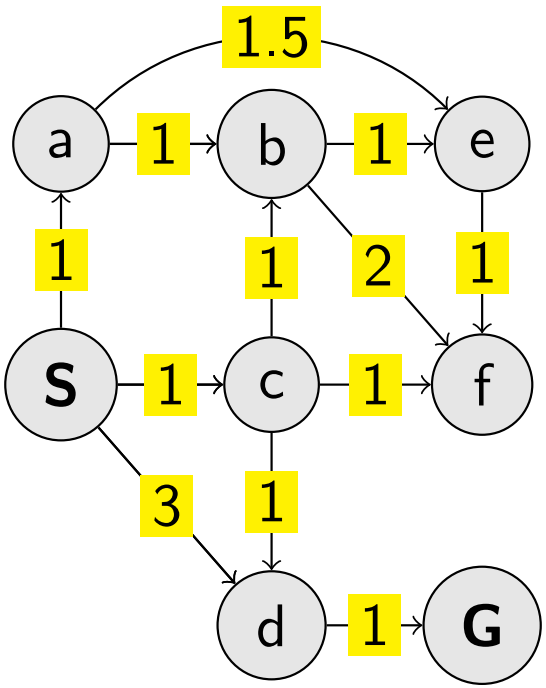

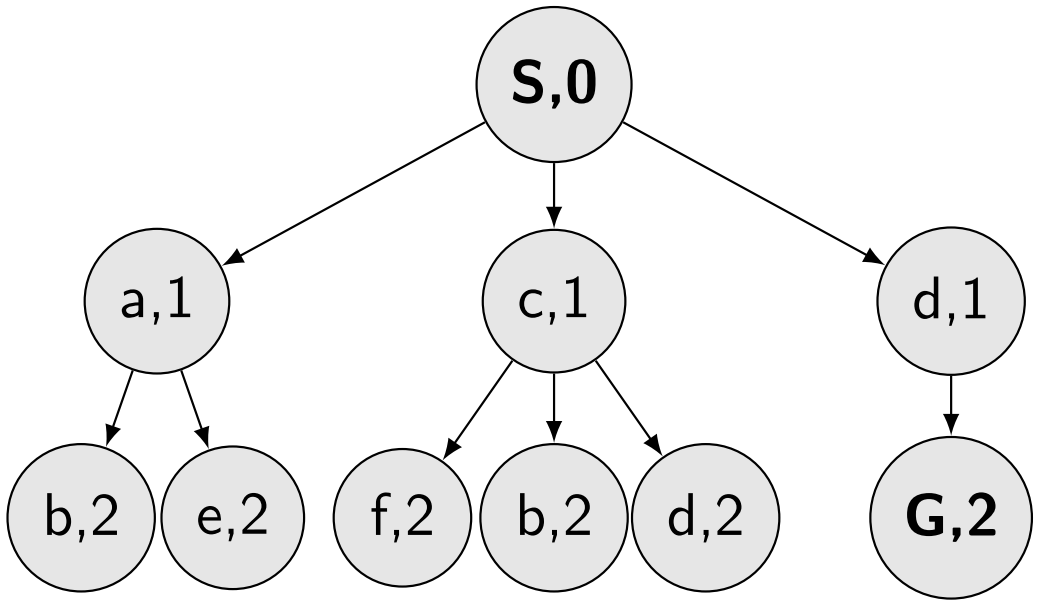

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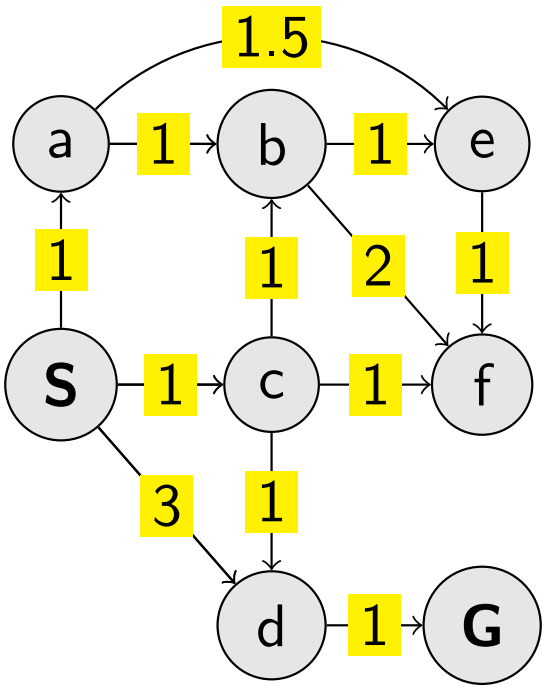

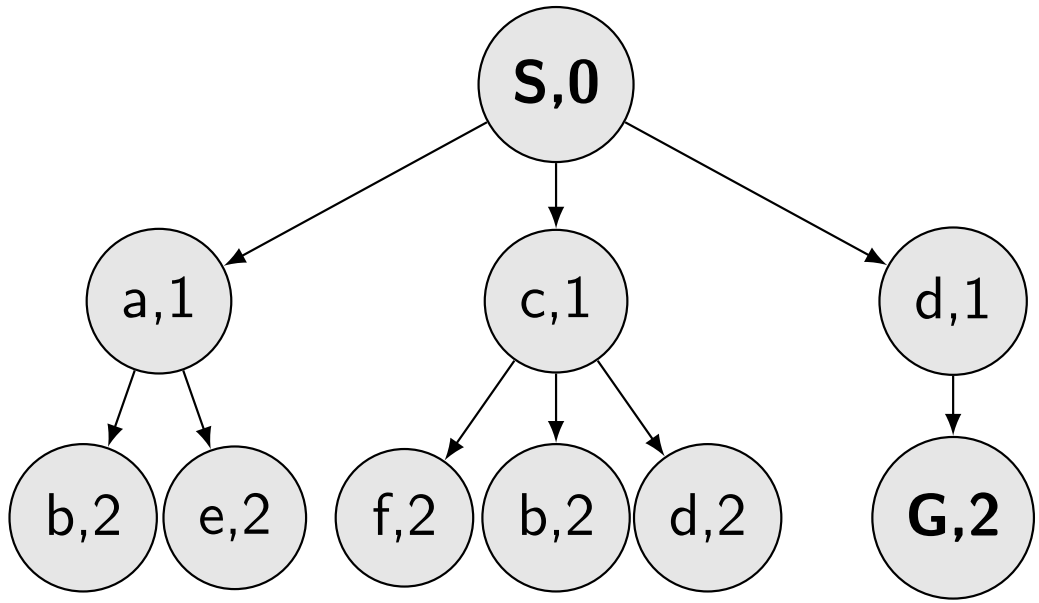

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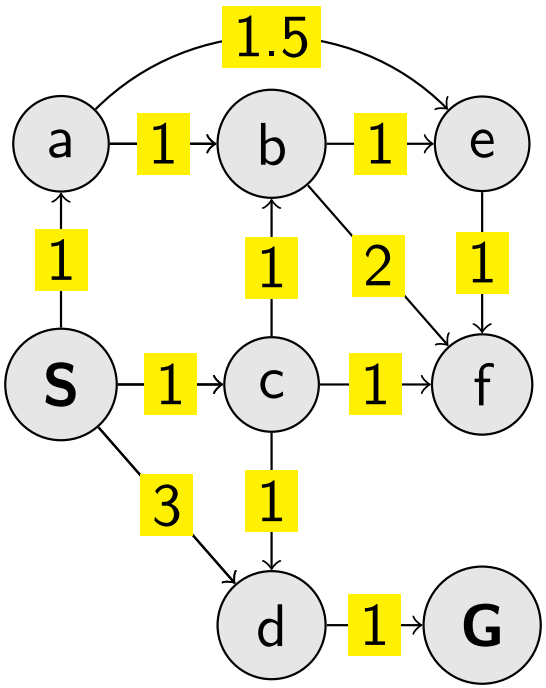

When to check the goal (and stop) the search? When visiting or expanding the node?

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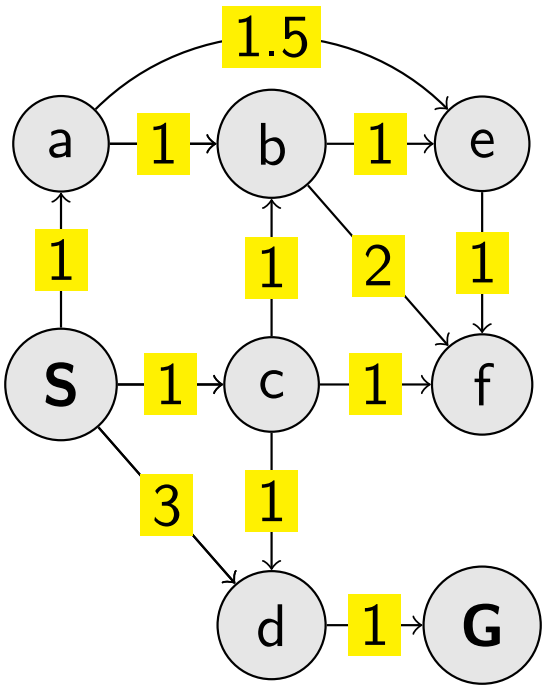

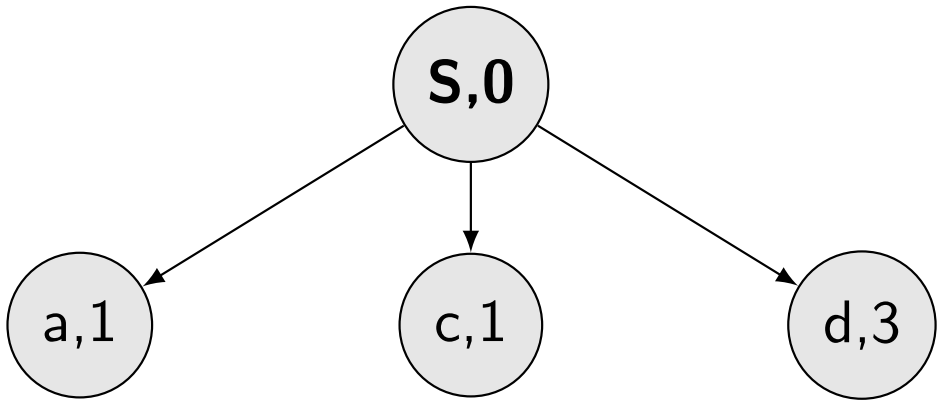

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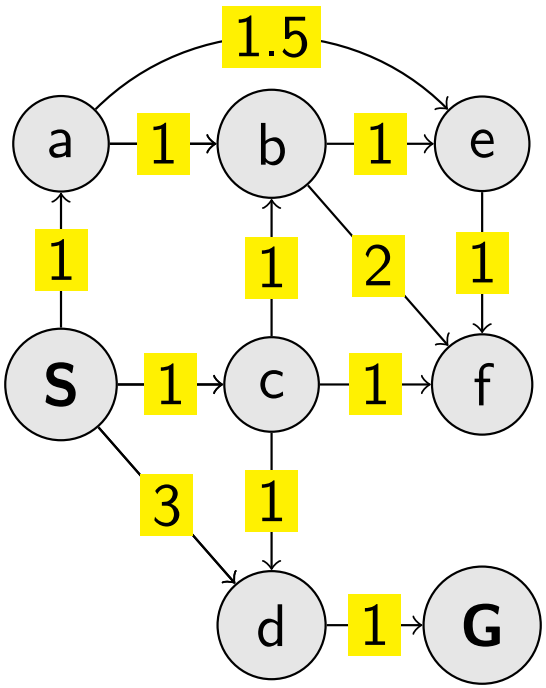

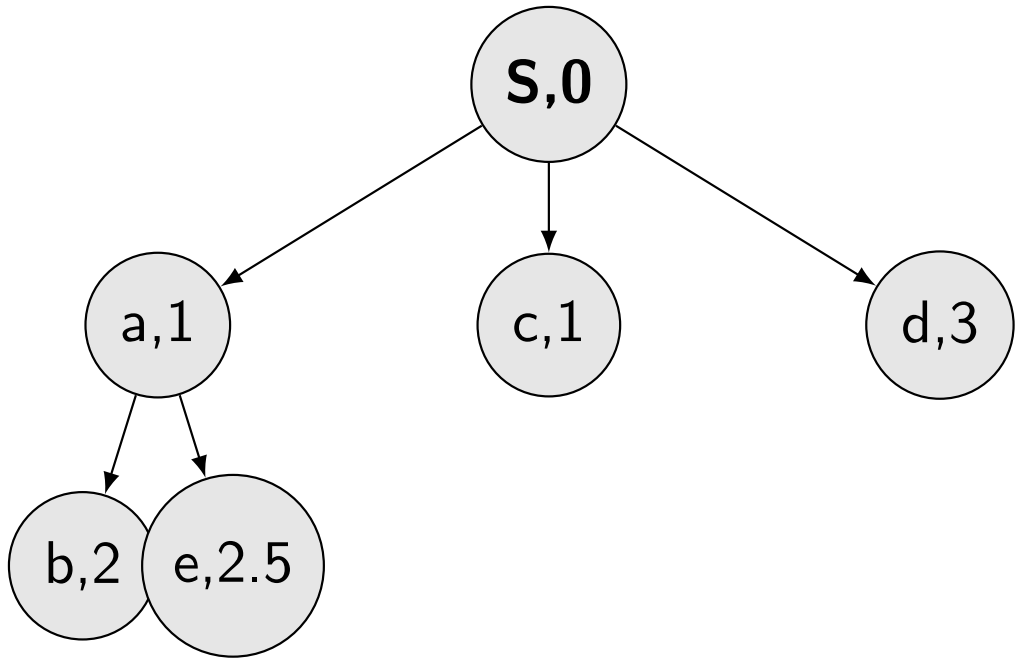

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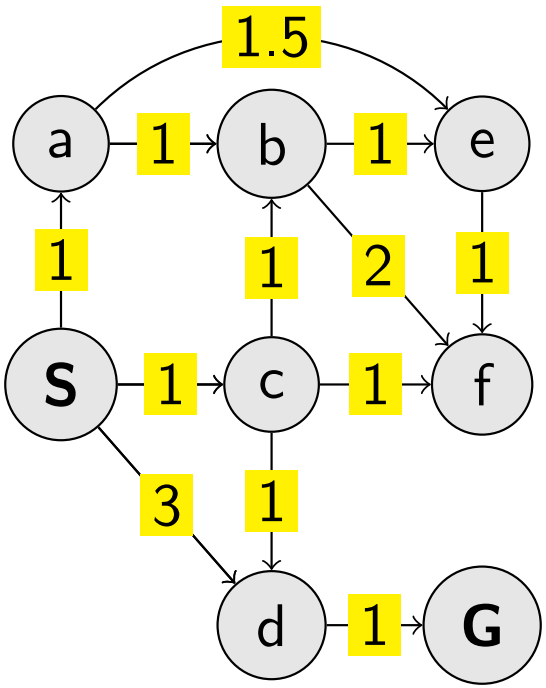

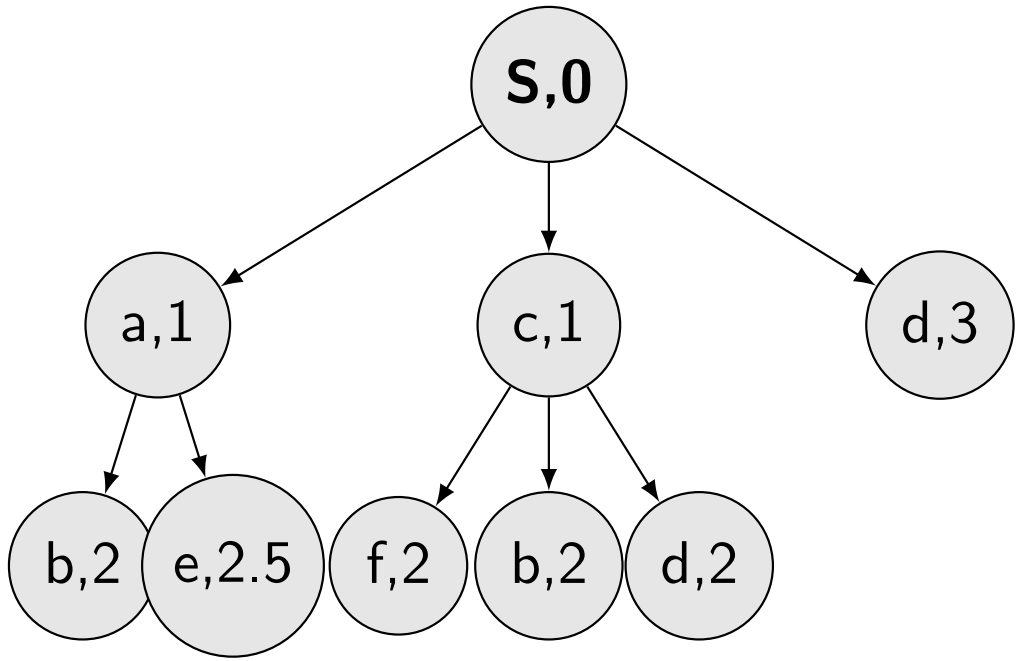

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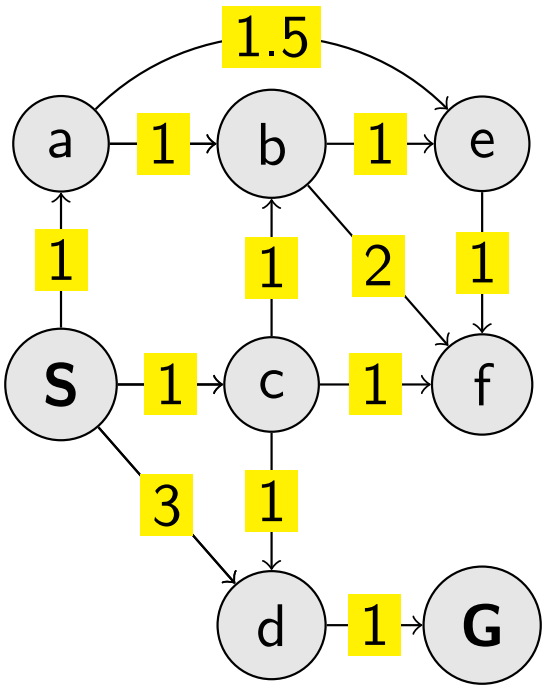

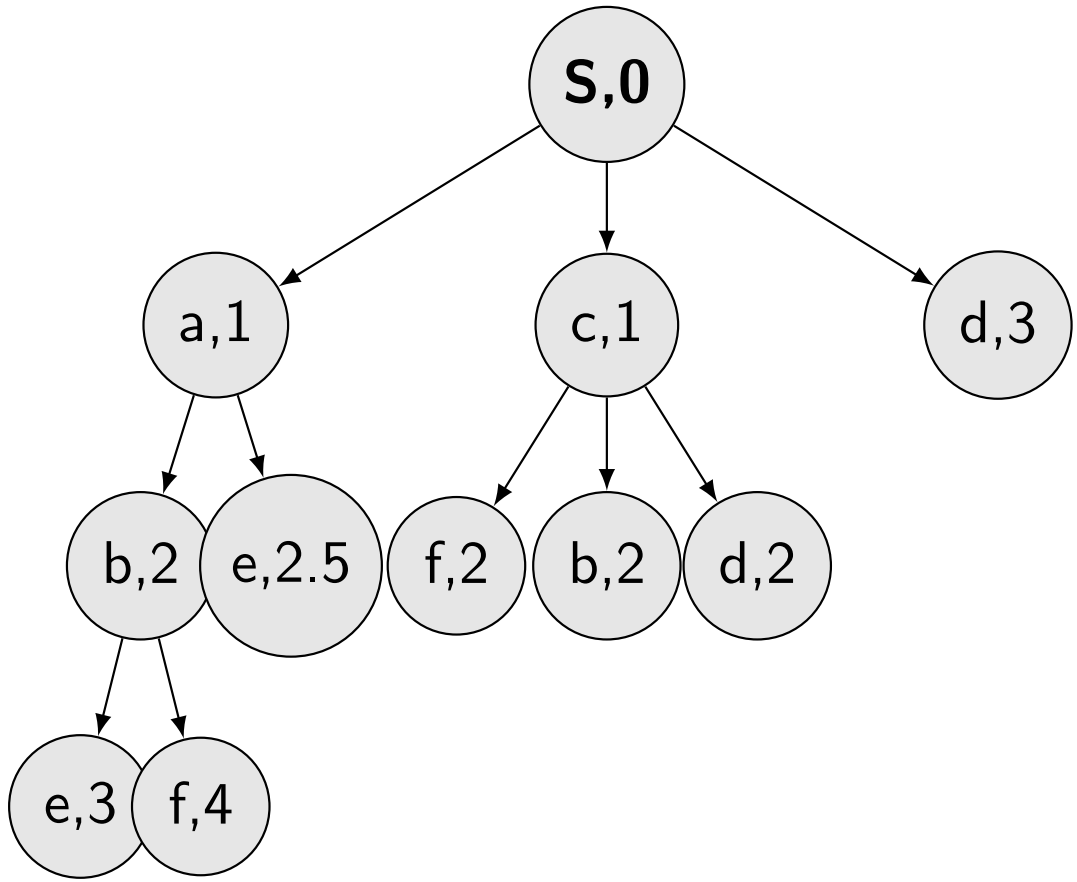

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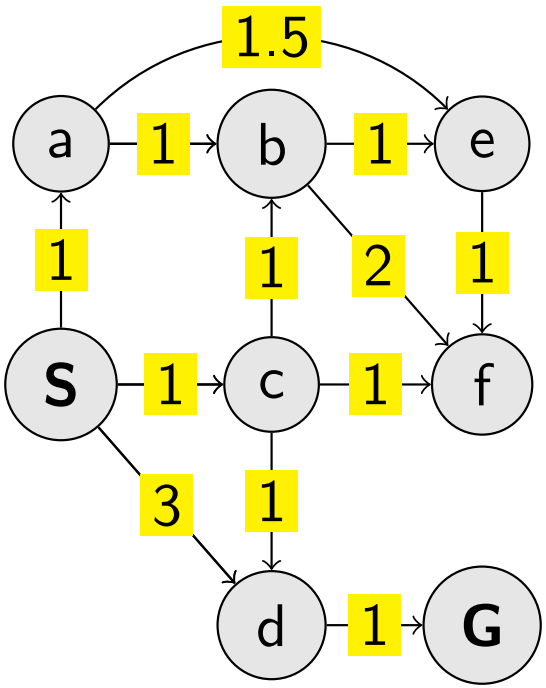

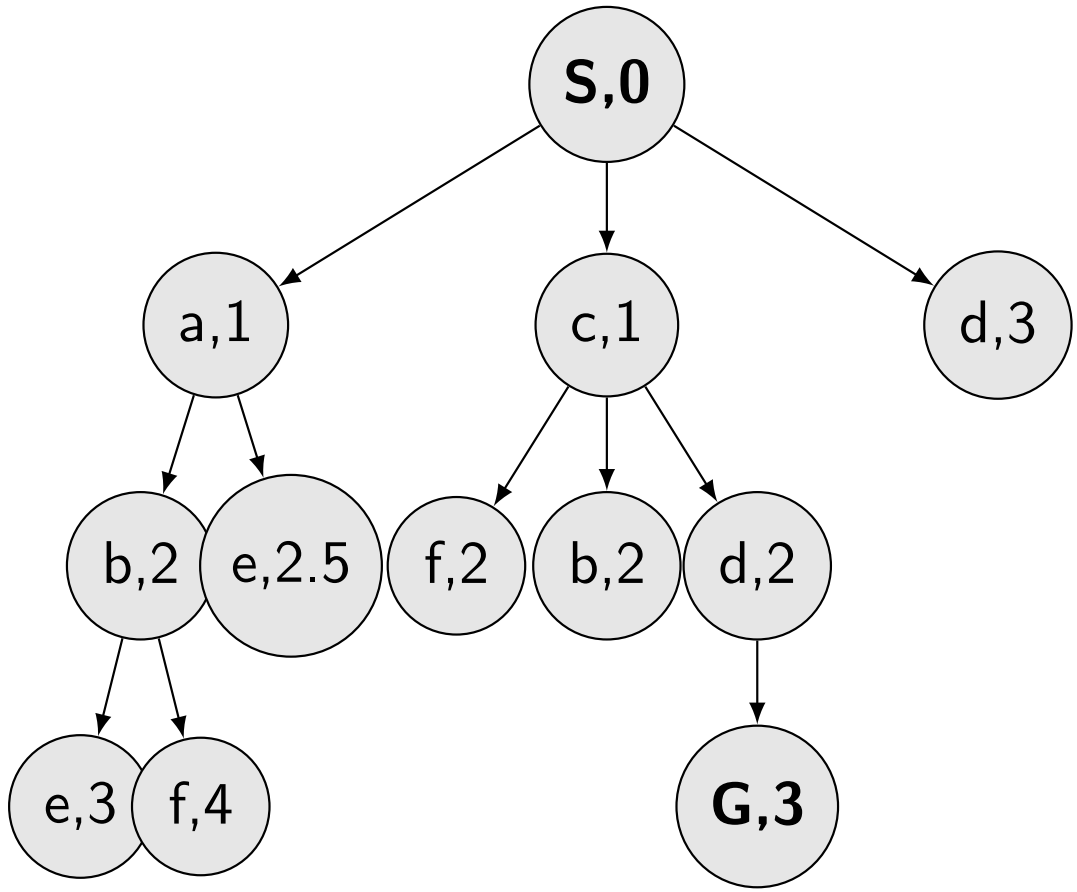

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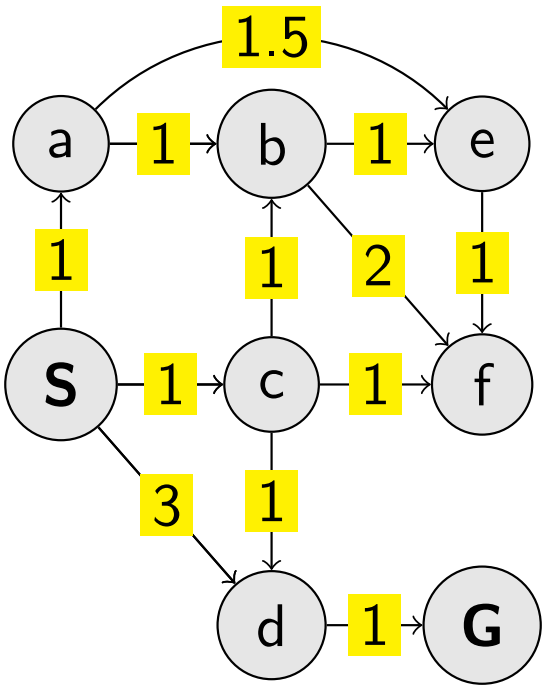

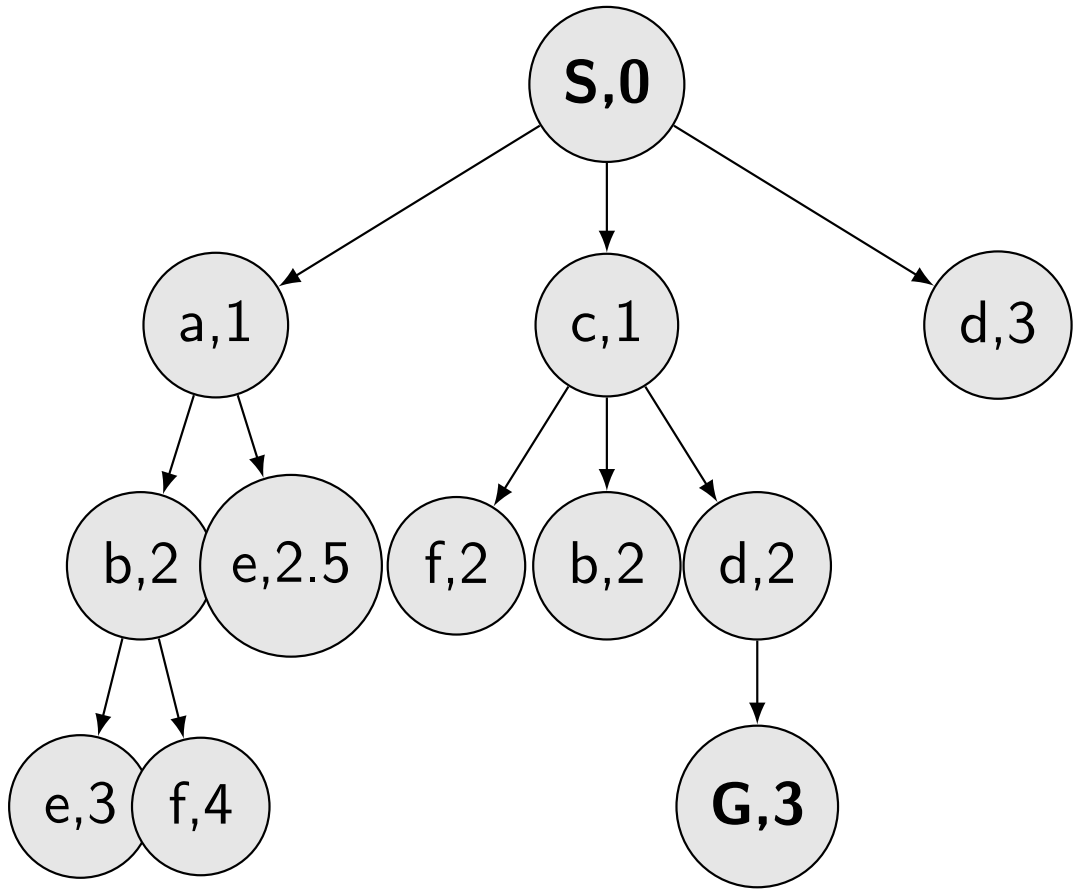

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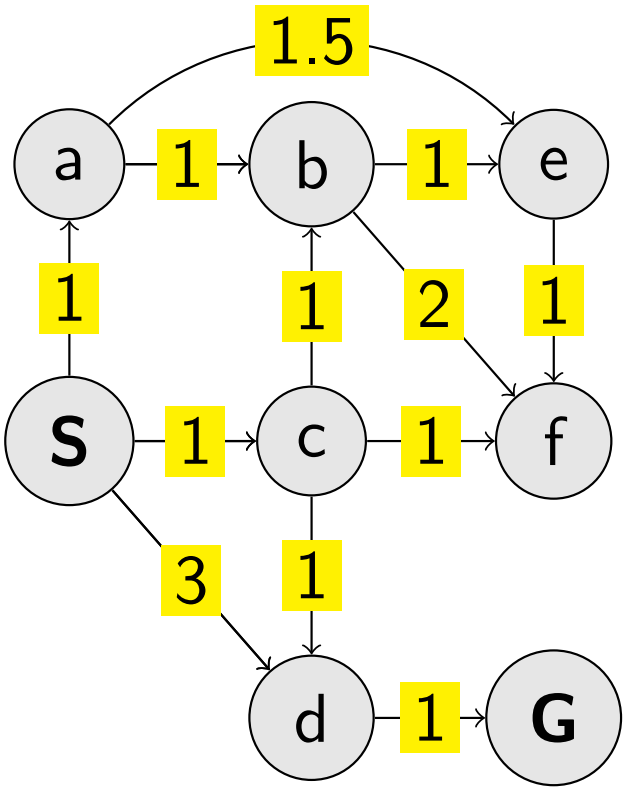

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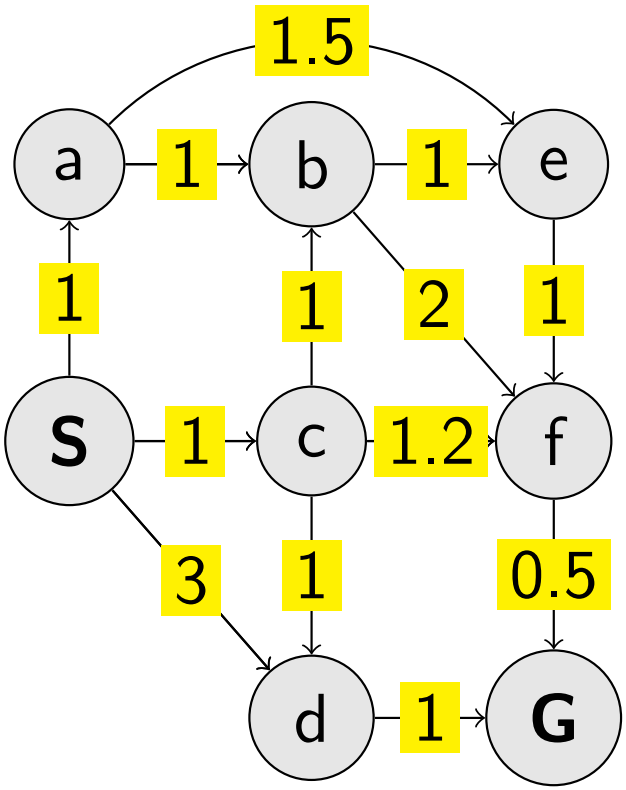

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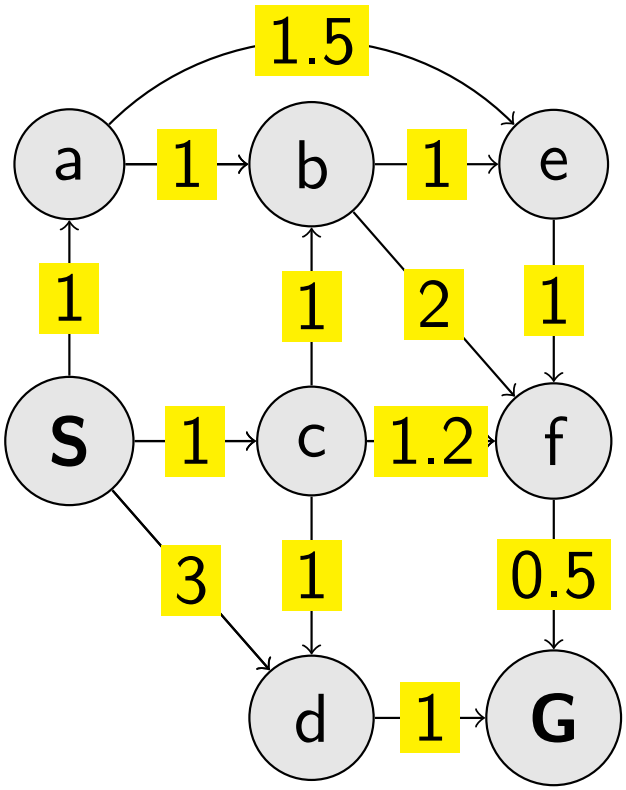

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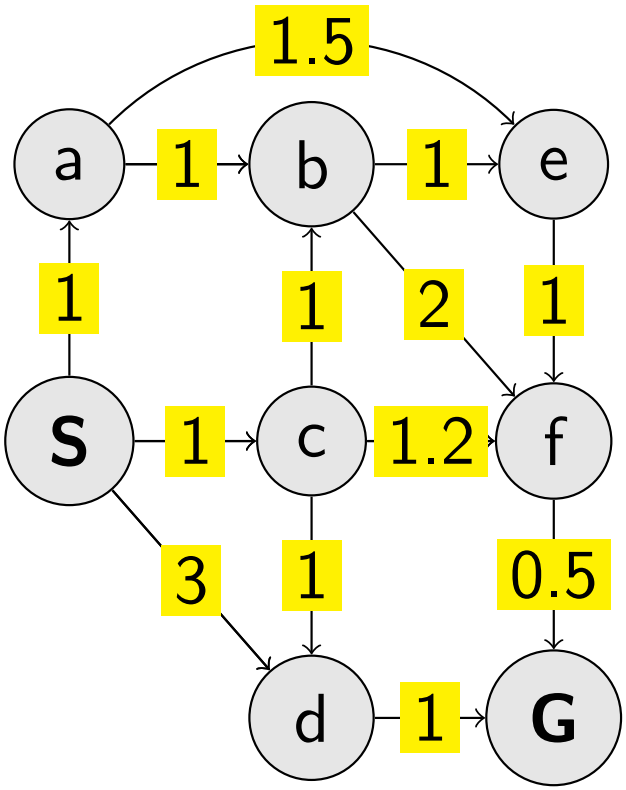

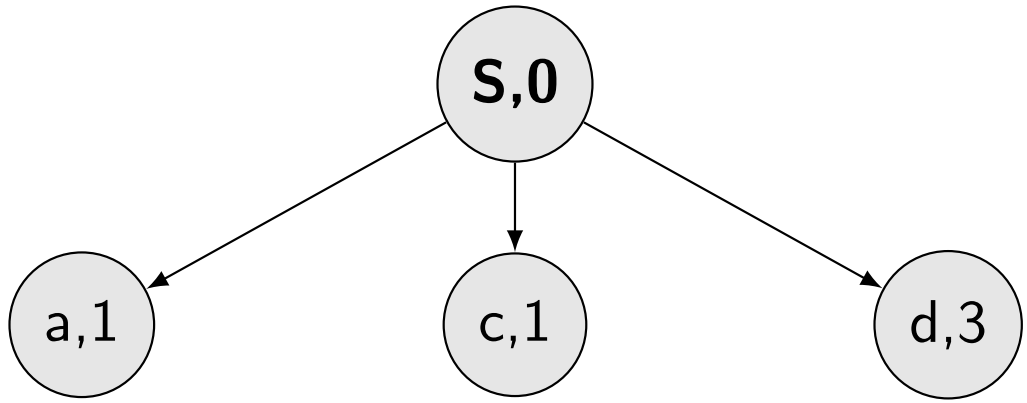

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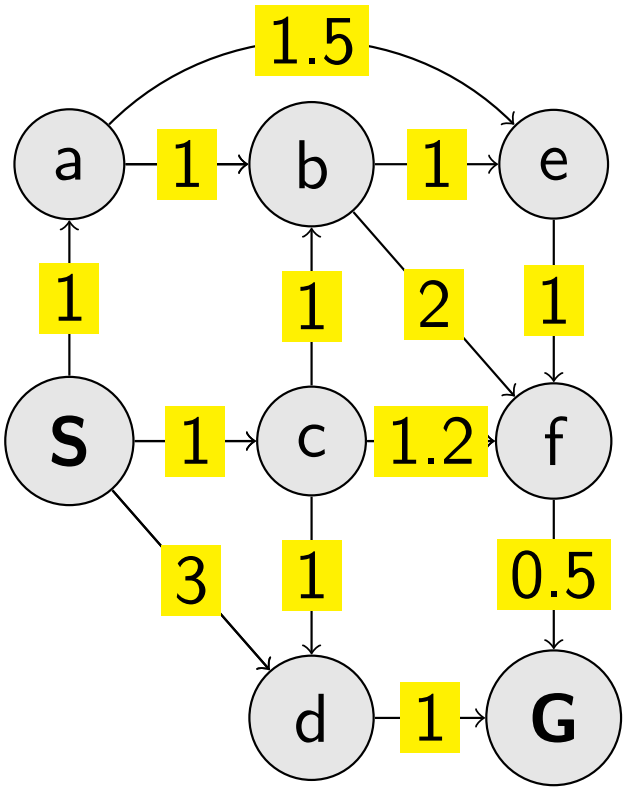

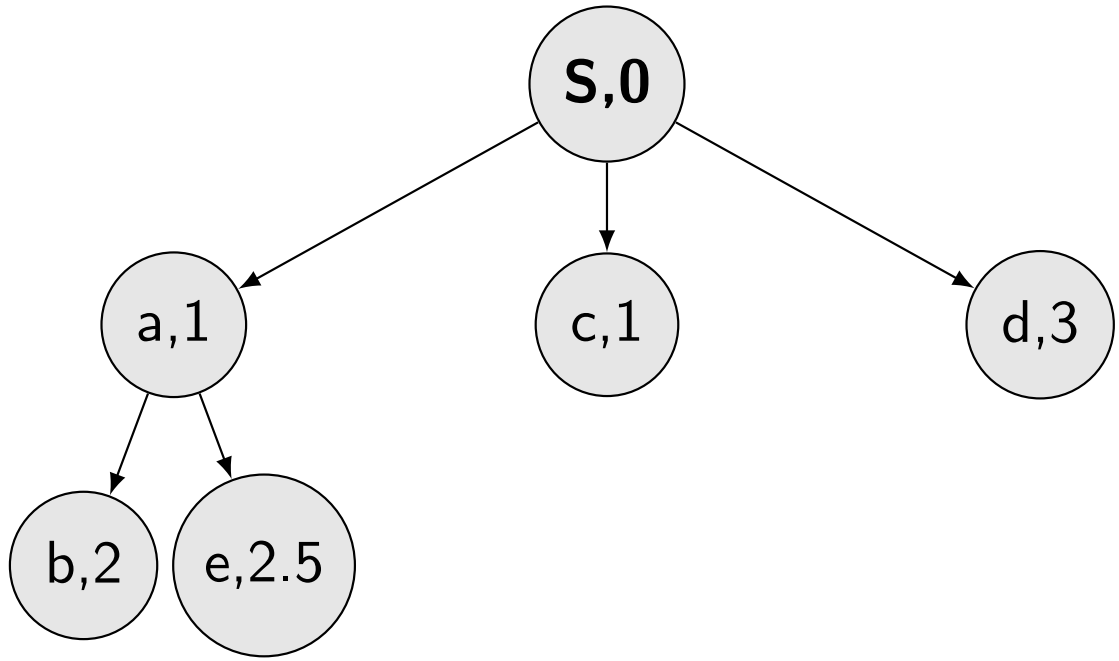

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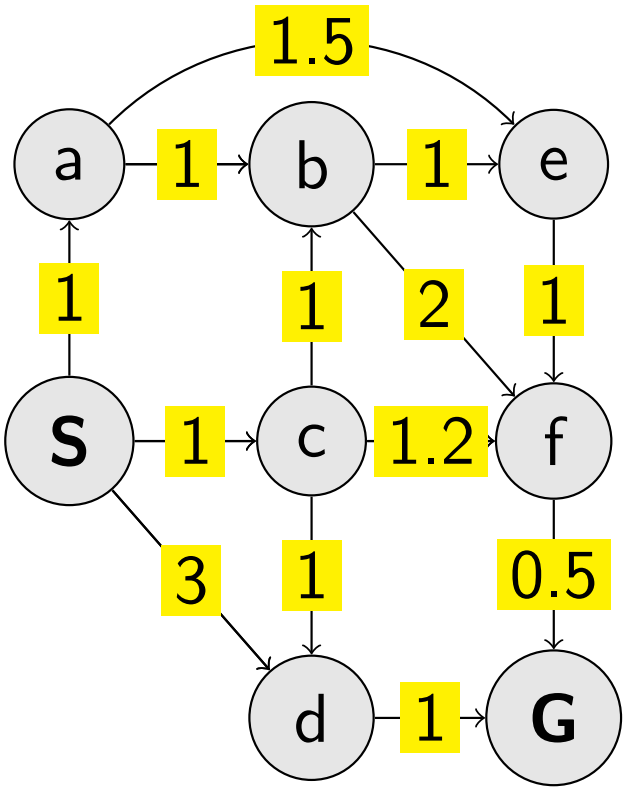

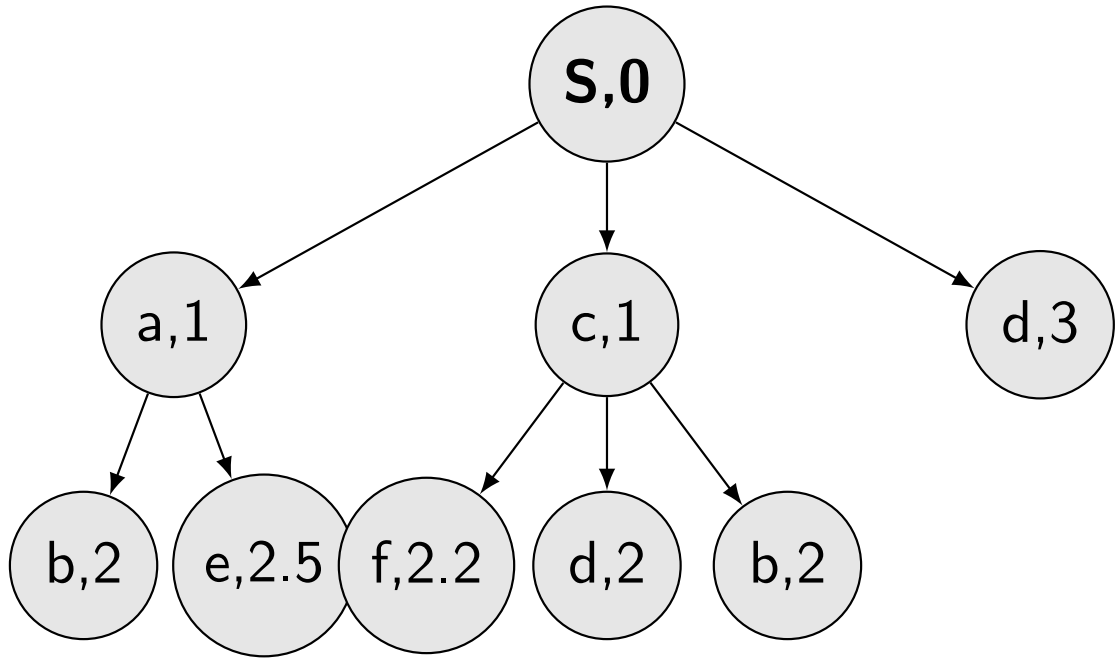

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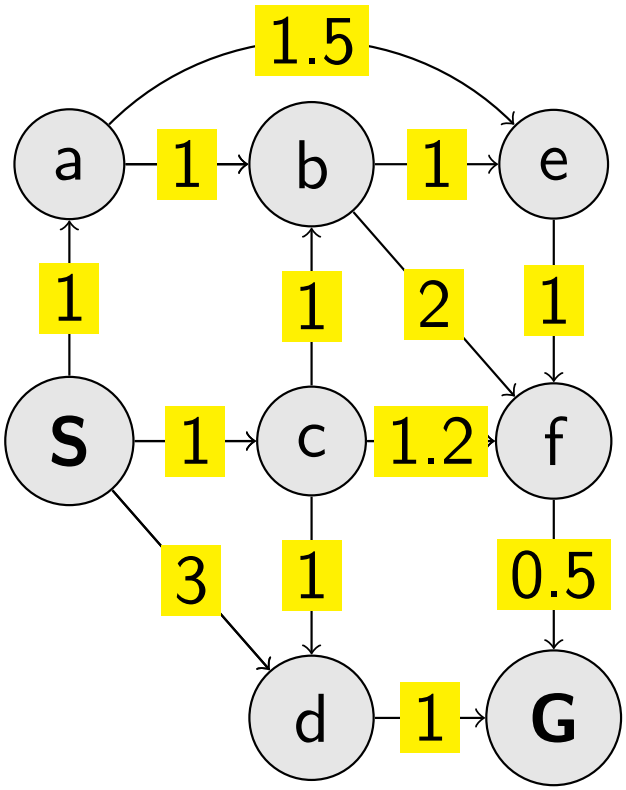

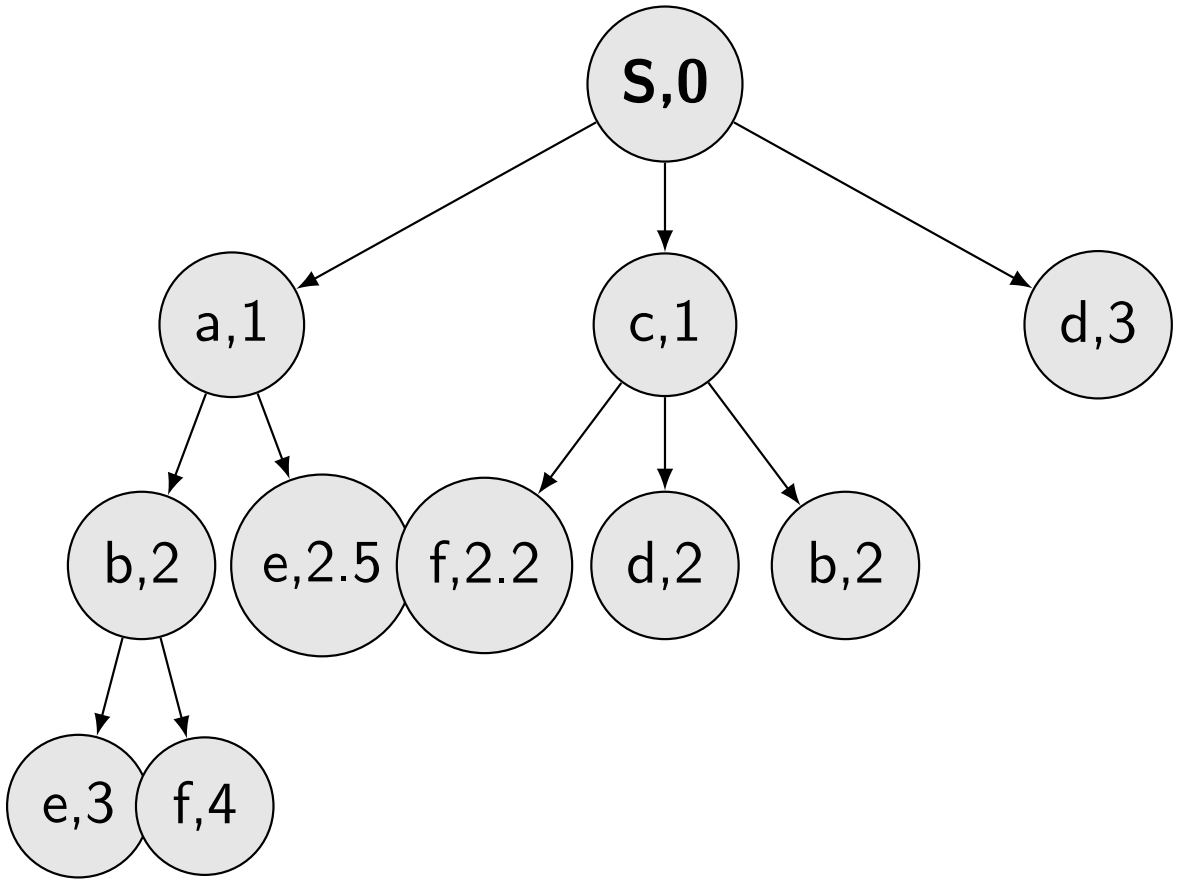

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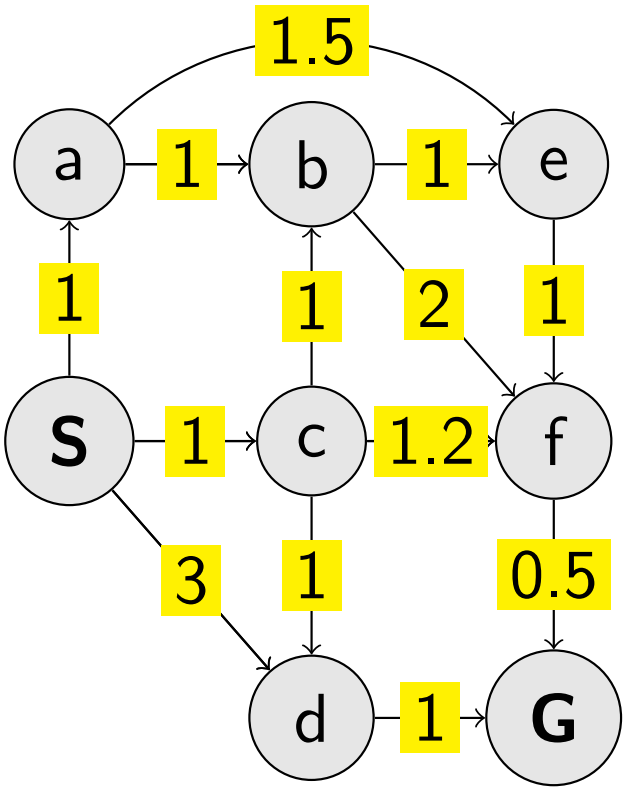

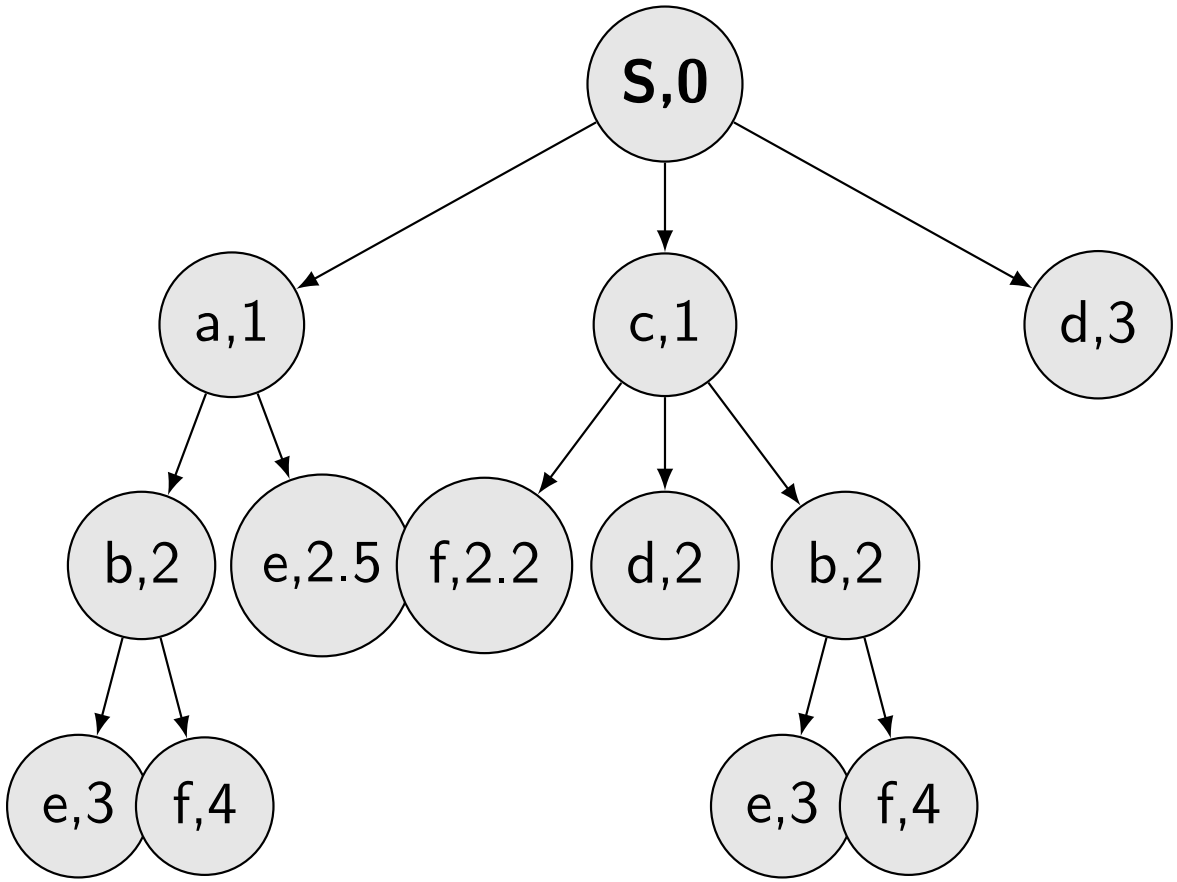

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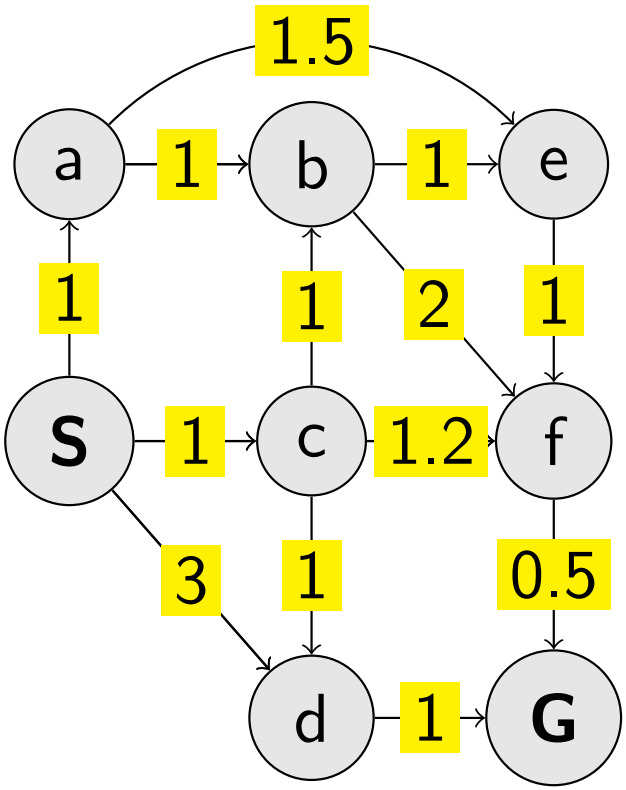

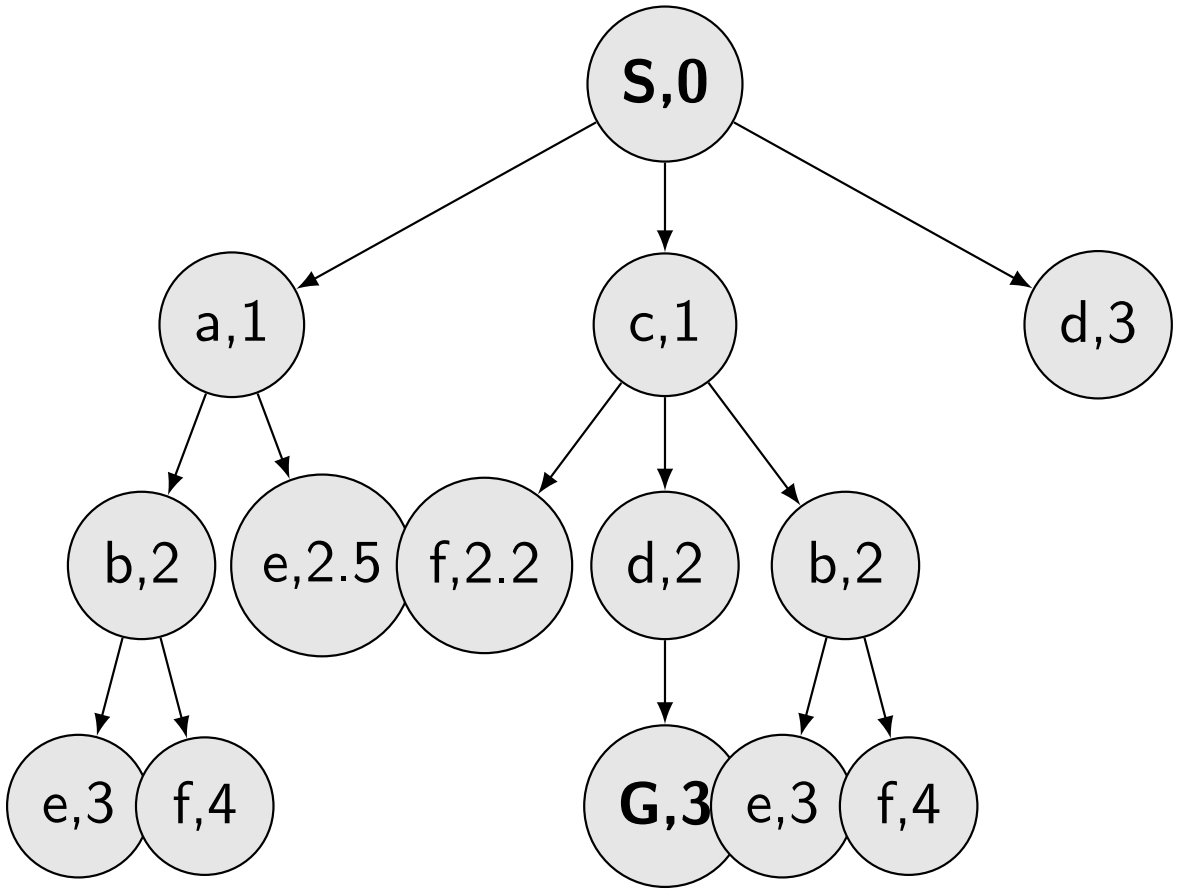

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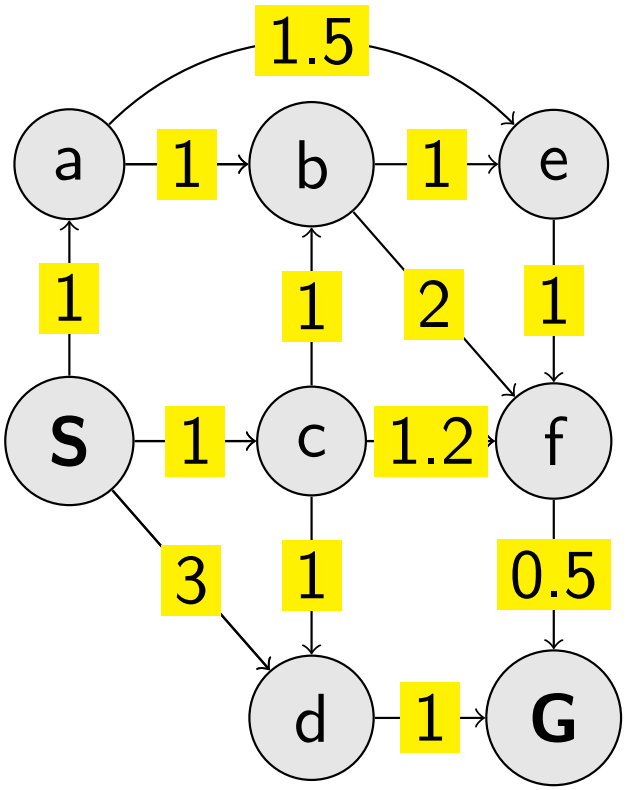

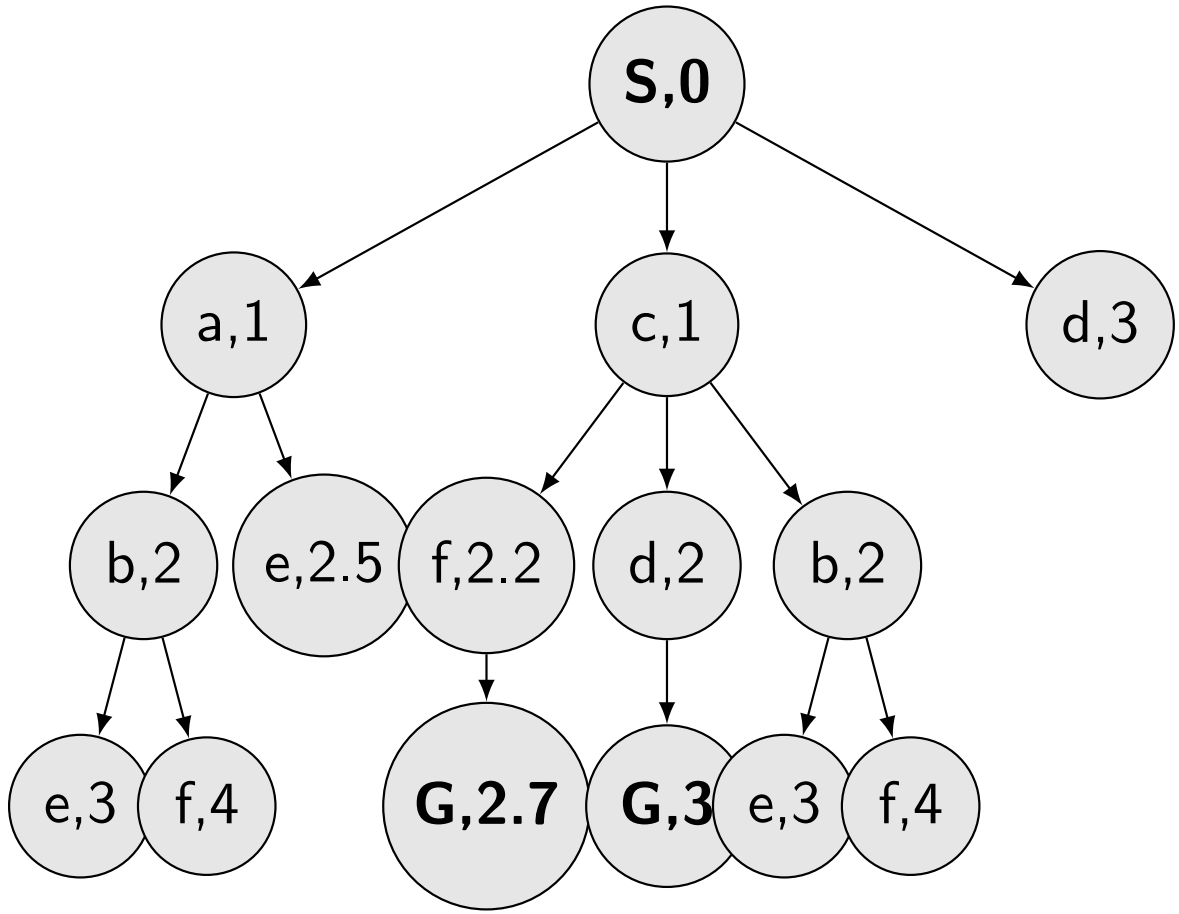

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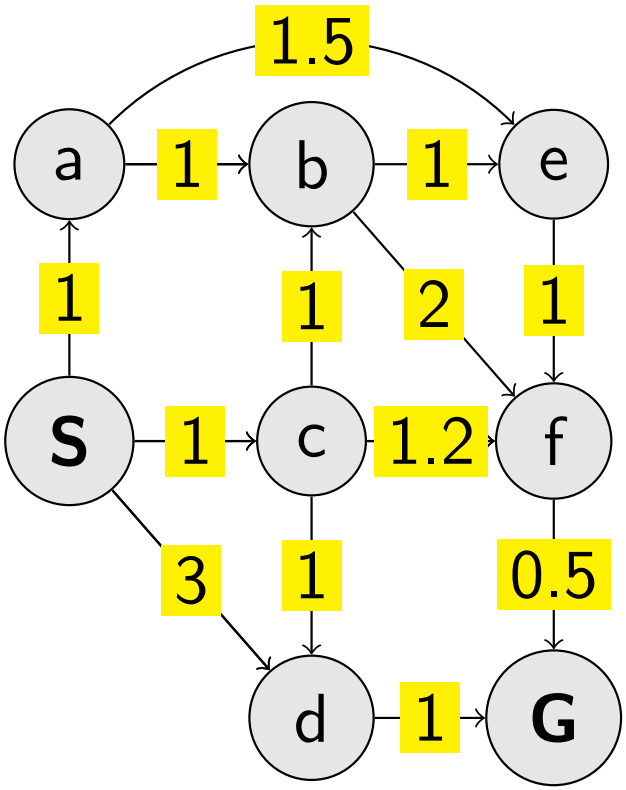

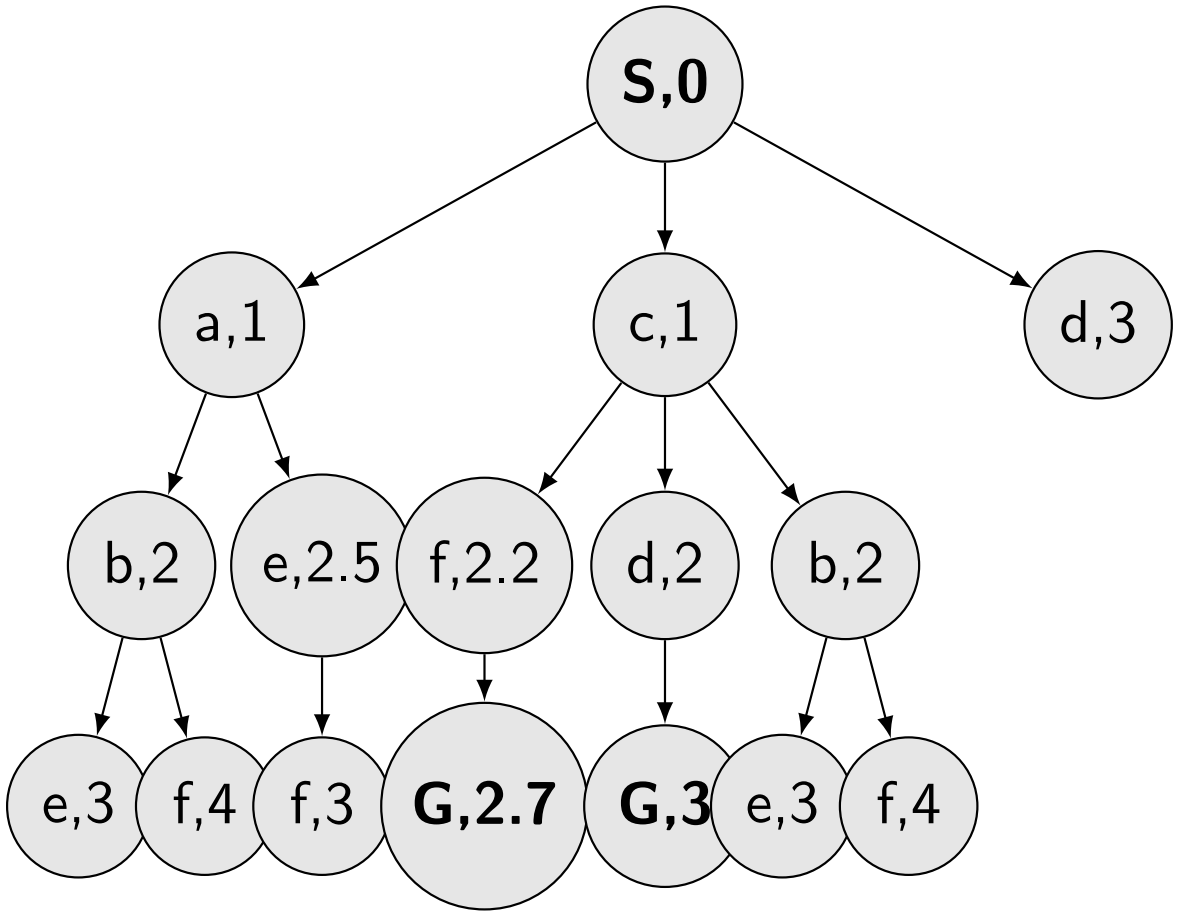

UCS properties

- Complete?
- Optimal?
- Time complexity?
- Space complexity?

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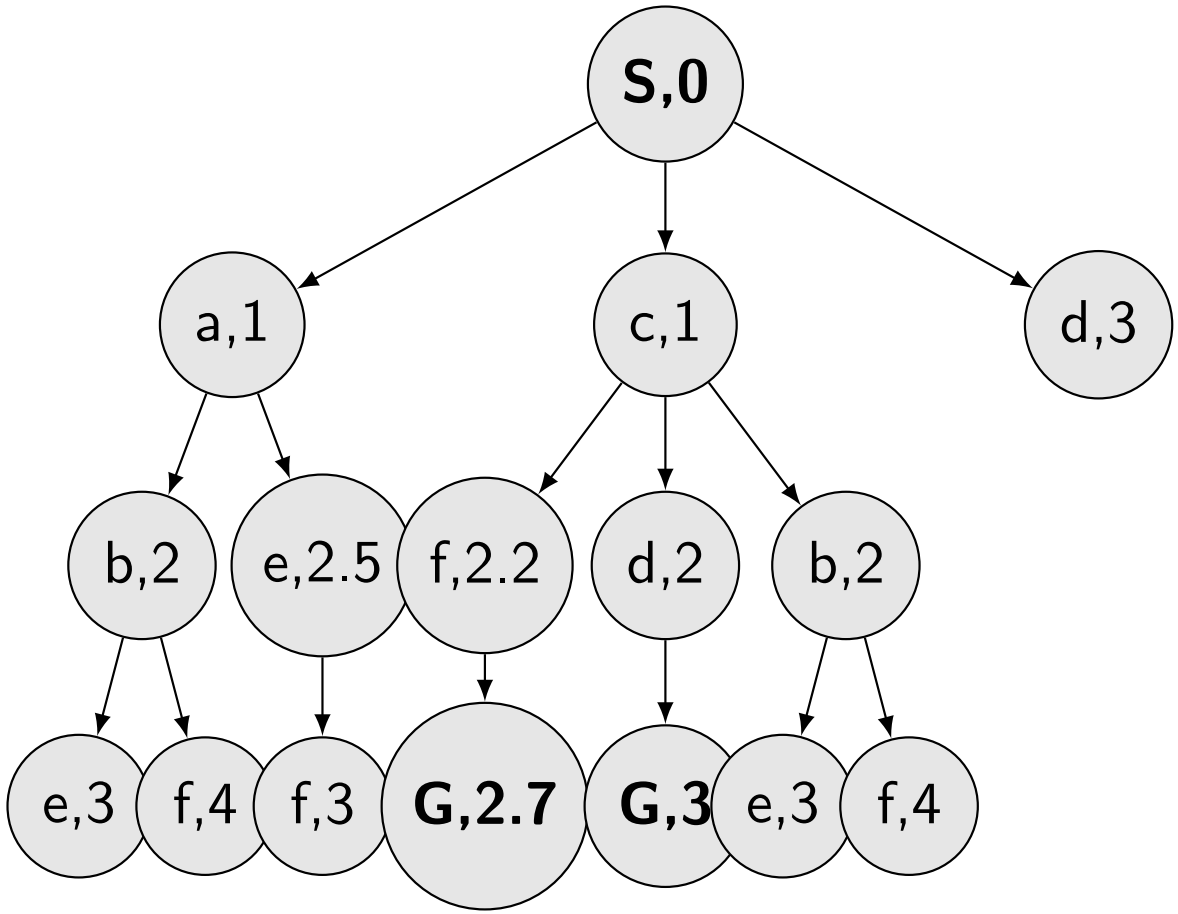
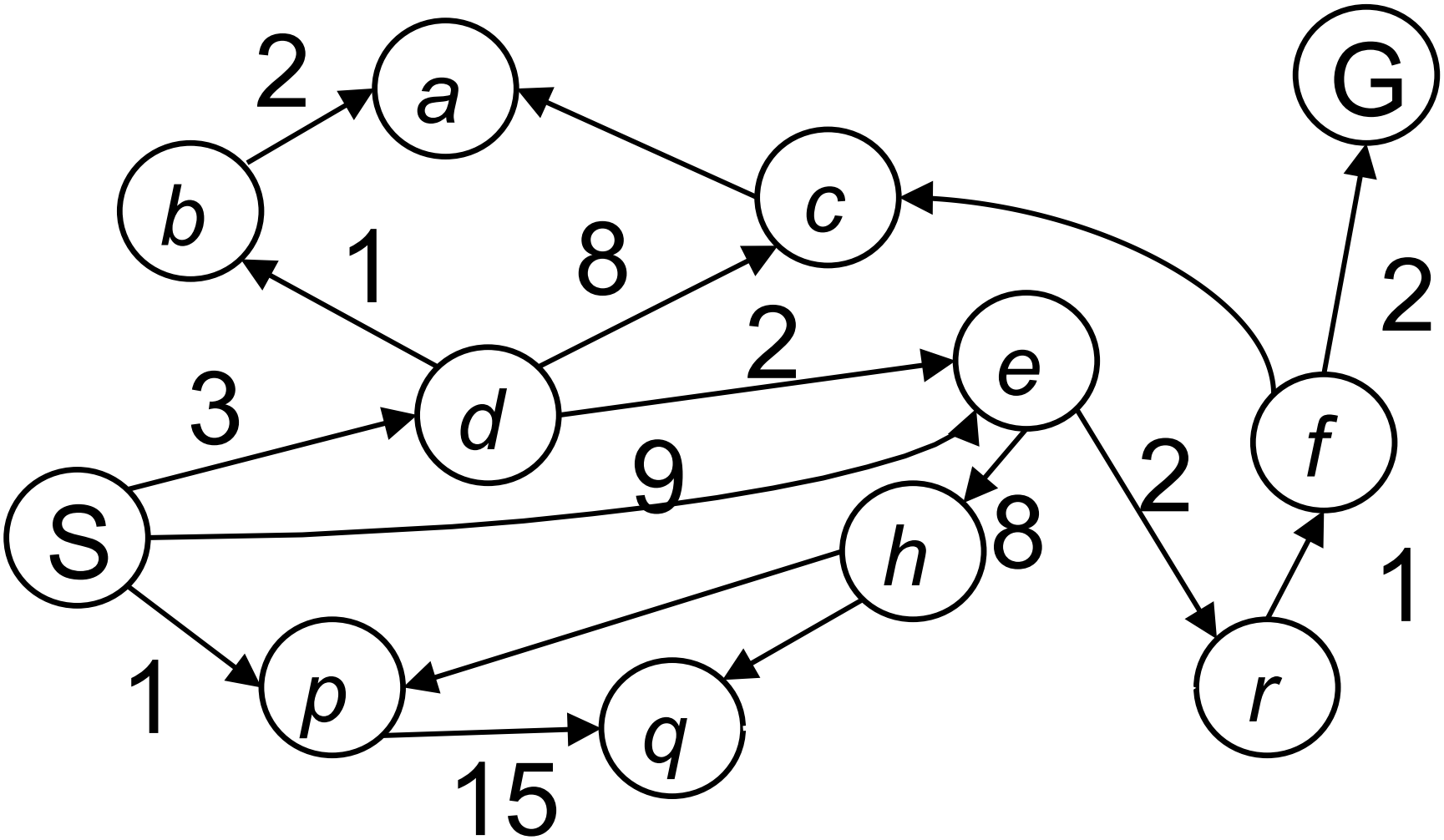


Example: Graph with costs



Programming a Tree Search

Infrastructure for (tree) search algorithms

What should a tree node `n` now?

- `n.state`
- `n.parent`
- `n.pathcost`

Perhaps we may add something later, if needed ...

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How to organize nodes?

The Python examples are just suggestions, ...

- A dynamically linked structure (`list()`).
- Add a node (`list.insert(node)`).
- Take a node and remove from the structure (`node=list.pop()`).
- Check the Python modules `heapq`[1] and `queue`[2] for inspiration.

[1] https://docs.python.org/3.5/library/heapq.html
[2] https://docs.python.org/3.5/library/queue.html

What is the solution?

- We stop when Goal is reached.
- How do we contruct the path?