Problem solving by search

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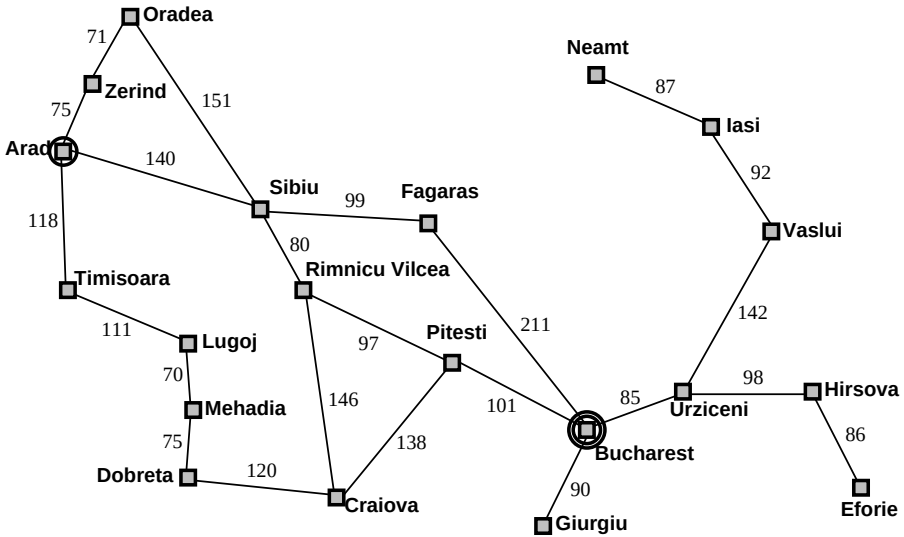
Outline

- Search problem.
- State space graphs.
- Search trees.
- Strategies, which tree branches to choose?
- Strategy/Algorithm properties?
- Programming instrastructure

Example: Romania



Ok, start with a simple one, almost everybody knows about the navigation - path planning problem. Waze, Garmin, ...

Can you think about more problems?

Example: Romania

Goal:

be in Bucharest

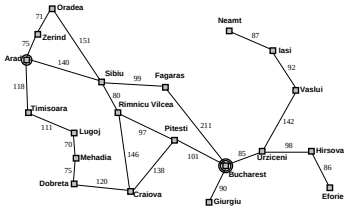
Problem formulation:

states: position in a city (cities)

actions: drive between cities

Solution:

Sequence of cities (path)



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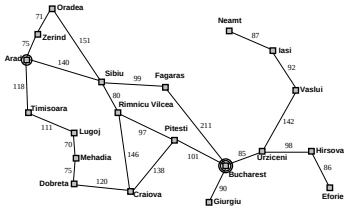

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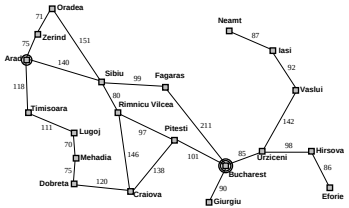

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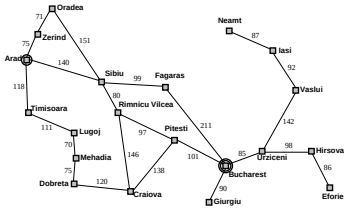

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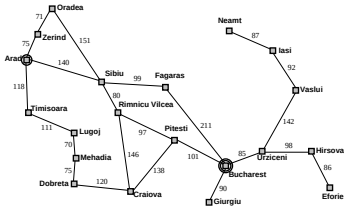

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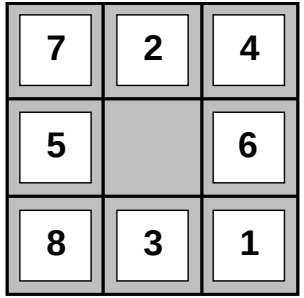
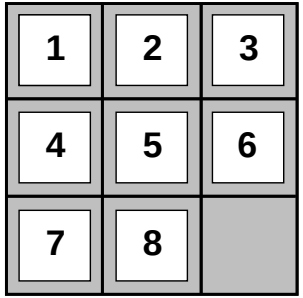
Example: The 8-puzzle



Start State

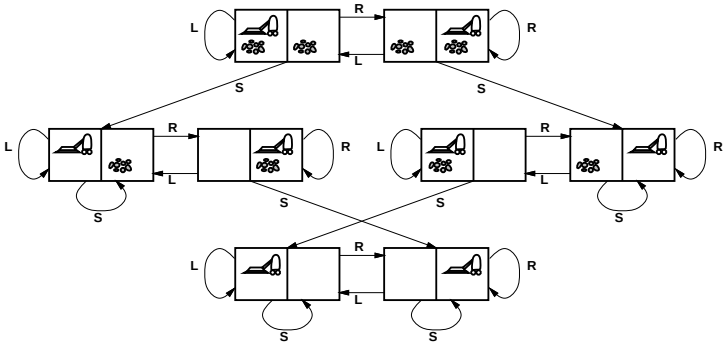


Goal State

states?
actions?
solution?
cost?

Also known as $n-1$ puzzle.

Example: Vacuum cleaner



states?
actions?
solution?
cost?

A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left ...
- Transition model : Given state and action return state (and cost)
- Goal test : Are we done?

We will use the terminology throught the next 5-6 lectures; also for Markov (Sequential) Decision Processes, Reinforcement Learning

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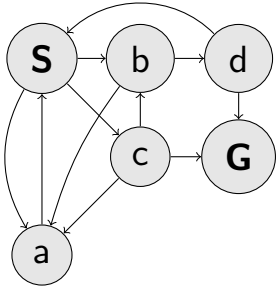
State Space Graphs

State space graph: a representation of a search problem

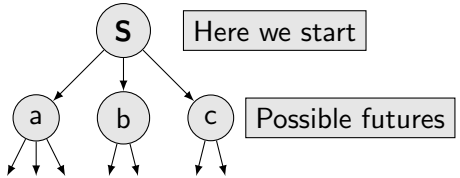
- Graph Nodes – states – are abstracted world configurations
- Arcs represent action results
- Goal test – a set of goal nodes

Each state occurs only *once* in a state (search) space.



Formalizing a real world problem – (creating) state space graph – could be a problem of itself. I put creating into brackets is it may be also infinite.

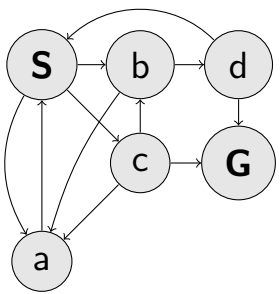
Search Trees



- A "what if" tree of plans and their outcomes
- Start node is the root
- Children are successors
- Nodes show/contains states, but correspond to *plans* that achieve those states

- What if decision about an action, repeats ...
- Nodes in the search tree are not the same as the nodes in the state space graph.

State Space Graphs vs. Search Trees



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- A search tree can be much bigger than the state space.
- Both items will be discussed next.

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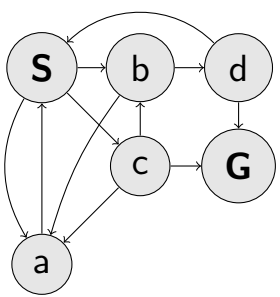

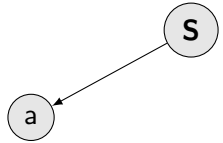

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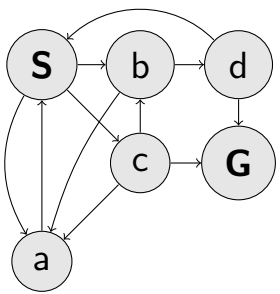

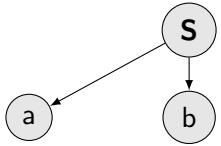

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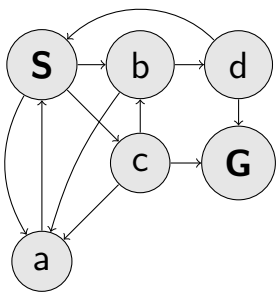

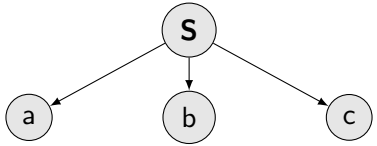

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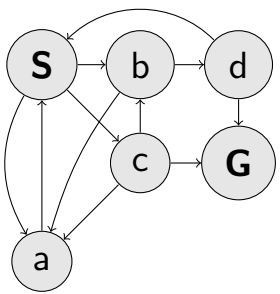

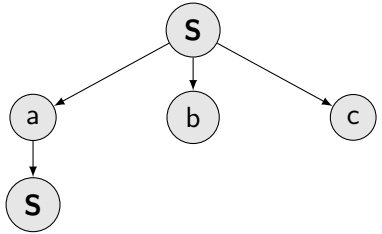

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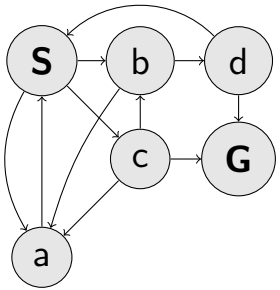

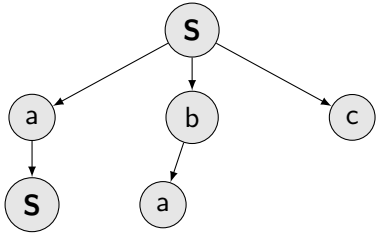

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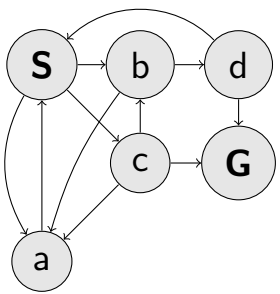

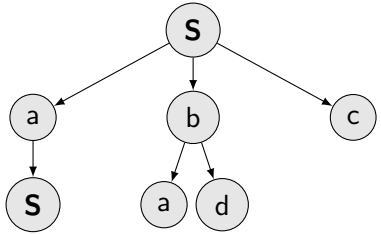

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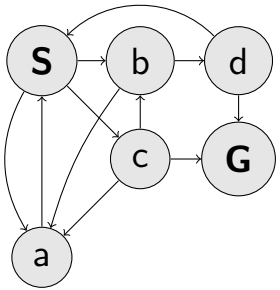

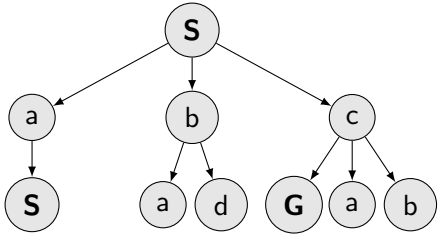

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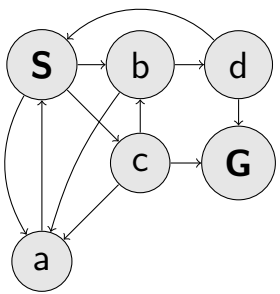

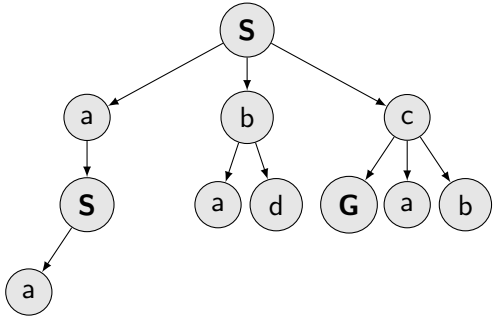

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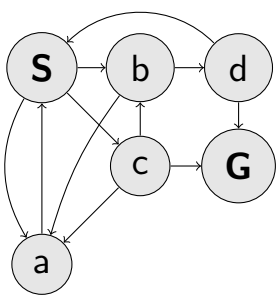

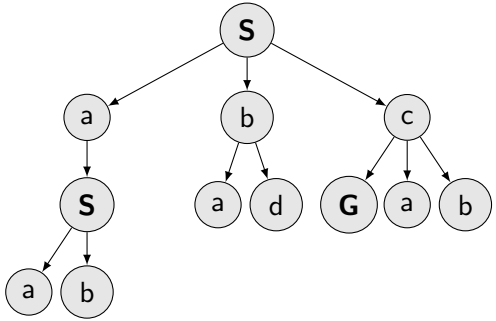

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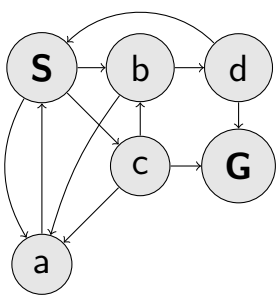

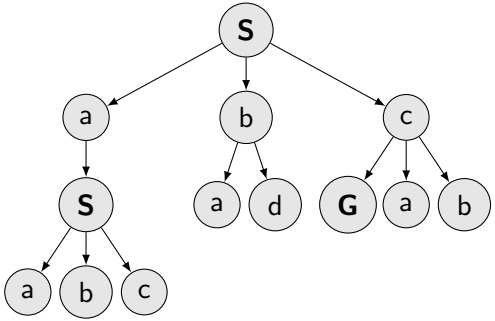

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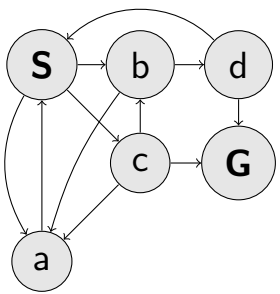

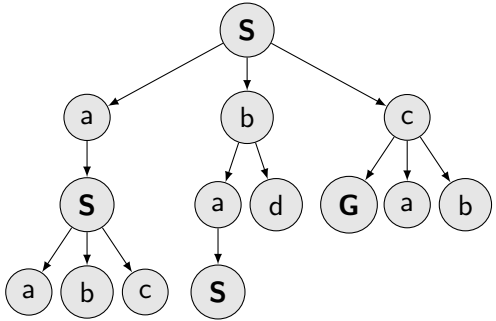

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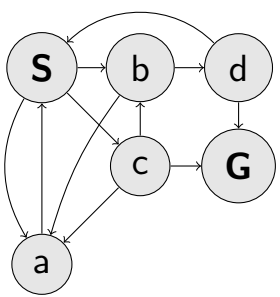

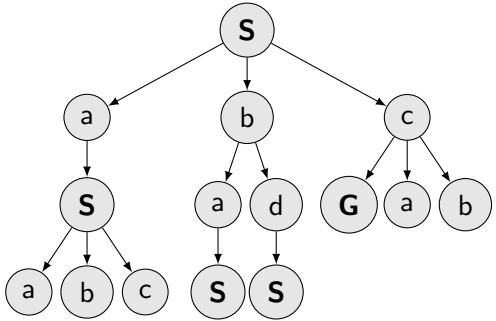

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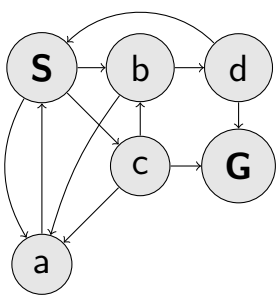

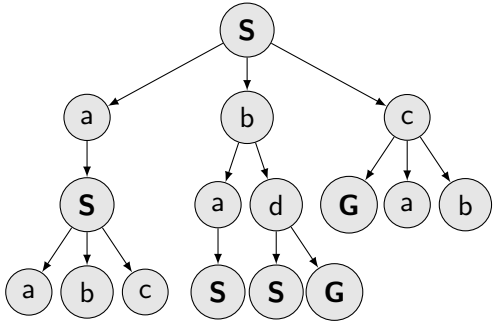

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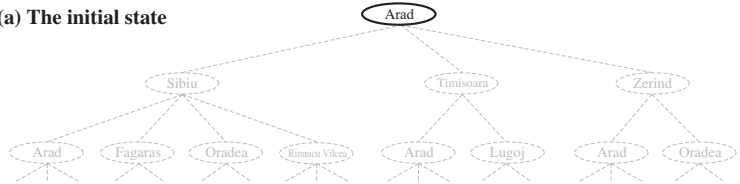
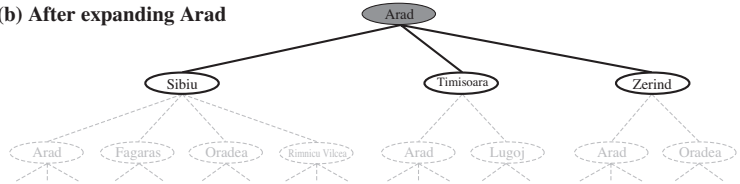
Search tree for Romania

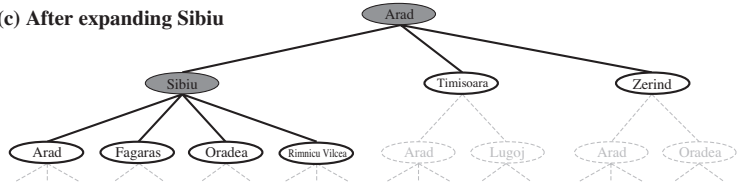
(a) The initial state



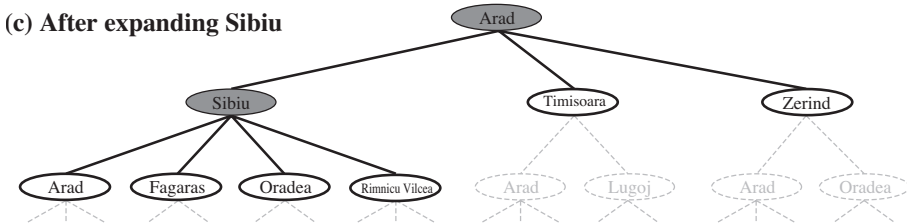
(b) After expanding Arad



(c) After expanding Sibiu

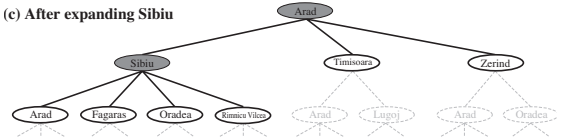


Search elements



- Expand plans - possible ways (tree nodes).
- Manage/Maintain fringe (or frontier) of plans under consideration.
- Expand new nodes *wisely(?).*

Tree search algorithm



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initialize by using the initial state of the problem

loop

if no candidates for expansion **then return** failure

else choose a leaf node for expansion

end if

if the node contains a goal state **then return** the solution

end if

Expand the node and add the resulting nodes to the tree

end loop

end function

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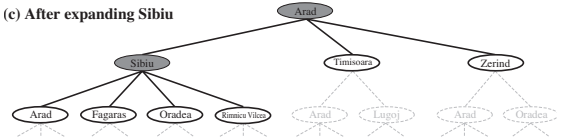

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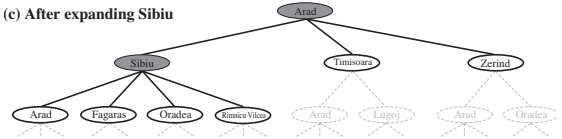

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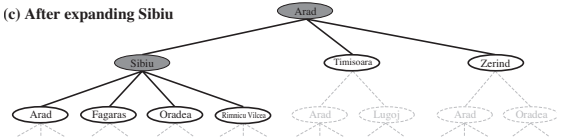

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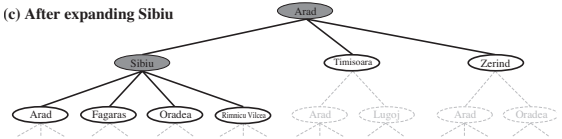

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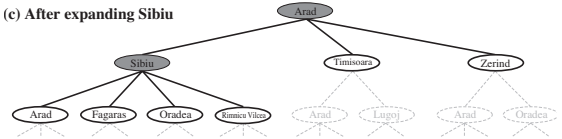

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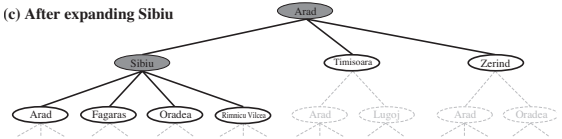

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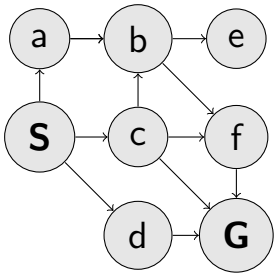

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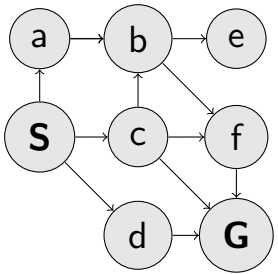

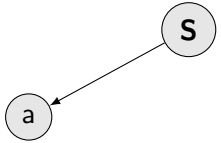

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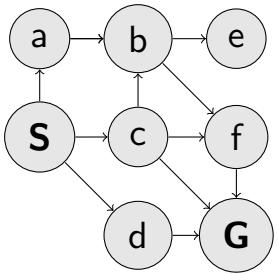

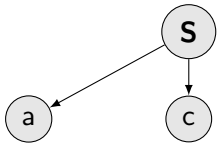

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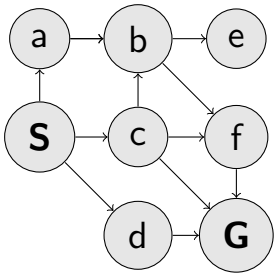

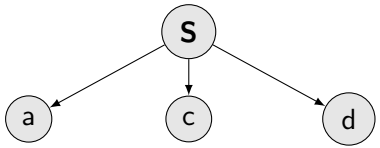

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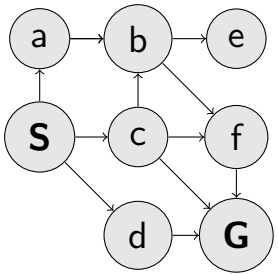

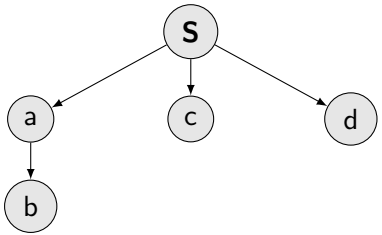

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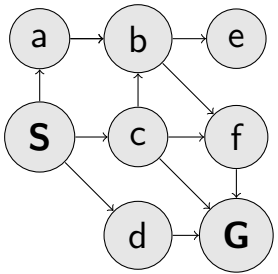

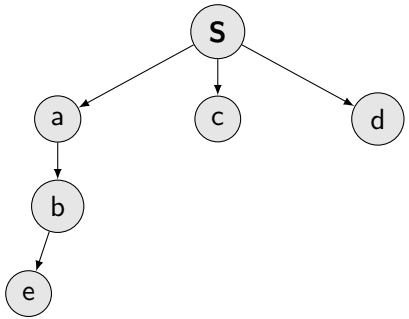

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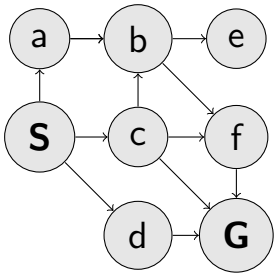

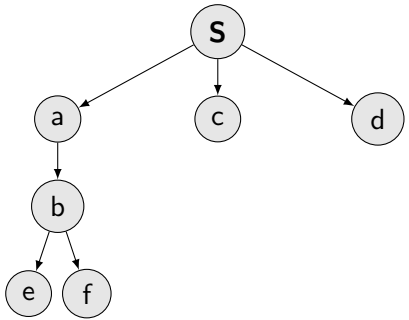

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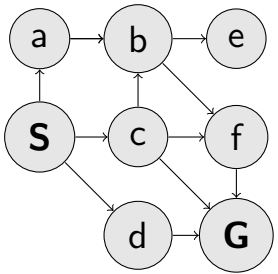

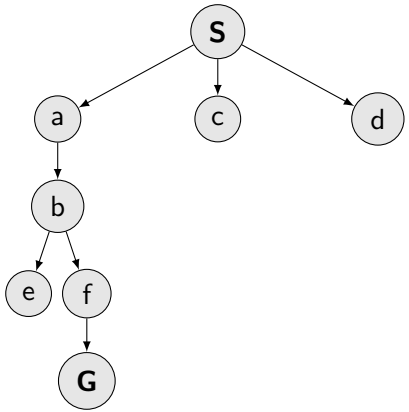

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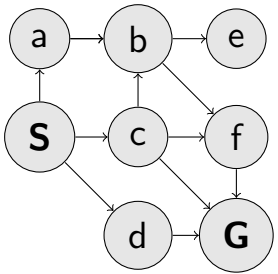

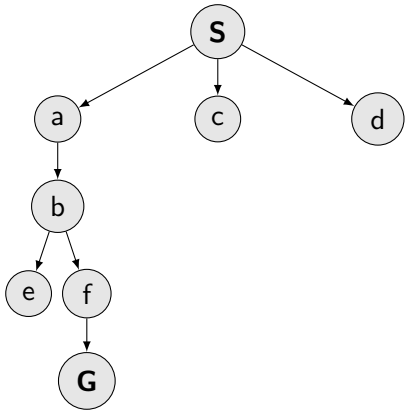

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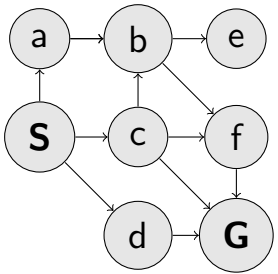

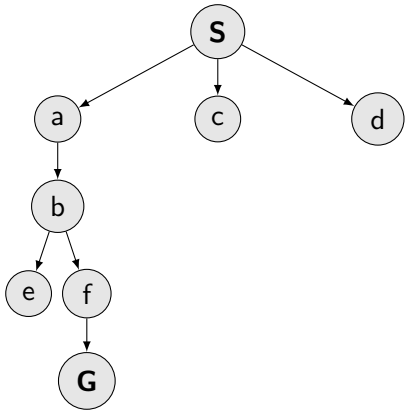

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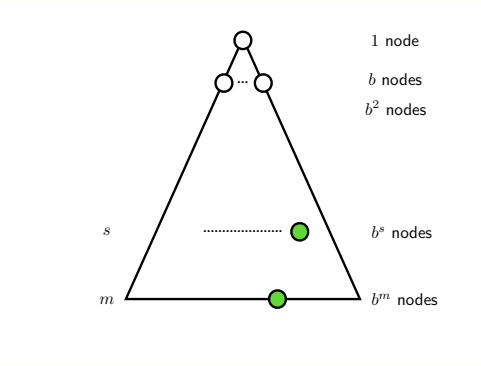

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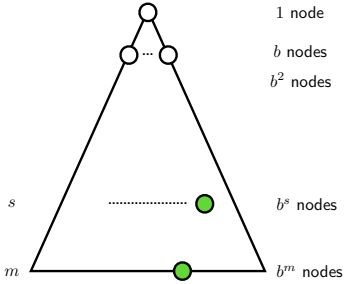

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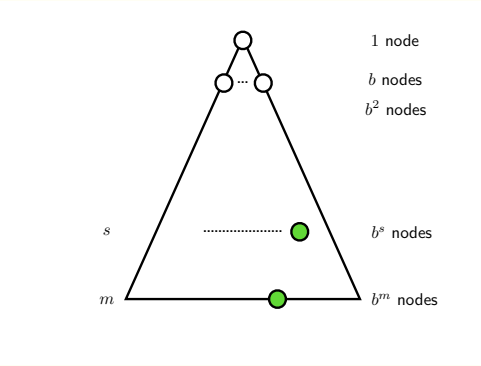

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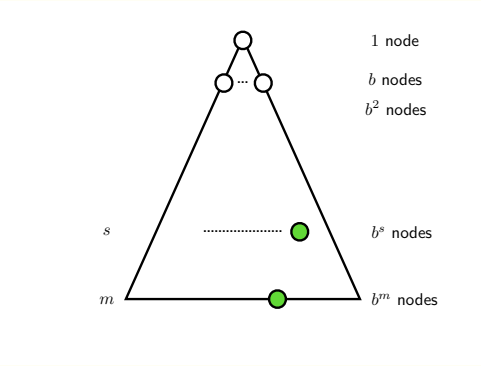

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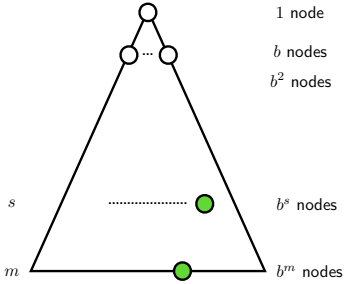

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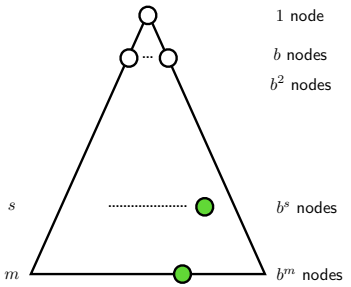

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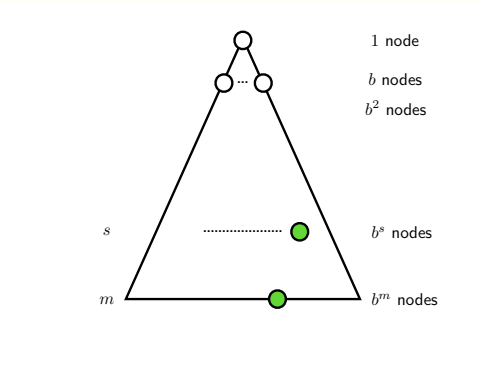

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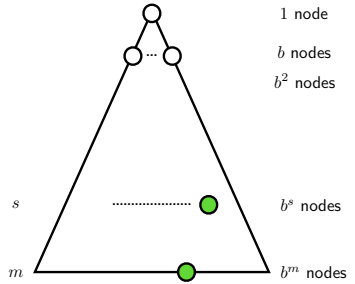

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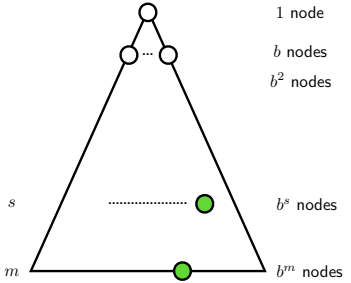

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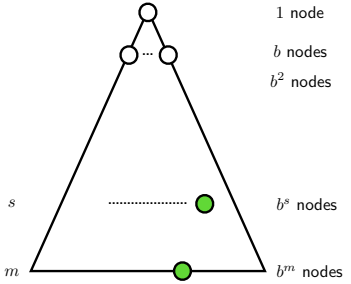

Strategies

How to traverse/build a search tree?

It is perhaps worth to remember that the search tree is built as the algorithm goes. Or better said, the tree is a human friendly representaion of the machine run.

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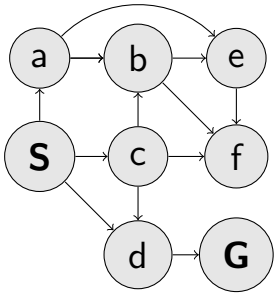

What are the DFS properties (complete, optimal, time, space)?

- In animation, we will do the expansion step at once.
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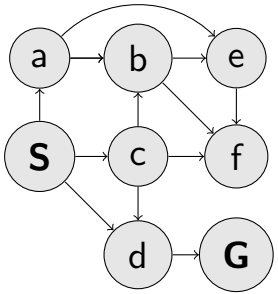

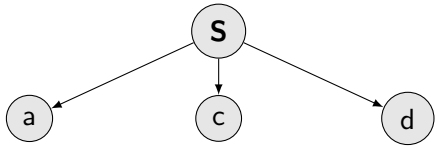

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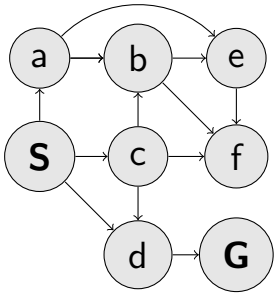

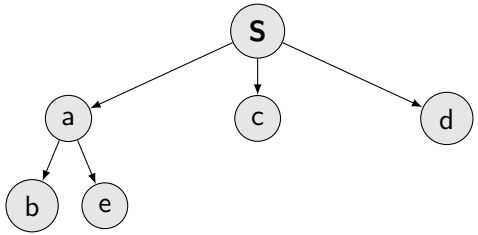

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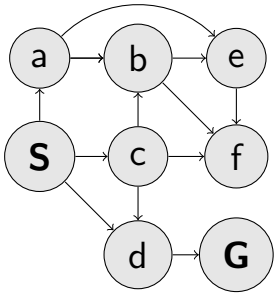

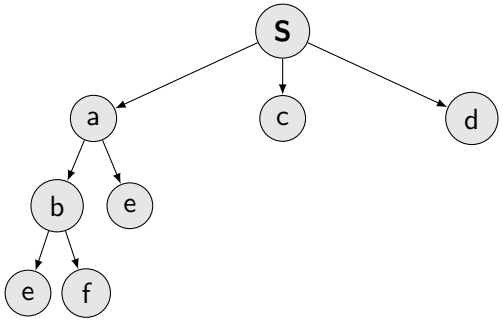

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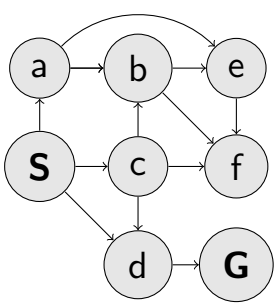

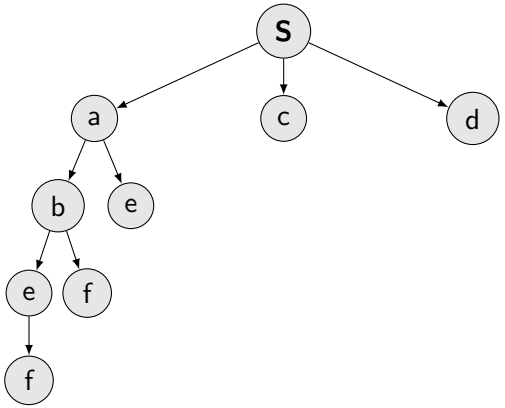

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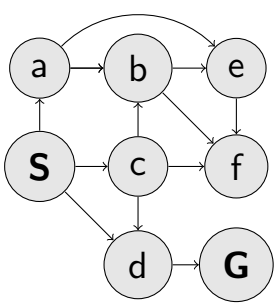

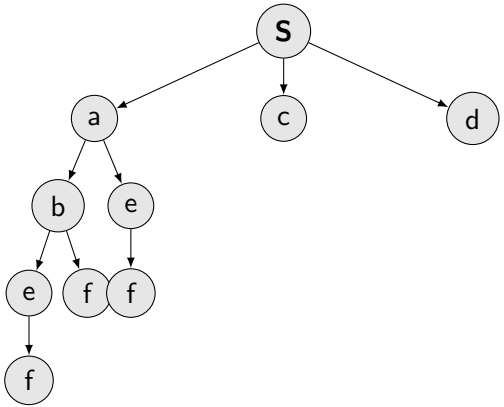

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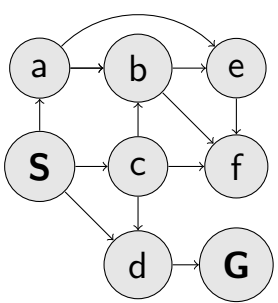

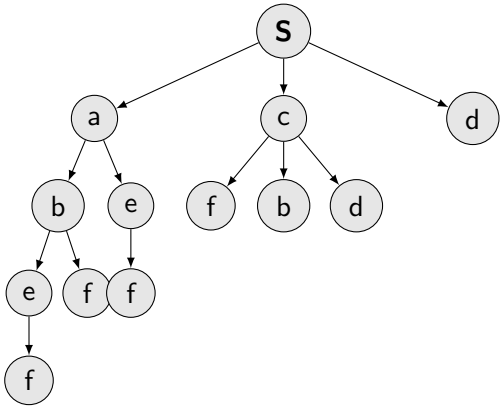

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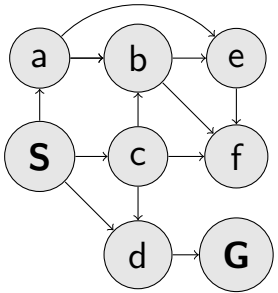

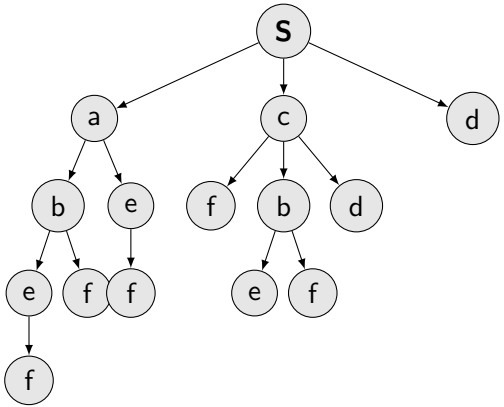

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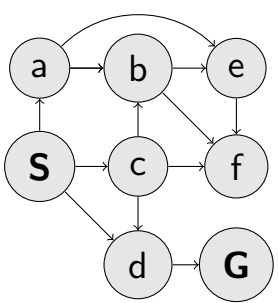

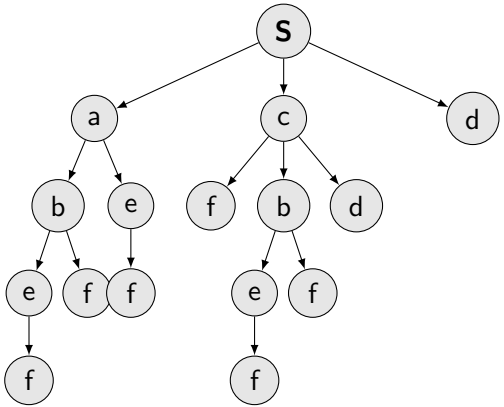

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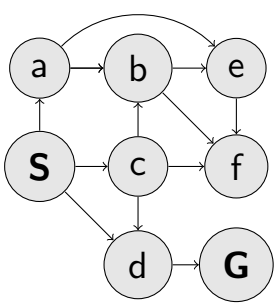

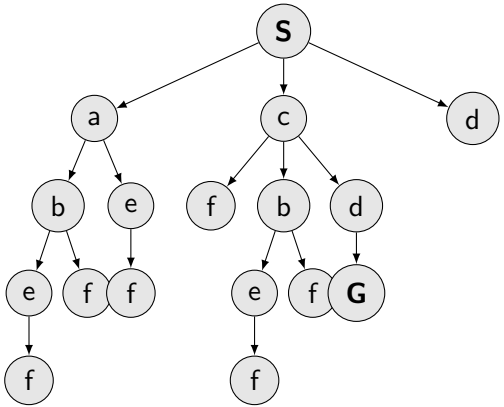

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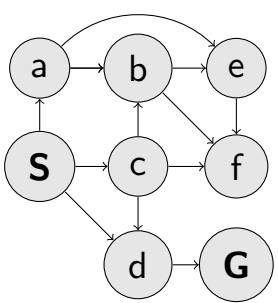

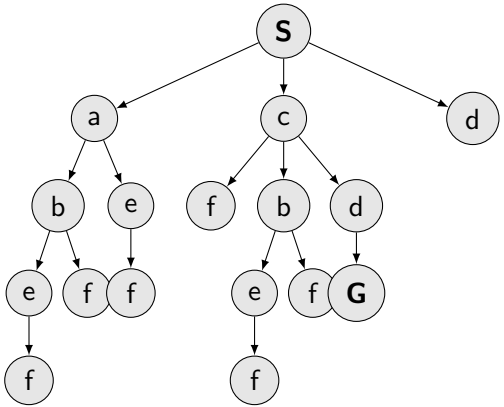

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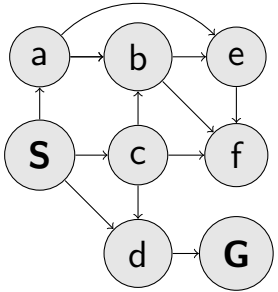

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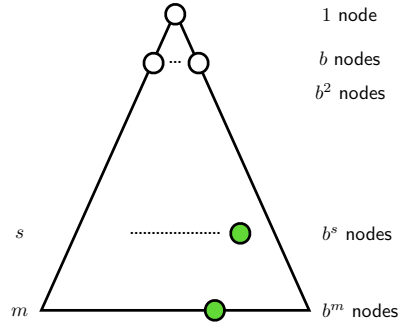

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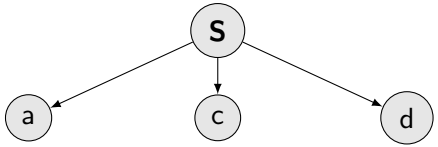

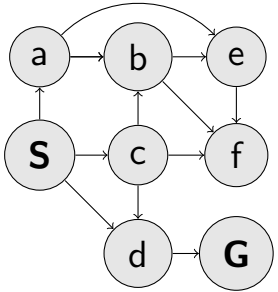

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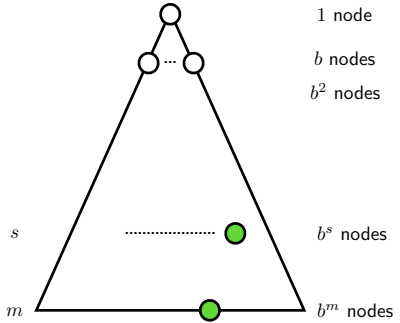

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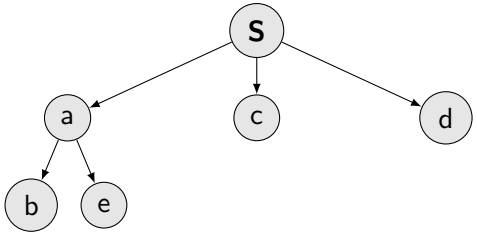

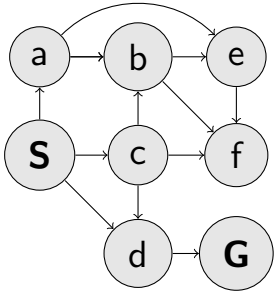

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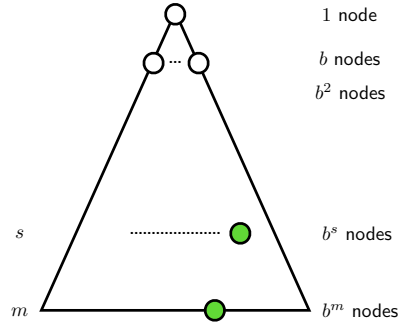

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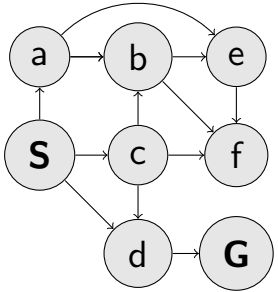

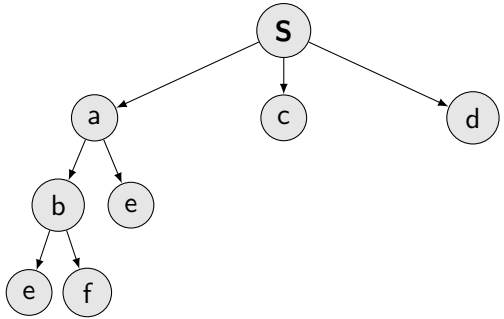

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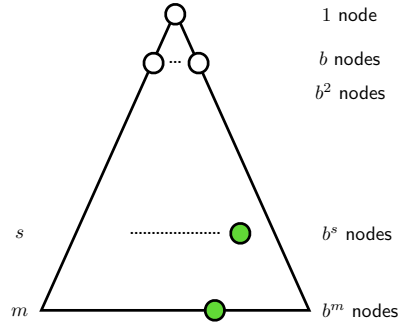

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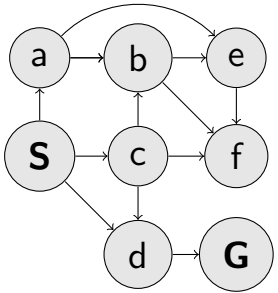

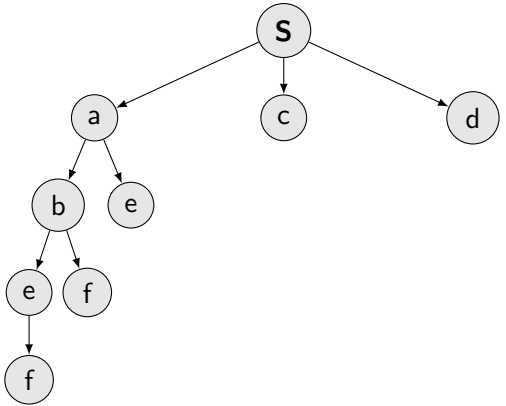

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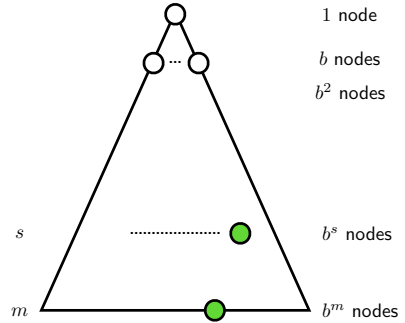

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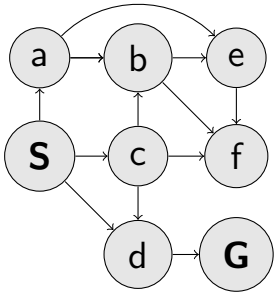

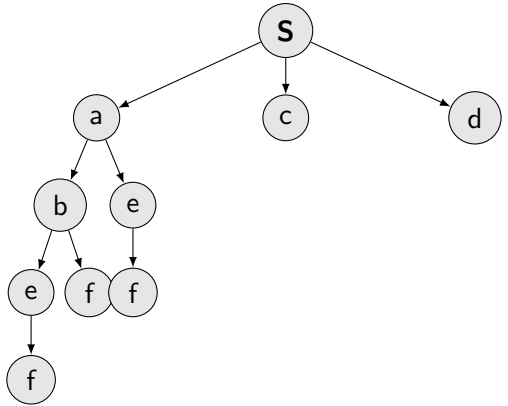

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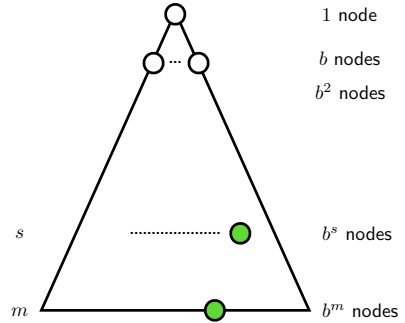

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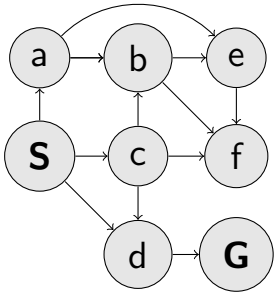

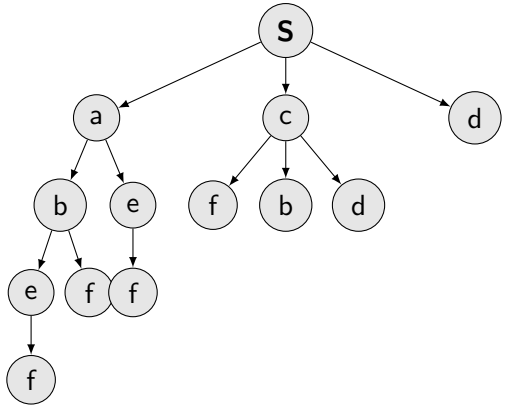

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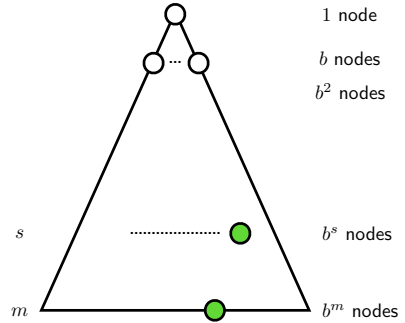

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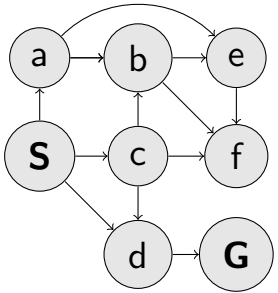

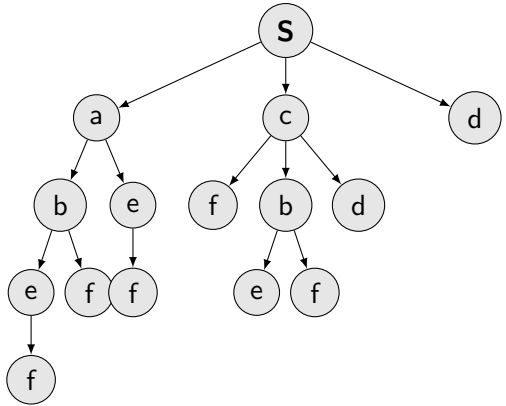

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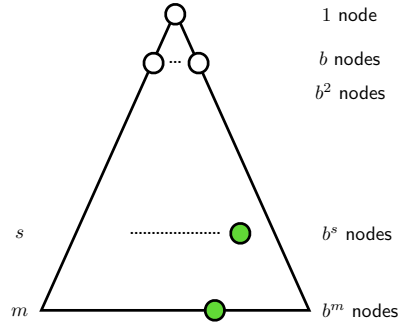

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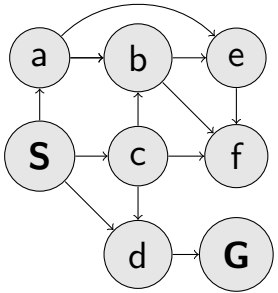

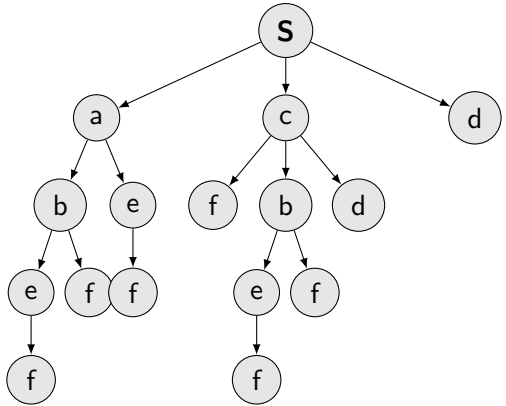

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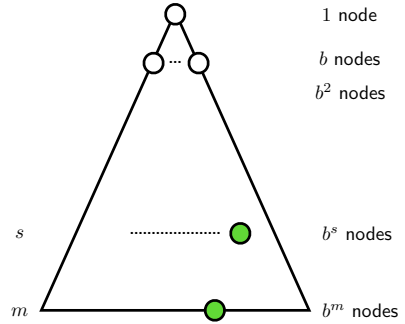

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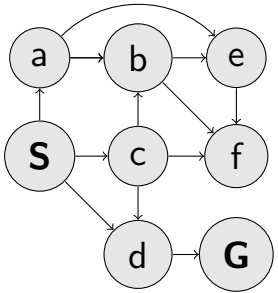

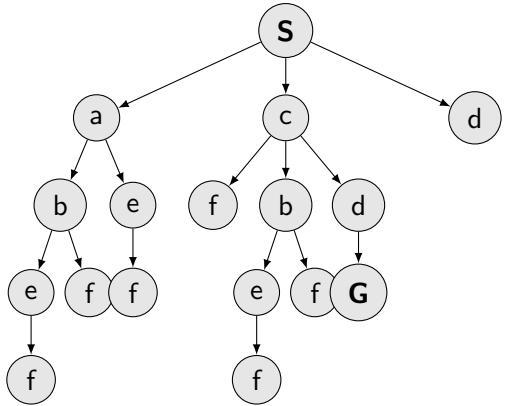

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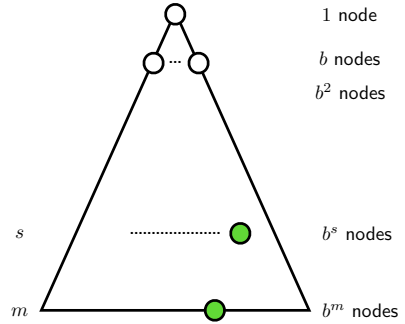
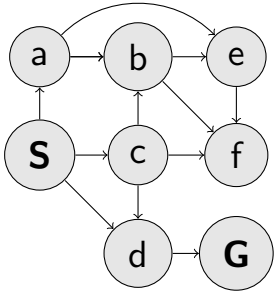


Breadth-First Search (BFS)



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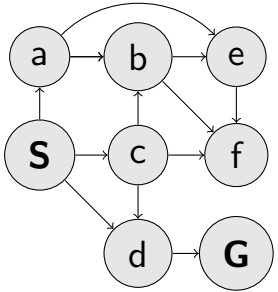

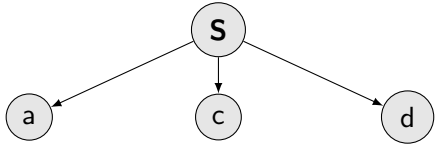

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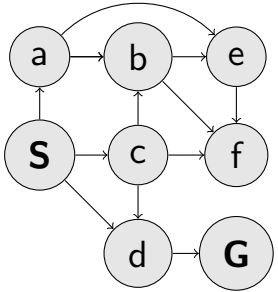
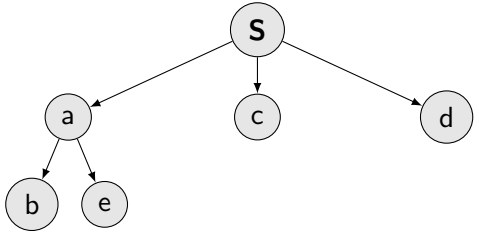
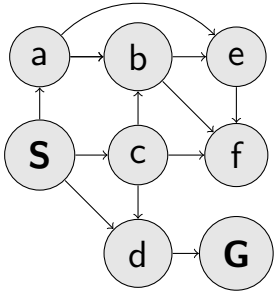
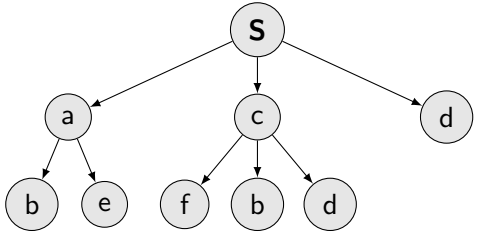




What are the BFS properties?

Breadth-First Search (BFS)





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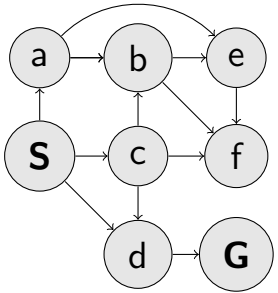

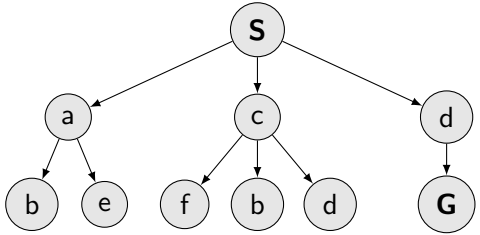

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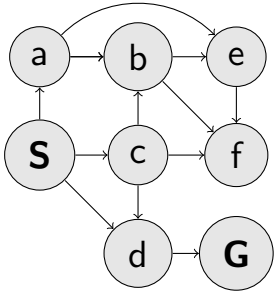
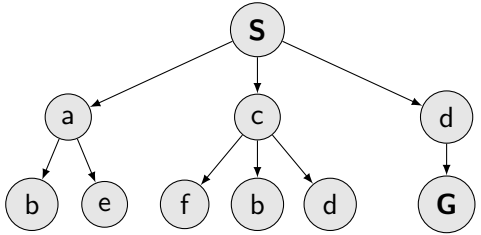
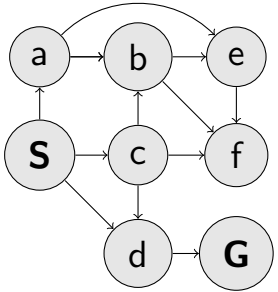




What are the BFS properties?

BFS properties

- Time complexity?
- Space complexity?
- Complete?
- Optimal?



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- Space, all the frontier: b^s
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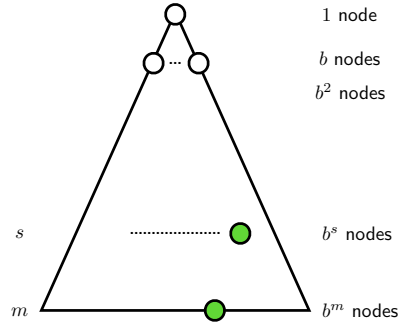

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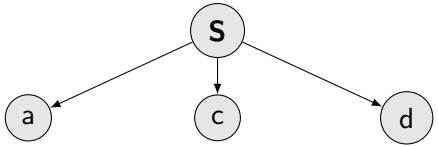

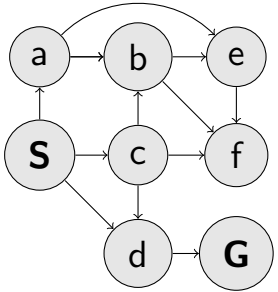

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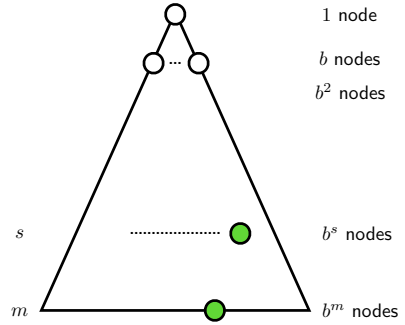

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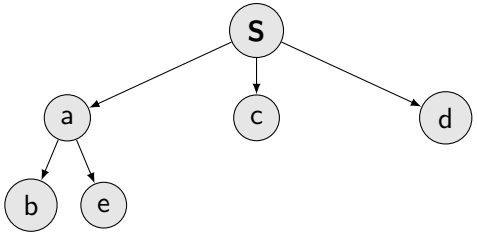

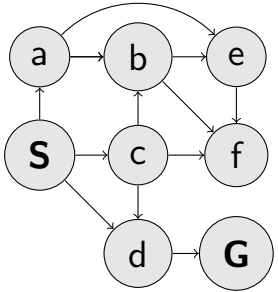

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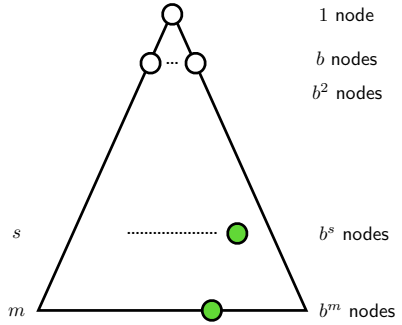

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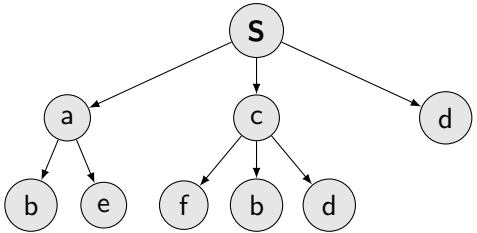

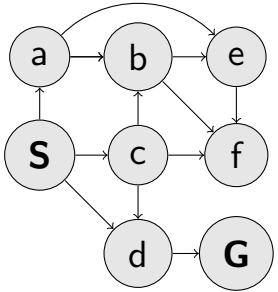

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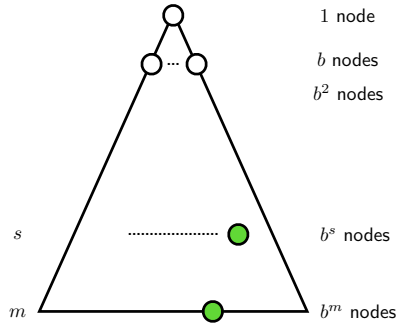

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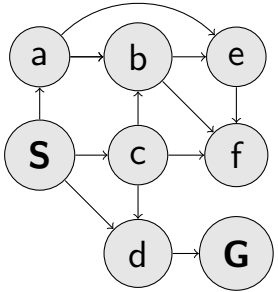

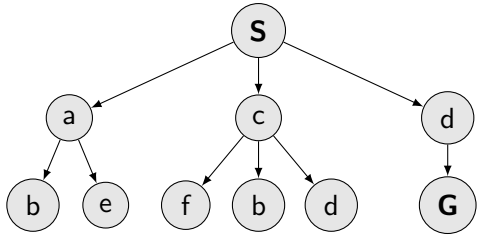

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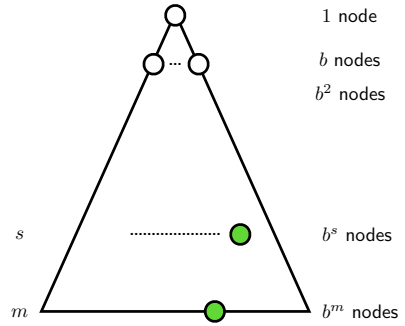

DFS vs BFS

What are (dis)advantages of the individual strategies?

- Do not overfit your thinking!
- Draw for yourself a different graph and contruct appropriate trees.
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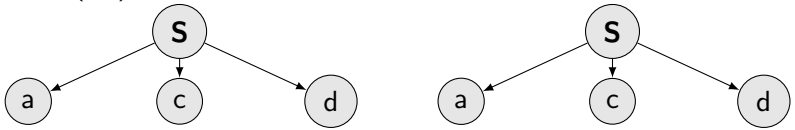

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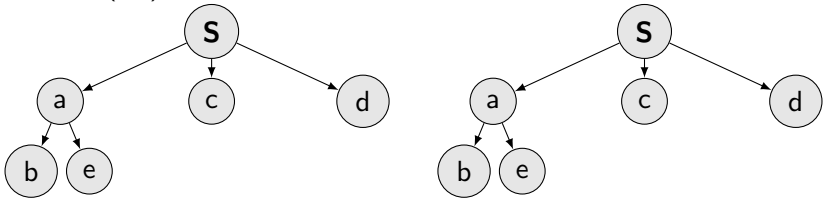

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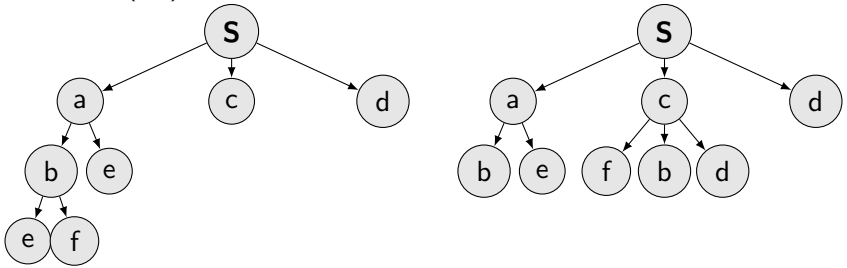

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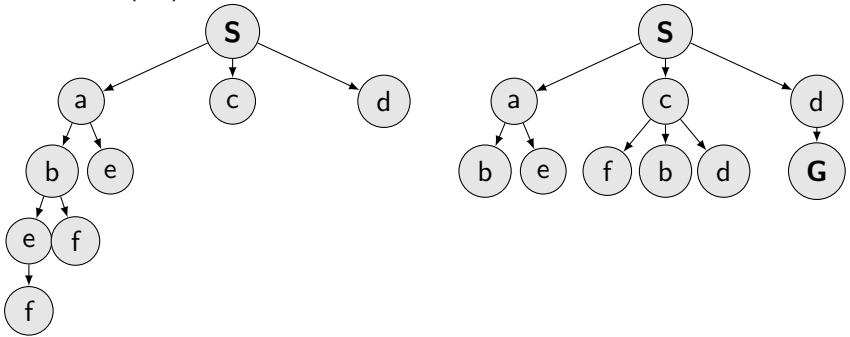

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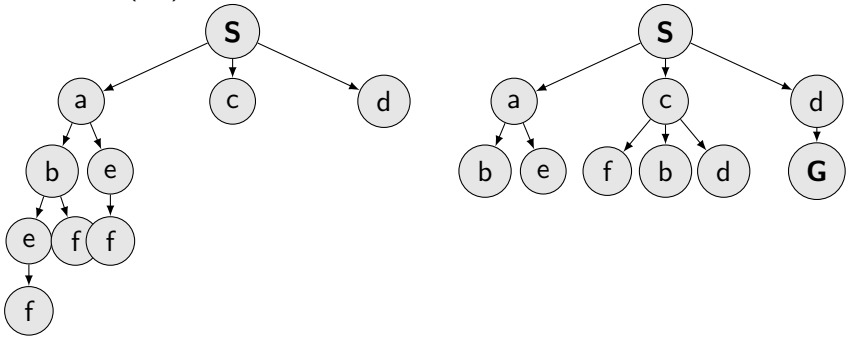

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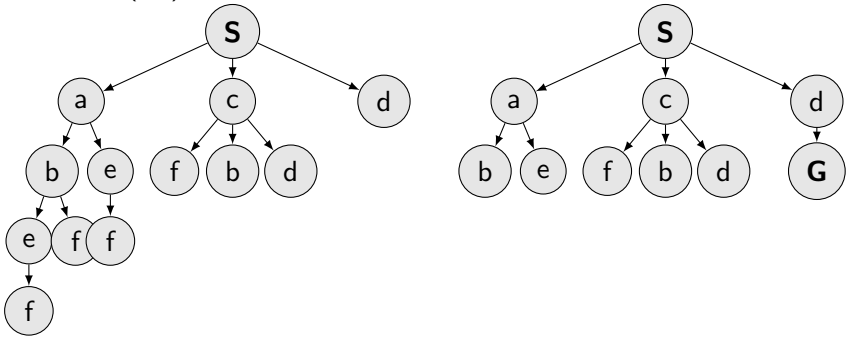

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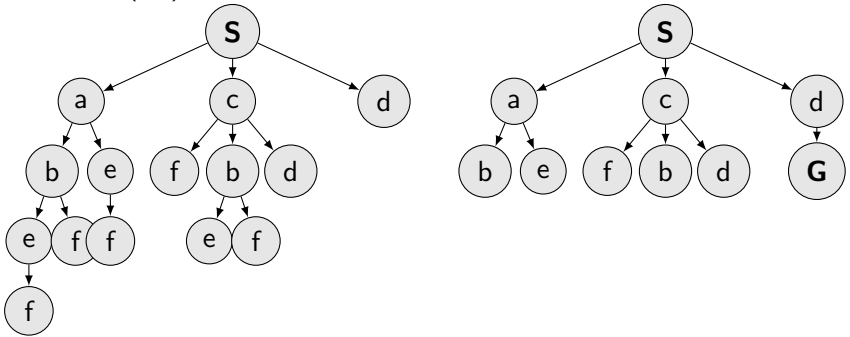

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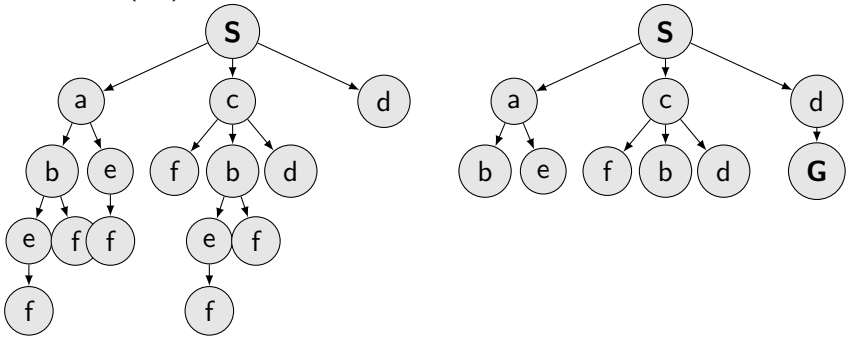

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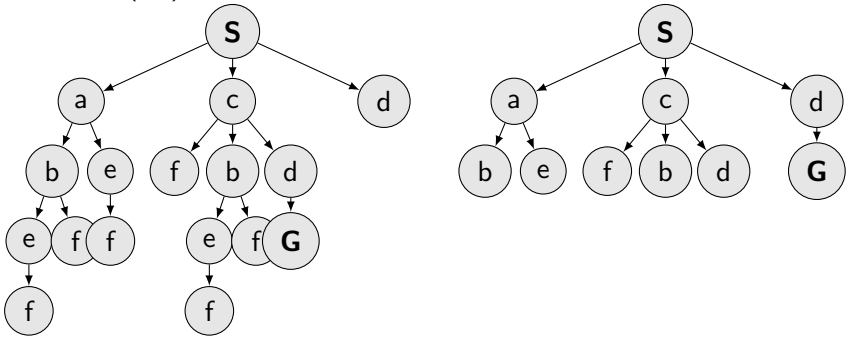

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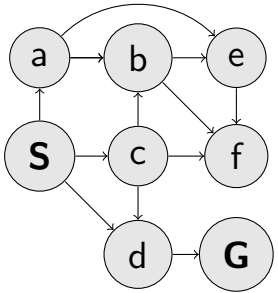
DFS with limited depth, `maxdepth=2`

Do not follow nodes with `depth` $>$ `maxdepth`



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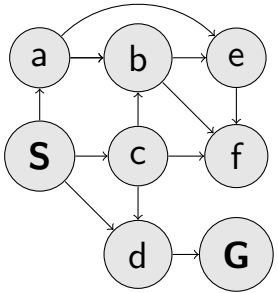

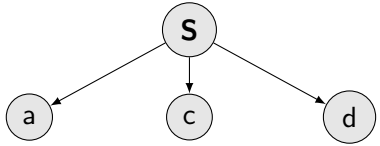

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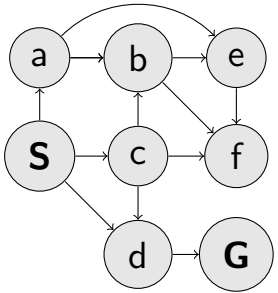

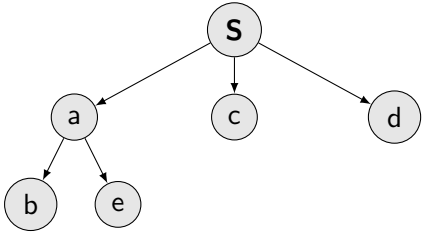

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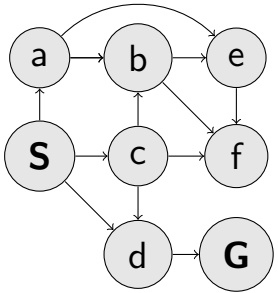

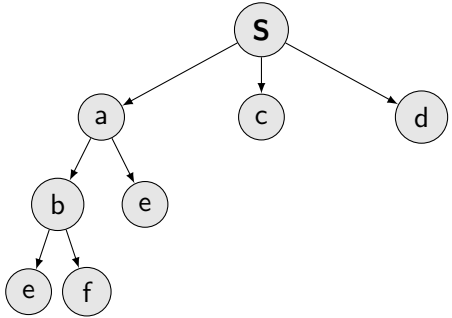

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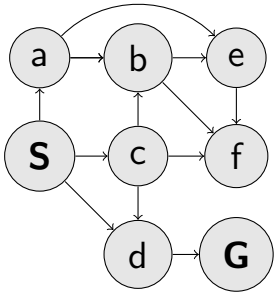

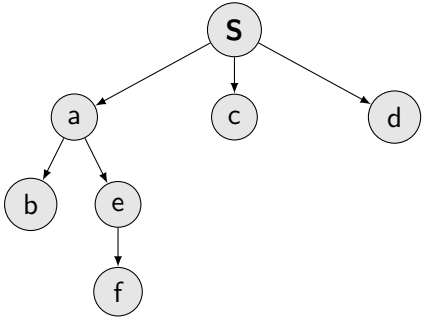

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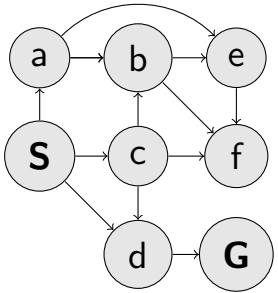

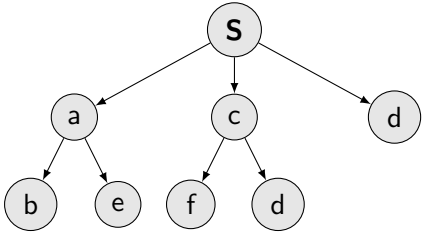

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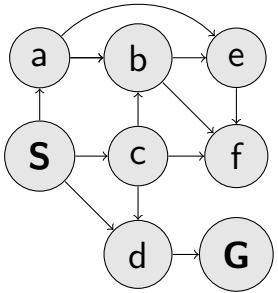

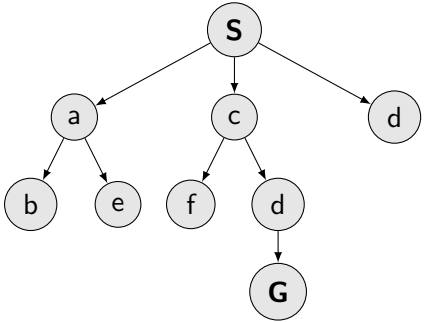

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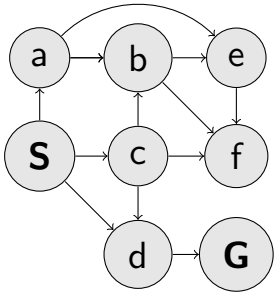

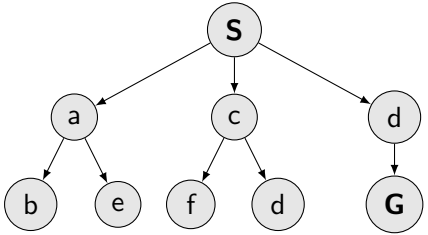

Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

Really, how much do we repeat/waste? Compare number fo nodes generated ID-DFS vs. BFS:

$$N(\text{ID-DFS}) = (s)b + (s-1)b^2 + (s-2)b^3 + \cdots + (1)b^s$$

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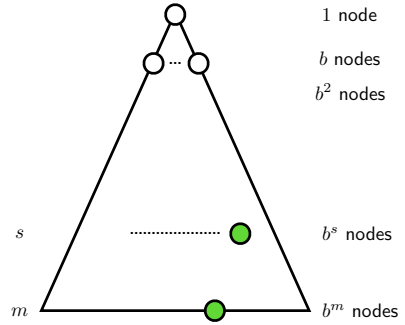

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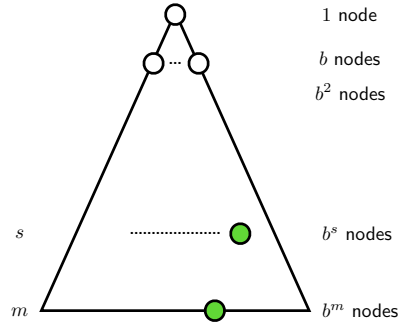

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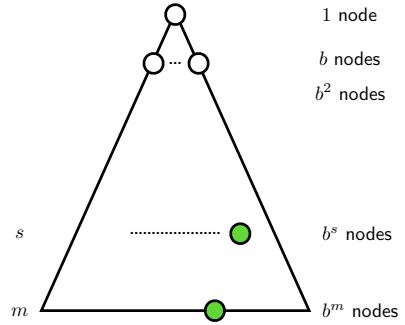

Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

Really, how much do we repeat/waste? Compare number fo nodes generated ID-DFS vs. BFS:

$$N(\text{ID-DFS}) = (s)b + (s-1)b^2 + (s-2)b^3 + \cdots + (1)b^s$$

$$N(\text{BFS}) = b + b^2 + b^3 + \cdots + b^s$$

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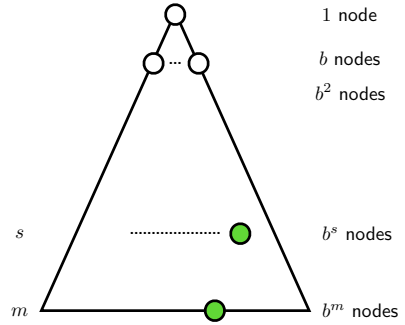
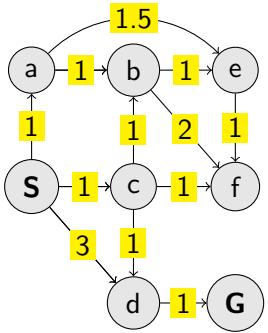


Cost sensitive search



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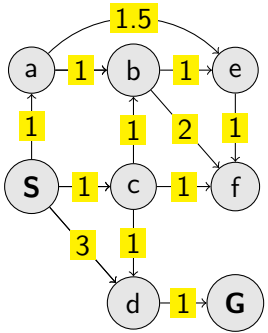

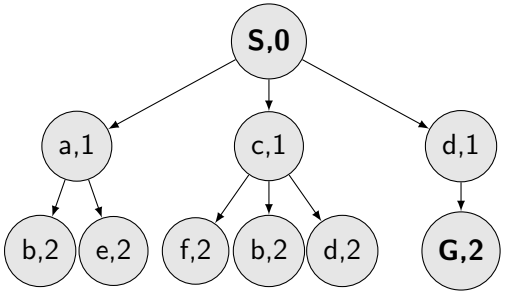

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- How was the depth actually computed?
- How to evaluate nodes with path cost?

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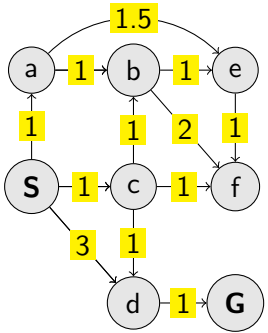

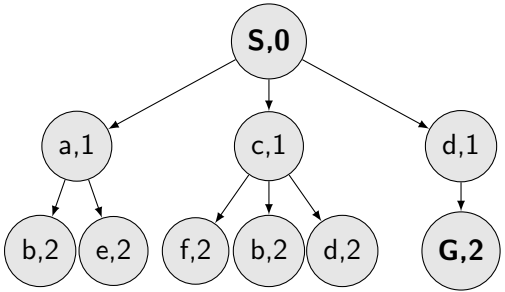

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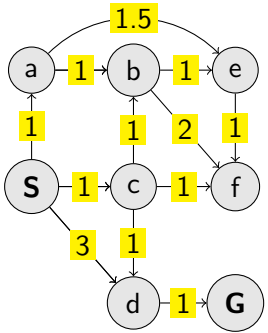

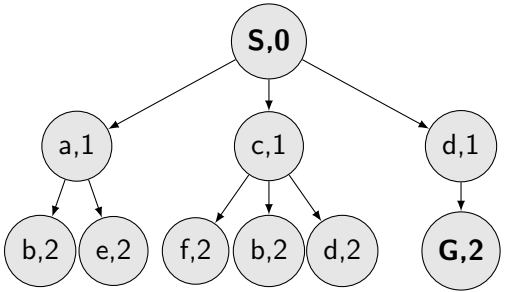

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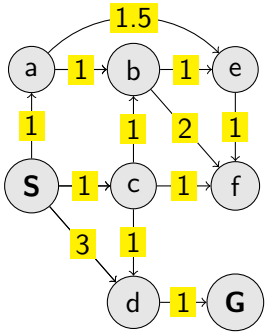
Uniform Cost Search (UCS)



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When to check the goal (and stop) the search? When visiting or expanding the node?

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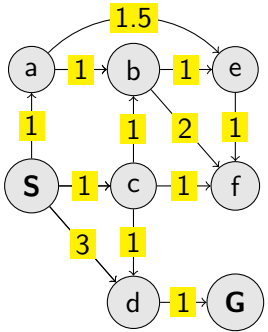

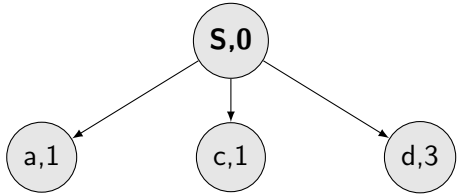

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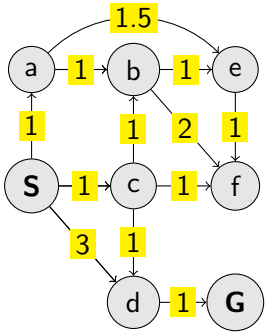

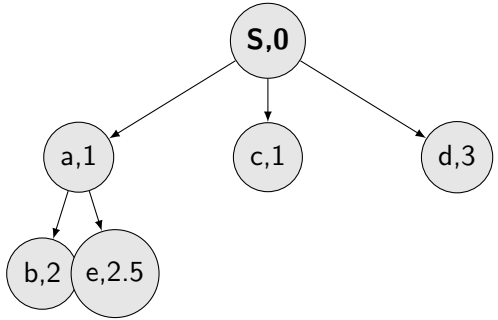

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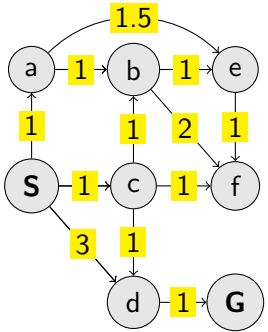

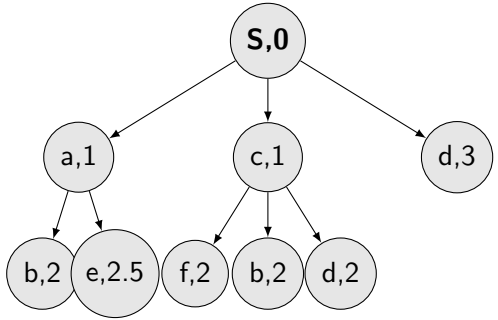

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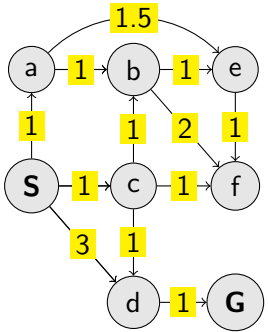

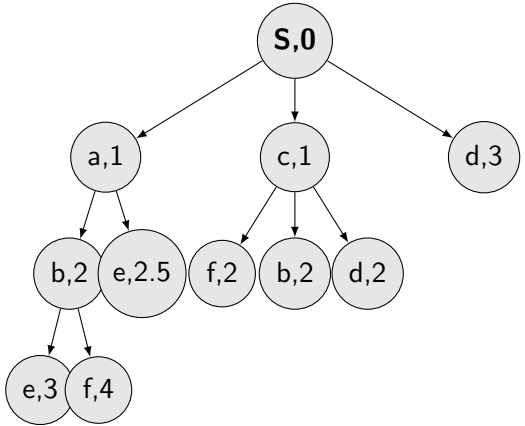

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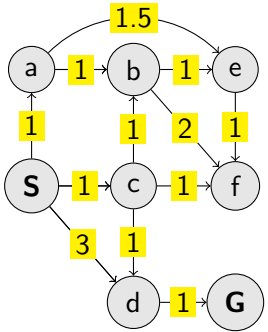

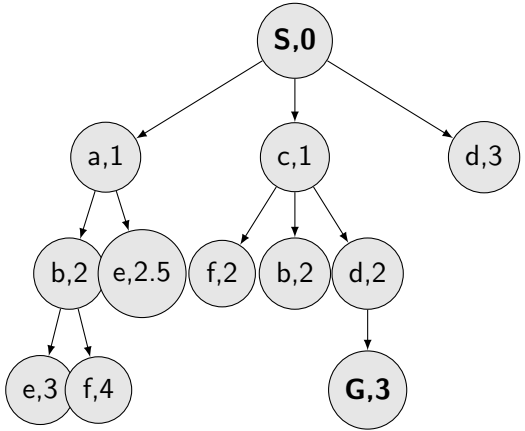

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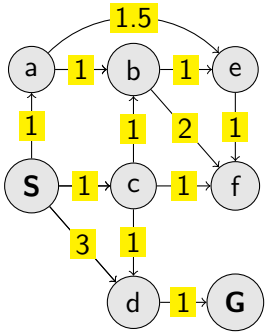

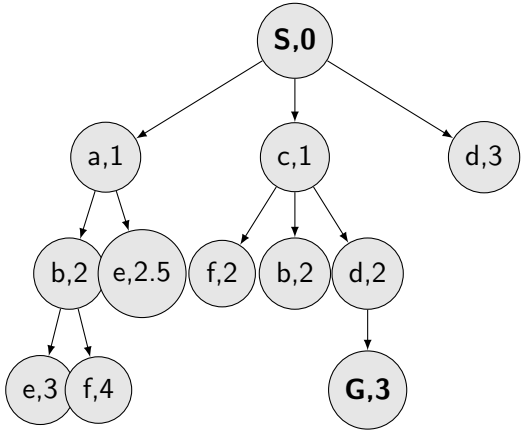

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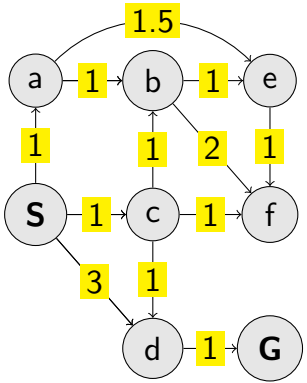

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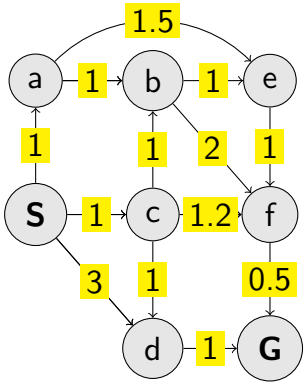

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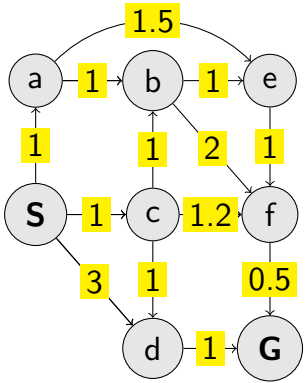

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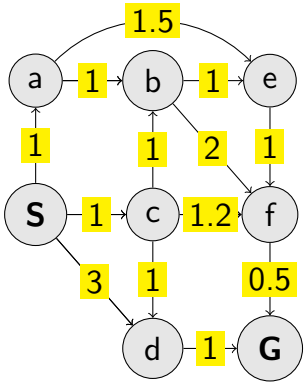

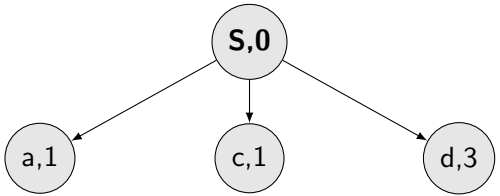

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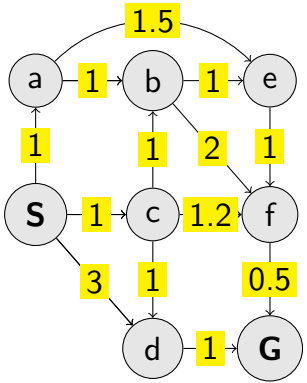

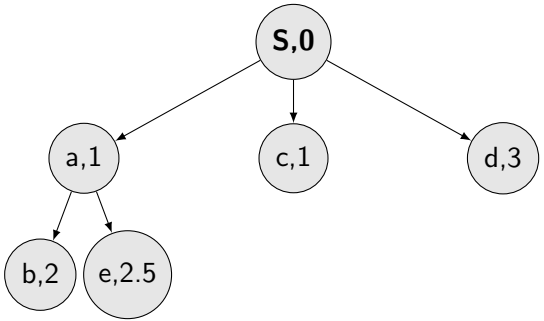

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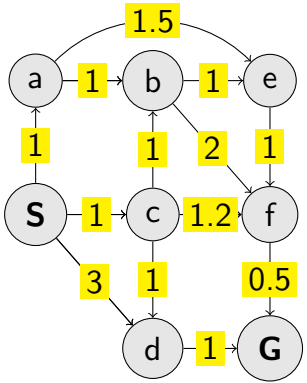

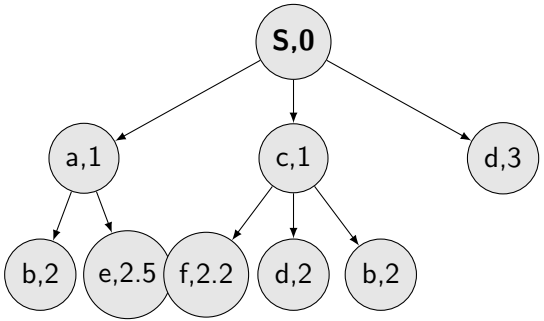

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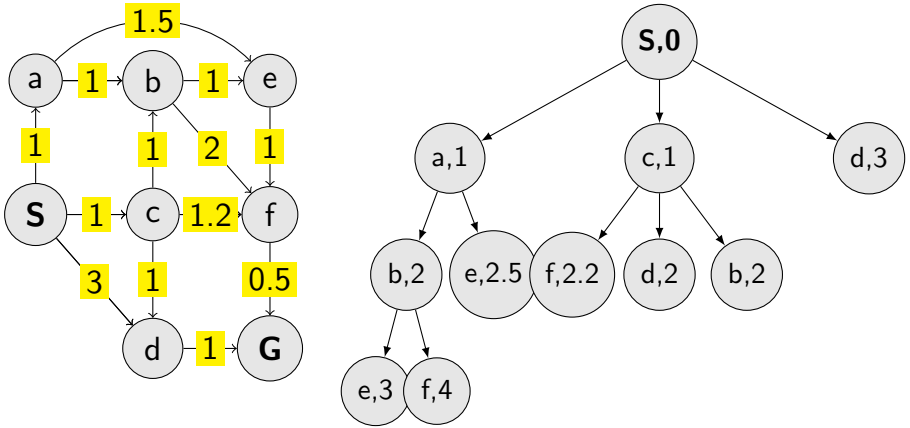

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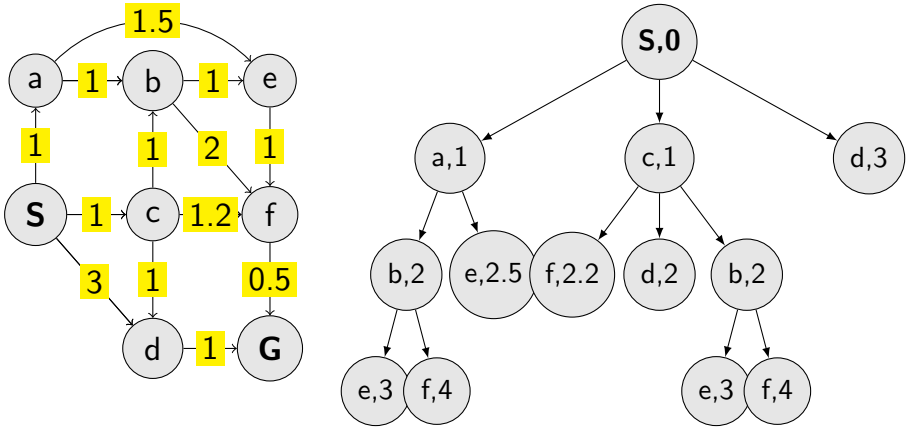

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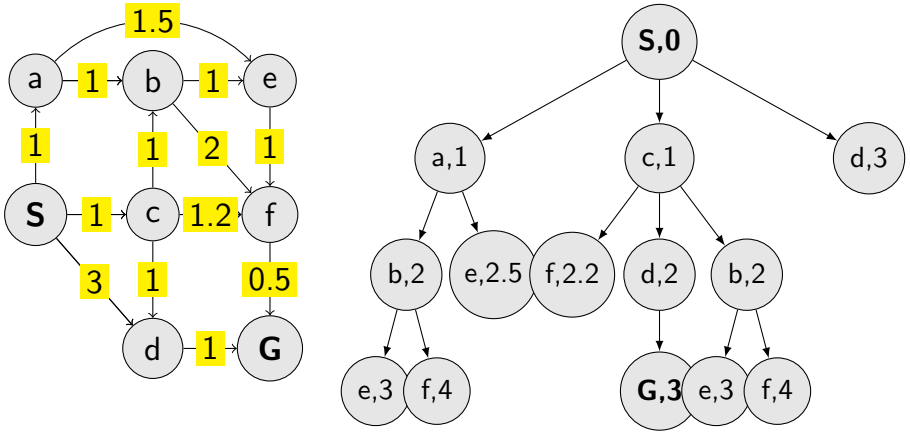

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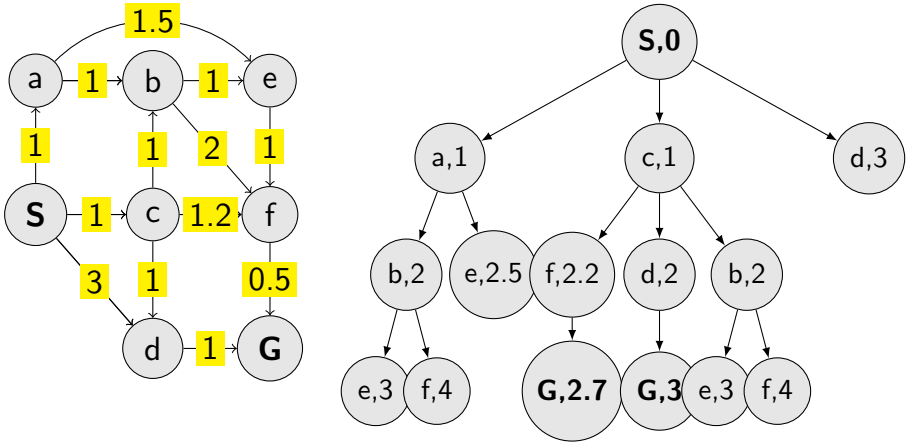

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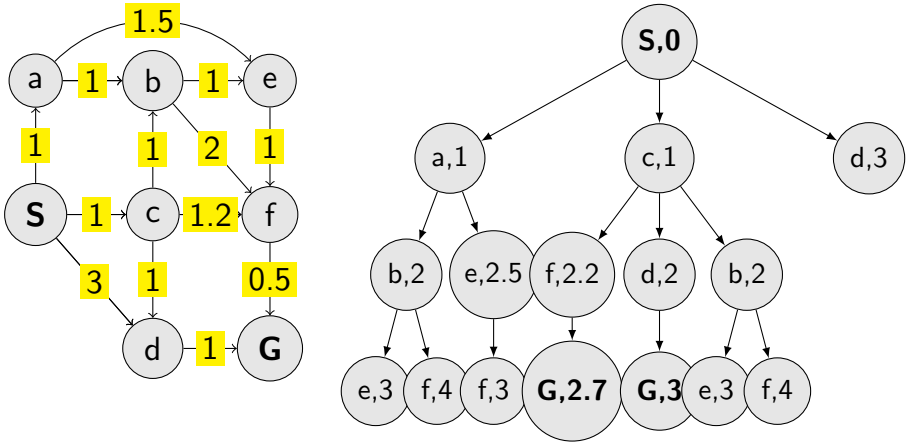
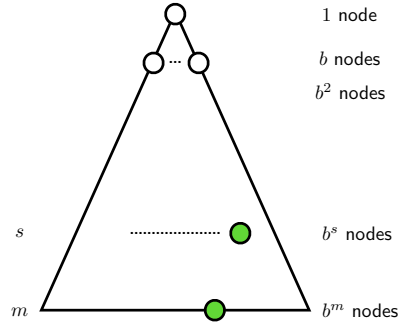


UCS properties

- Time complexity?
- Space complexity?
- Complete?
- Optimal?

Solution cost C^*, transition cost at least ϵ. Effective depth, roughly C^*/ϵ.

- Time: $b^{C^*/\epsilon}$
- Space: $b^{C^*/\epsilon}$
- Completness: Yes!
- Optimality: Yes! Why?



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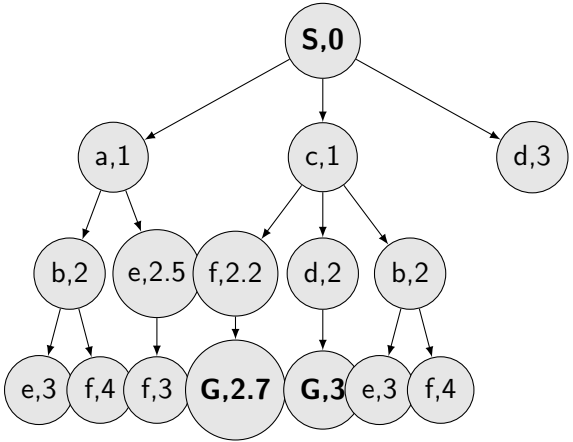

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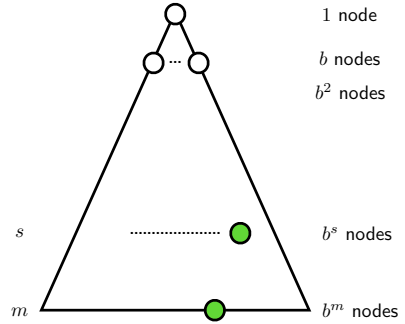
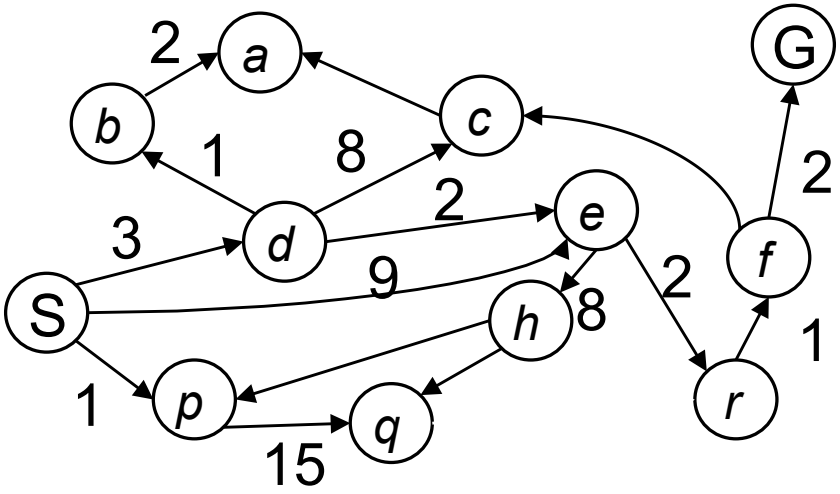


Example: Graph with costs



Try it on paper, mark which nodes are in frontier, mark lines of equal cost.

Programming a Tree Search

Infrastructure for (tree) search algorithms

What should a tree node `n` now?

- `n.state`
- `n.parent`
- `n.pathcost`

Perhaps we may add something later, if needed . . .

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How to organize nodes?

The Python examples are just suggestions, ...

- A dynamically linked structure (`list()`).
- Add a node (`list.insert(node)`).
- Take a node and remove from the structure (`node=list.pop()`).
- Check the Python modules `heapq`[1] and `queue`[2] for inspiration.

[1] https://docs.python.org/3.5/library/heapq.html
[2] https://docs.python.org/3.5/library/queue.html

What is the solution?

- We stop when Goal is reached.
- How do we contruct the path?

References

Some figures if from [1].

[1] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.