DCGI

DEPARTMENT OF COMPUTER GRAPHICS AND INTERACTION

INTERSECTIONS OF LINE SEGMENTS AND POLYGONS

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Based on [Berg], [Mount], [Kukral], and [Drtina]

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Talk overview

- Intersections of line segments
  - Motivation
  - Sweep line algorithm recapitulation
  - Sweep line intersections of line segments
- Intersection of polygons or planar subdivisions
  - See assignment [21] or [Berg, Section 2.3]
- Intersection of axis parallel rectangles

Geometric intersections – what are they for?

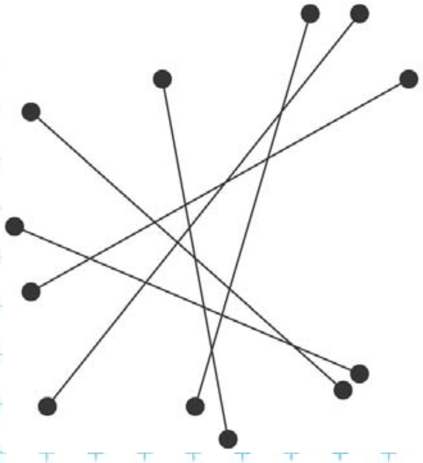
One of the most basic problems in computational geometry

- Solid modeling
  - Intersection of object boundaries in CSG
- Overlay of subdivisions, e.g. layers in GIS
  - Bridges on intersections of roads and rivers
  - Maintenance responsibilities (road network X county boundaries)
- Robotics
  - Collision detection and collision avoidance
- Computer graphics
  - Rendering via ray shooting (intersection of the ray with objects)
- …

Line segment intersection

- Intersection of complex shapes is often reduced to simpler and simpler intersection problems
- Line segment intersection is the most basic intersection algorithm
- Problem statement:

  Given *n* line segments in the plane, report all points where a pair of line segments intersect.
- Problem complexity
  - Worst case – $I = O(n^2)$ intersections
  - Practical case – only some intersections
  - Use an output sensitive algorithm
    - $O(n \log n + I)$ optimal randomized algorithm
    - $O(n \log n + I \log n)$ sweep line algorithm - %



[Berg]

Plane sweep line algorithm recapitulation

- Horizontal line (sweep line, *scan line*) l moves top-down (or vertical line: left to right) over the set of objects
- The move is not continuous, but l jumps from one event point to another
  - Event points are in priority queue or sorted list
  - The left-most event point is removed first
  - New event points may be created (usually as interaction of neighbors on the sweep line) and inserted in the queue
- Scan-line status
  - Stores information about the objects intersected by SL
  - It is updated while halting on event point

Line segment intersection - Sweep line alg.

- Avoid testing of pairs of segments far apart
- Compute intersections of neighbors on the sweep line only
- O(n log n + I log n) time in O(n) memory

  2n steps for end points, I steps for intersections, log n search the tree

- Ignore “nasty cases” (most of them will be solved later on)
  - No segment is parallel to a sweep line ()
  - Segments intersect in one point and do not overlap
  - No three segments meet in a common point

Line segment intersections

- *Status* = ordered sequence of segments intersecting the sweep line **l**
- *Events* (waiting in the priority queue)

  = points, where the algorithm actually does something

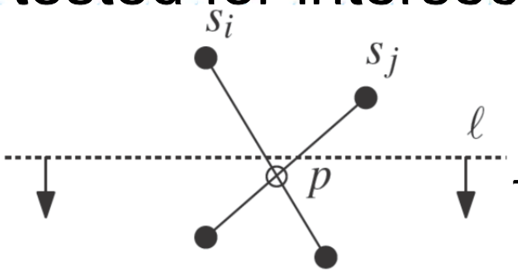
  – Segment *end-points*
    - known at algorithm start

  – Segment *intersections* between neighboring segments along SL
    - Discovered as the sweep executes

Detecting intersections

- Intersection events must be detected and inserted to the event queue before they occur
- Given two segments *a, b* intersecting in a point *p*, there must be a placement of the **l** prior to *p*, such that segments *a, b* are adjacent along **l** (only adjacent will be tested for intersection)
  - segments *a, b* are not adjacent when the alg. starts
  - segments *a, b* are adjacent just before *p*

  => there must be an event point when *a,b* become adjacent and therefore are tested for intersection



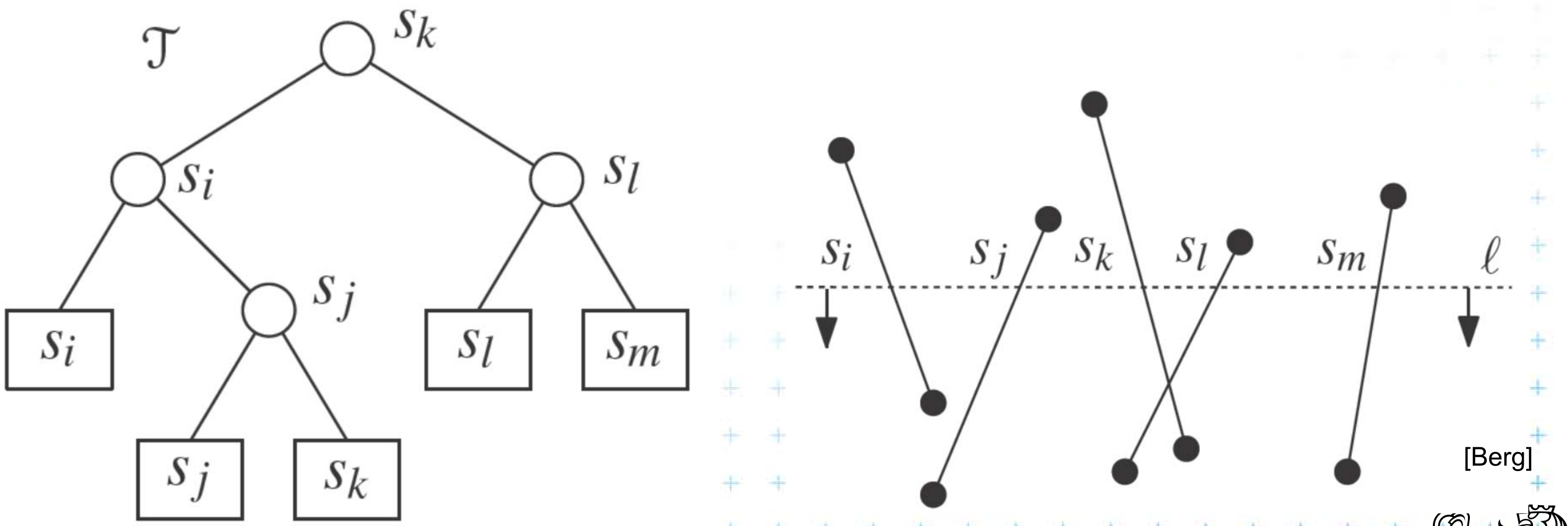
[Berg]

Data structures

Sweep line **l** status = order of segments along **l**

- Balanced binary search tree
- Coords of intersections with **l** vary as **l** moves

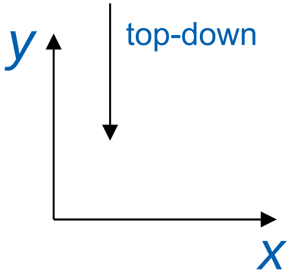
  => BST keys are pointers to the line segments
  - – Position of **l** is plugged in the *y=mx+b* to get the key



[Berg]

Data structures

Event queue (postupový plán, časový plán)



- Define: Order ⁄

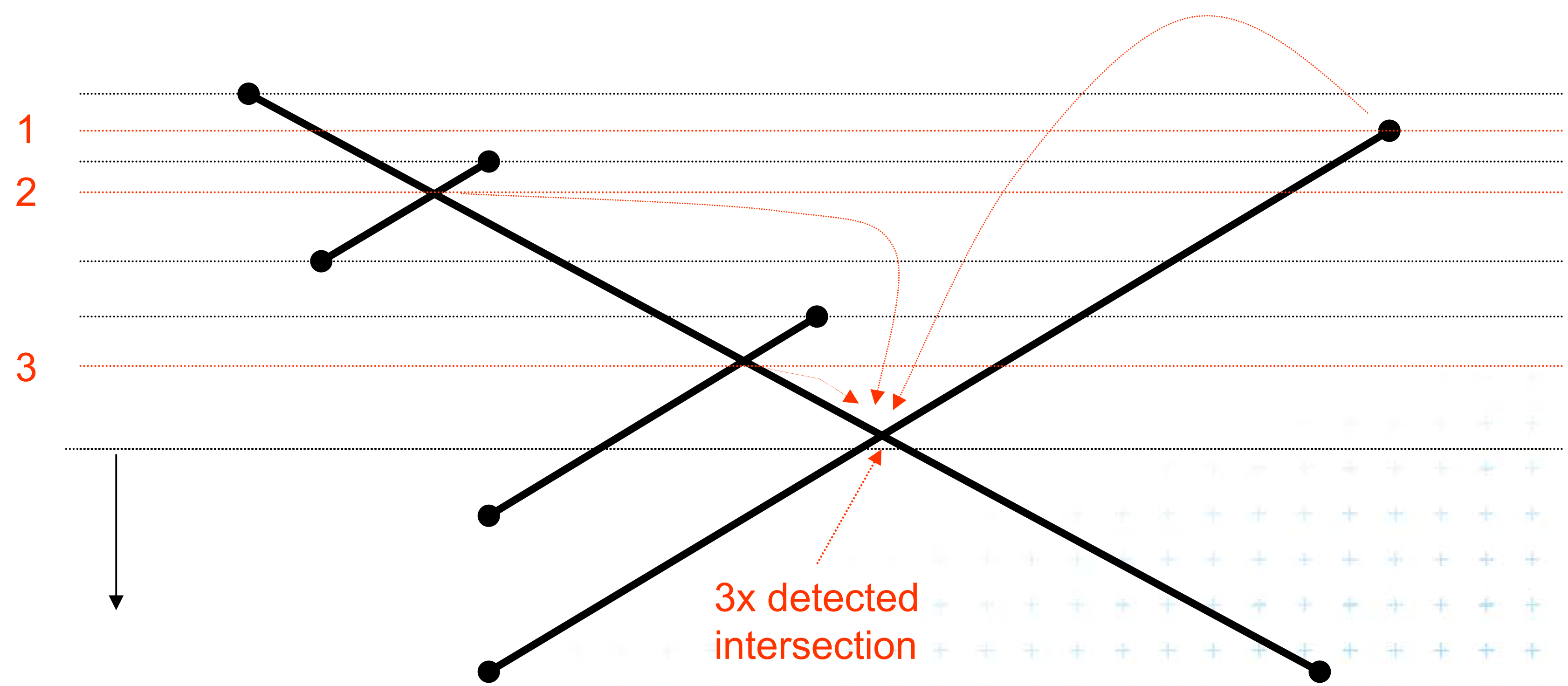
  p ⁄ q **iff** $p_y > q_y$ **or** $p_y = q_y$ and $p_x < q_x$

  top-down, left-right approach

  (points on l treated left to right)

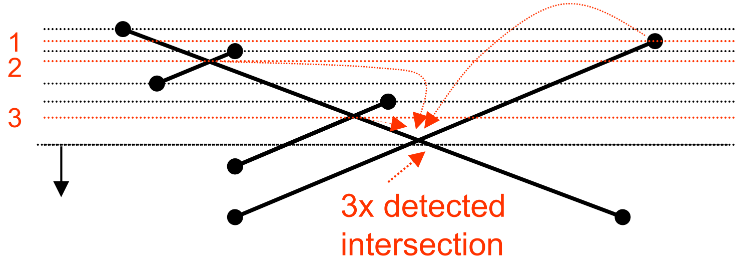
- Operations
  - Insertion of computed intersection points
  - Fetching the next event (highest *y* below l)
  - Test, if the segment is already present in the queue
  - (Delete intersection event in the queue)

Problem with duplicities of intersections



Data structures

Event queue data structure



- Heap
  - Problem: can not check duplicated intersection events (reinvented more than once)
  - Processed twice or more
  - Memory complexity up to O(n^2)
- Ordered dictionary (balanced binary tree)
  - Can check duplicated events (adds just constant factor)
  - Nothing inserted twice
  - If non-neighbor intersections are deleted
    i.e., only intersection of neighbors is stored
    then memory complexity just O(n)

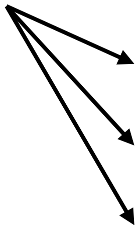
Line segment intersection algorithm

FindIntersections(*S*)

Input: A set *S* of line segments in the plane

Output: The set of intersection points + pointers to segments in each

```
init an empty event queue $Q$ and insert the segment endpoints
init an empty status structure $T$
while Q in not empty
    remove next event $p$ from $Q$
    handleEventPoint($p$)
```



Upper endpoint

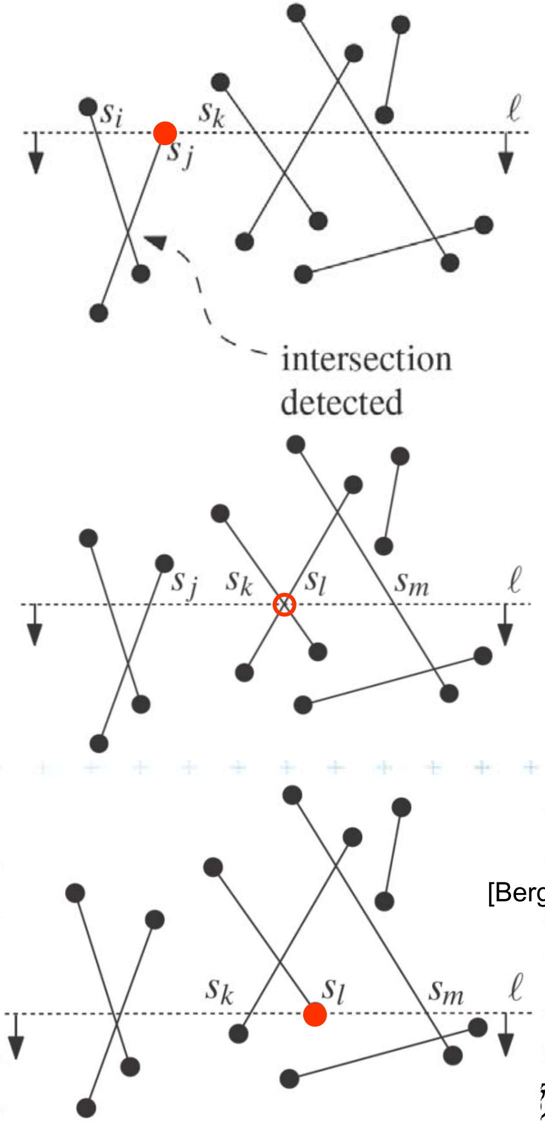
Intersection

Lower endpoint

Note: Upper-end-point events store info about the segment

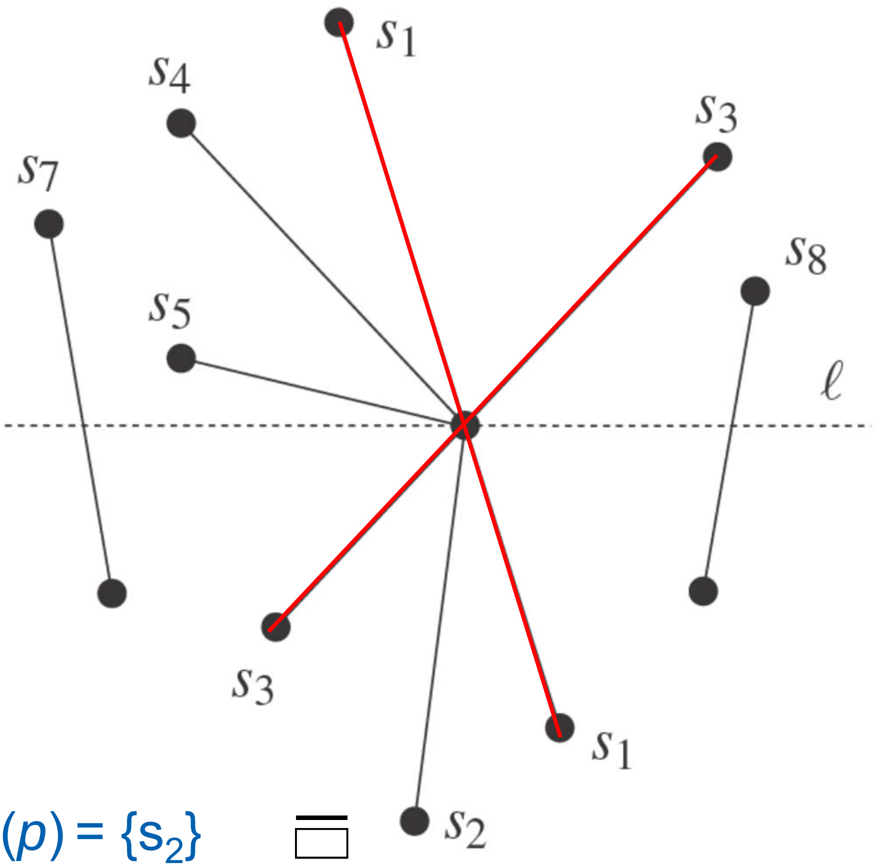
handleEventPoint principle

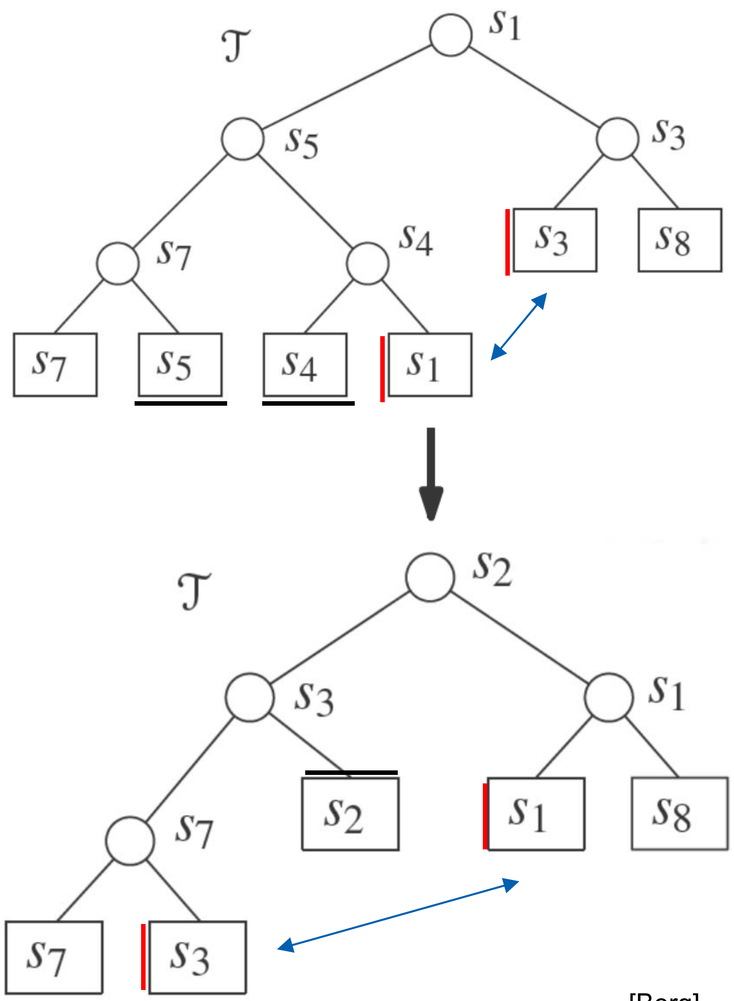
- Upper endpoint *U*(*p*)
  - insert *p* (on s_j) to *T*
  - add intersections with left and right neighbors to *Q*
- Intersection *C*(*p*)
  - switch order of segments in *T*
  - add intersections of left and right neighbors to *Q*
- Lower endpoint *L*(*p*)
  - remove *p* (on s_l) from *T*
  - add intersections of left and right neighbors to *Q*



[Berg]

More than two segments incident





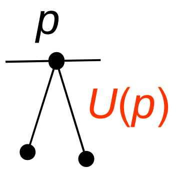
$U(p) = \{s_2\}$ ▭

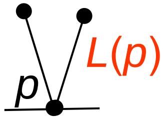
$C(p) = \{s_1, s_3\}$ ▭

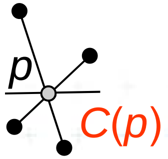
$L(p) = \{s_4, s_5\}$ ▭

[Berg]

Handle Events [Berg, page 25]







handleEventPoint(p)

```
1.  Let $U(p)$ = set of segments whose upper point is $p$.
    These segmets are stored with the event point $p$ (will be added to $T$)
2.  Search $T$ for all segments $S(p)$ that contain $p$ (are adjacent in $T$):
        Let $L(p)$ ð $S(p)$ = segments whose lower endpoint is $p$
        Let $C(p)$ ð $S(p)$ = segments that contains $p$ in interior
3.  if( $L(p)$ Ã $U(p)$ Ã $C(p)$ contains more than one segment )
4.          report $p$ as intersection together with $L(p)$, $U(p)$, $C(p)$
5.  Delete the segments in $L(p)$ Ã $C(p)$ from $T$
6.  Insert  the segments in $U(p)$ Ã $C(p)$ from $T$
    (horizontal segment as the last)
7.  if( $U(p)$ Ã $C(p)$ = þ ) then FindNewEvent($s_l$, $s_r$, $p$) // left & right neighbors
8.  else  s' = leftmost segment of $U(p)$ Ã $C(p)$;   findNewEvent($s_l$ , s', $p$)
          s'' = rightmost segment of $U(p)$ Ã $C(p)$; findNewEvent(s'', $s_r$ , $p$)
```

Reverse order of $C(p)$ in T

Detection of new intersections

findNewEvent(s_l, s_r, p)

Input: two segments and a current event point *p*

Output: updated event queue *Q* with new intersections

```
1.  if [ ( $s_l$ and $s_r$ intersect below the sweep line l ) or
       ( intersect on l and to the right of $p$ ) ] and        // horizontal segments
       ( the intersection is not present in $Q$ )
2.  then
       insert $p$ as an event into $Q$
```

Line segment intersections

- Memory O(I) = O(n^2) with duplicities in Q
  or O(n) with duplicities in Q deleted
- Operational complexity
  - – $n + I$ stops
  - – log n each

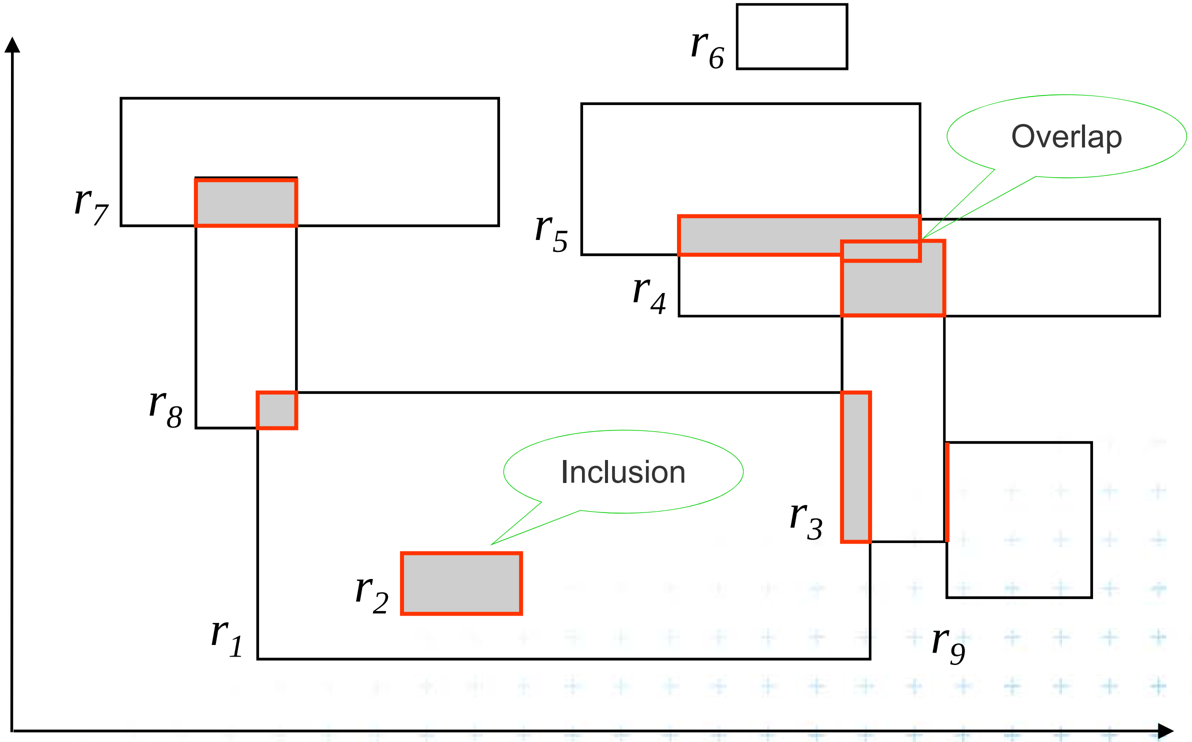
  => O($I + n$) log n total
- The algorithm is by Bentley-Ottmann

Bentley, J. L.; Ottmann, T. A. (1979), "Algorithms for reporting and counting geometric intersections", *IEEE Transactions on Computers* **C-28** (9): 643-647, doi:10.1109/TC.1979.1675432 .

See also http://wapedia.mobi/en/Bentley%E2%80%93Ottmann_algorithm

Intersection of axis parallel rectangles

- Given the collection of *n isothetic* rectangles, report all intersecting parts



[?]

Answer: (r_1, r_3) (r_1, r_8) (r_3, r_4) (r_3, r_5) (r_4, r_5) (r_7, r_8) (r_1, r_2) (r_3, r_9)

Brute force intersection

Brute force algorithm

Input: set *S* of axis parallel rectangles

Output: pairs of intersected rectangles

```
1. For every pair $(r_i, r_j)$ of rectangles $\in S$, $i \neq j$
2.     if $(r_i \cap r_j \neq \emptyset)$ then
3.         report $(r_i, r_j)$
```

Analysis

Preprocessing: None.

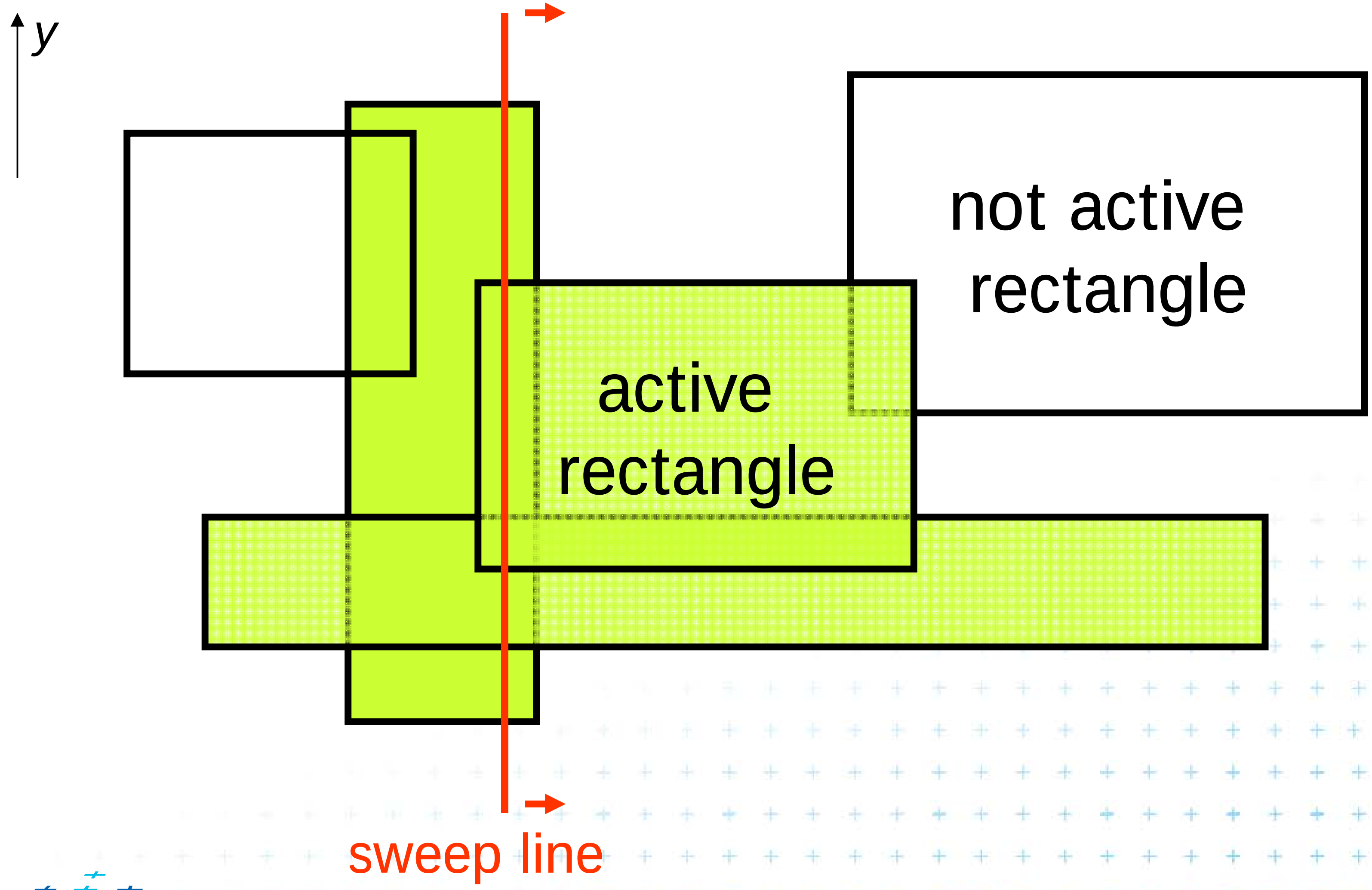
Query: $O(N^2)$; $\binom{N}{2} = (N(N-1))/2 \in O(N^2)$.

Storage: $O(N)$.

Plane sweep intersection algorithm

- Vertical sweep line moves from left to right
- Stops at every x-coordinate of a rectangle (either its left side or its right side).
- active rectangles – a set
  = rectangles currently intersecting the sweep line
  - left side event of a rectangle
    => the rectangle is added to the active set.
  - right side
    => the rectangle is deleted from the active set.
- The active set used to detect rectangle intersection

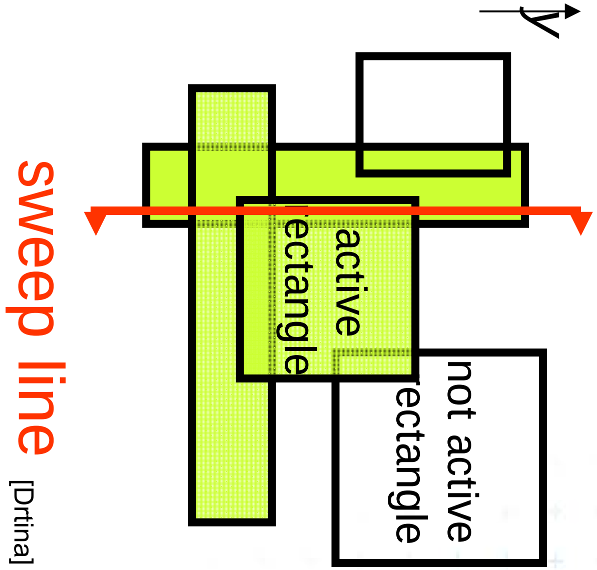
Example rectangles and sweep line



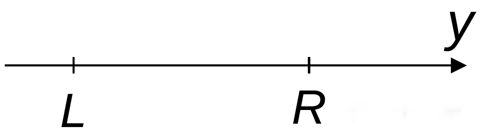
[Drtina]

Interval tree as sweep line status structure

- Vertical sweep-line => Only *y*-coordinates along it
- Turn our view in slides 90° right
- Sweep line (*y*-axis) will be drawn as horizontal



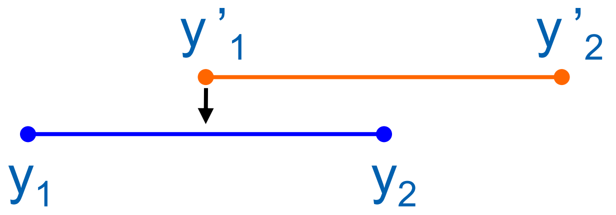
[Drtina]



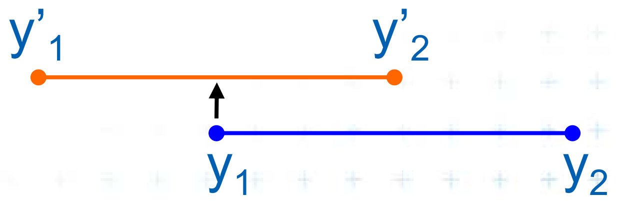
Intersection test – between pair of intervals

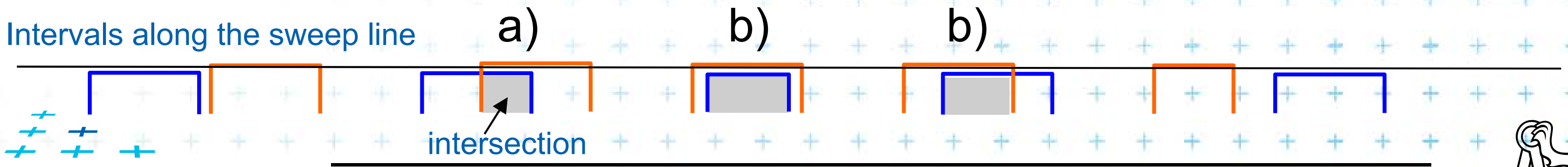
- Given two intervals R = [y_1, y_2] and R’ = [$y’_1$, $y’_2$] the condition R ∩ R’ is equivalent to one of these mutually exclusive conditions:

a) $y_1 \le y'_1 \le y_2$



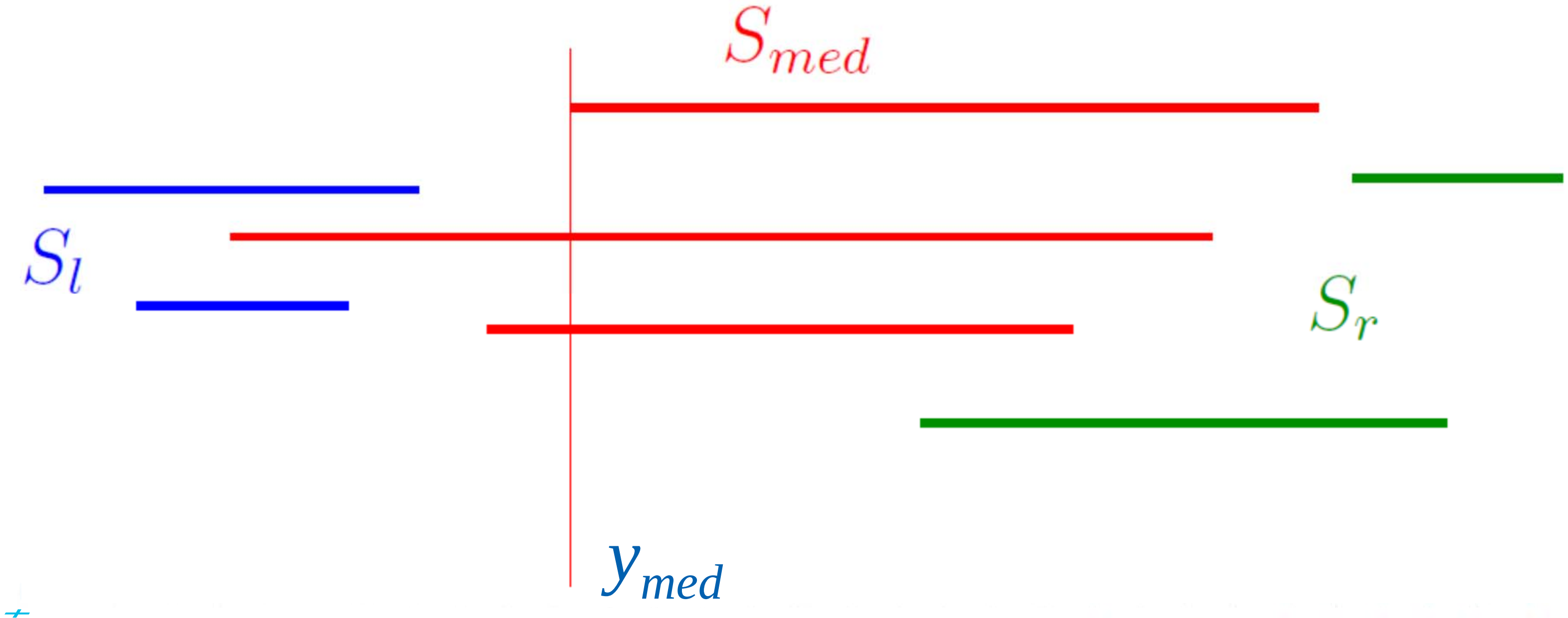
b) $y'_1 \le y_1 \le y'_2$





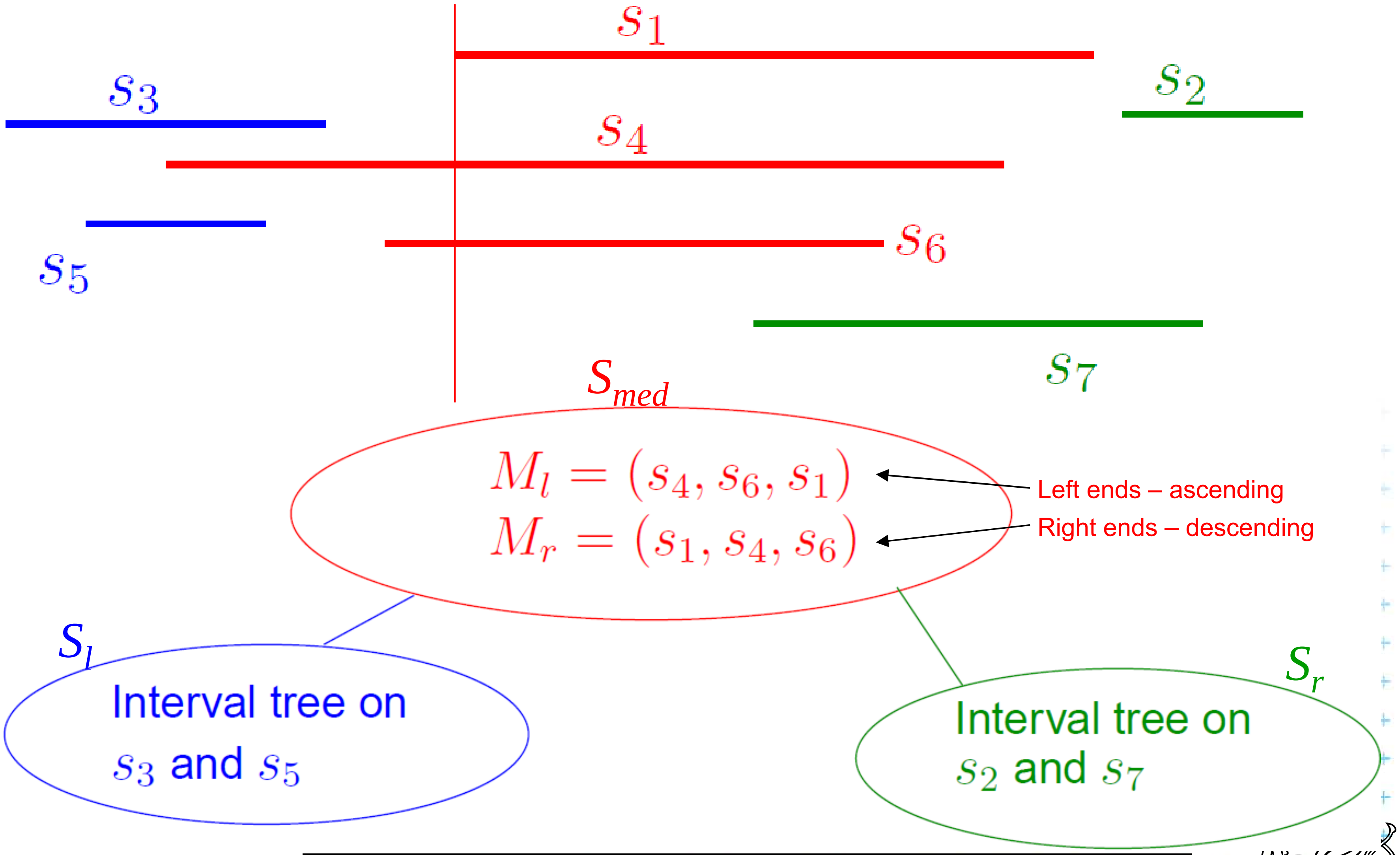
Static interval tree – stores all end points

- Let *v* = y_{med} be the median of end-points of segments
- S_l : segments of S that are completely to the left of y_{med}
- S_{med} : segments of S that contain $\mathrm{y}_{\mathrm{med}}$
- S_r : segments of S that are completely to the right of $\mathrm{y}_{\mathrm{med}}$



[Vigneron]

Static interval tree – Example



$M_l = (s_4, s_6, s_1)$

$M_r = (s_1, s_4, s_6)$

Left ends – ascending

Right ends – descending

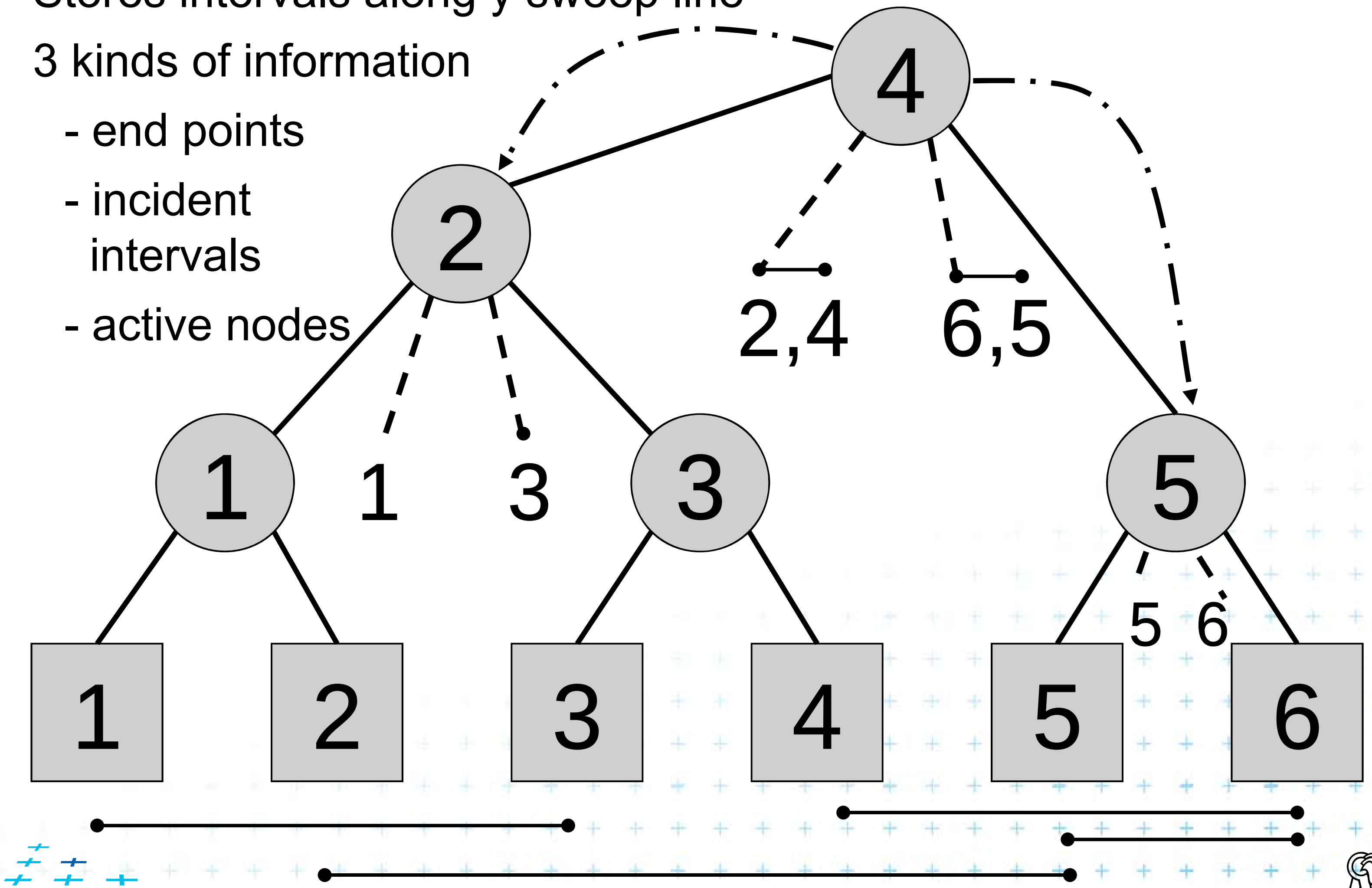
Interval tree on s_3 and s_5

Interval tree on s_2 and s_7

[Vigneron]

Static interval tree [Edelsbrunner80]

- Stores intervals along y sweep line
- 3 kinds of information
  - end points
  - incident intervals
  - active nodes

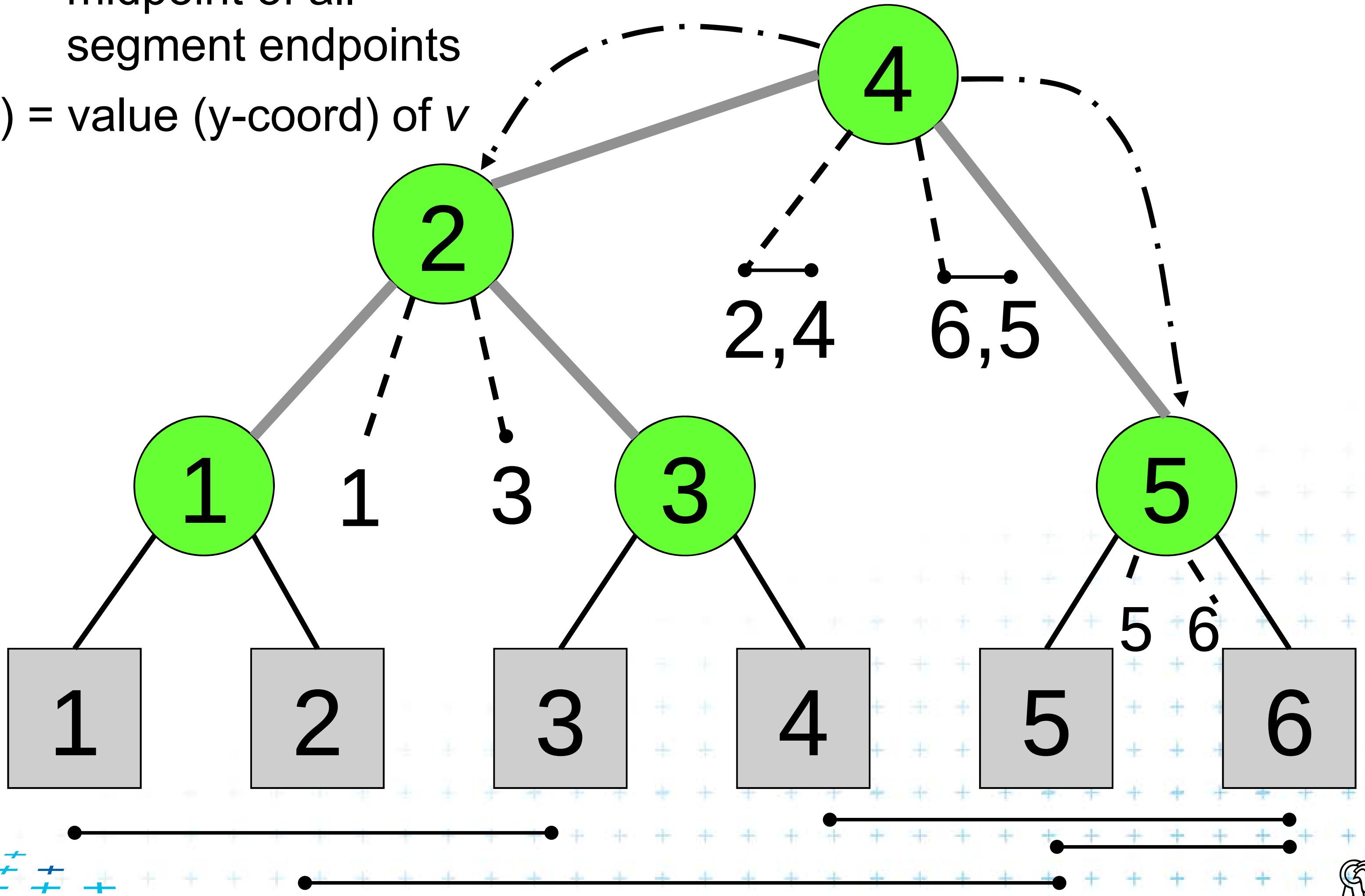


[Kukral]

Primary structure – static tree for endpoints

v = midpoint of all segment endpoints

H(v) = value (y-coord) of *v*

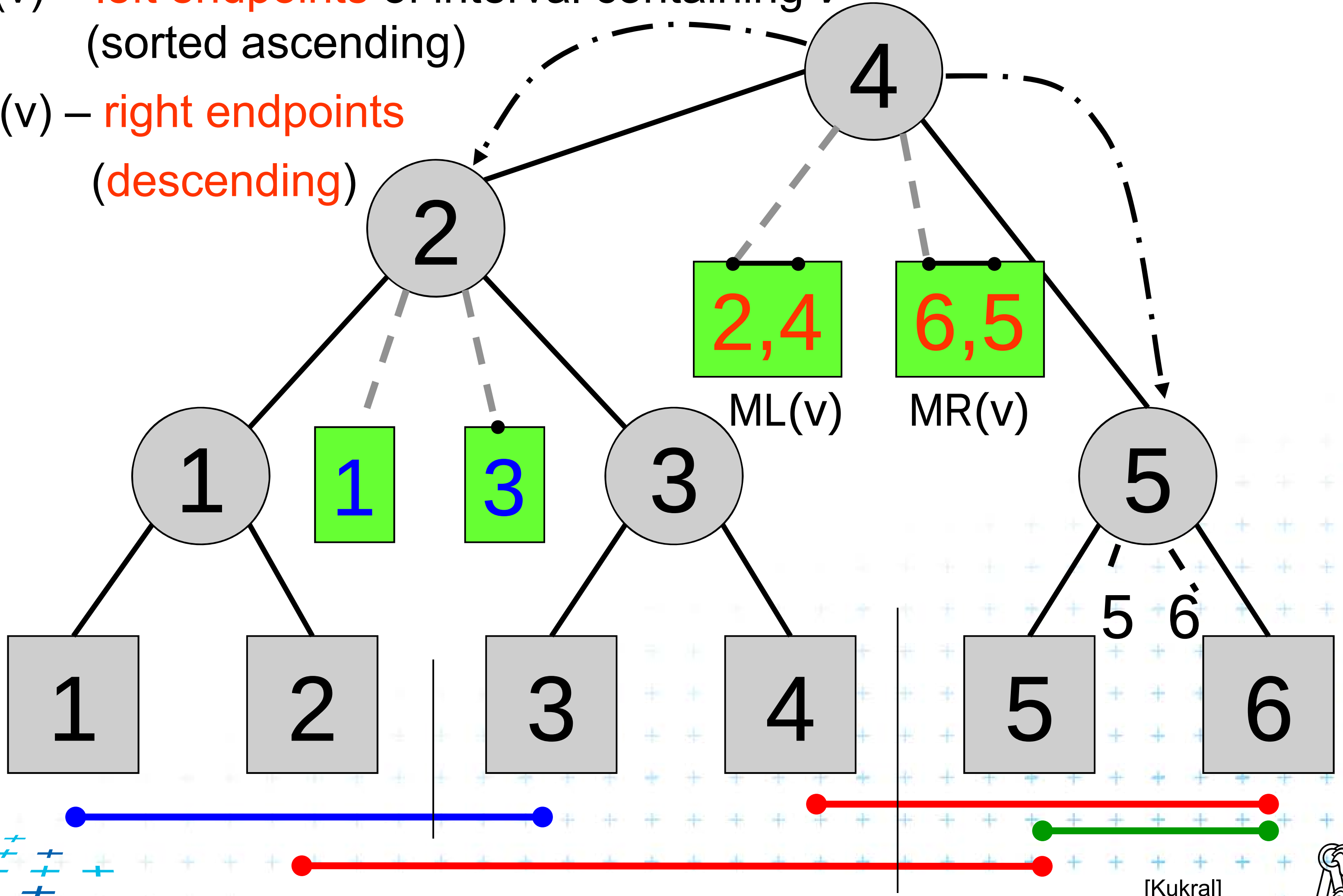


[Kukral]

Secondary lists of incident interval end-pts.

ML(v) – left endpoints of interval containing *v* (sorted ascending)

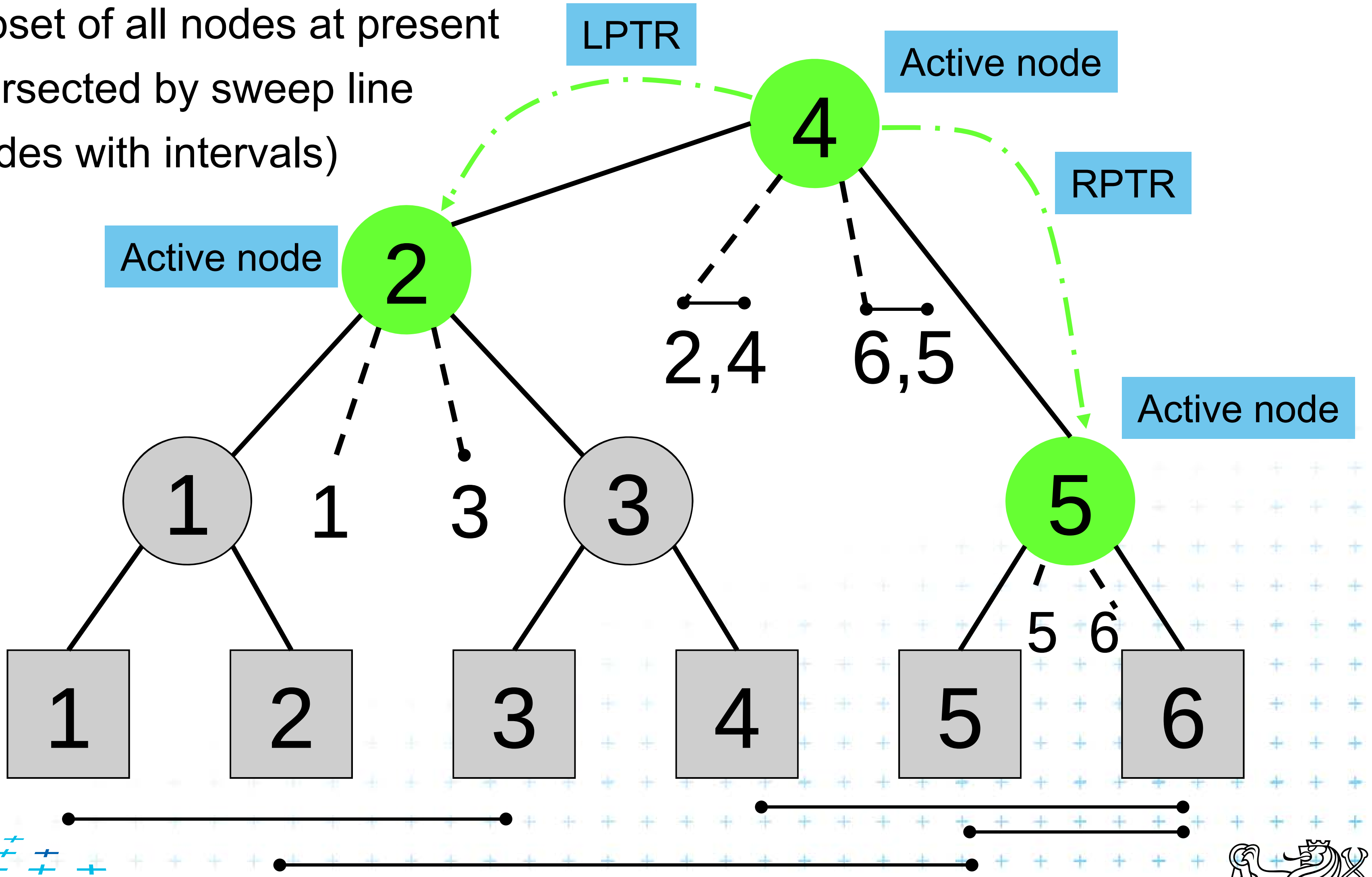
MR(v) – right endpoints (descending)



[Kukral]

Active nodes – intersected by the sweep line

Subset of all nodes at present intersected by sweep line (nodes with intervals)



[Kukral]

Query = sweep and report intersections

RectangleIntersections(*S*)

Input: Set *S* of rectangles

Output: Intersected rectangle pairs

```
Preprocess( S )          // create the interval tree T (for y-coords)
                         // and event queue Q          (for x-coords)
while ( Q û þ ) do
    Get next entry ($x_{il}$ , $y_{il}$ , $y_{ir}$ , $t$) from Q        // t @ { left | right }
    if ( t = left )   // left edge
            a) QueryInterval ( $y_{il}$ , $y_{ir}$, root(T))   // report intersections
            b) InsertInterval  ( $y_{il}$ , $y_{ir}$, root(T))  // insert new interval
    else              // right edge
            c) DeleteInterval ( $y_{il}$ , $y_{ir}$, root(T) )
```

Preprocessing

Preprocess(S)
Input: Set *S* of rectangles
Output: Primary structure of the interval tree *T* and the event queue *Q*

```
1.  T = PrimaryTree(S)      // Construct the static primary structure
                            // of the interval tree -> sweep line STATUS T

2.  // Init event queue Q with vertical rectangle edges in ascending order.
    // Put the left edges with the same x ahead of right ones.
3.  for i = 1 to n
4.      insert( ( $x_{il}$ , $y_{il}$ , $y_{ir}$ , left ), Q)      // left edges of i-th rectangle
5.      insert( ( $x_{ir}$, $y_{il}$ , $y_{ir}$ , right ), Q)      // right edges
```

Interval tree – primary structure construction

PrimaryTree(*S*)

Input: Set *S* of rectangles

Output: Primary structure of an interval tree *T*

```
$S_y$ = Sort endpoints of all segments in $S$ according to $y$-coordinate
$T$ = BST( $S_y$ )
return $T$
```

BST(S_y)

```
if( $|S_y|$ = 0 ) return null
$yMed$ = median of $S_y$
$L$ = endpoints $p_y$ ˛ $yMed$
R = endpoints $p_y$ A $yMed$
$t$ = new IntervalTreeNode( $yMed$ )
$t.left$ = BST($L$)
$t.right$ = BST($R$)
return $t$
```

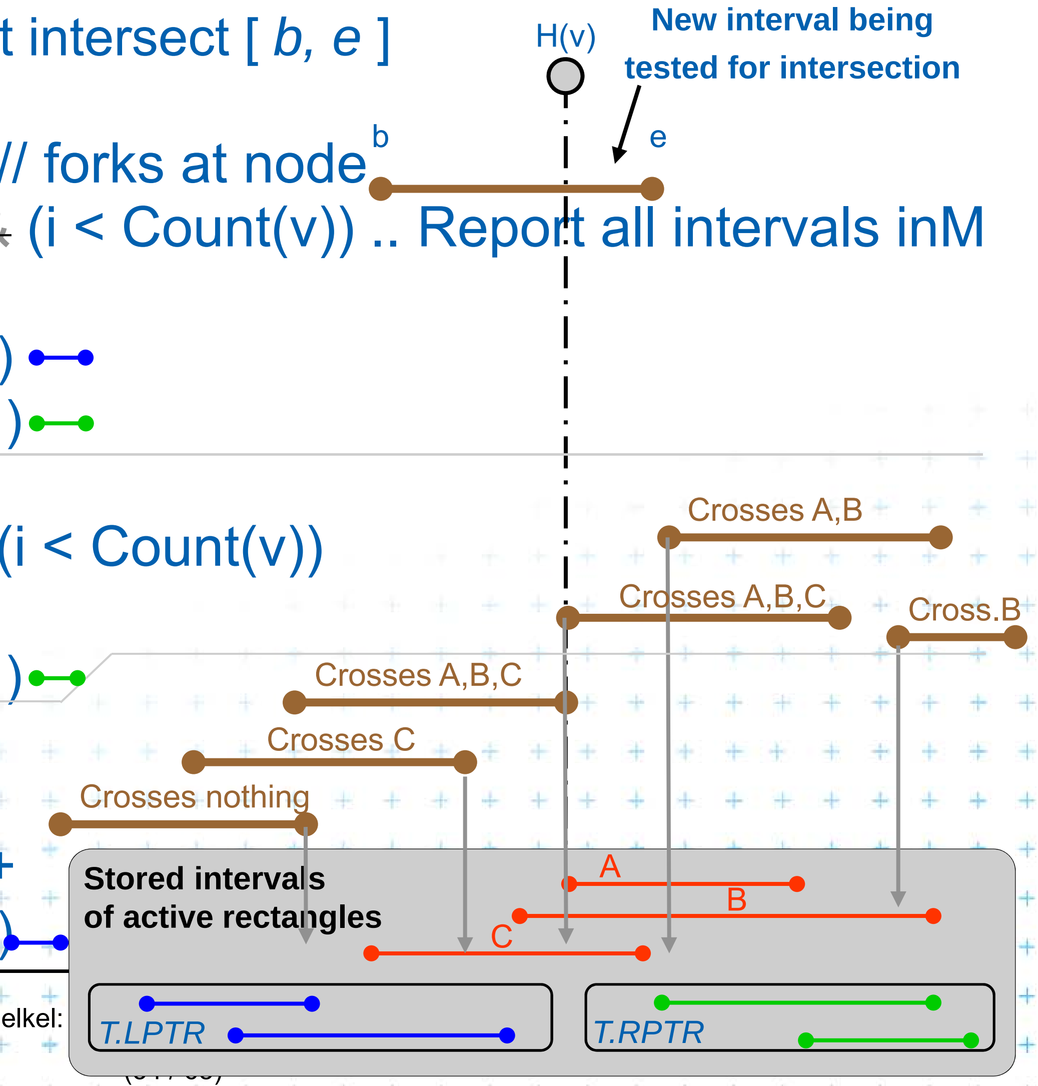
Interval tree – search the intersections

QueryInterval (*b, e, T*)

Input: Interval of the edge and current tree *T*

Output: Report the rectangles that intersect [*b, e*]

```
if( T = null ) return
i=0; if( b < H(v) < e )            // forks at node
    while ~~( MR(v).[i] >= b ) &&~~ (i < Count(v)) .. Report all intervals inM
        ReportIntersection; i++
    QueryInterval( b,e,T.LPTR ) ●—●
    QueryInterval( b,e,T.RPTR ) ●—●
else if (H(v) ¸ b < e)
    while (MR(v).[i] >= b) && (i < Count(v))
        ReportIntersection; i++
    QueryInterval( b,e,T.RPTR ) ●—●
else  // b < e ¸ H(v)
    while (ML(v).[i] <= e)
        ReportIntersection; i++
    QueryInterval( b,e,T.LPTR ) ●—●
```



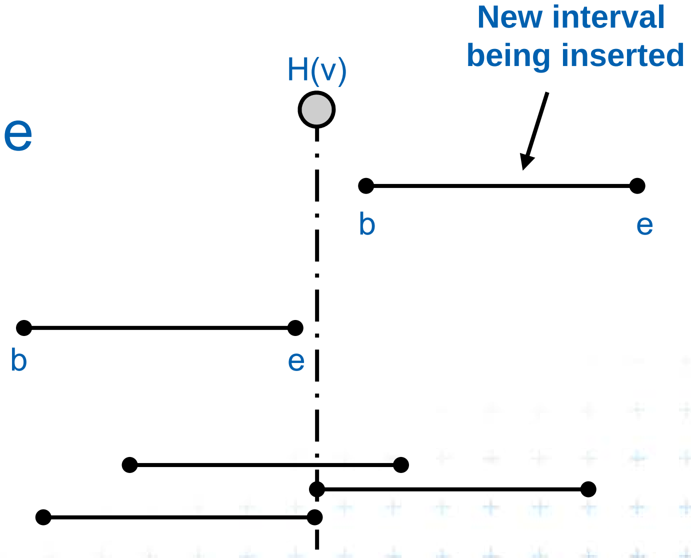
Interval tree - interval insertion

InsertInterval (*b, e, T*)

Input: Interval [*b*,*e*] and interval tree *T*

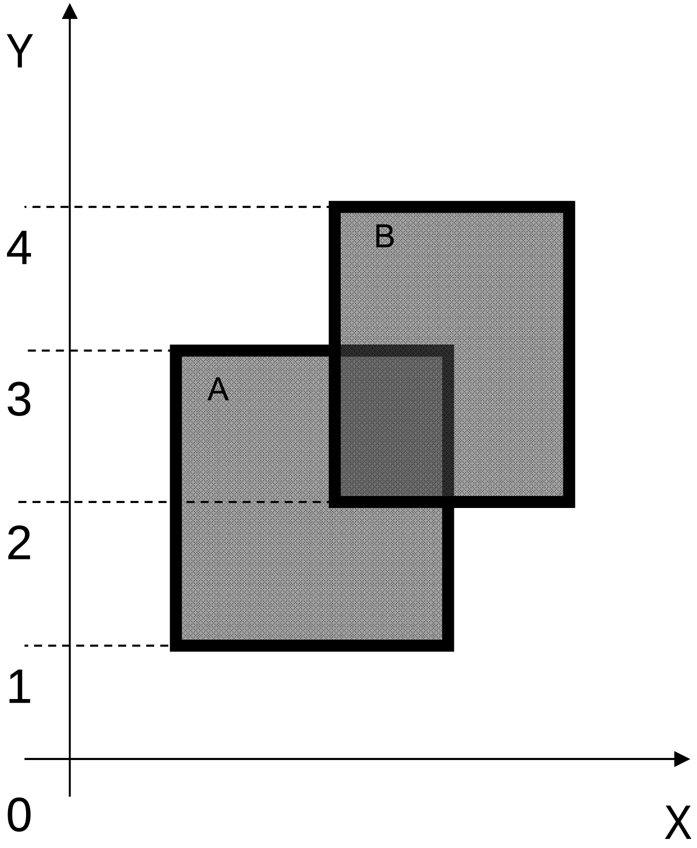
Output: *T* after insertion of the interval

```
v = root(T )
while( v != null )        // find the fork node
    if (H(v) < b < e)
        v = v.right       // continue right
    else if (b < e < H(v))
        v = v.left        // continue left
    else  // b ≤ H(v) ≤ e  // insert interval
        set v node to active
        connect LPTR resp. RPTR to its parent
        insert [b,e] into list ML(v) – sorted in ascending order of b's
        insert [b,e] into list MR(v) – sorted in descending order of e's
        break
endwhile
return T
```

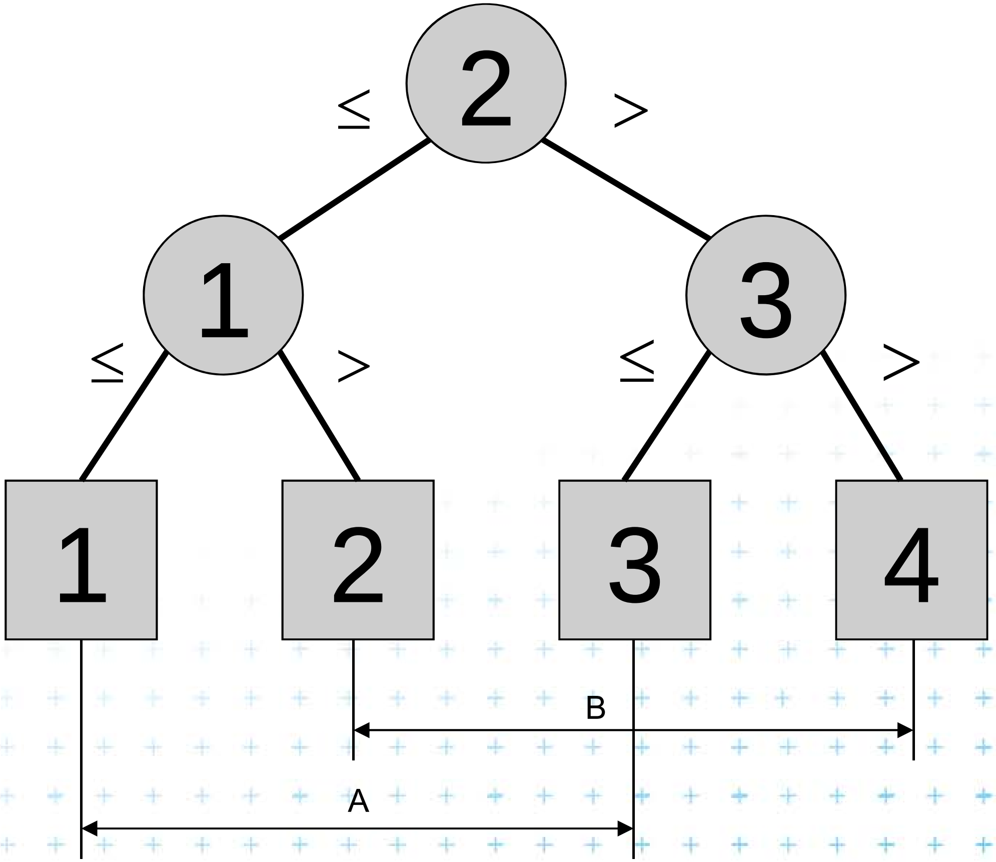


Example 1

Example 1 – static tree on endpoints



H(v) – value of node *v*



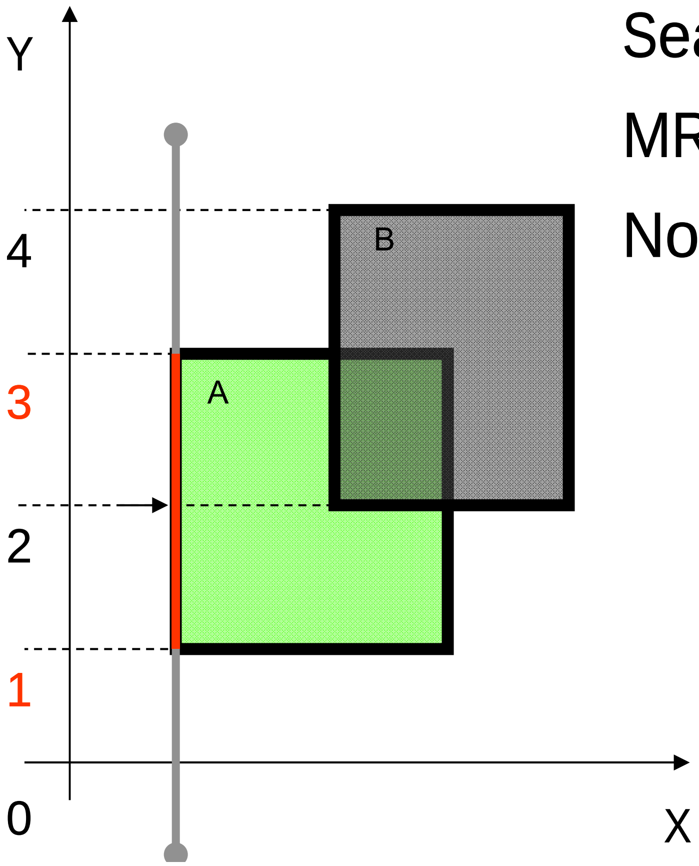
[Drtina]

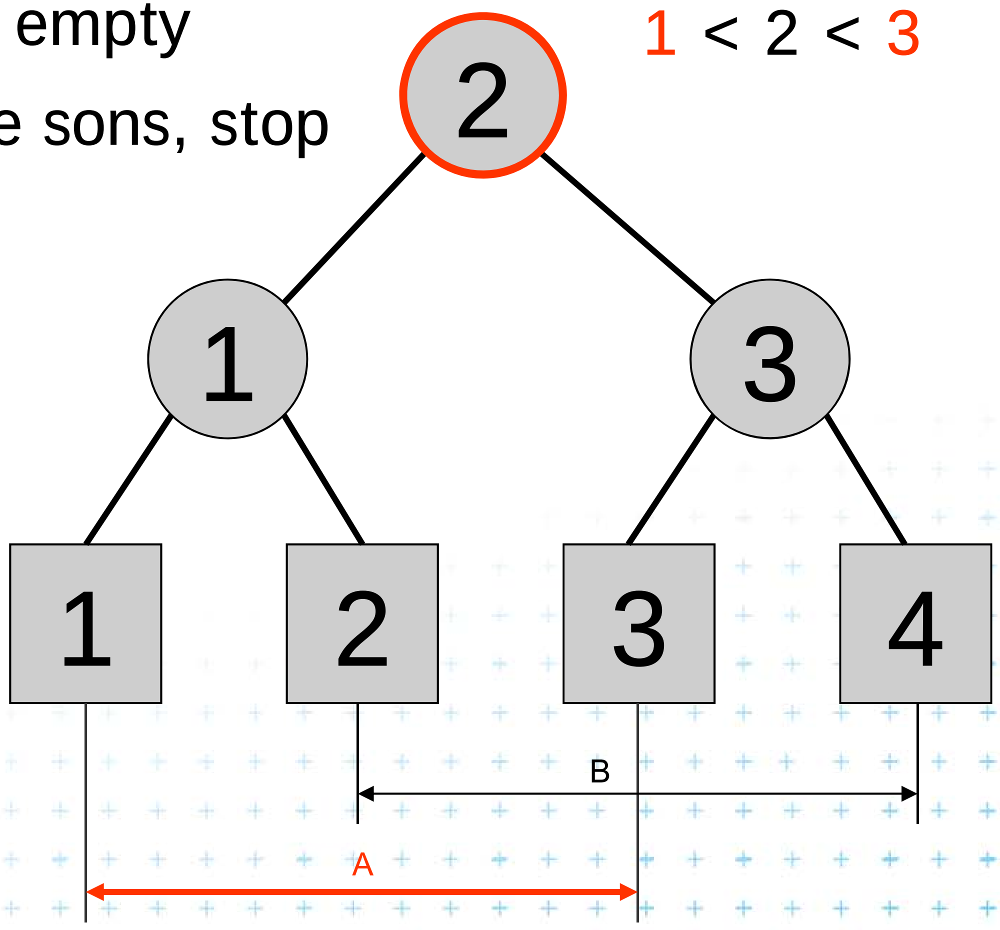
Interval insertion [1,3]   a) Query Interval

Search MR(v) or ML(v): ← b < H(v) < e

MR(v) is empty

No active sons, stop





1 < 2 < 3

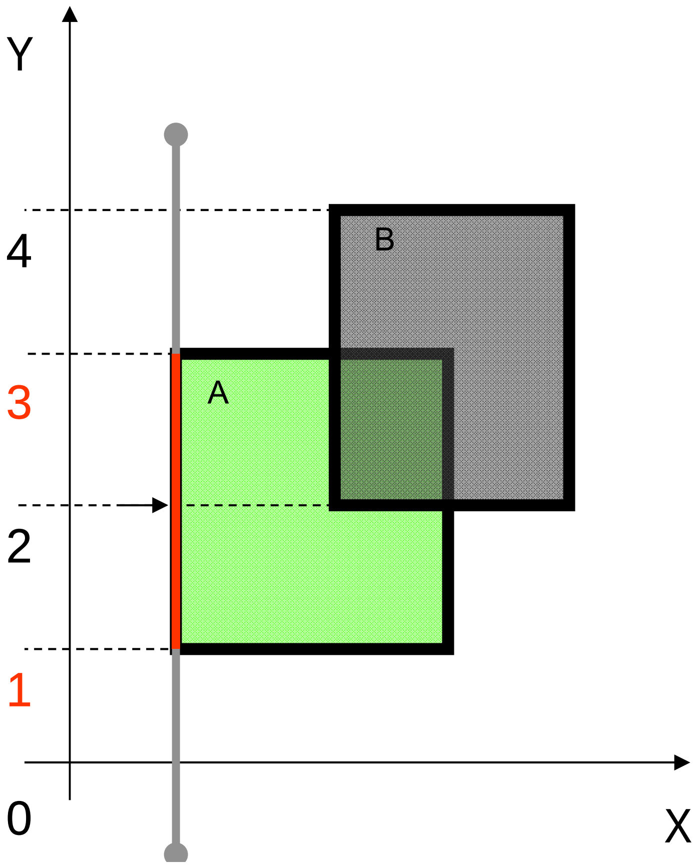
🟩 Active rectangle

○ Current node

● Active node

[Drtina]

Interval insertion [1,3]   b) Insert Interval



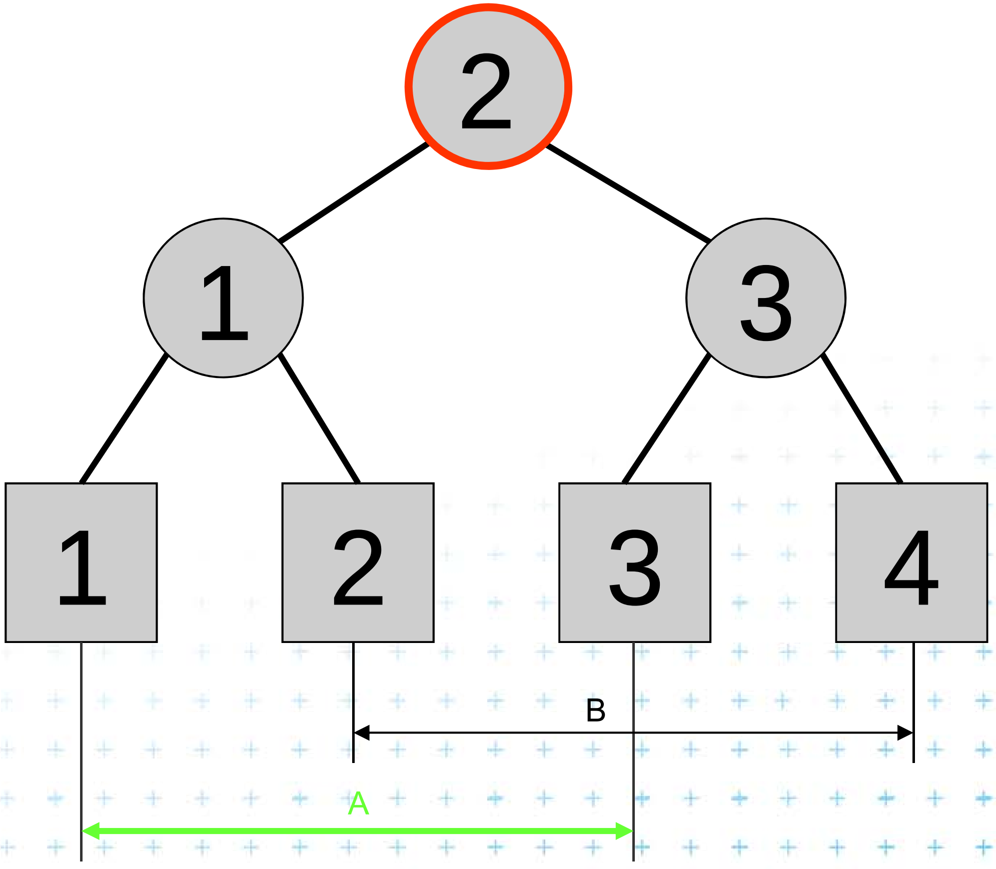
🟩 Active rectangle

○ Current node

● Active node

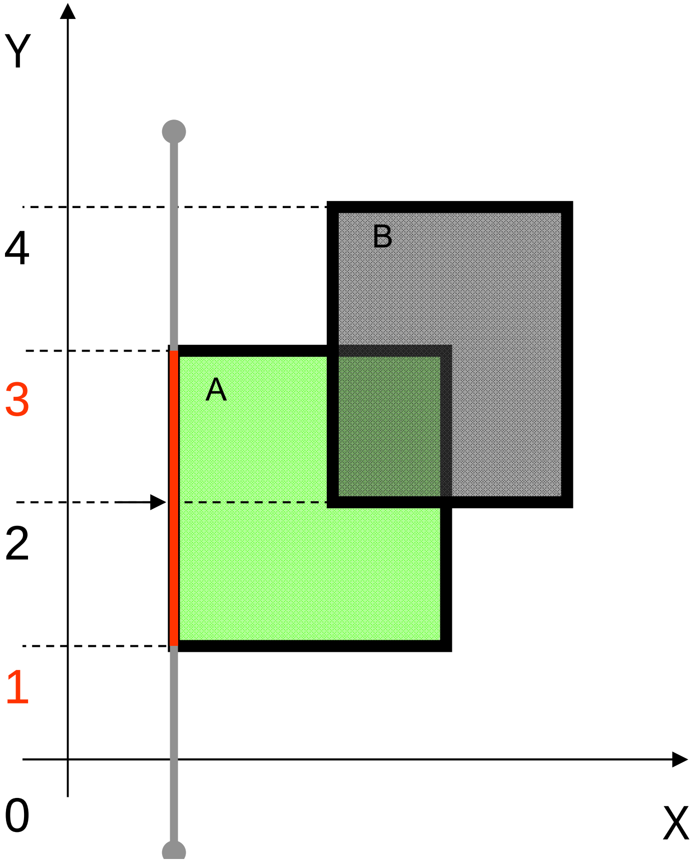
b ¸ H(v) ¸ e

? 1 ¸ 2 ¸ 3 ?



[Drtina]

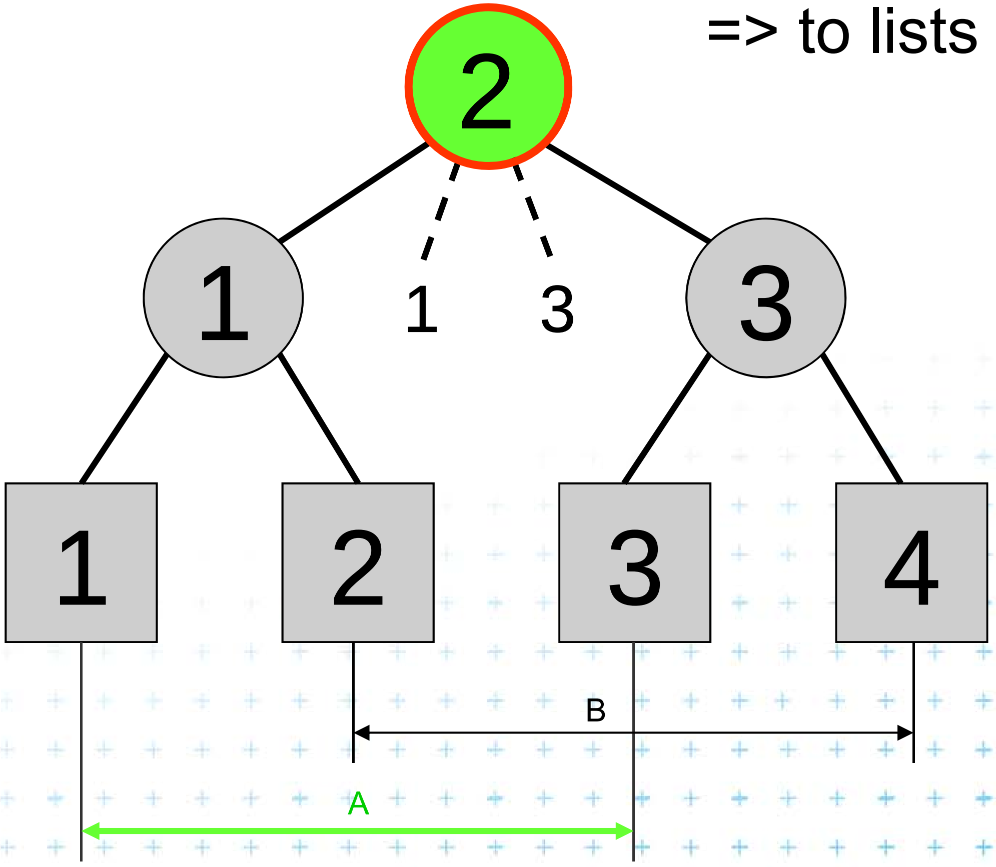
Interval insertion [1,3]   b) Insert Interval



b ¸ H(v) ¸ e

1 ¸ 2 ¸ 3

fork
=> to lists



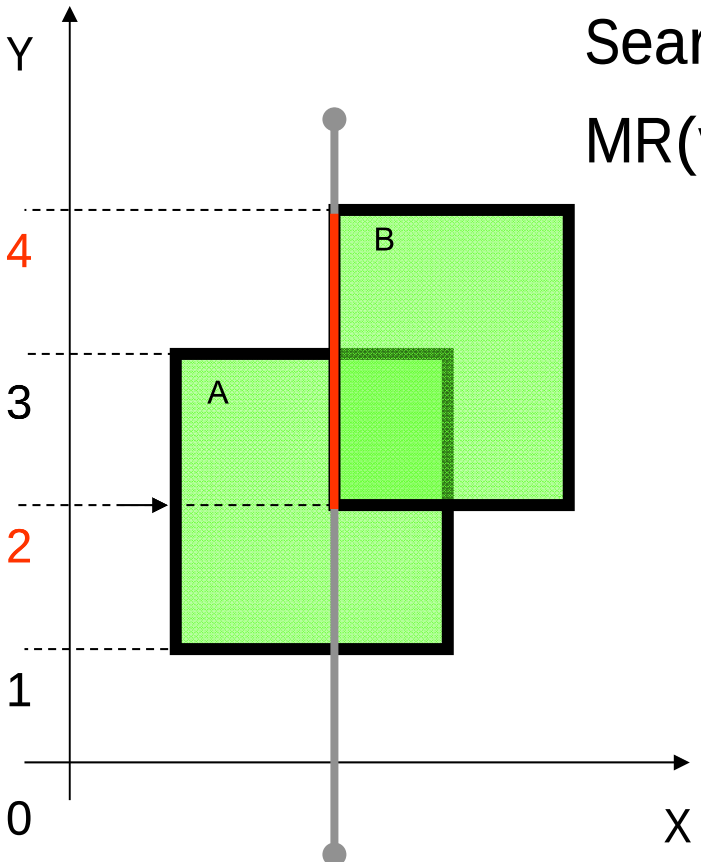
🟩 Active rectangle

○ Current node

● Active node

[Drtina]

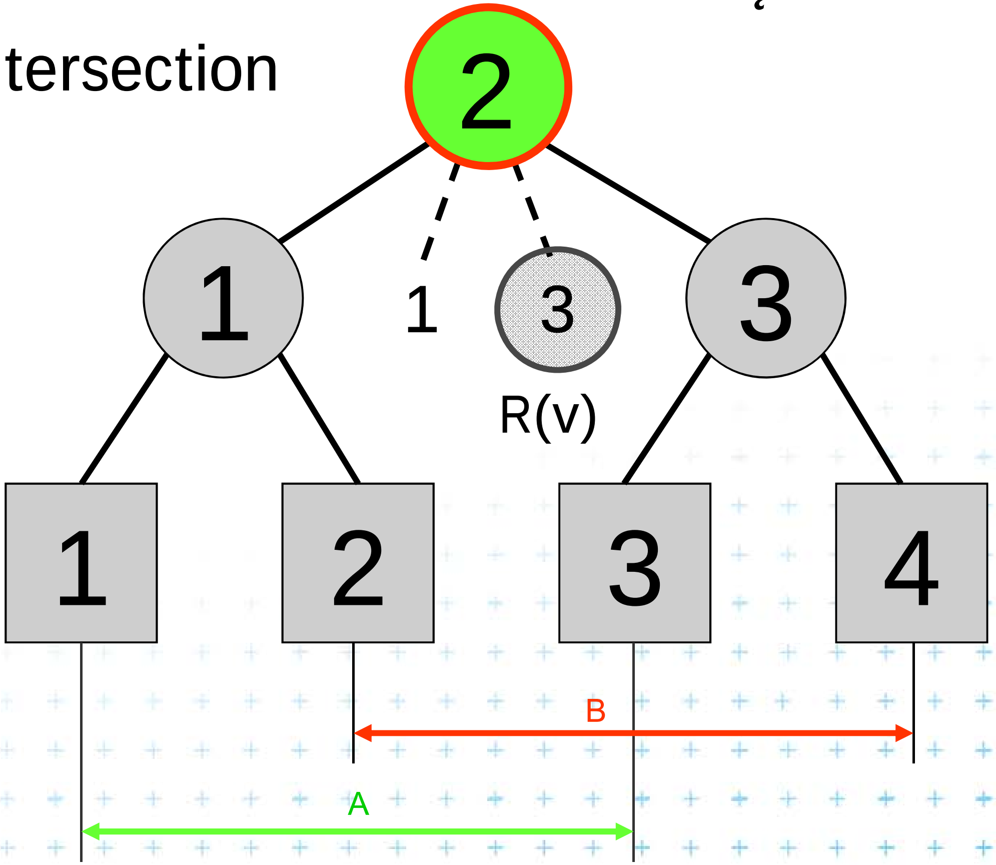
Interval insertion [2,4]   a) Query Interval



Search MR(v) only: ← H(v) ¸ b < e

MR(v)[1] = 3 – 2?   2 ¸ 2 < 4

=> intersection



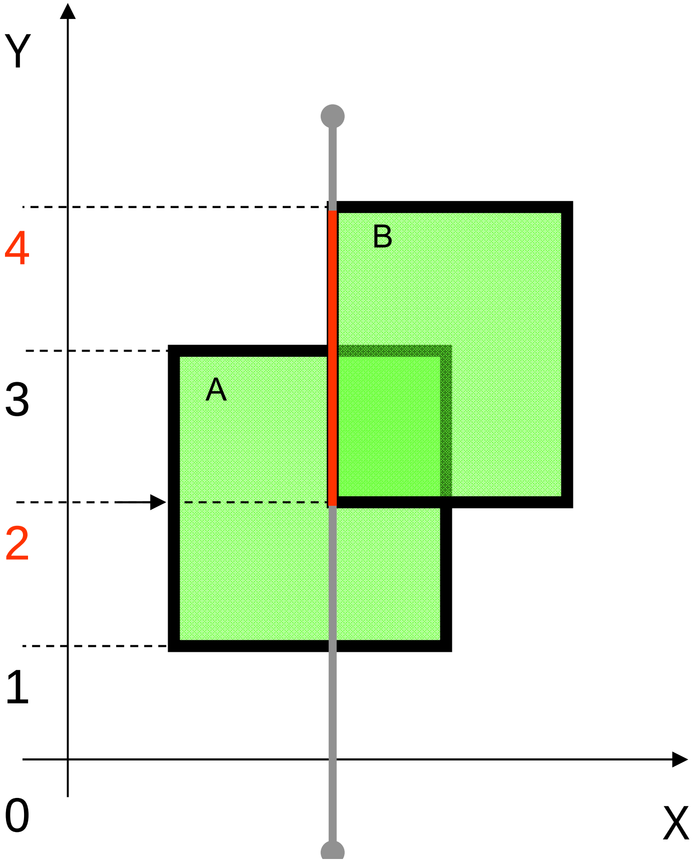
🟩 Active rectangle

○ Current node

● Active node

[Drtina]

Interval insertion [2,4] b) Insert Interval

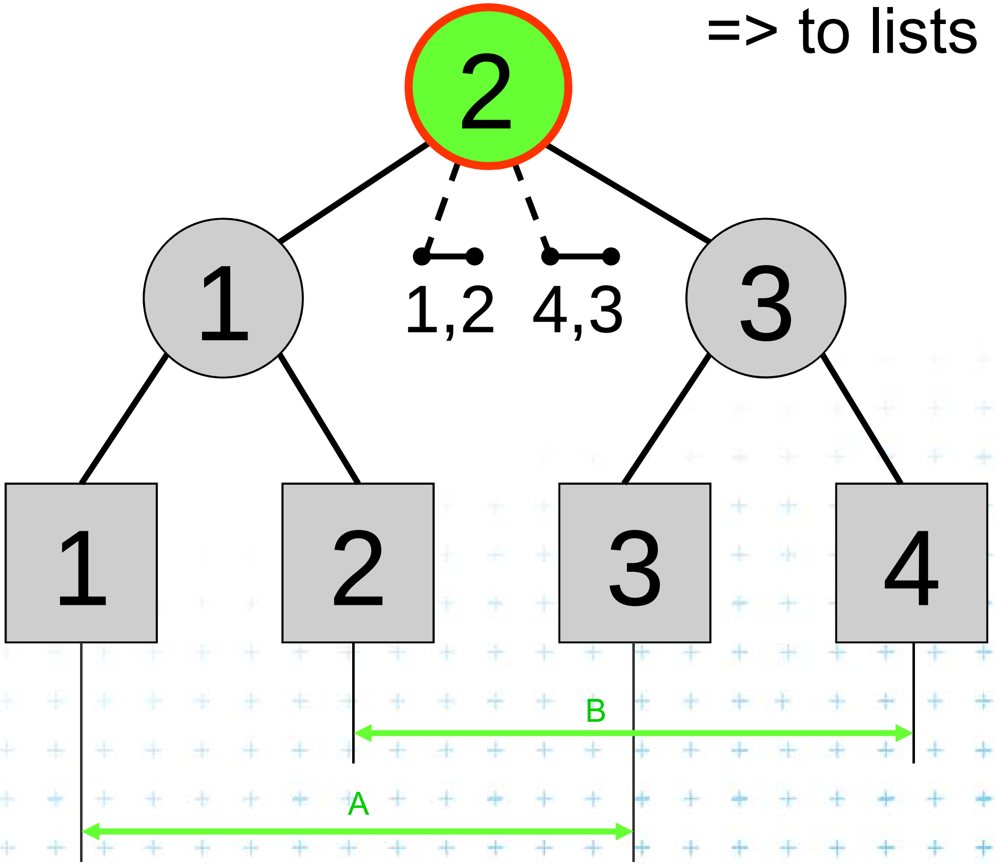


- 🟩 Active rectangle
- ○ Current node
- ● Active node

$b \le H(v) \le e$

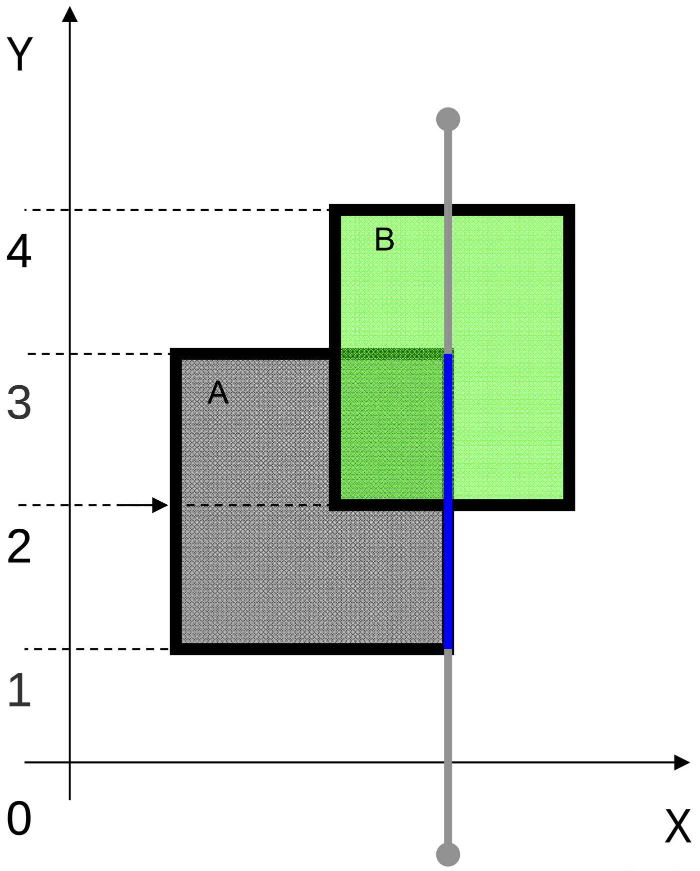
$2 \le 2 \le 4$

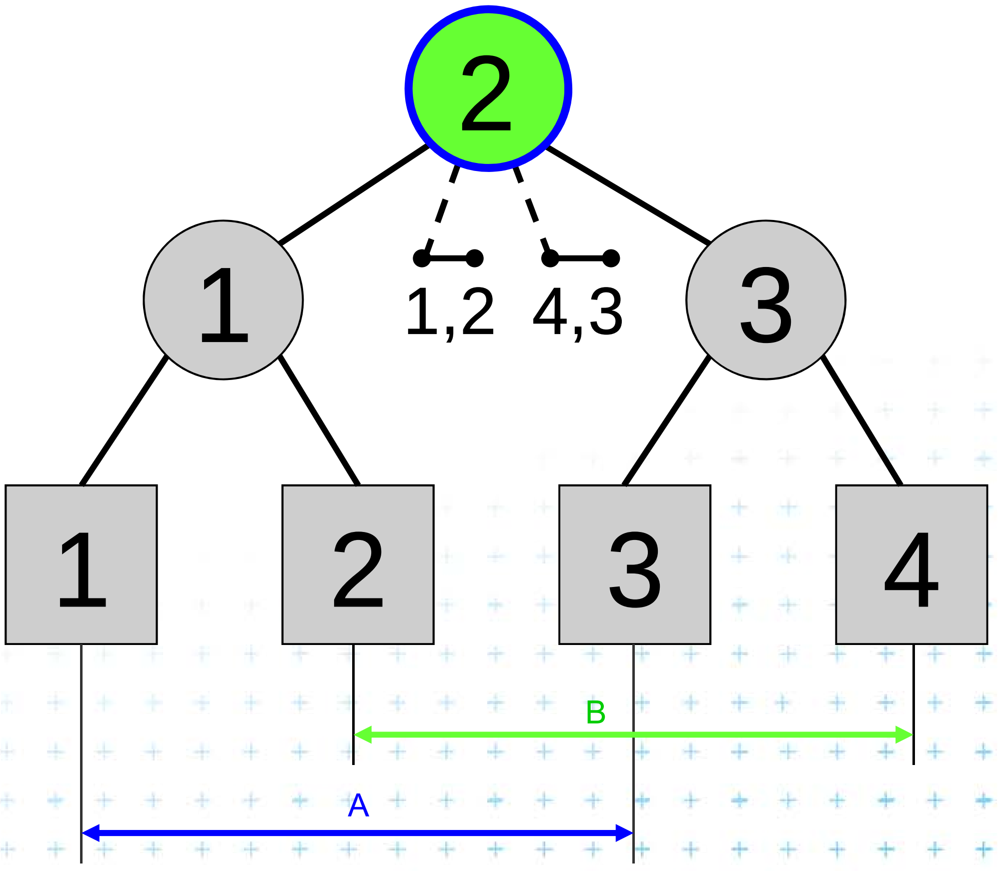
fork
=> to lists



[Drtina]

Interval delete [1,3]





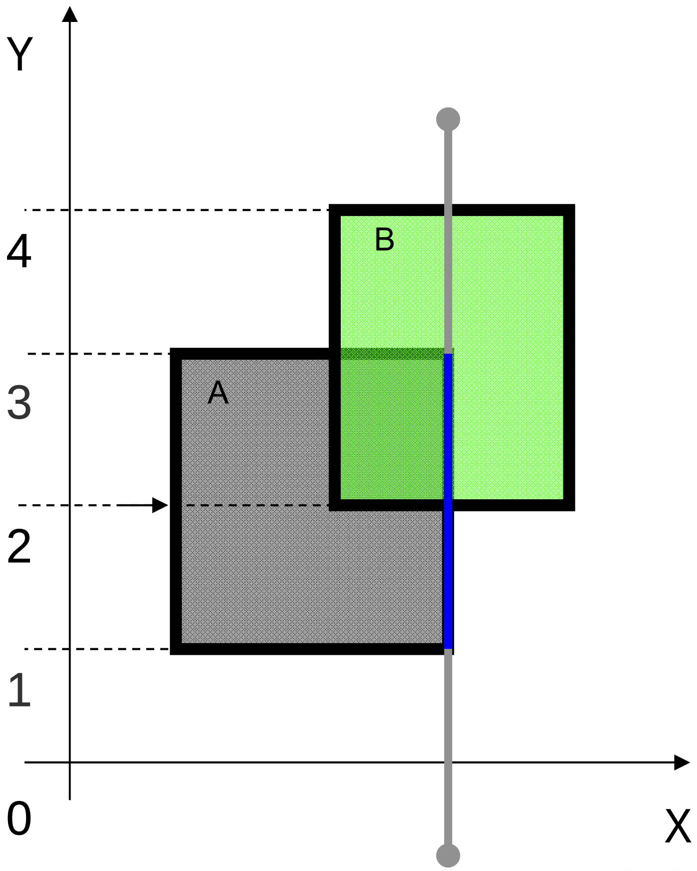
🟩 Active rectangle

○ Current node

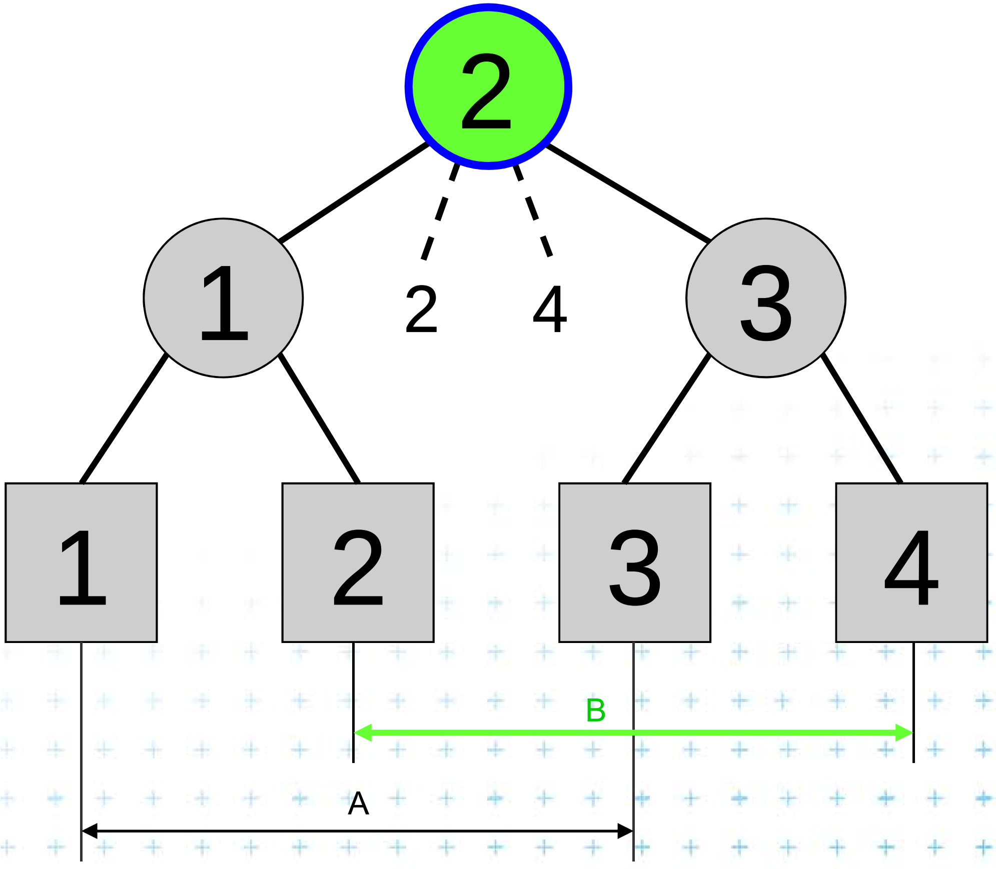
● Active node

[Drtina]

Interval delete [1,3]

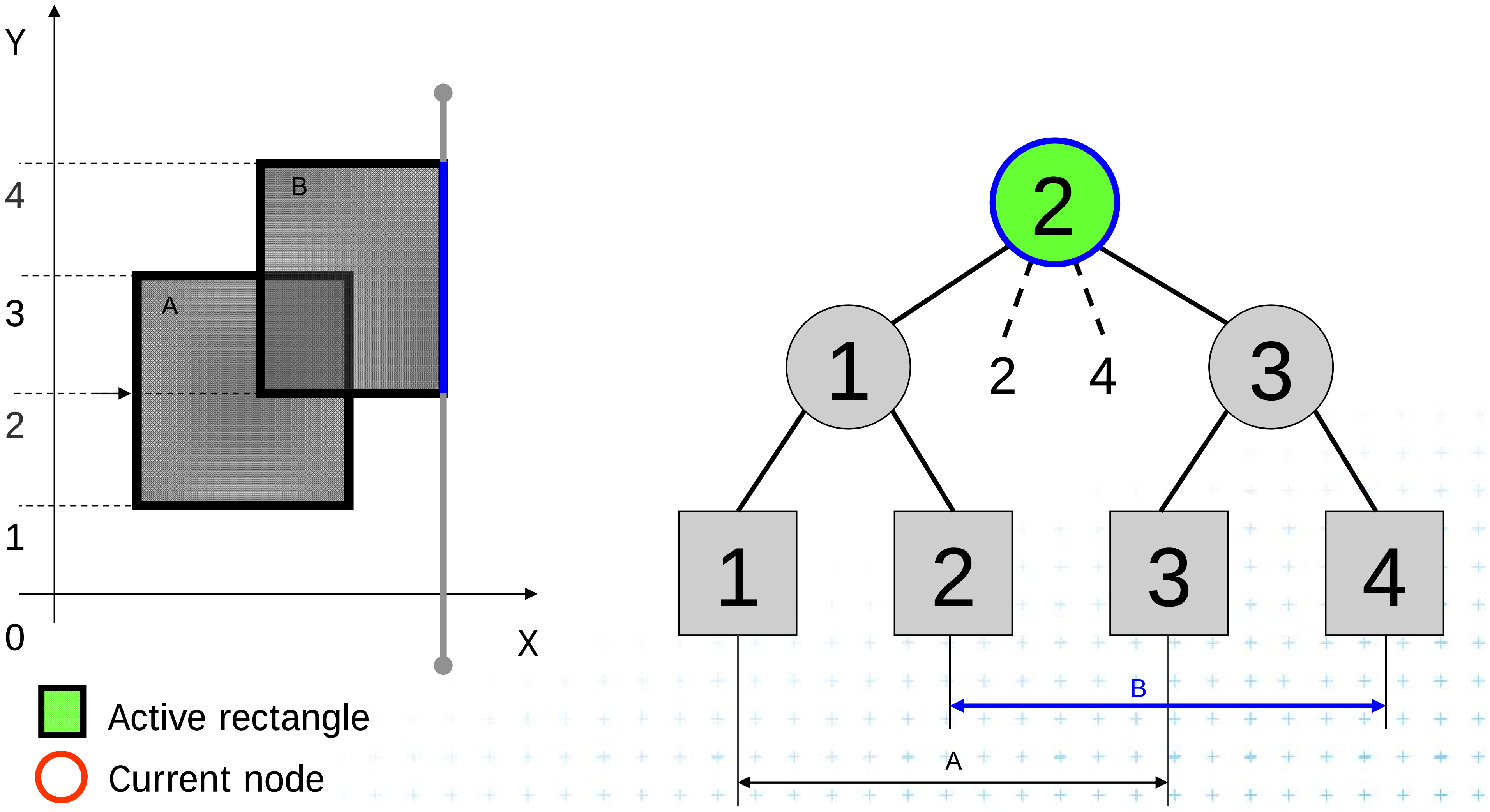


- 🟩 Active rectangle
- ○ Current node
- ● Active node



[Drtina]

Interval delete [2,4]



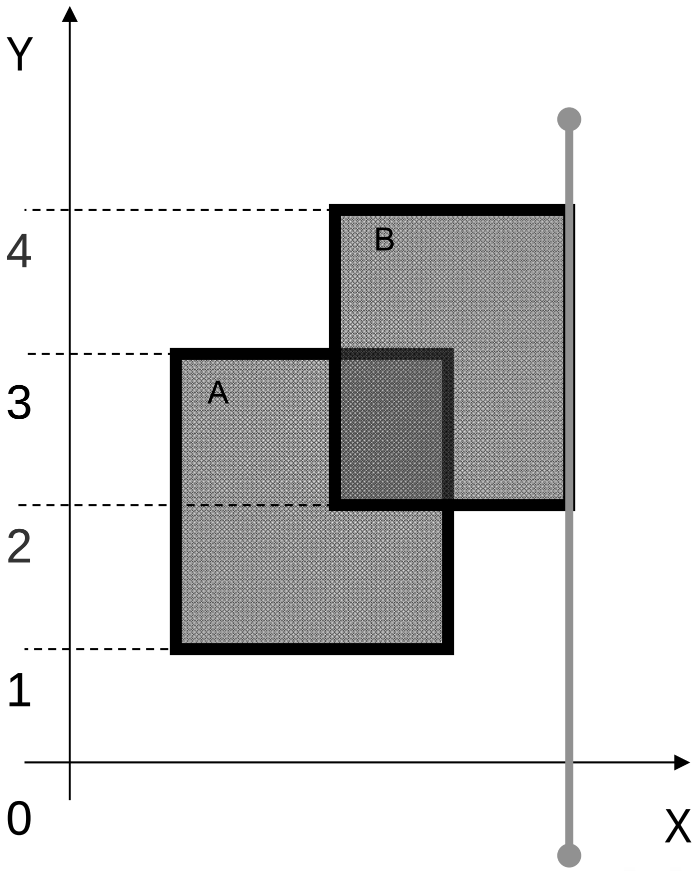
🟩 Active rectangle

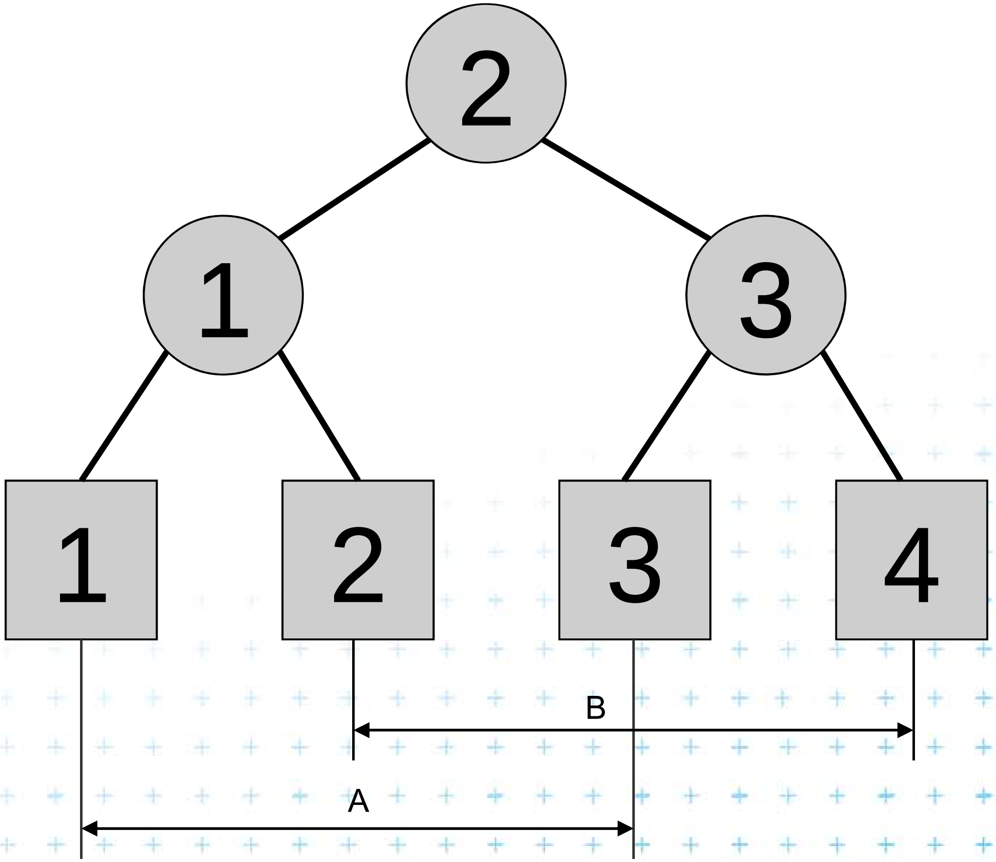
○ Current node

● Active node

[Drtina]

Interval delete [2,4]





[Drtina]

Example 2

RectangleIntersections(*S*) **// this is copy of the slide before**

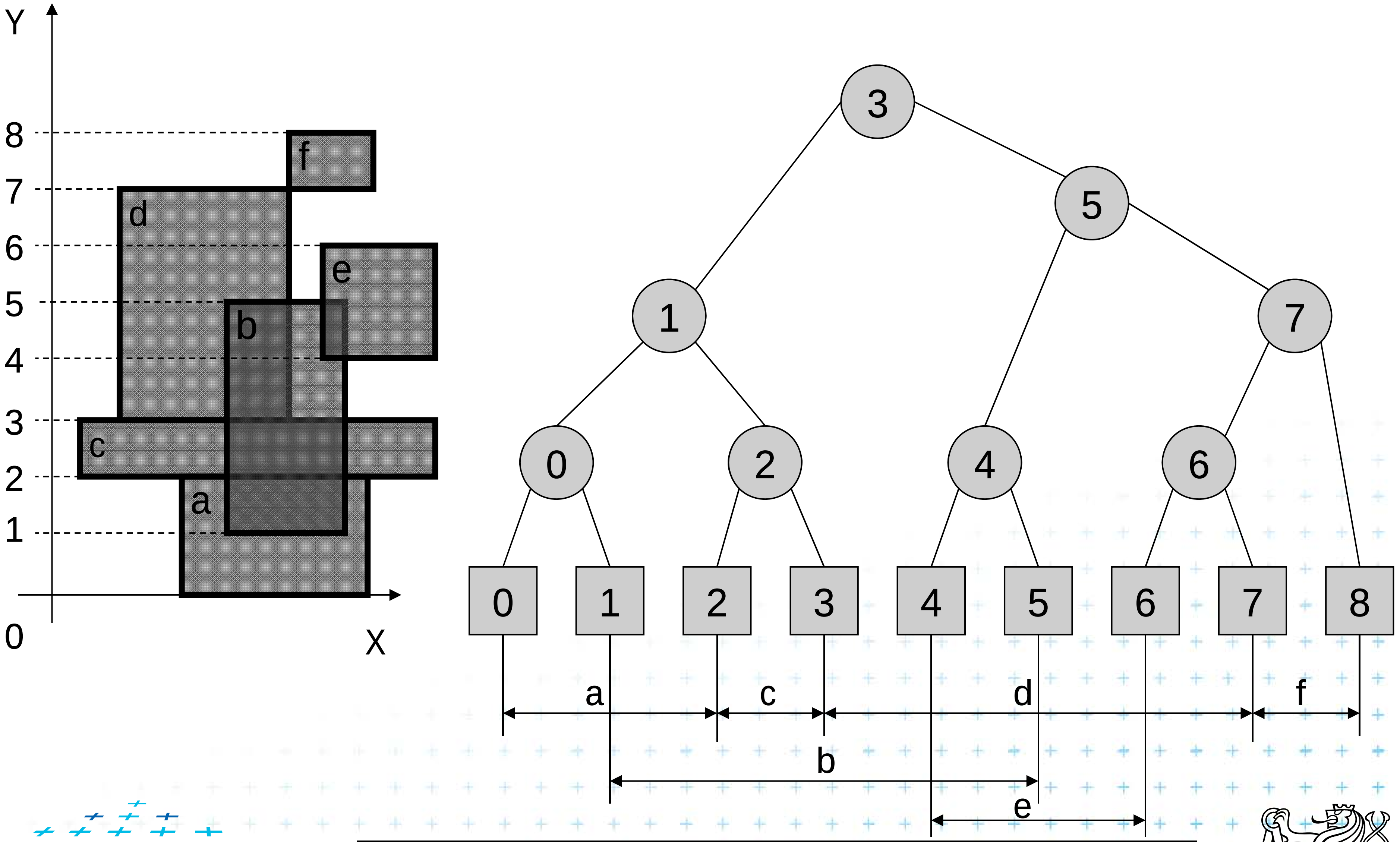
Input: Set *S* of rectangles **// just to remember the algorithm**

Output: Intersected rectangle pairs

```
Preprocess( S )            // create the interval tree $T$ and event queue $Q$

while ( $Q$ û þ ) do
    Get next entry ($x_{il}, y_{il}, y_{ir}, t$) from $Q$      // $t$ @ { left | right }
    if ( $t$ = left )   // left edge
            a) QueryInterval ( $y_{il}, y_{ir}$, root($T$))          // report intersections
            b) InsertInterval ( $y_{il}, y_{ir}$, root($T$))   // insert new interval
    else                // right edge
            c) DeleteInterval ( $y_{il}, y_{ir}$, root($T$) )
```

Example 2



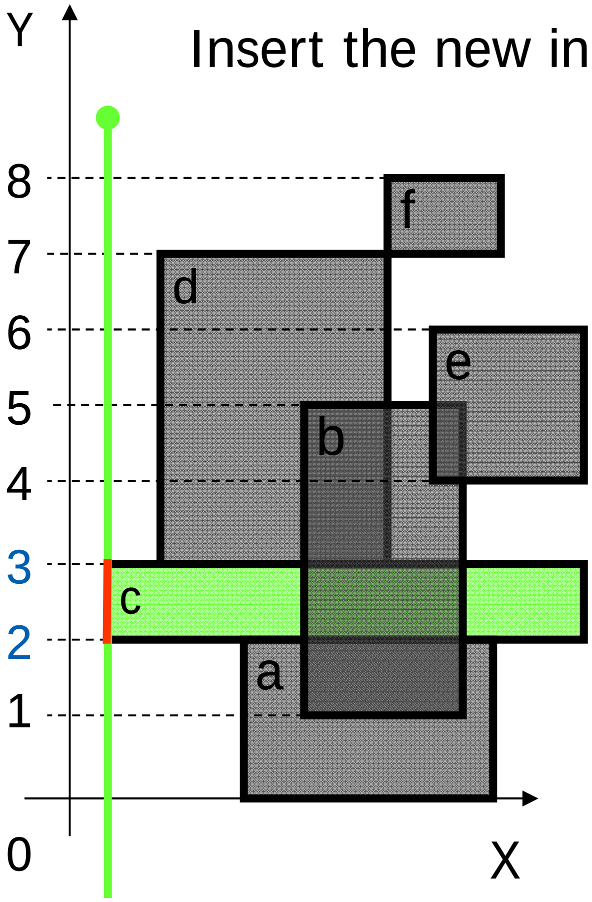
[Drtina]

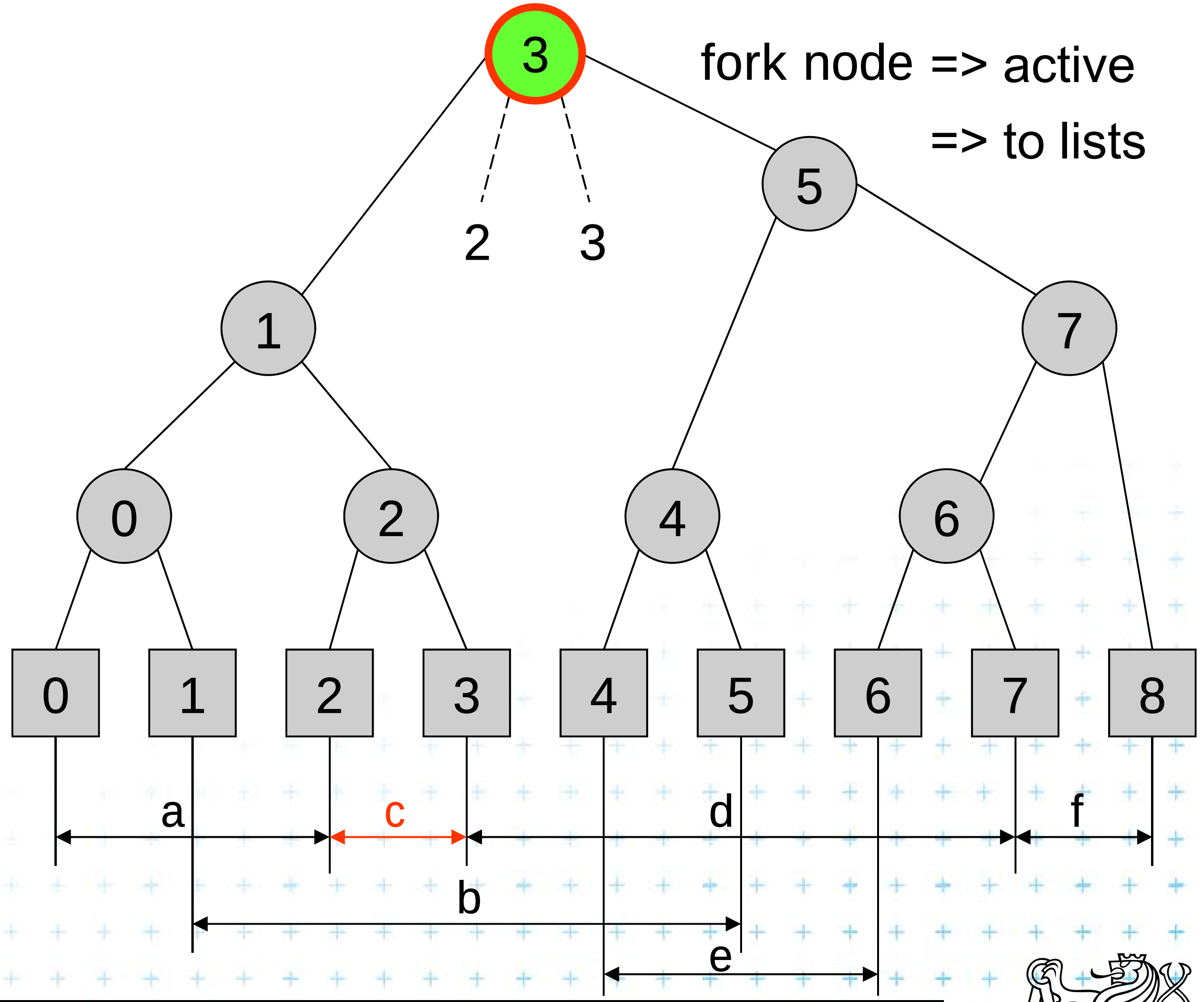
Insert [2,3] – empty => b) Insert Interval

b ¸ H(v) ¸ e

Insert the new interval to secondary lists

? 2 ¸ 3 ¸ 3 ?





fork node => active

=> to lists

🟩 Active rectangle

○ Current node

● Active node

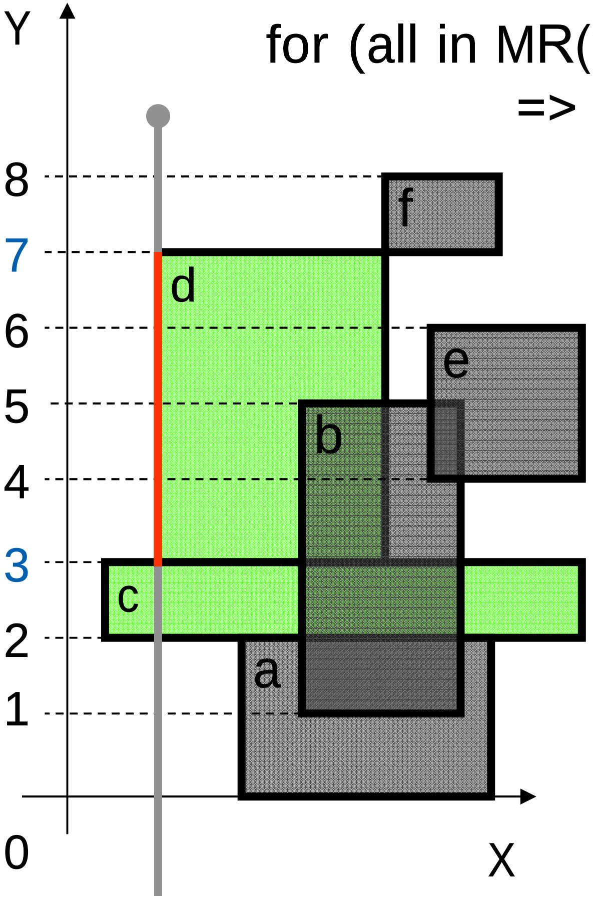
[Drtina]

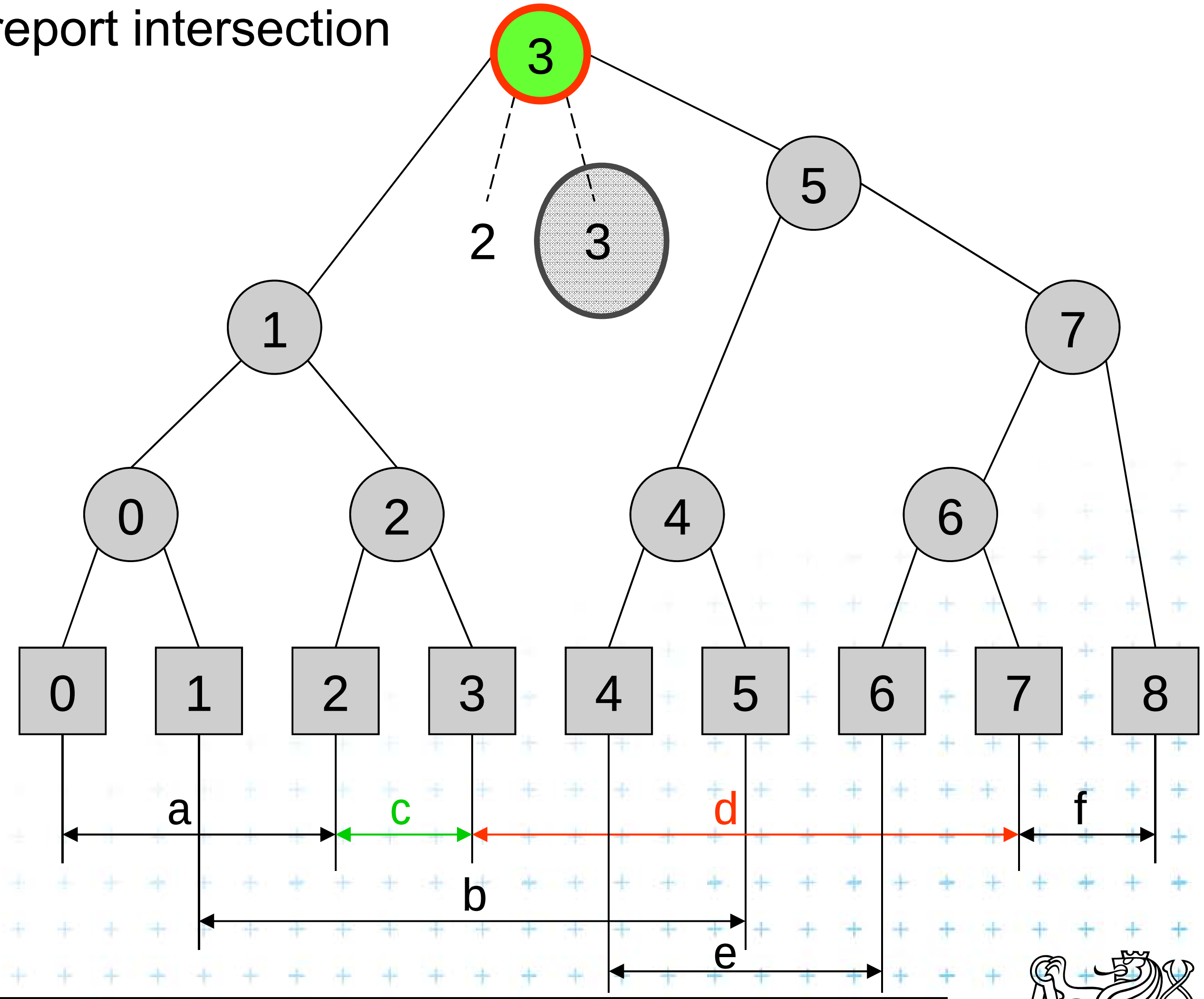
Insert [3,7] a) Query Interval

H(v) ¸ b ? e

for (all in MR(v)) test MR(v)[i] >= 3
=> report intersection

? 3 ¸ 3 ? 7 ?





🟩 Active rectangle

○ Current node

● Active node

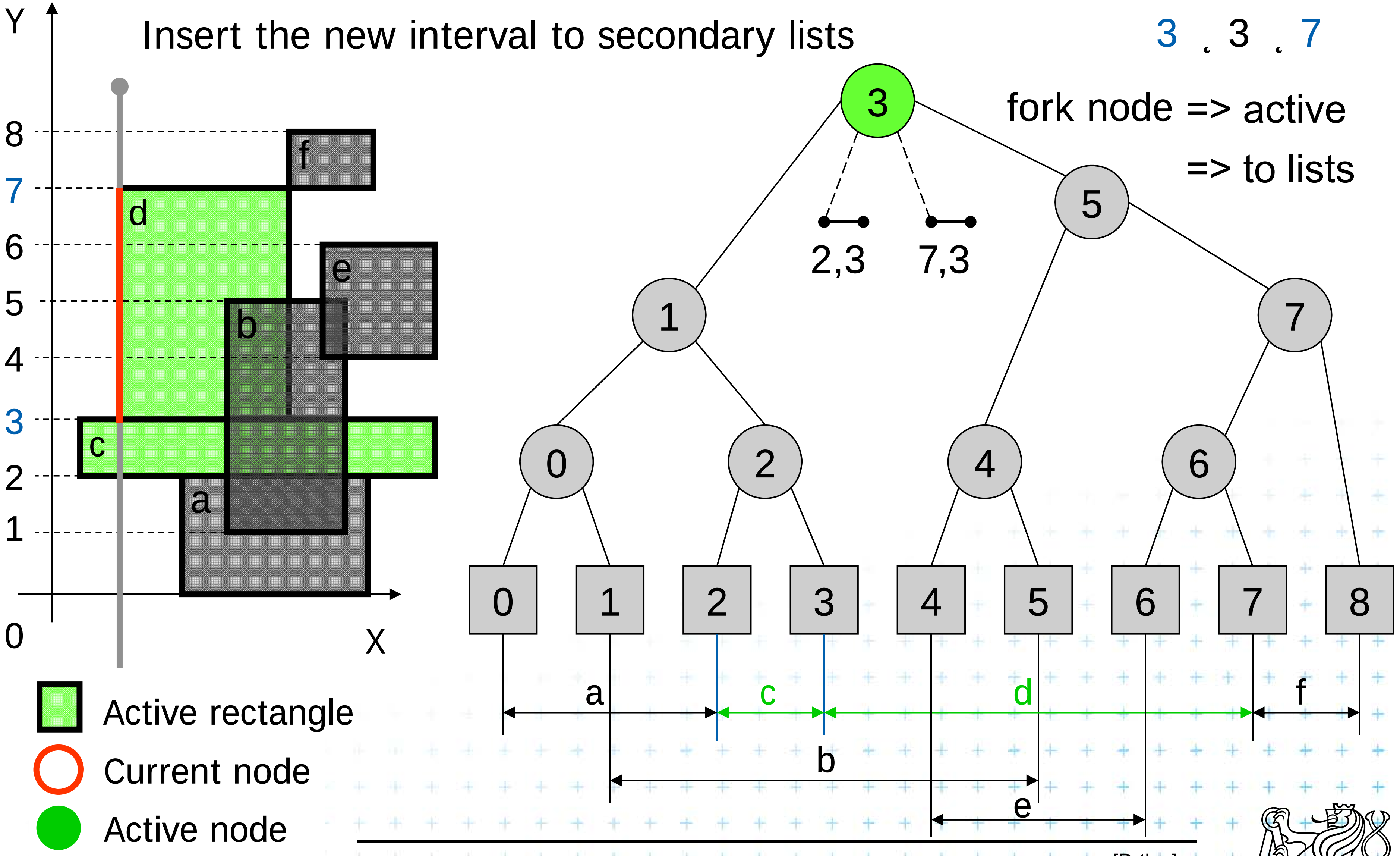
[Drtina]

Insert [3,7] b) Insert Interval

b ¸ H(v) ¸ e

Insert the new interval to secondary lists

3 ¸ 3 ¸ 7



fork node => active

=> to lists

🟩 Active rectangle

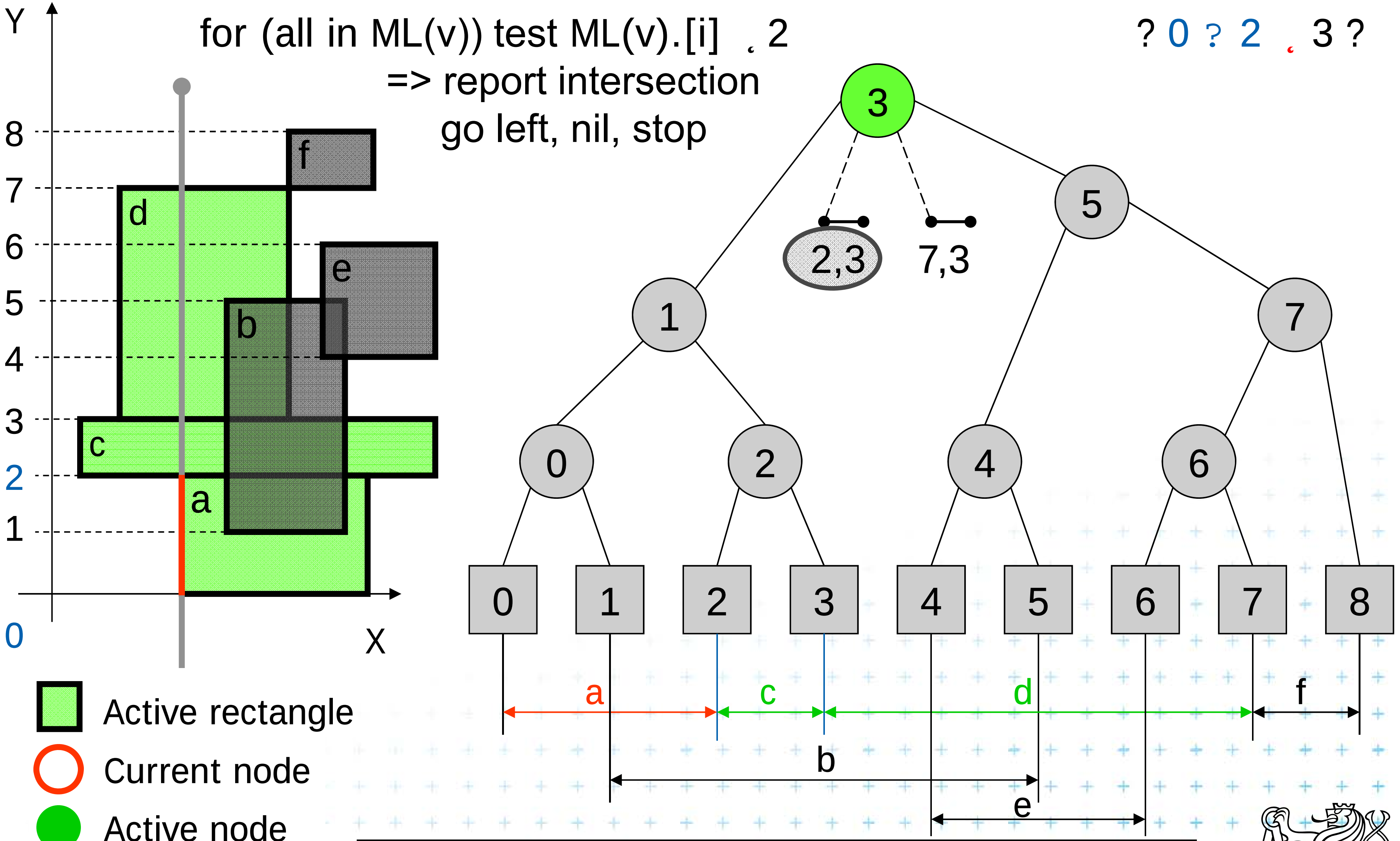
○ Current node

● Active node

[Drtina]

Insert [0,2] a) Query Interval

b ? e ¸ H(v)



for (all in ML(v)) test ML(v).[i] ¸ 2
=> report intersection
go left, nil, stop

? 0 ? 2 ¸ 3 ?

🟩 Active rectangle

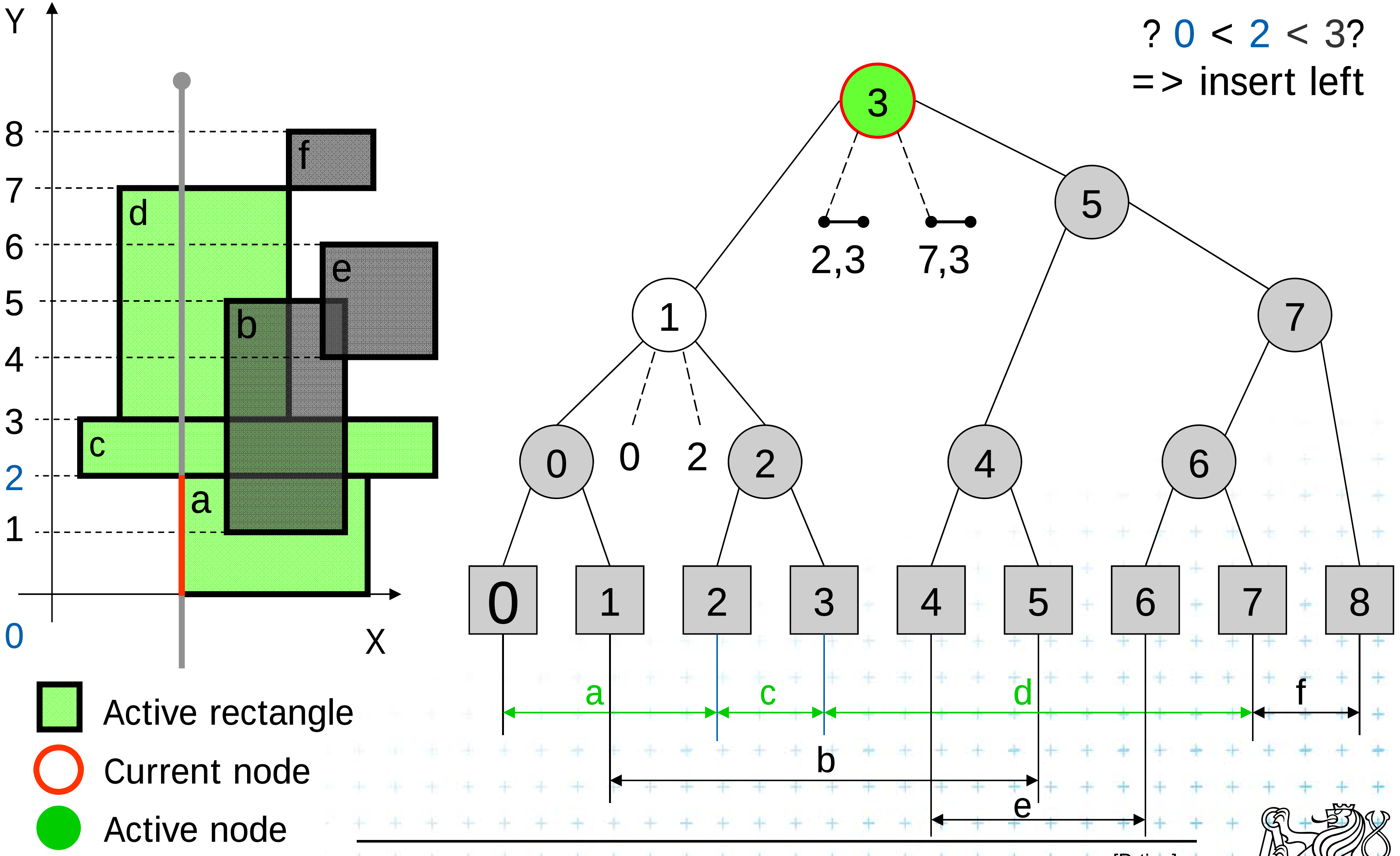
○ Current node

● Active node

[Drtina]

Insert [0,2] b) Insert Interval 1/2

b ? e <H(v)



? 0 < 2 < 3?
=> insert left

- 🟩 Active rectangle
- ○ Current node
- ● Active node

[Drtina]

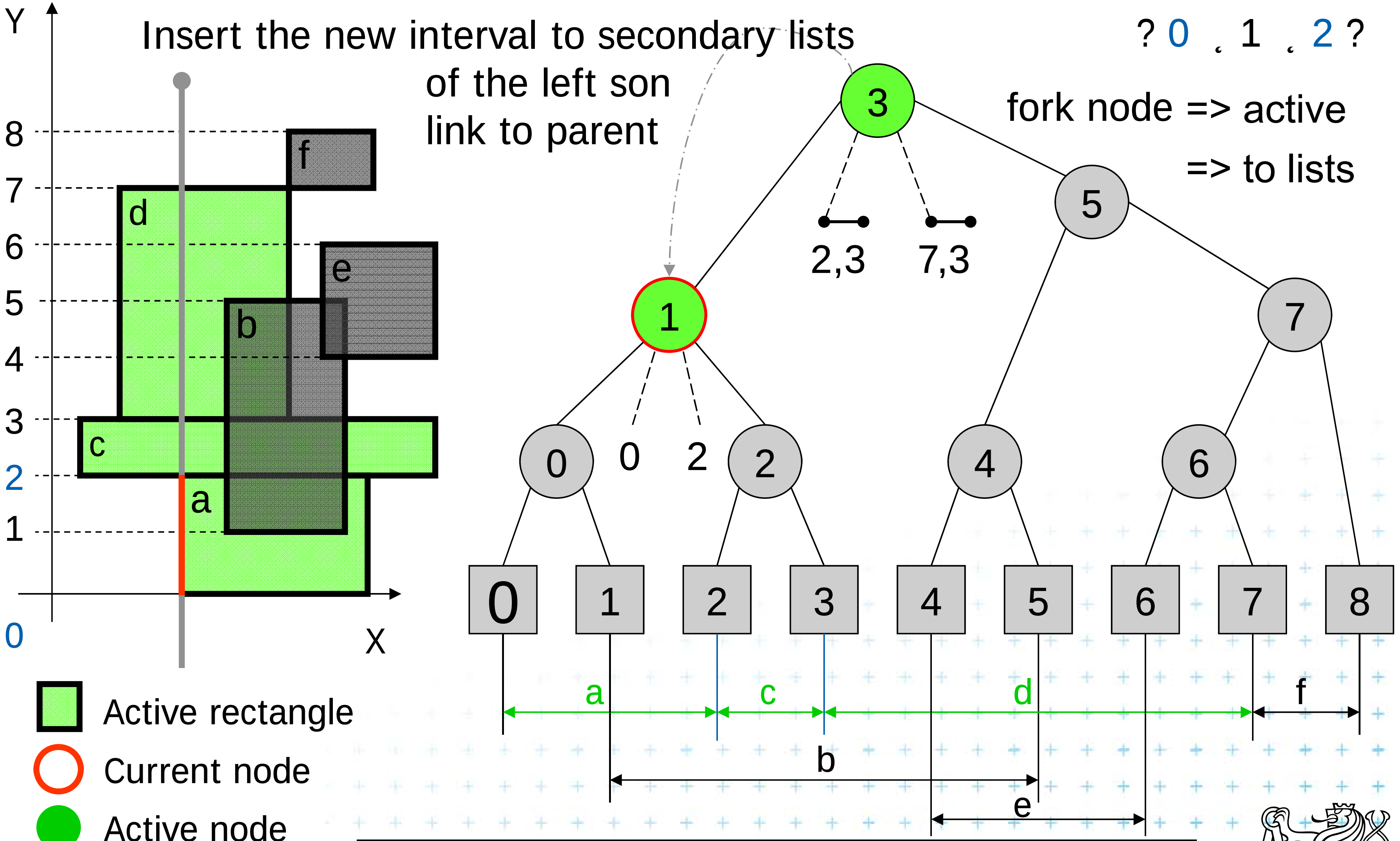
Insert [0,2] b) Insert Interval 2/2

b ˛ H(v) ˛ e

? 0 ˛ 1 ˛ 2 ?

Insert the new interval to secondary lists of the left son link to parent

fork node => active => to lists



🟩 Active rectangle

○ Current node

● Active node

[Drtina]

Insert [1,5] a) Query Interval 1/2

b ? H(v) < e

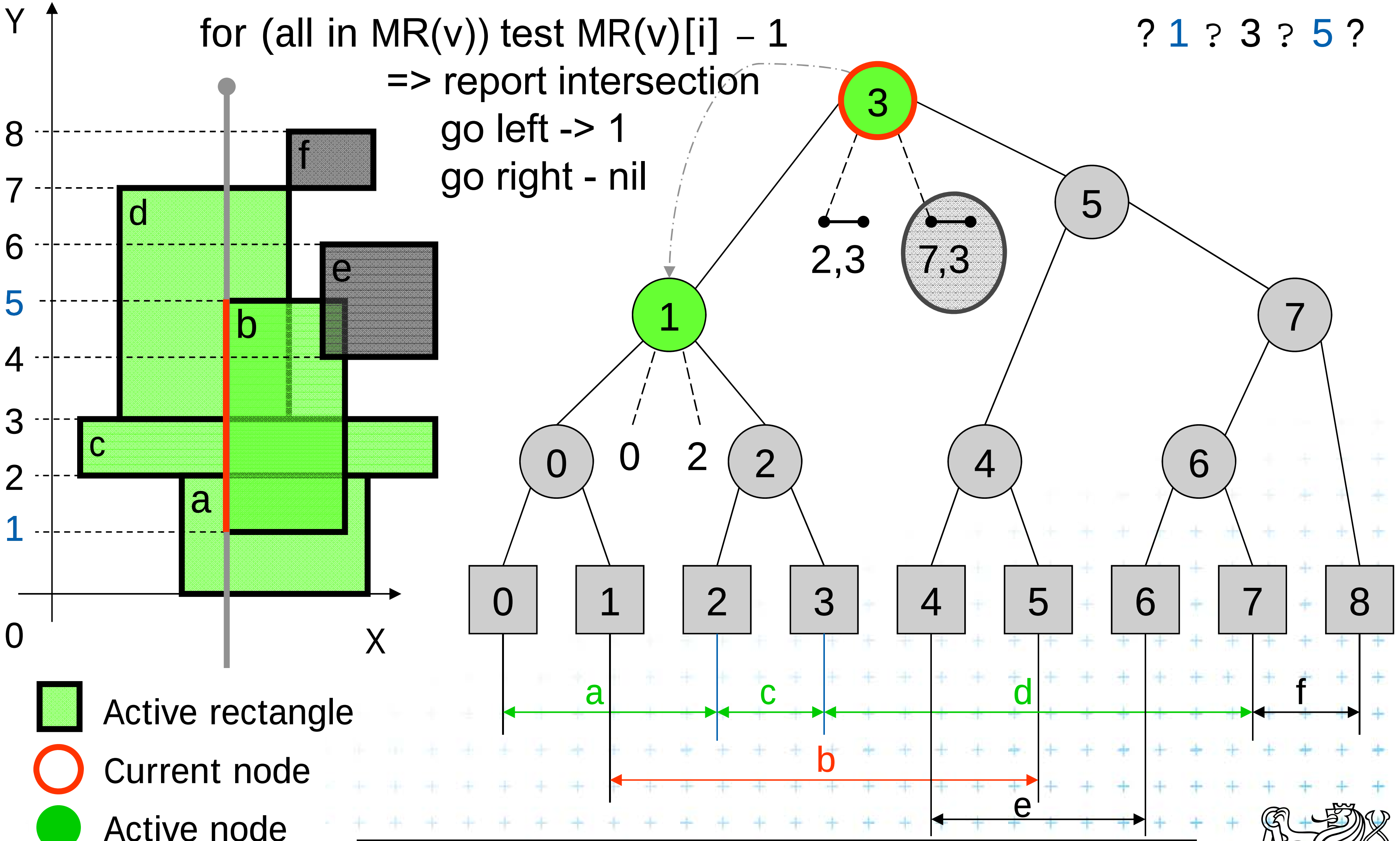
? 1 ? 3 ? 5 ?

for (all in MR(v)) test MR(v)[i] – 1

=> report intersection

go left -> 1

go right - nil



🟩 Active rectangle

○ Current node

● Active node

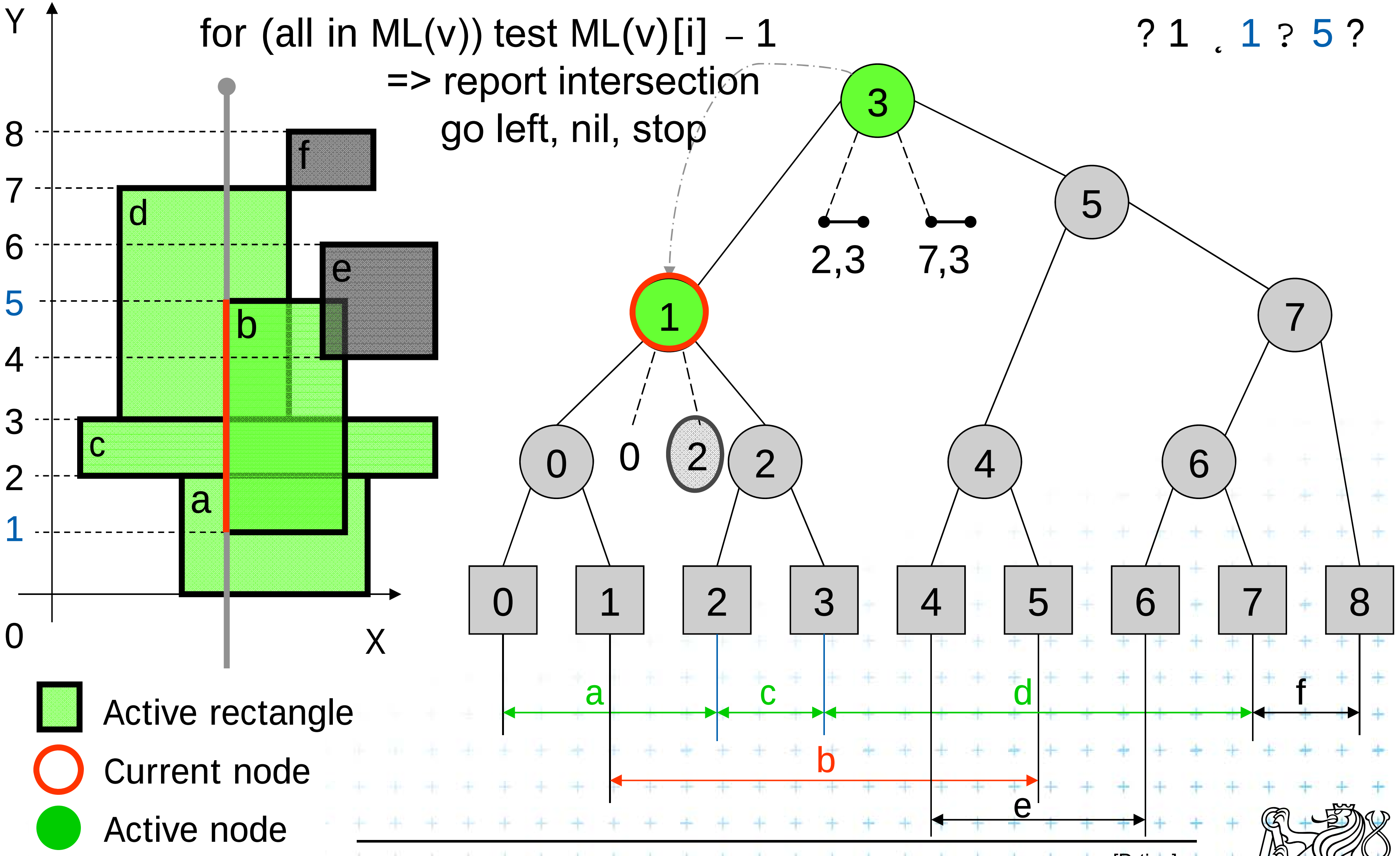
[Drtina]

Insert [1,5] a) Query Interval 2/2

H(v) ¸ b ? e

? 1 ¸ 1 ? 5 ?

for (all in ML(v)) test ML(v)[i] – 1
=> report intersection
go left, nil, stop



🟩 Active rectangle

○ Current node

● Active node

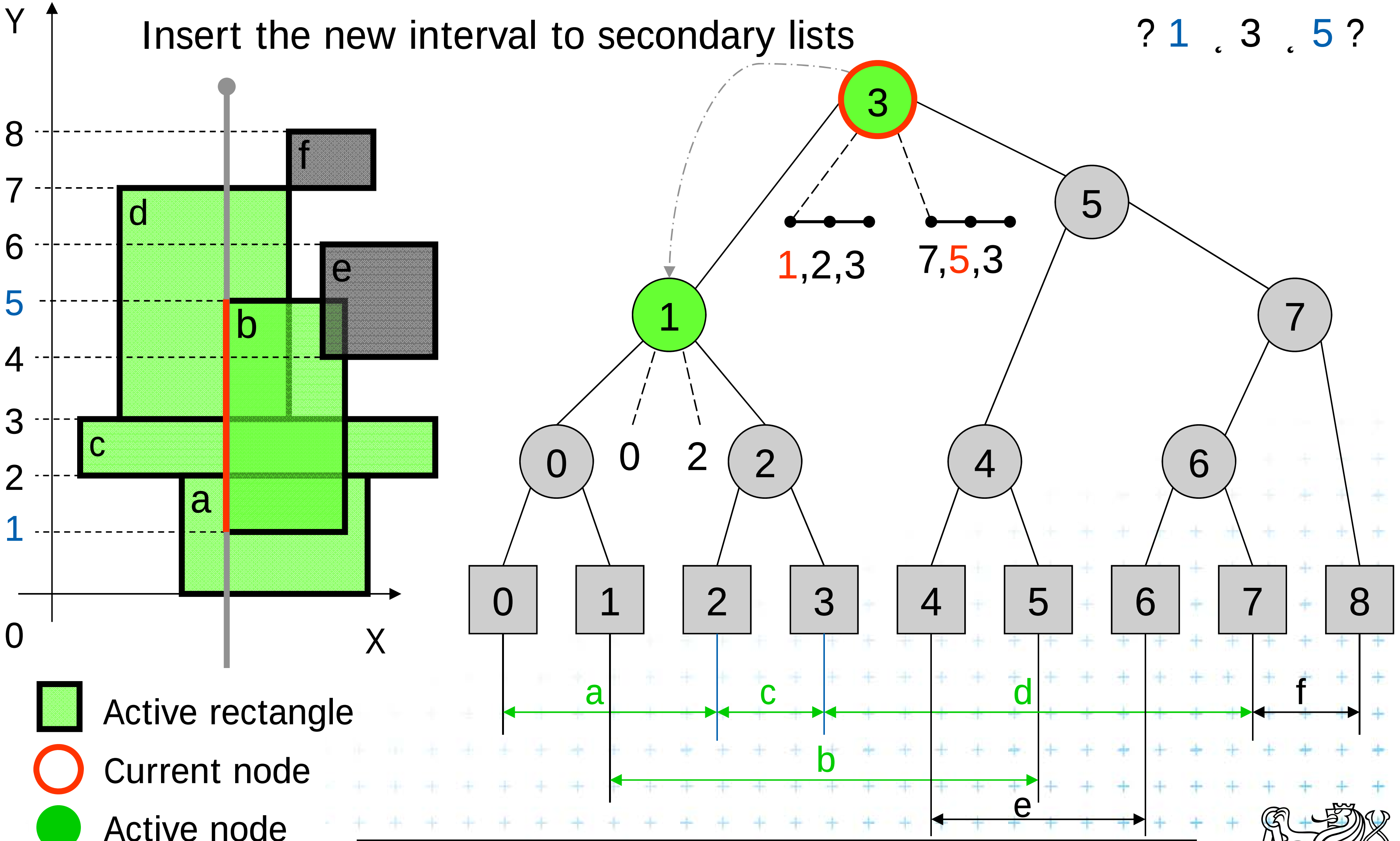
[Drtina]

Insert [1,5] b) Insert Interval

b ¸ H(v) ¸ e

Insert the new interval to secondary lists

? 1 ¸ 3 ¸ 5 ?



🟩 Active rectangle

○ Current node

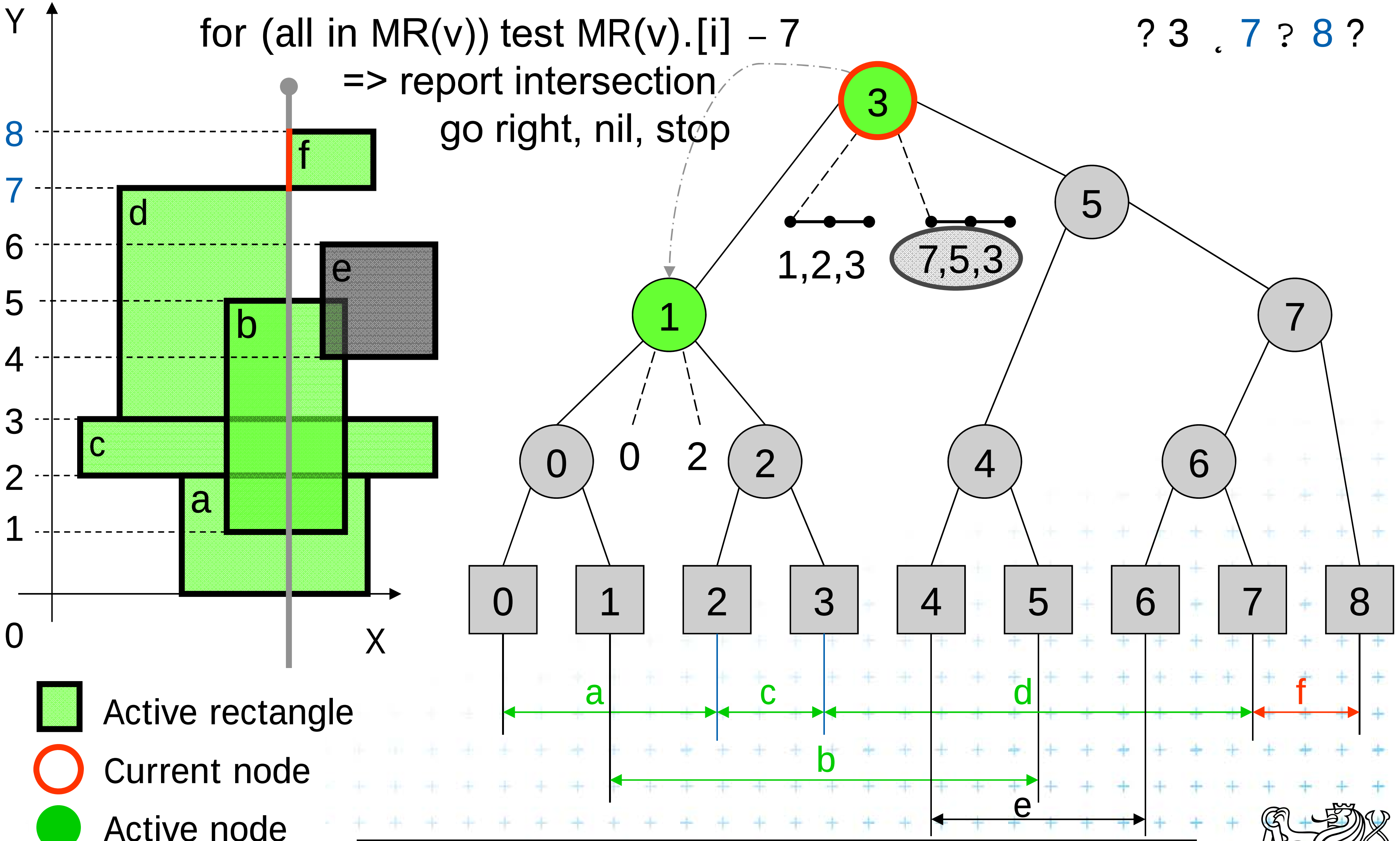
● Active node

[Drtina]

Insert [7,8] a) Query Interval

H(v) ˛ b ? e

? 3 ˛ 7 ? 8 ?



for (all in MR(v)) test MR(v).[i] – 7

=> report intersection

go right, nil, stop

🟩 Active rectangle

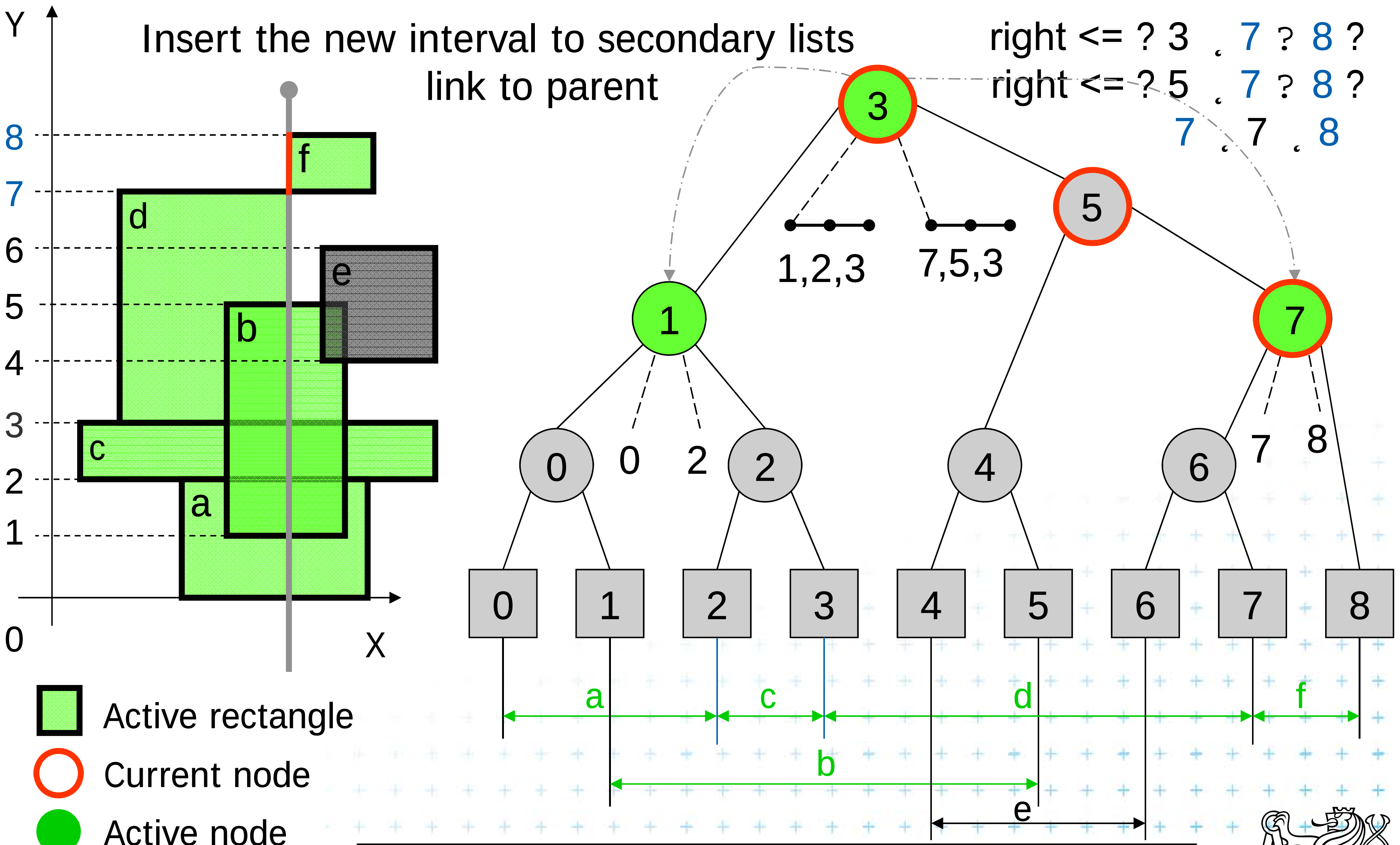
○ Current node

● Active node

[Drtina]

Insert [7,8] b) Insert Interval

b ¸ H(v) ¸ e



Insert the new interval to secondary lists

link to parent

right <= ? 3 ¸ 7 ? 8 ?

right <= ? 5 ¸ 7 ? 8 ?

7 ¸ 7 ¸ 8

🟩 Active rectangle

○ Current node

● Active node

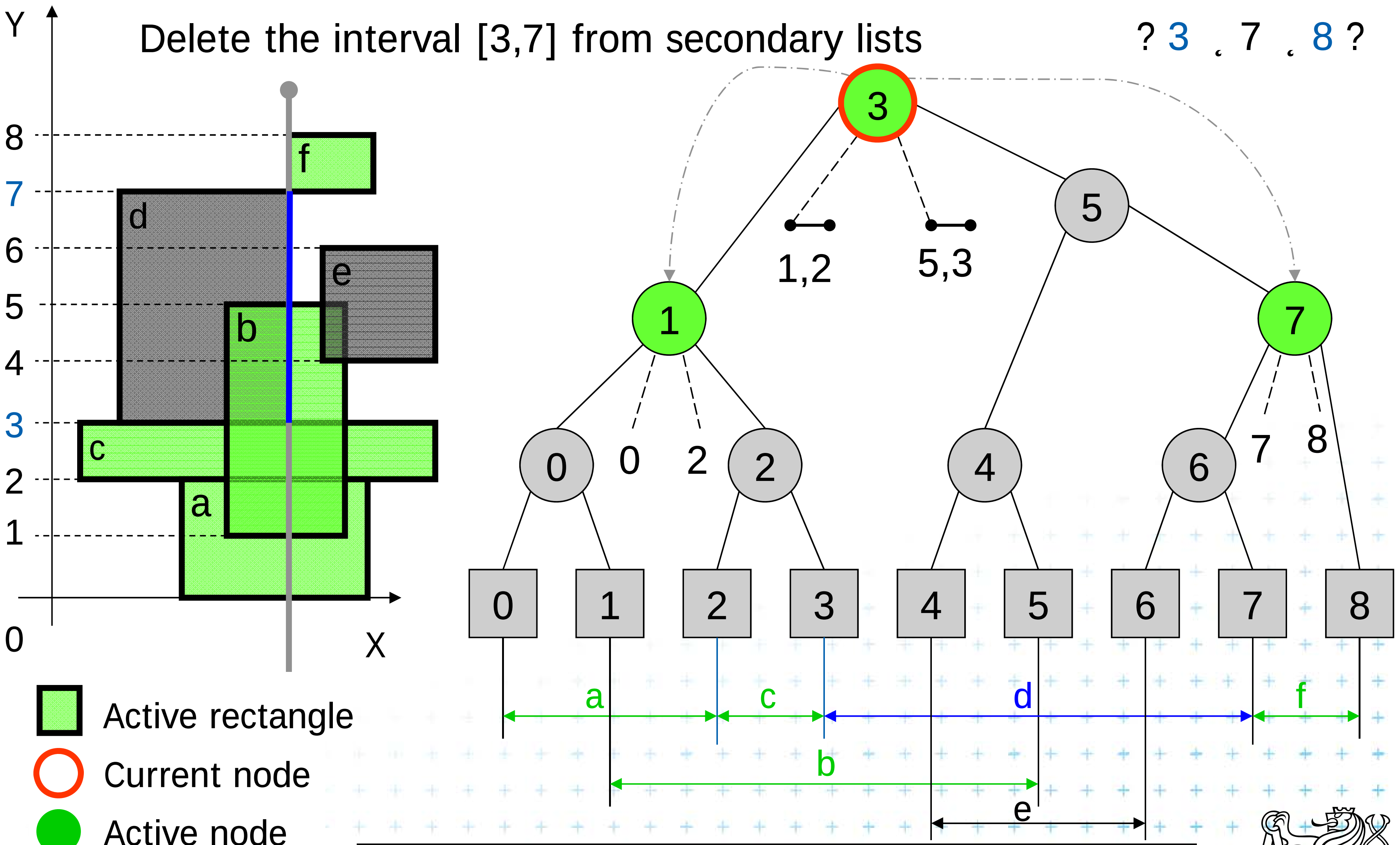
[Drtina]

Delete [3,7] Delete Interval

b ¸ H(v) ¸ e

Delete the interval [3,7] from secondary lists

? 3 ¸ 7 ¸ 8 ?



🟩 Active rectangle

○ Current node

● Active node

[Drtina]

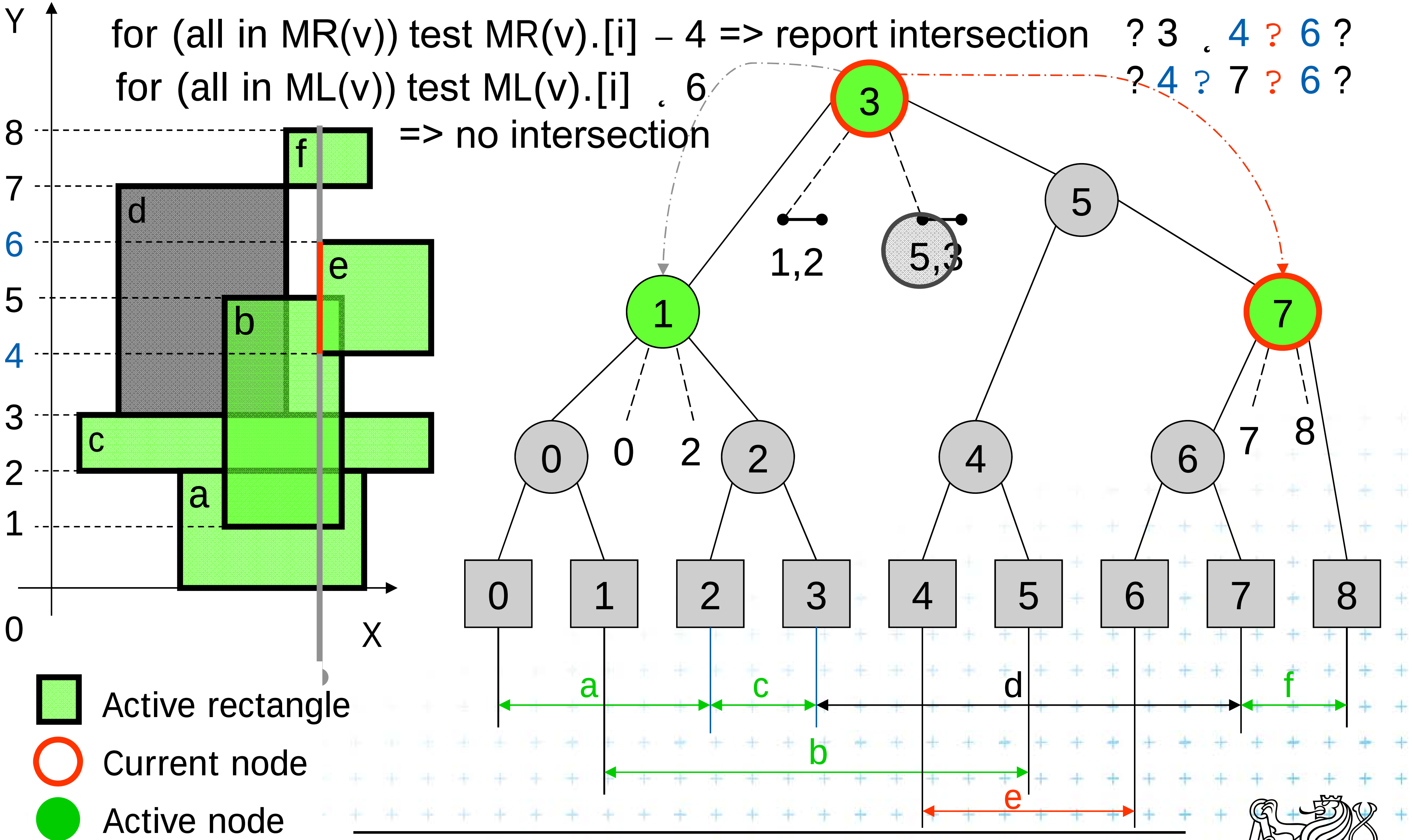
Insert [4,6] a) Query Interval

H(v) ¸ b ? e

for (all in MR(v)) test MR(v).[i] – 4 => report intersection ? 3 ¸ 4 ? 6 ?

for (all in ML(v)) test ML(v).[i] ¸ 6 ? 4 ? 7 ? 6 ?

=> no intersection



[Drtina]

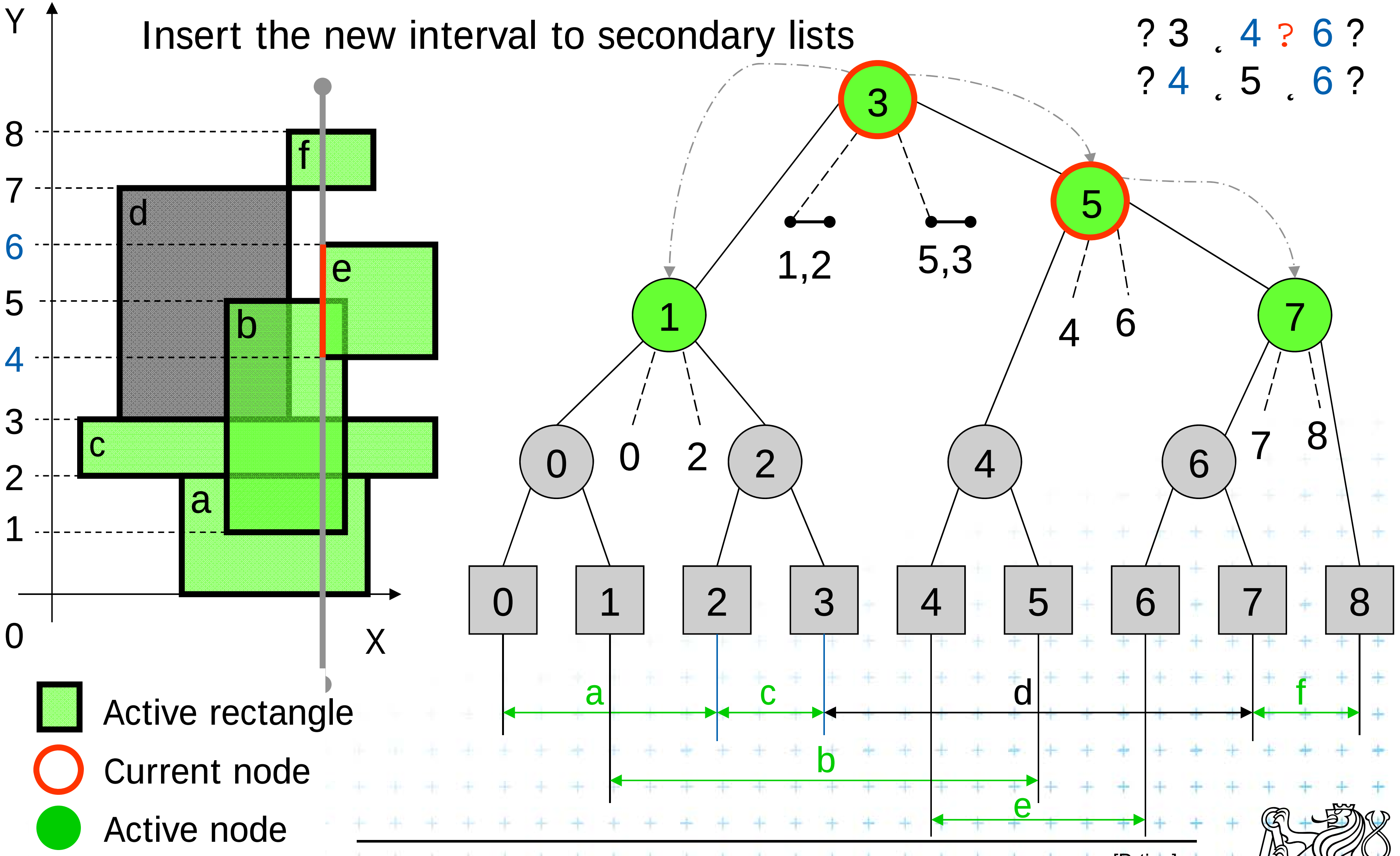
Insert [4,6] b) Insert Interval

s

Insert the new interval to secondary lists

? 3 ¸ 4 ? 6 ?

? 4 ¸ 5 ¸ 6 ?



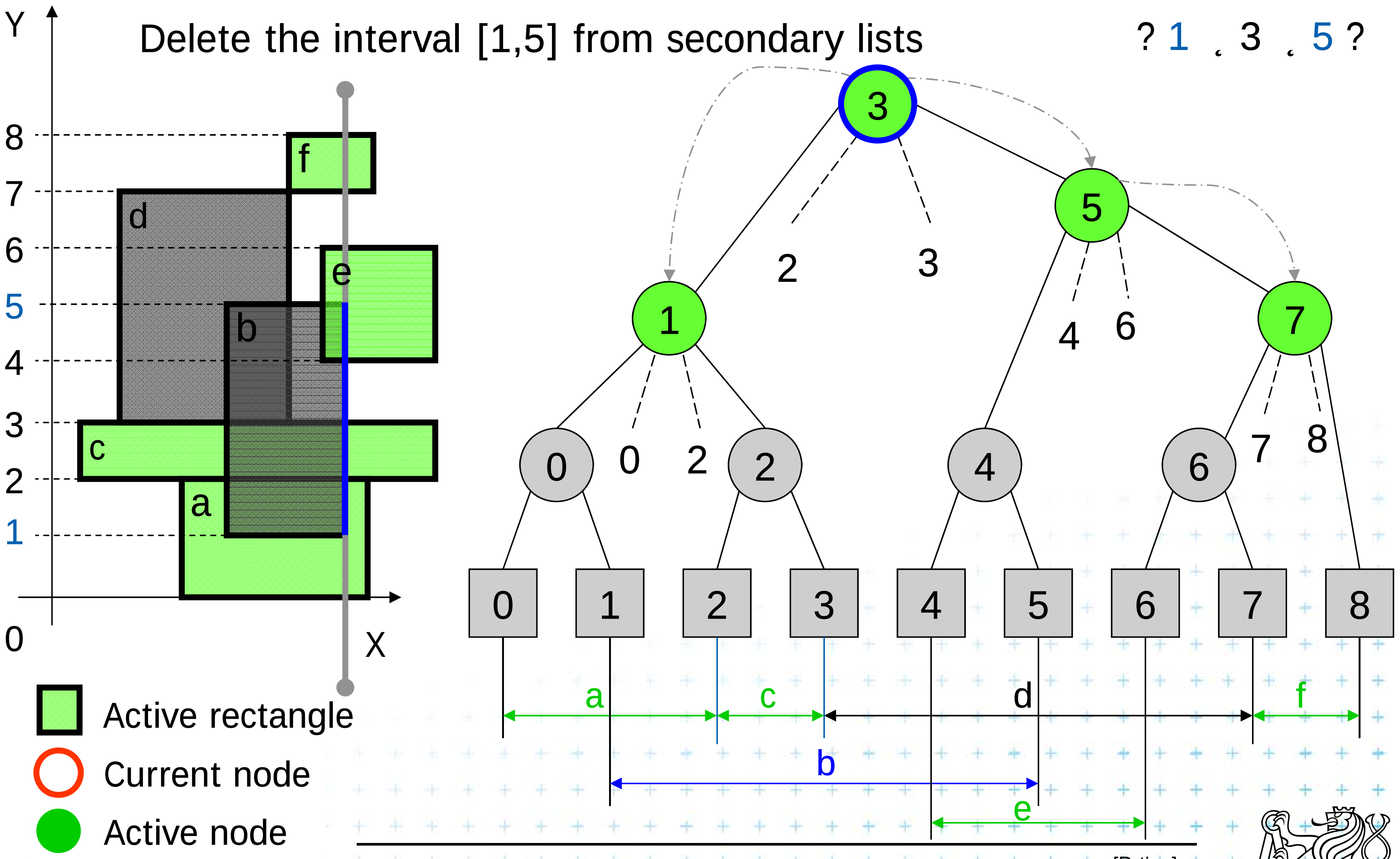
[Drtina]

Delete [1,5] Delete Interval

b ¸ H(v) ¸ e

Delete the interval [1,5] from secondary lists

? 1 ¸ 3 ¸ 5 ?



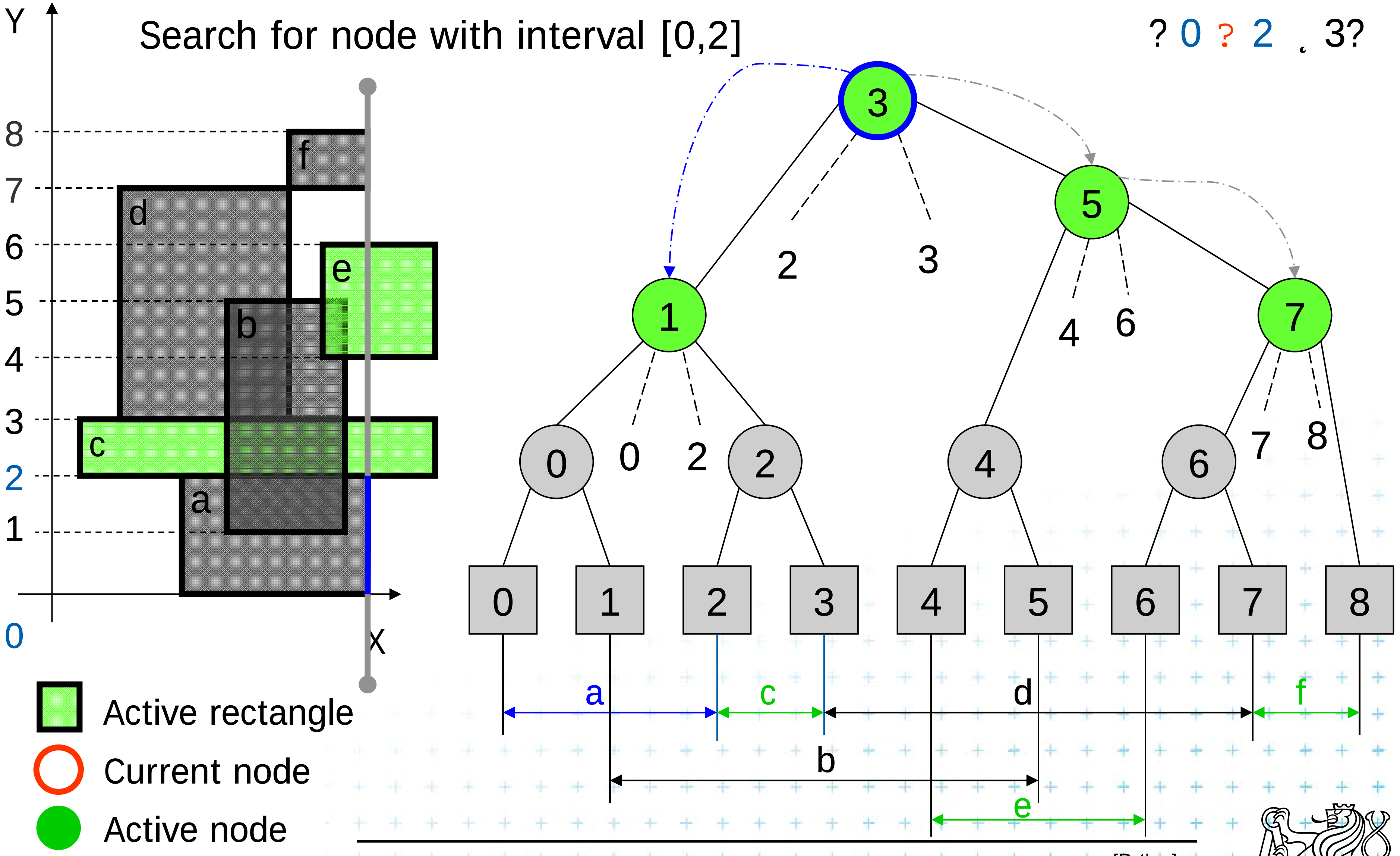
[Drtina]

Delete [0,2] Delete Interval 1/2

b ? e ¸ H(v)

Search for node with interval [0,2]

? 0 ? 2 ¸ 3?

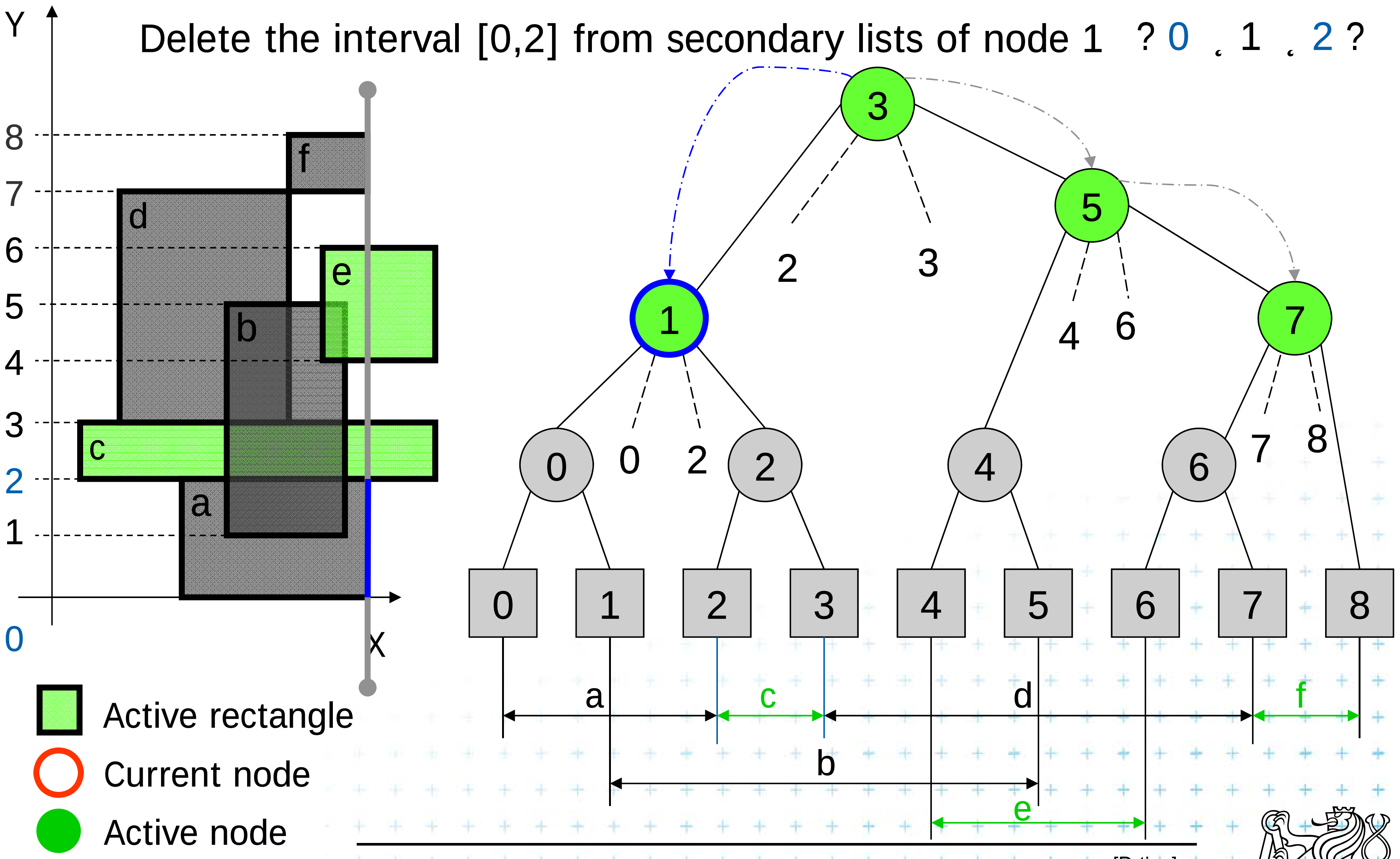


[Drtina]

Delete [0,2] Delete Interval 2/2

b ¸ H(v) ¸ e

Delete the interval [0,2] from secondary lists of node 1 ? 0 ¸ 1 ¸ 2 ?



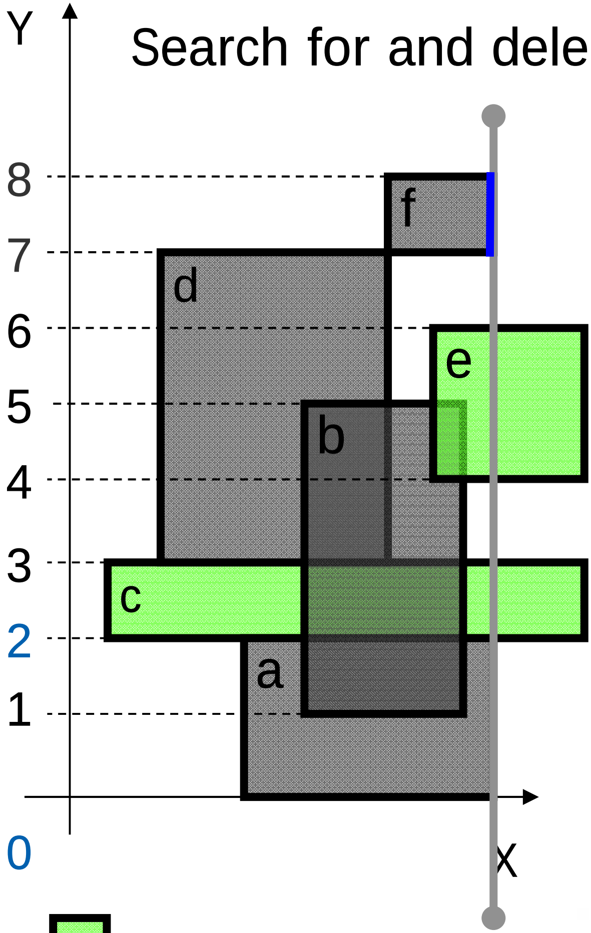
[Drtina]

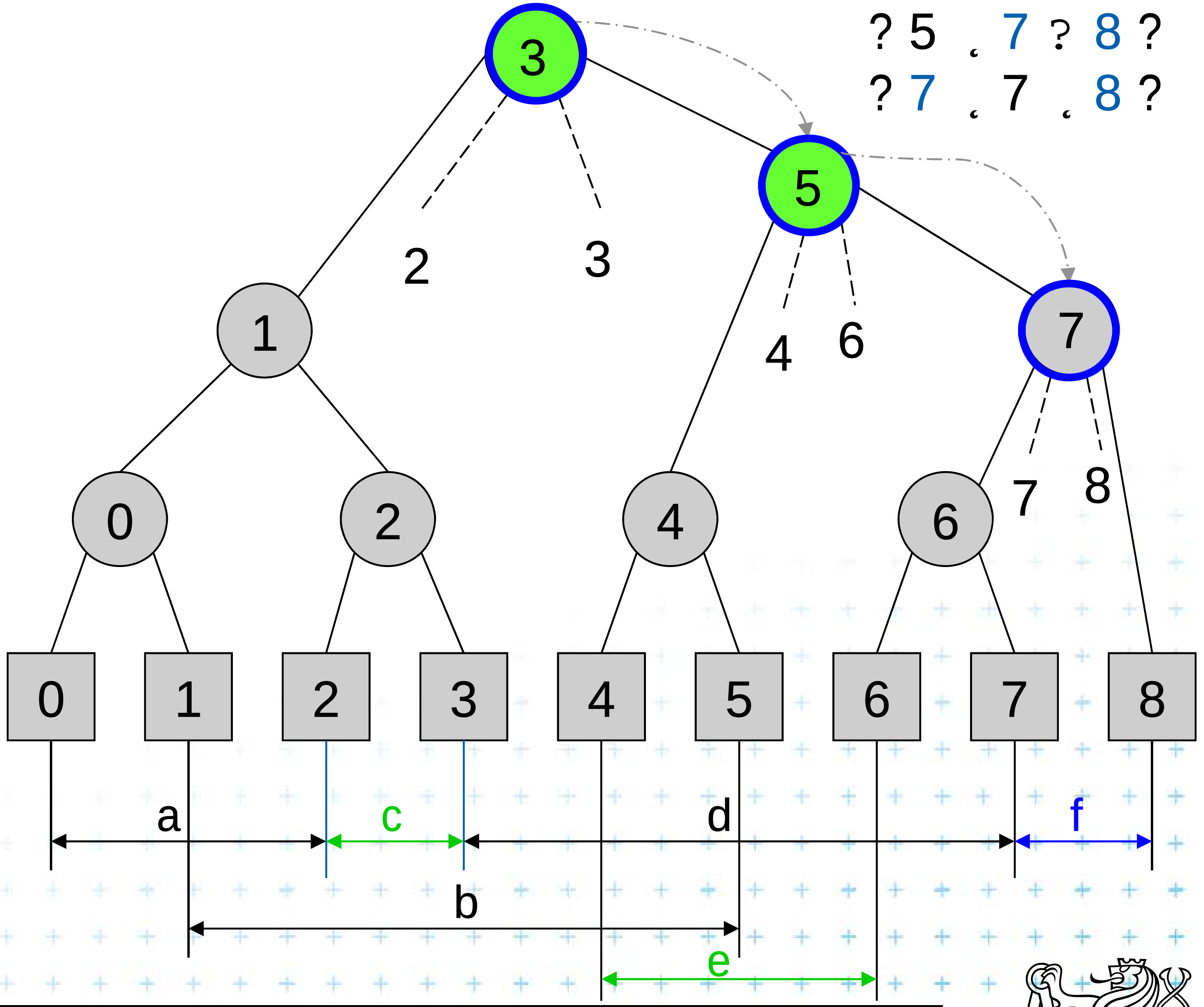
Delete [7,8] Delete Interval

b ¸ H(v) ¸ e

Search for and delete node with interval [7,8]

? 3 ¸ 7 ? 8 ?
? 5 ¸ 7 ? 8 ?
? 7 ¸ 7 ¸ 8 ?





🟩 Active rectangle

○ Current node

● Active node

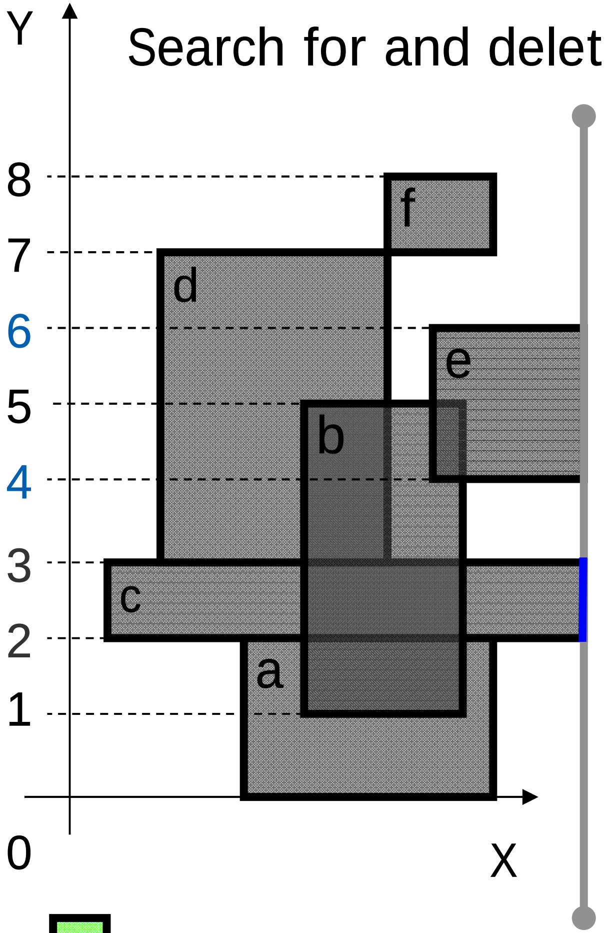
[Drtina]

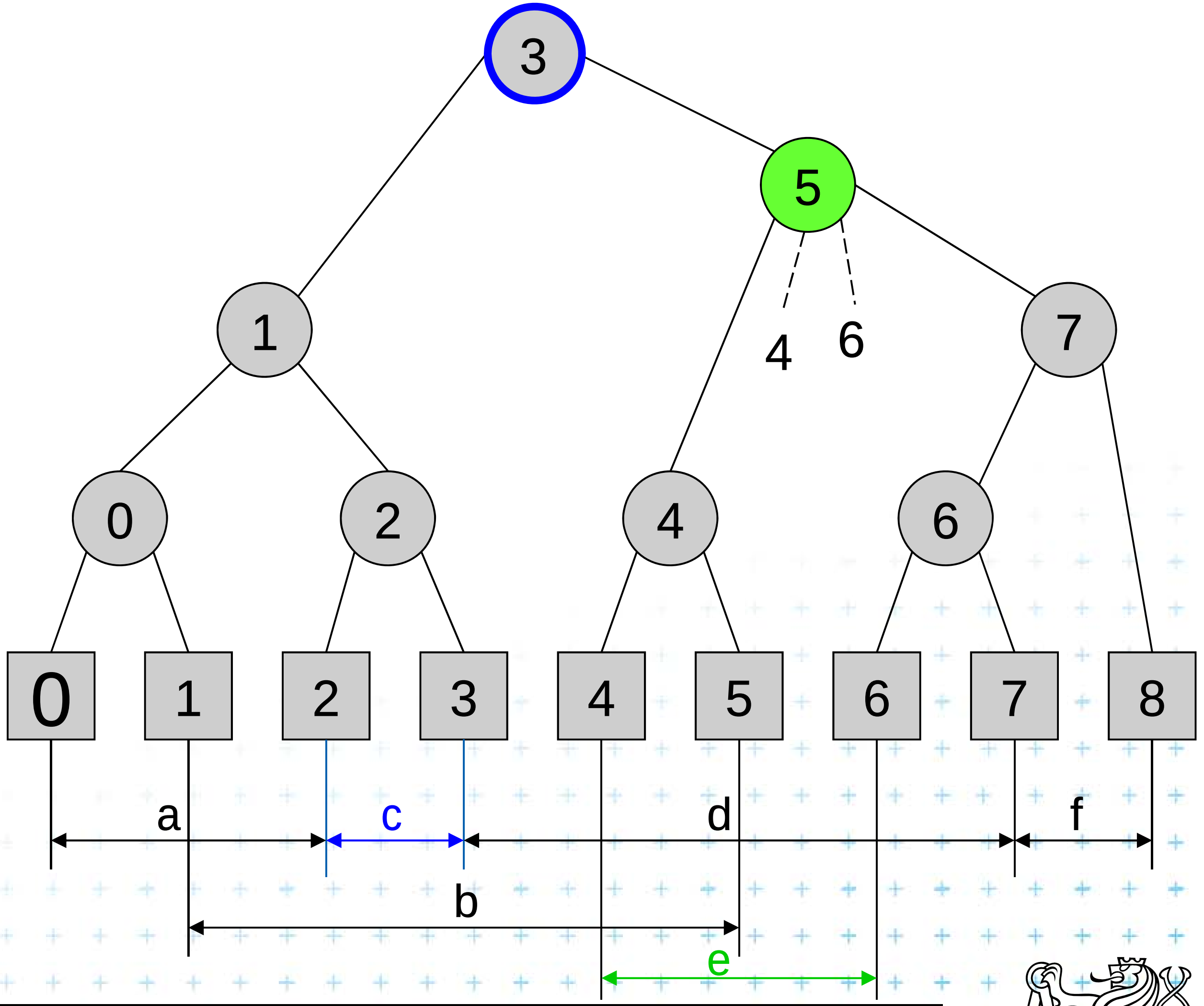
Delete [2,3] Delete Interval

b ¸ H(v) ¸ e

Search for and delete node with interval [2,3]

? 2 ¸ 3 ¸ 3 ?





🟩 Active rectangle

○ Current node

● Active node

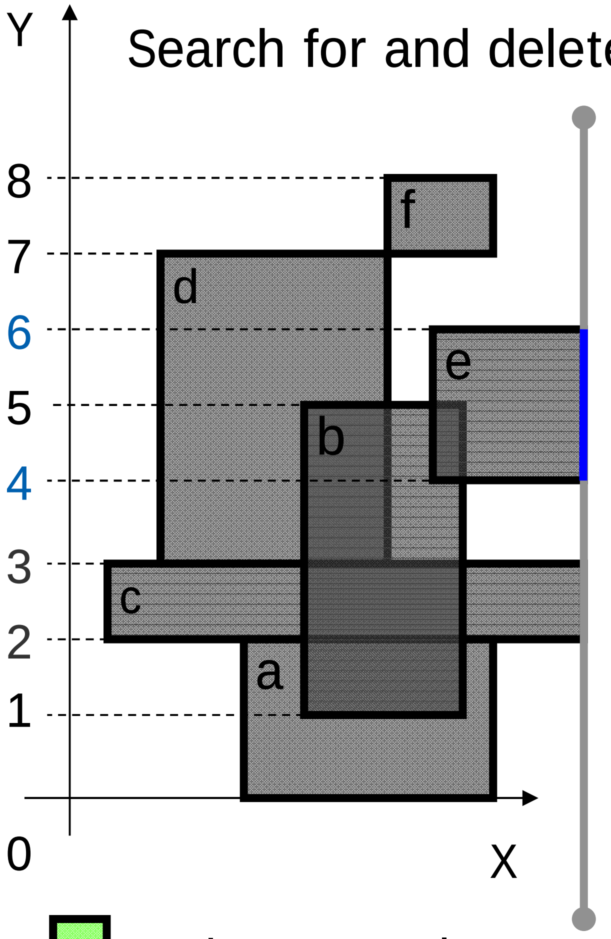
[Drtina]

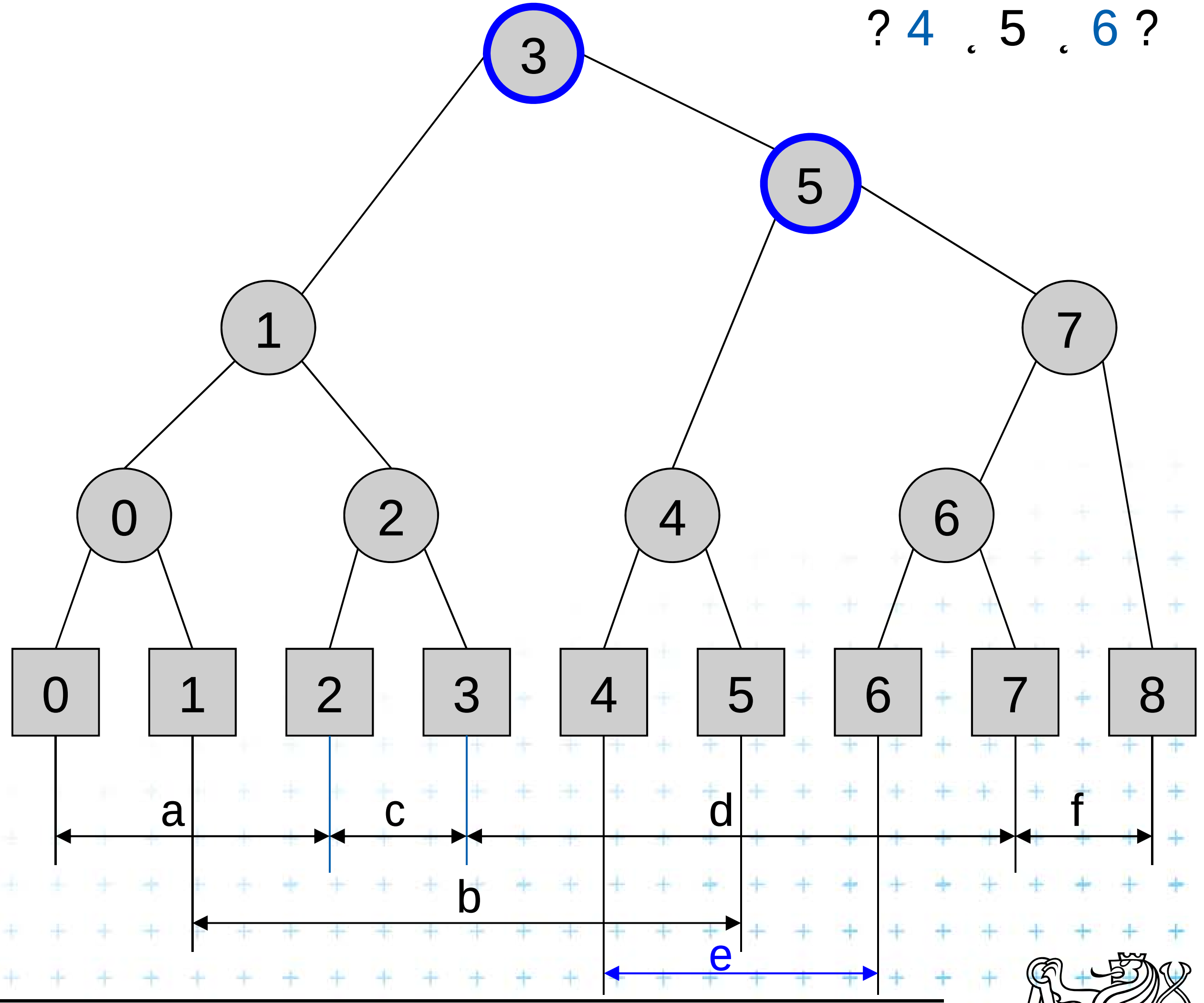
Delete [4,6] Delete Interval

b ¸ H(v) ¸ e

Search for and delete node with interval [4,6]

? 3 ¸ 4 ? 6 ?
? 4 ¸ 5 ¸ 6 ?





🟩 Active rectangle

○ Current node

● Active node

[Drtina]

Complexities of rectangle intersections

- *n* rectangles, *s* intersected pairs found
- O(*n* log *n*) preprocessing time to separately sort
  - x-coordinates of the rectangles for the plane sweep
  - the y-coordinates for initializing the interval tree.
- The plane sweep itself takes O(*n* log *n* + *s*) time, so the overall time is O(*n* log *n* + *s*)
- O(*n*) space
- This time is optimal for a decision-tree algorithm (i.e., one that only makes comparisons between rectangle coordinates).

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