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Melkman algorithm

for convex hull of a 2D simple polyline

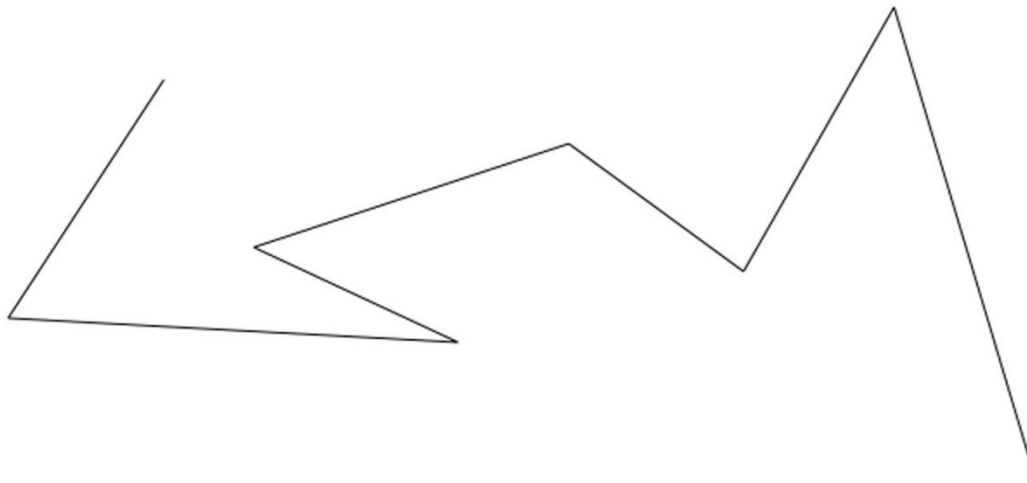
Summary

- ▶ Basic information
- ▶ Example of the Melkman algorithm
- ▶ IsLeft
- ▶ Pseudocode

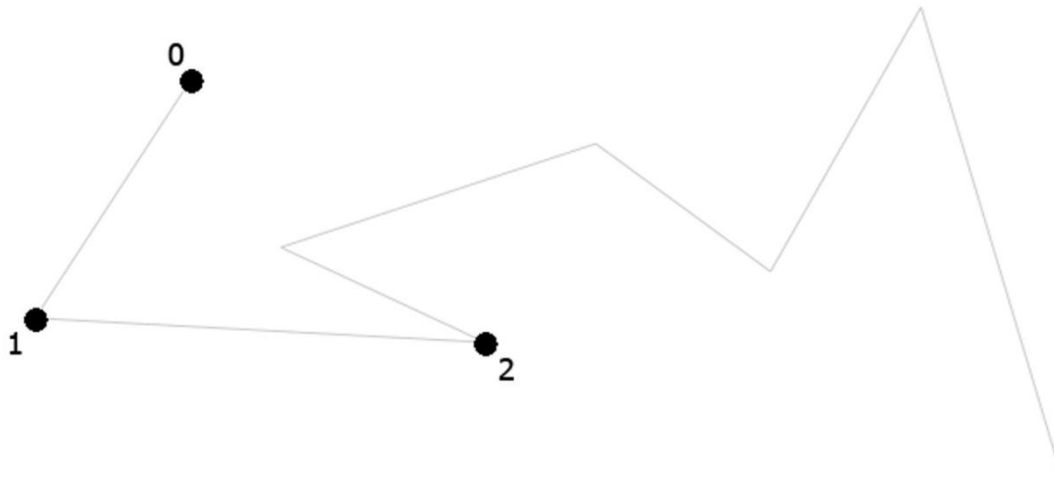
Basic information

- ▶ By Avraham A. Melkman, 1987
- ▶ Algorithm for convex hull of a simple polyline
 - ▶ Input: ordered vertex set $\Omega = \{P_0, P_1, \dots, P_n\}$
 - ▶ Output: convex hull of Ω
- ▶ Complexity time
 - ▶ $O(n)$
- ▶ Structures
 - ▶ Double-ended queue (= deque)

Example of the Melkman algorithm

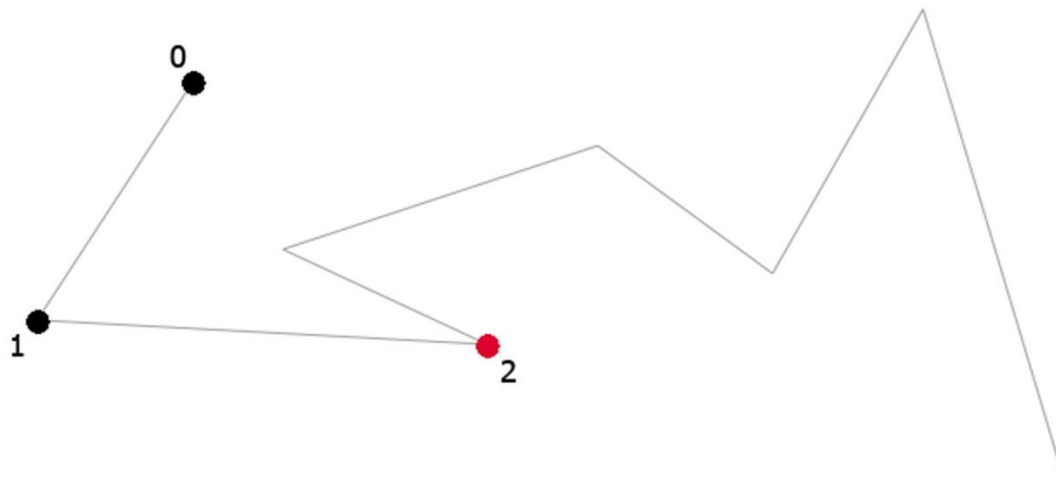


Example of the Melkman algorithm



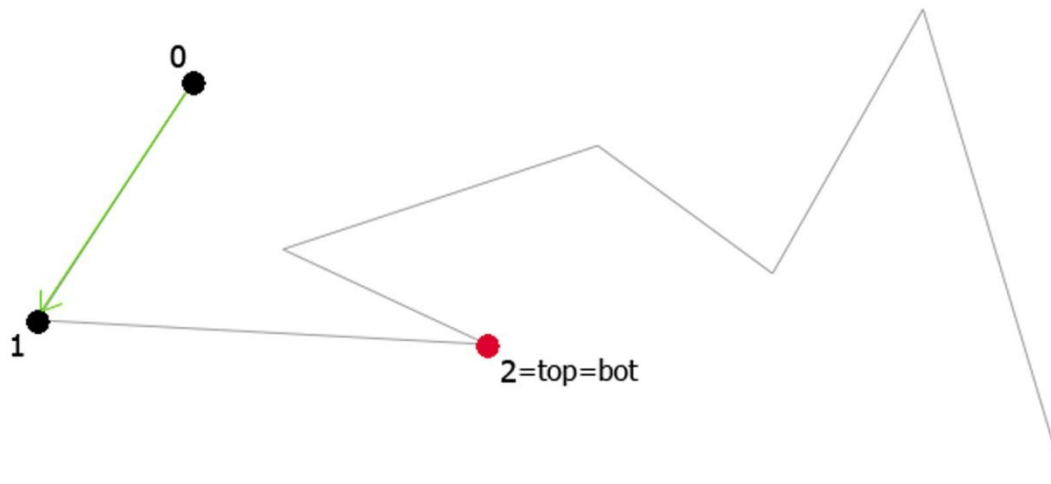
$H = \{ \}$

Example of the Melkman algorithm



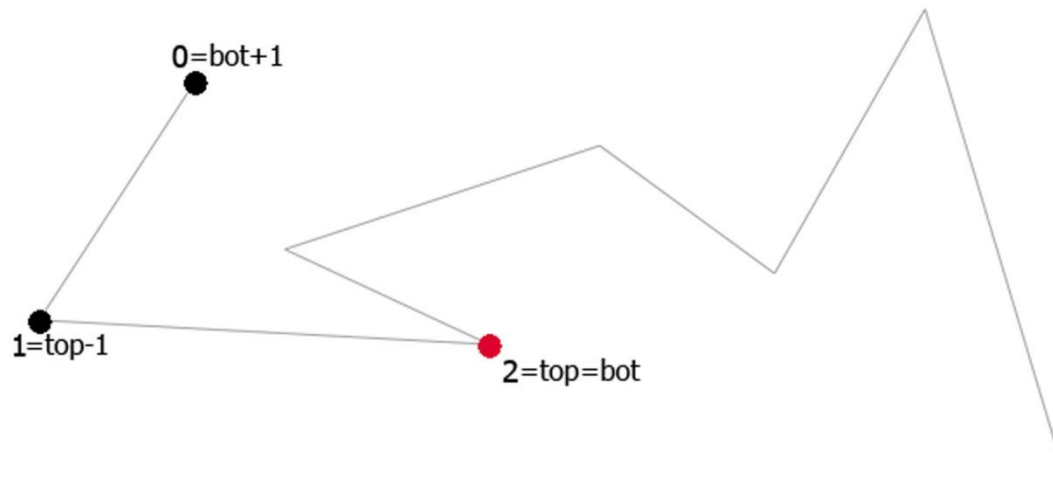
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Example of the Melkman algorithm



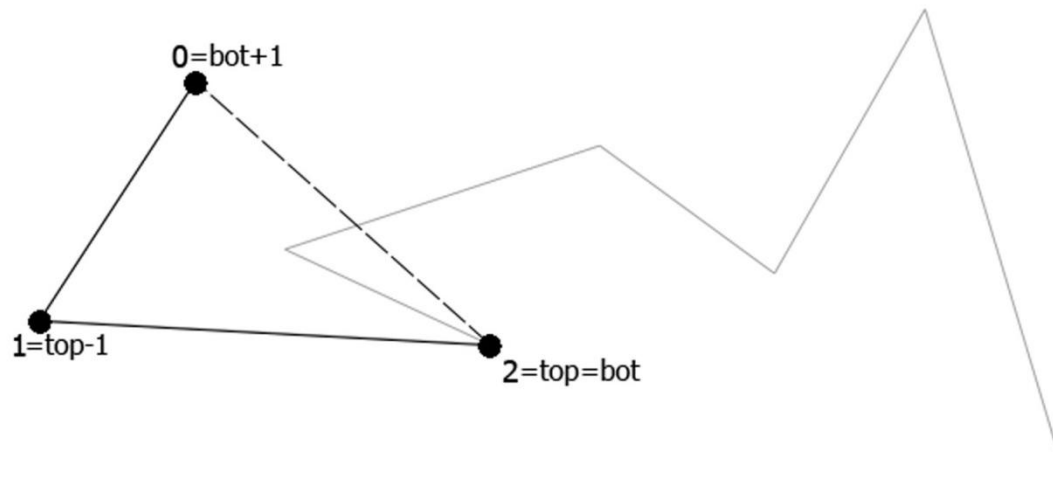
$$H = \{ 2, , , 2 \}$$

Example of the Melkman algorithm



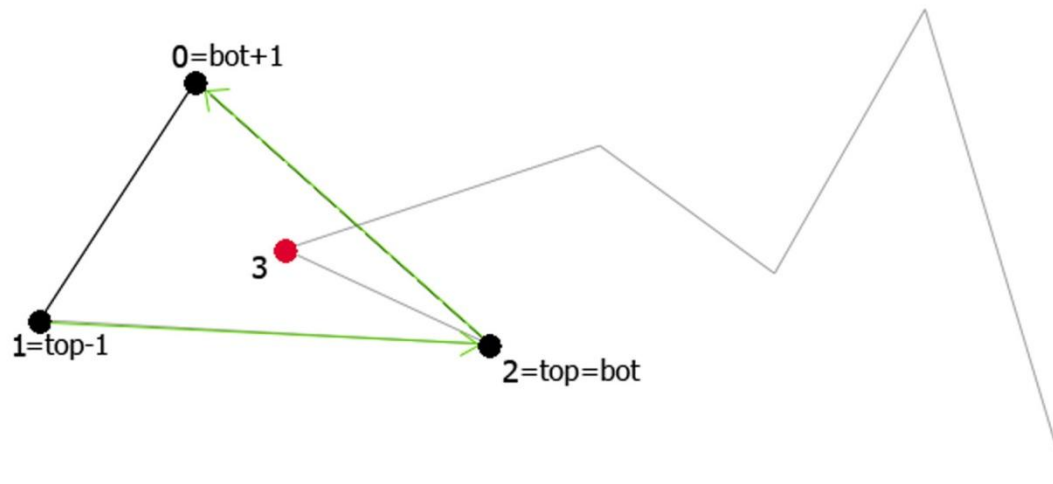
$$H = \{ 2, 0, 1, 2 \}$$

Example of the Melkman algorithm



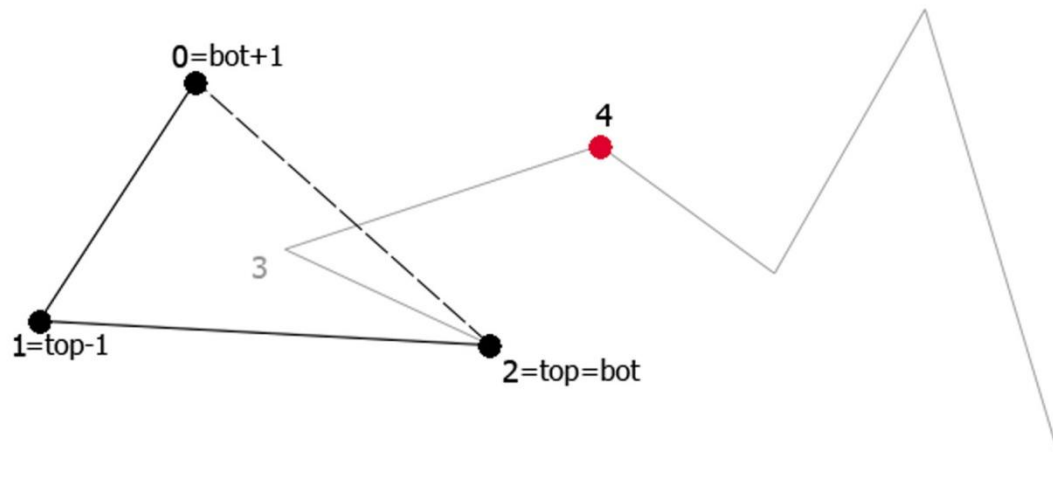
$$H = \{ 2, 0, 1, 2 \}$$

Example of the Melkman algorithm



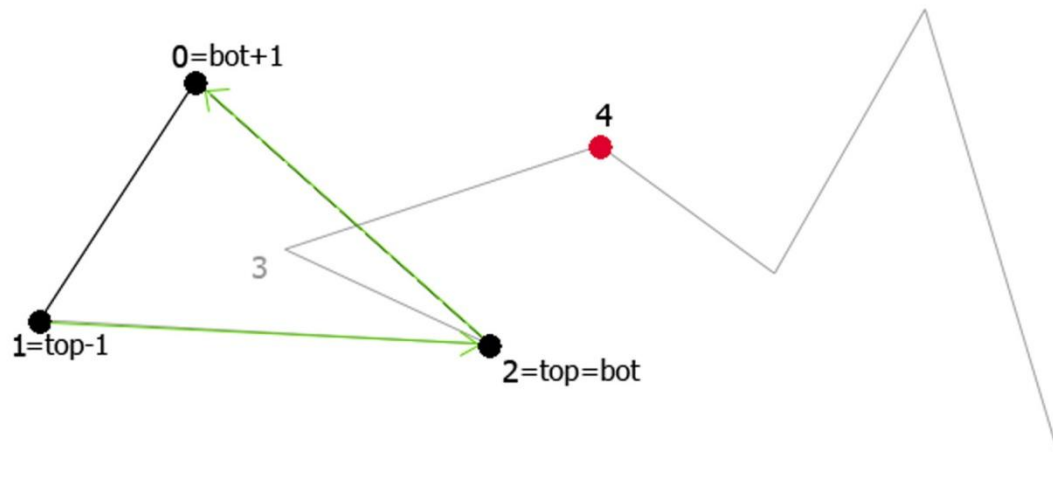
$$H = \{ 2, 0, 1, 2 \}$$

Example of the Melkman algorithm



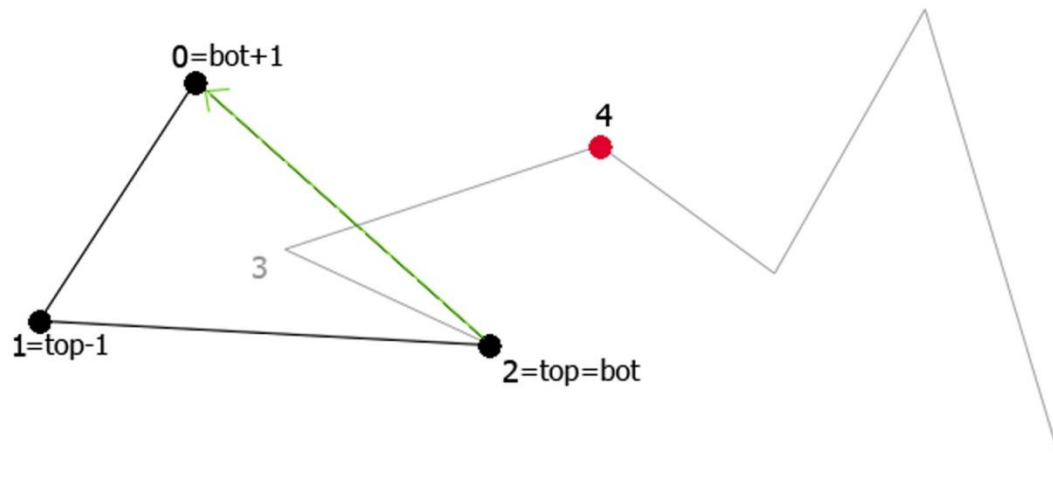
$$H = \{ 2, 0, 1, 2 \}$$

Example of the Melkman algorithm



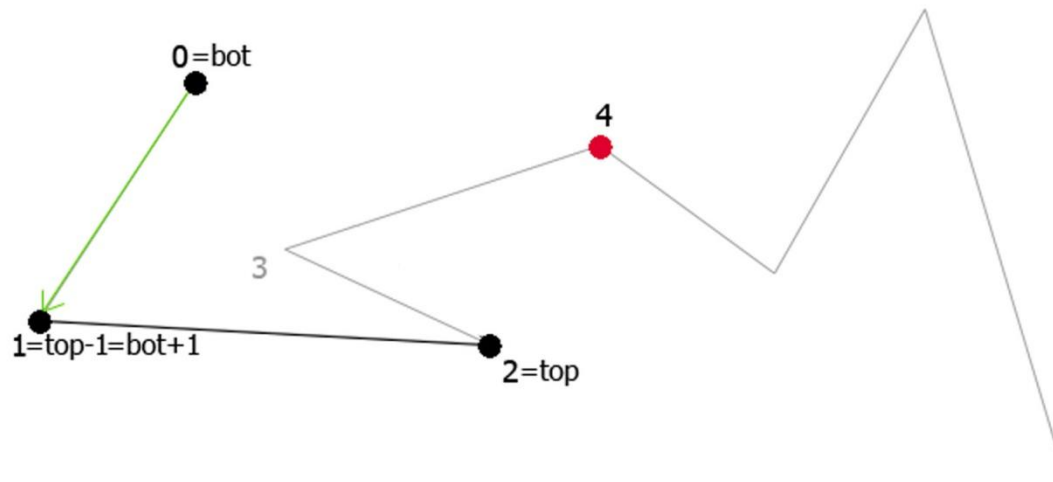
$$H = \{ 2, 0, 1, 2 \}$$

Example of the Melkman algorithm



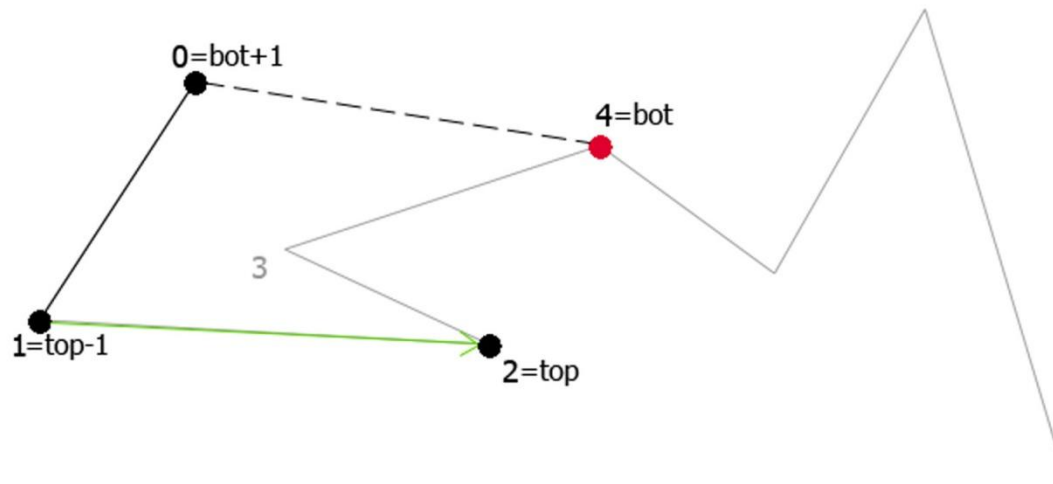
$$H = \{ 2, 0, 1, 2 \}$$

Example of the Melkman algorithm



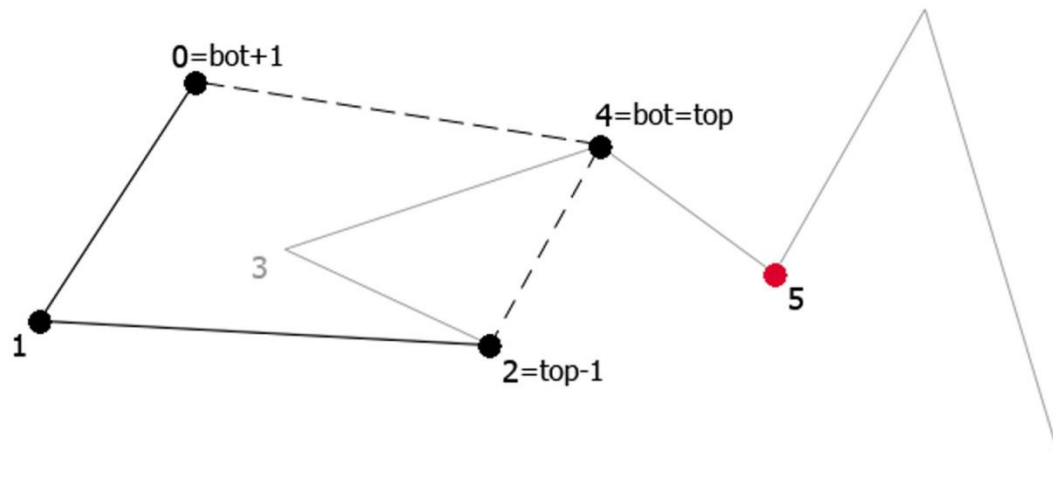
$$H = \{ 0, 1, 2 \}$$

Example of the Melkman algorithm



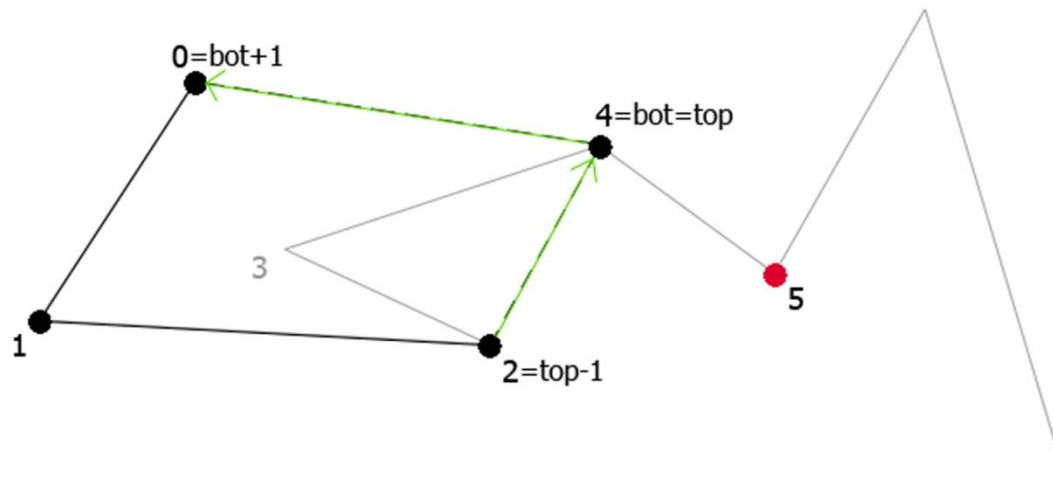
$$H = \{ 4, 0, 1, 2 \}$$

Example of the Melkman algorithm



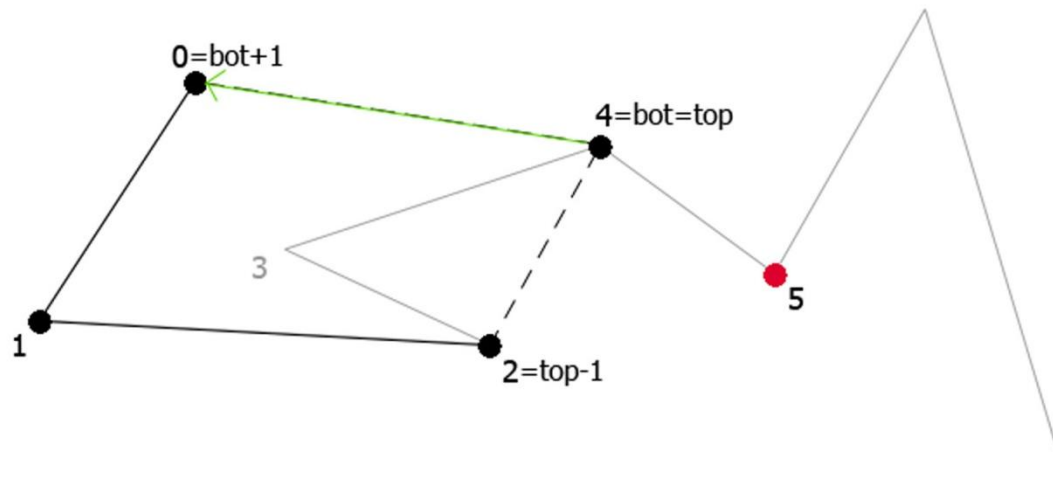
$H = \{ 4, 0, 1, 2, 4 \}$

Example of the Melkman algorithm



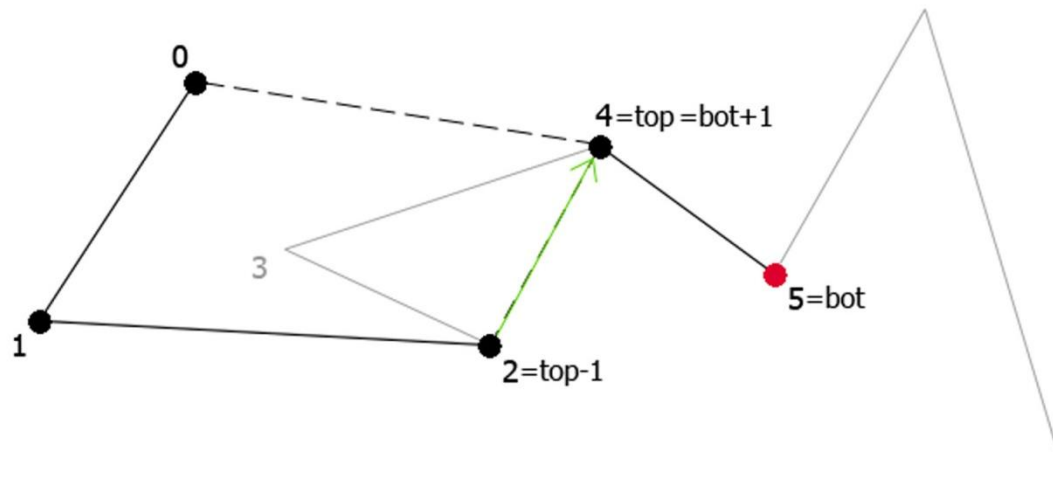
$$H = \{ 4, 0, 1, 2, 4 \}$$

Example of the Melkman algorithm



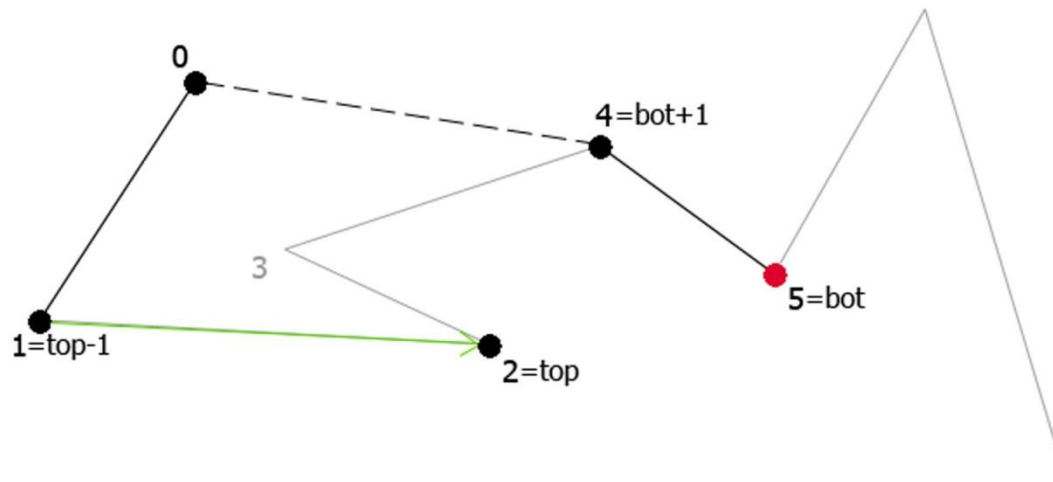
$$H = \{ 4, 0, 1, 2, 4 \}$$

Example of the Melkman algorithm



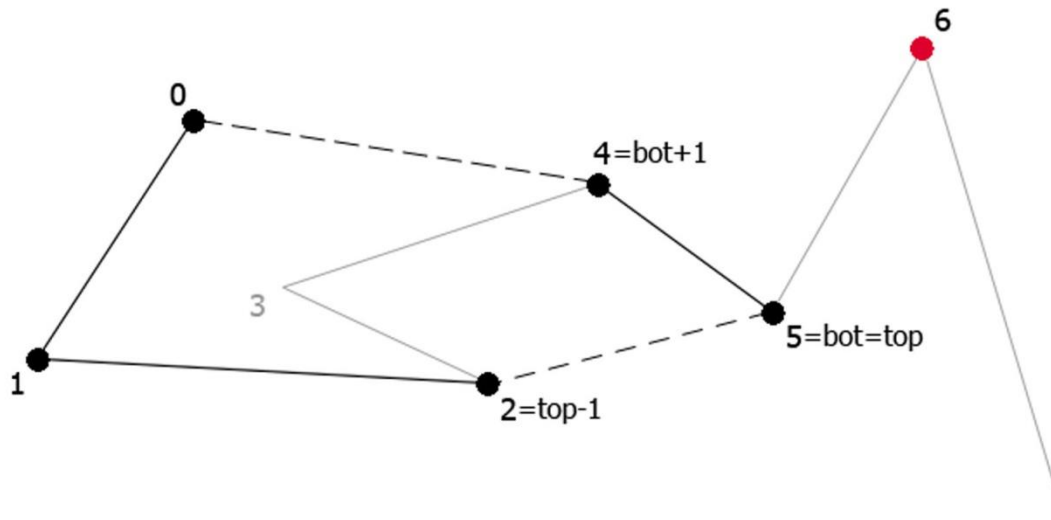
$H = \{ 5, 4, 0, 1, 2, 4 \}$

Example of the Melkman algorithm



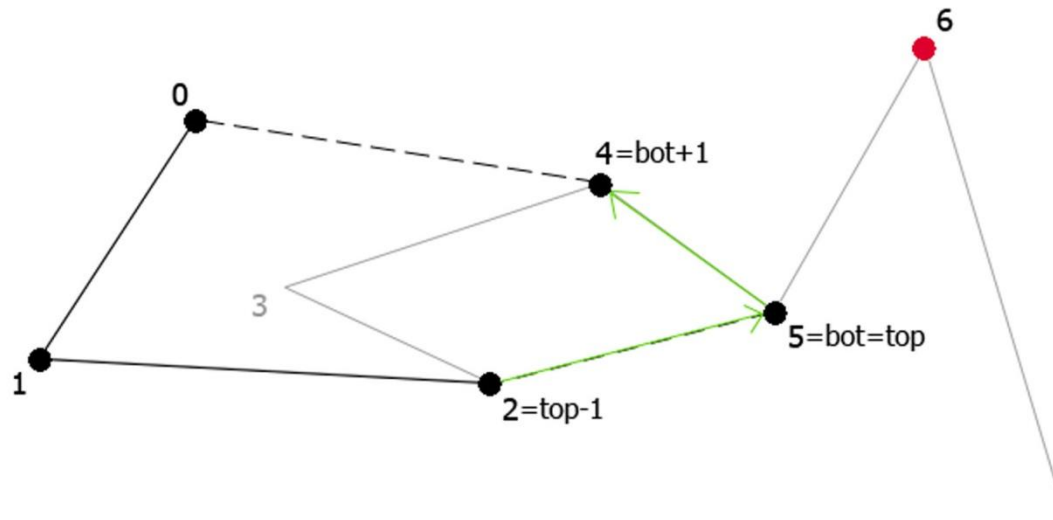
$$H = \{ 5, 4, 0, 1, 2 \}$$

Example of the Melkman algorithm



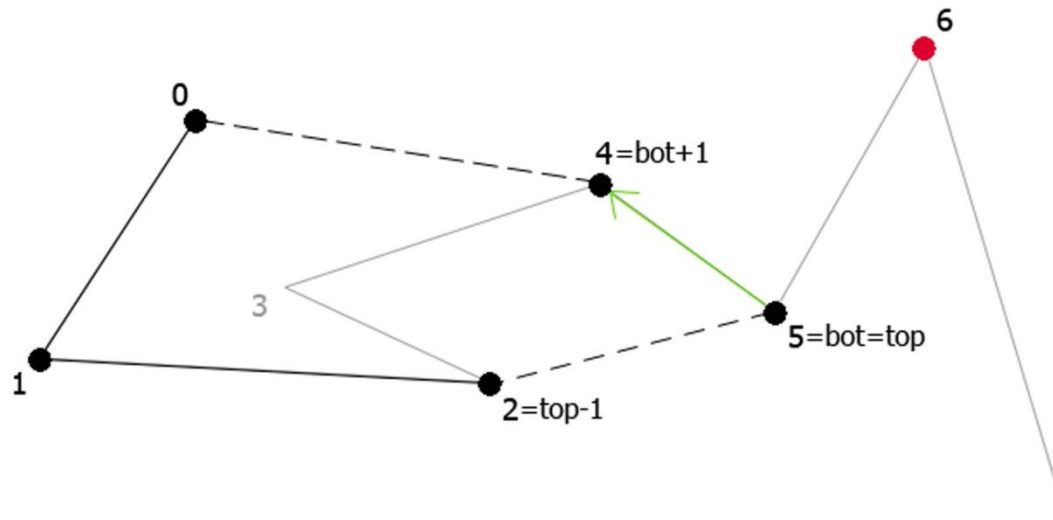
$H = \{ 5, 4, 0, 1, 2, 5 \}$

Example of the Melkman algorithm



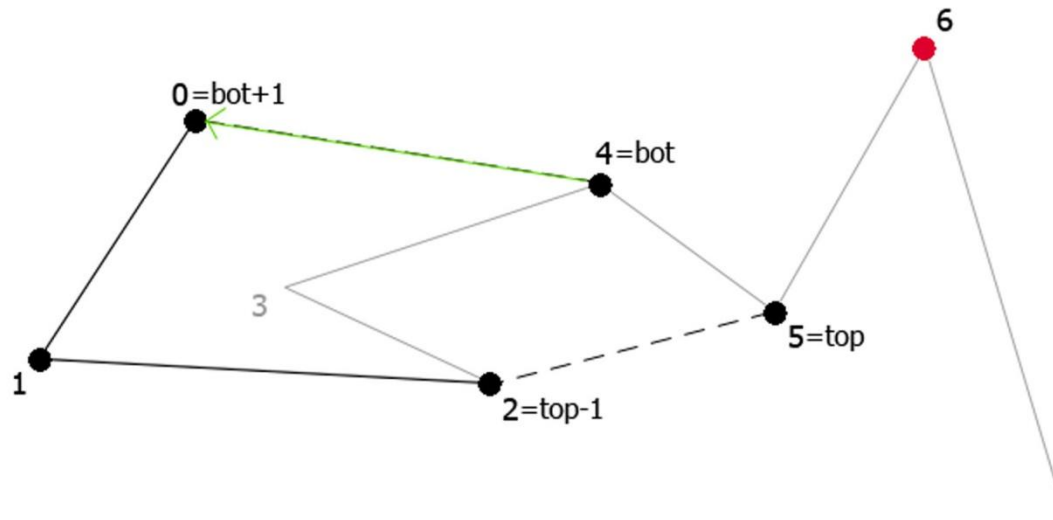
$H = \{ 5, 4, 0, 1, 2, 5 \}$

Example of the Melkman algorithm



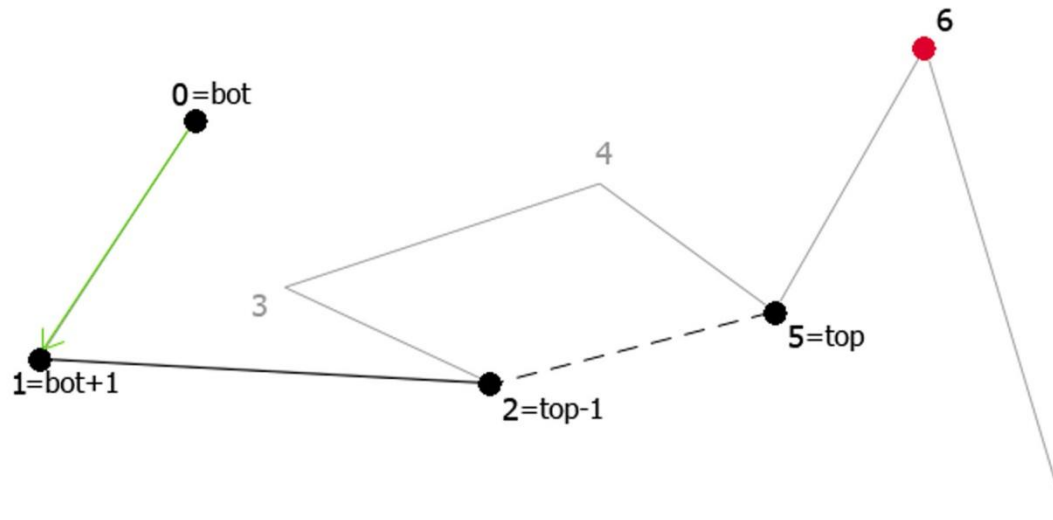
$H = \{ 5, 4, 0, 1, 2, 5 \}$

Example of the Melkman algorithm



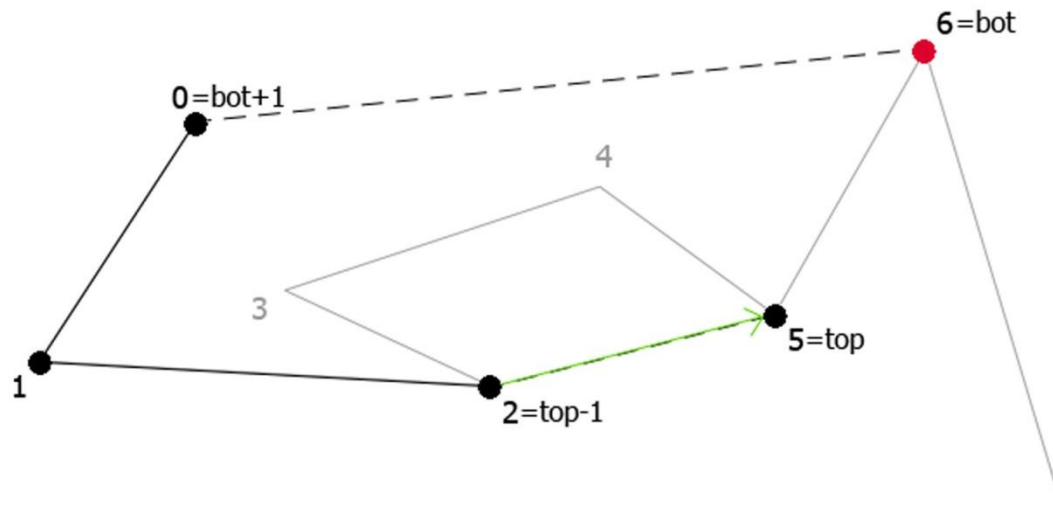
$$H = \{ 4, 0, 1, 2, 5 \}$$

Example of the Melkman algorithm



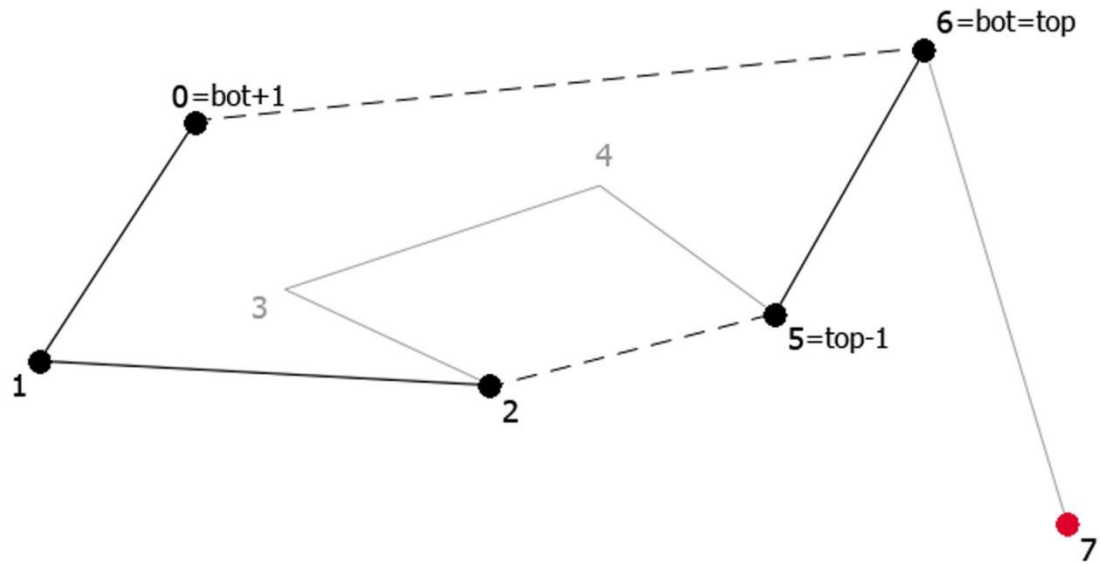
$$H = \{ 0, 1, 2, 5 \}$$

Example of the Melkman algorithm



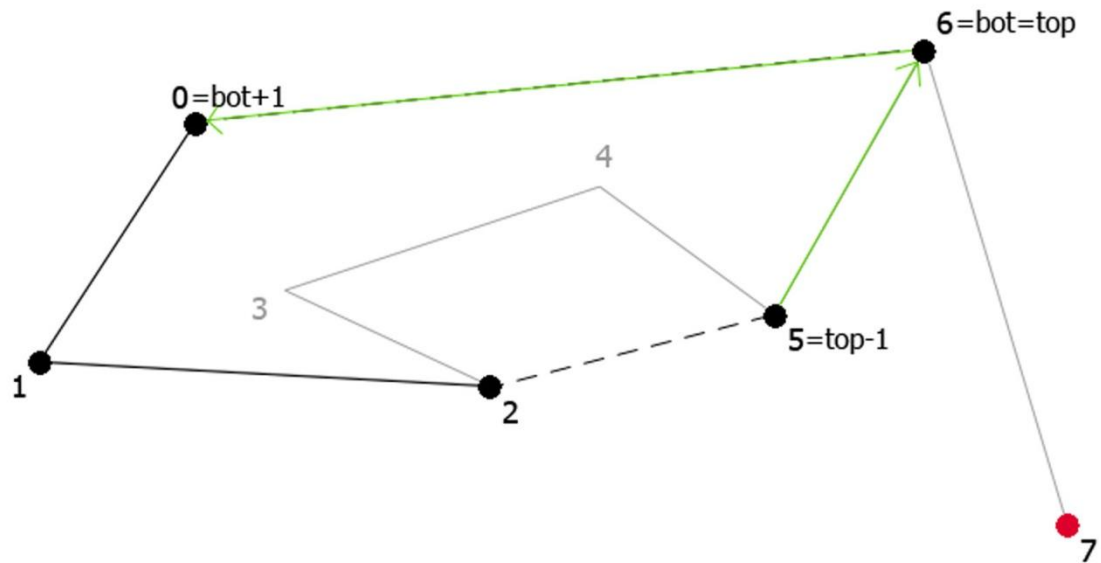
$H = \{ 6, 0, 1, 2, 5 \}$

Example of the Melkman algorithm



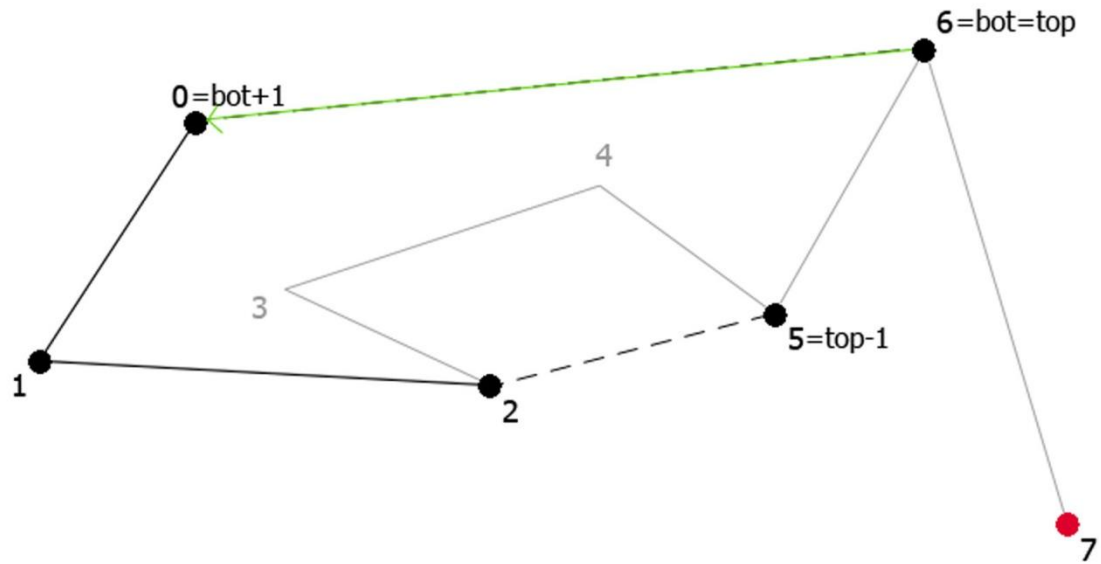
$H = \{ 6, 0, 1, 2, 5, 6 \}$

Example of the Melkman algorithm



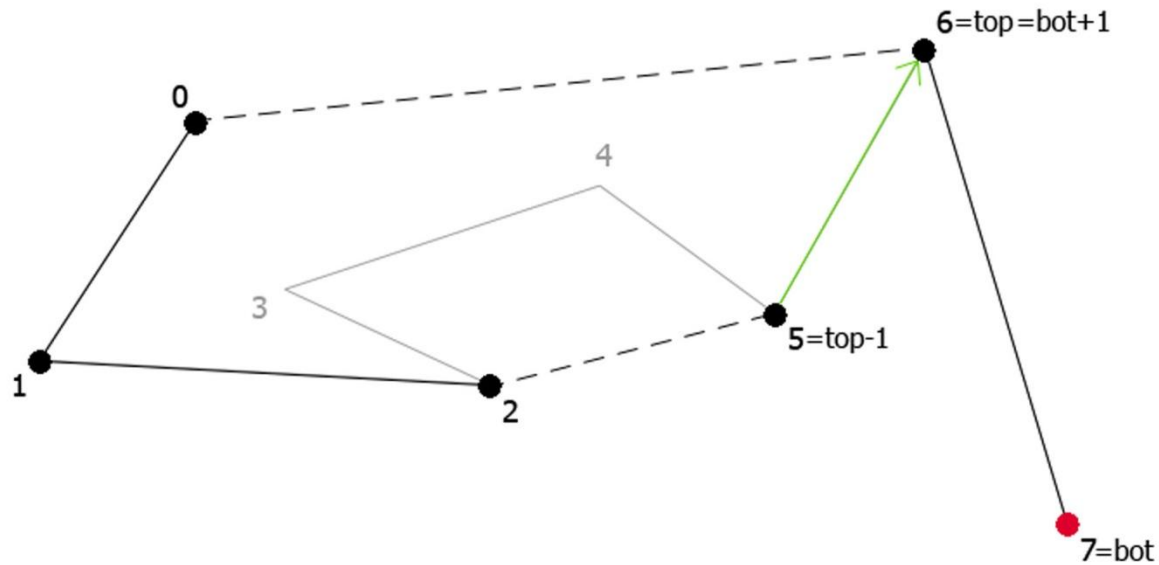
$H = \{ 6, 0, 1, 2, 5, 6 \}$

Example of the Melkman algorithm



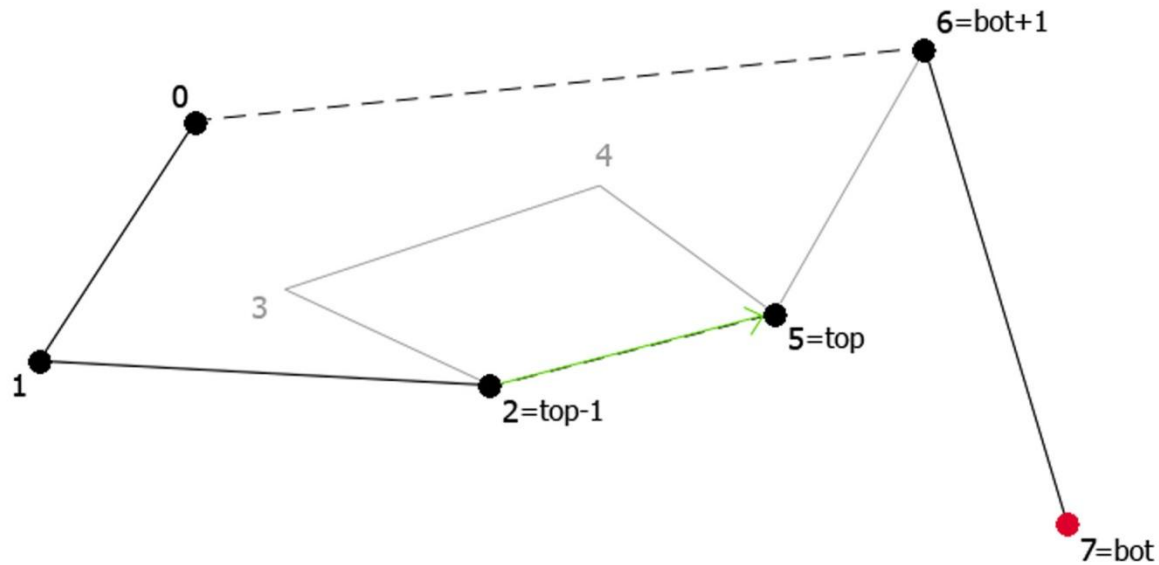
$H = \{ 6, 0, 1, 2, 5, 6 \}$

Example of the Melkman algorithm



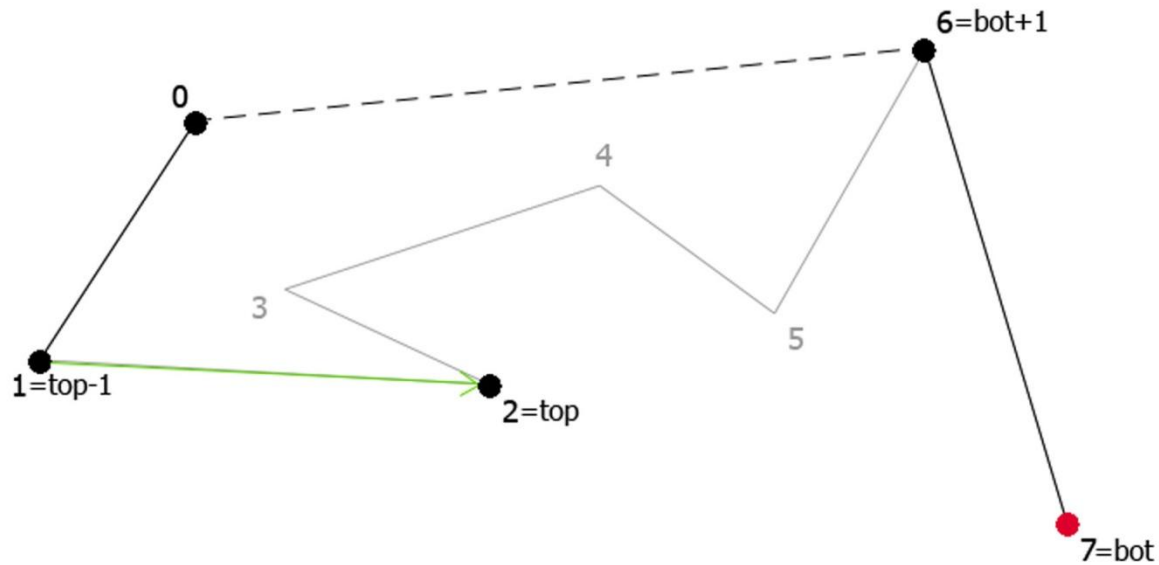
$H = \{ 7, 6, 0, 1, 2, 5, 6 \}$

Example of the Melkman algorithm



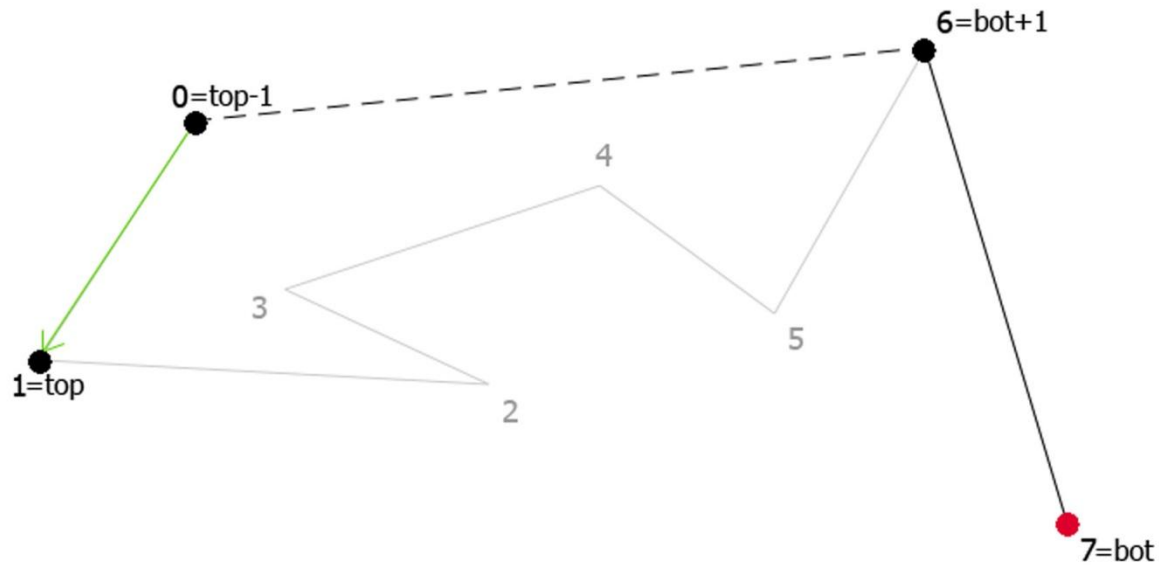
$H = \{ 7, 6, 0, 1, 2, 5 \}$

Example of the Melkman algorithm



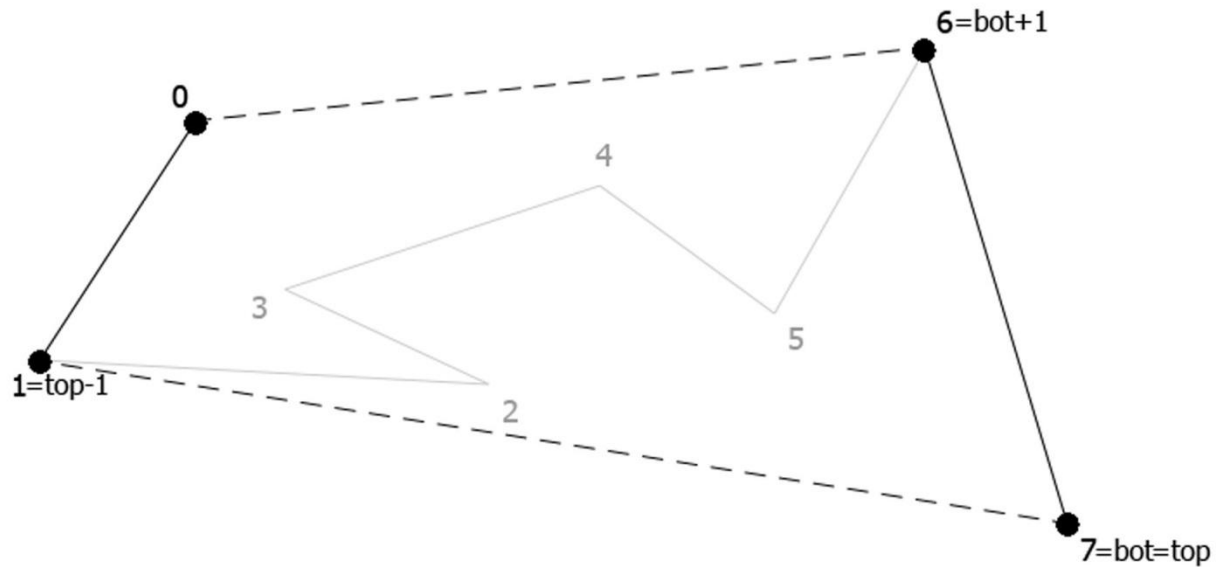
$H = \{ 7, 6, 0, 1, 2 \}$

Example of the Melkman algorithm



$$H = \{ 7, 6, 0, 1 \}$$

Example of the Melkman algorithm



$$H = \{ 7, 6, 0, 1, 7 \}$$

IsLeft

- ▶ Test if a point is left|on|right of an infinite line
 - ▶ Input: three points P_0, P_1, P_2
 - ▶ Return: > 0 for P_2 left of the line trough P_0 and P_1
 $= 0$ for P_2 on the line
 < 0 for P_2 right of the line
 - ▶ `isLeft(P_0, P_1, P_2)`
 - {
 - return $(P_1.x - P_0.x) * (P_2.y - P_0.y) - (P_2.x - P_0.x) * (P_1.y - P_0.y);$
 - }

Pseudocode

- ▶ **Input:** a simple polyline Ω with n vertices $V[n]$
- ▶ **Init:** put first 3 vertices onto deque D so that:
 - a) 3rd vertex $V[2]$ is at bottom and top of D
 - b) on D they form a counterclockwise triangle
- ▶ While there are more vertices of Ω to process:
 - Get the next vertex $V[i]$
 - //Test if $V[i]$ is inside D
 - if $V[i]$ is left of $D[bot]D[bot+1]$ and $D[top-1]D[top]$
 - skip $V[i]$ and continue with next vertex
 - //Get the tangent to the bottom
 - while $V[i]$ is right of $D[bot]D[bot+1]$
 - remove $D[bot]$
 - insert $V[i]$ at the bottom of D
 - //Get the tangent to the top
 - while $V[i]$ is right of $D[top-1]D[top]$
 - remove $D[top]$
 - insert $V[i]$ at the top of D
- ▶ **Output:** $D =$ the ccw convex hull of Ω

Sources

- ▶ http://softsurfer.com/Archive/algorithm_0203/algorithm_0203.htm
- ▶ <http://www.ams.sunysb.edu/~jsbm/courses/545/melkman.pdf>
- ▶ <http://w3.impa.br/~rdcastan/Cgeometry/>

Thank You for your attention!

▶ Questions?



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