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A4M33MAS - Multiagent Systems

Agents and their behaviour modeling by means of formal logic

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Selected graphics taken from Valentin Goranko and Wojtek Jamroga: Modal Logics for Multi-Agent Systems, 8th European Summer School in Logic Language and Information

Multi-agent systems & Logic

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- Multi-agent systems
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  - Provides methodology for specification and verification of complex programs

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- Logic
  - Provides a paradigm for modeling and reasoning about the complex world in a precise and exact manner
  - Provides methodology for specification and verification of complex programs
- Can be used for practical things (also in MAS):
  - automatic verification of multi-agent systems
  - and/or executable specifications of multi-agent systems

Best logic for MAS?

Modal logic

Modal logic is an extension of classical logic by new connectives $\Box$ and $\Diamond$: necessity and possibility.

- $\Box\varphi$ means that φ is necessarily true
- $\Diamond\varphi$ means that φ is possibly true

Independently of the precise definition, the following holds:

$$\Diamond\varphi \leftrightarrow \neg\Box\neg\varphi$$

Modal logic

01

Definition 1.1 (Modal Logic with n modalities)

The language of modal logic with n modal operators $\Box_1, \ldots, \Box_n$ is the smallest set containing:

- atomic propositions $p, q, r, \ldots$;
- for formulae φ, it also contains $\neg\varphi, \Box_1\varphi, \ldots, \Box_n\varphi$;
- for formulae φ, ψ, it also contains $\varphi \wedge \psi$.

We treat $\vee, \rightarrow, \leftrightarrow, \Diamond$ as macros (defined as usual).

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We treat $\vee, \rightarrow, \leftrightarrow, \Diamond$ as macros (defined as usual).

Note that the modal operators can be nested:

$$(\Box_1\Box_2\Diamond_1 p) \ \vee \ \Box_3\neg p$$

Modal logic

More precisely, necessity/possibility is interpreted as follows:

- p is necessary $\Leftrightarrow$ p is true in all possible scenarios
- p is possible $\Leftrightarrow$ p is true in at least one possible scenario

$\rightsquigarrow$ possible worlds semantics

Modal logic

Definition 1.2 (Kripke Structure)

A Kripke structure is a tuple $\langle \mathcal{W}, \mathcal{R} \rangle$, where $\mathcal{W}$ is a set of possible worlds, and $\mathcal{R}$ is a binary relation on worlds, called accessibility relation.

Definition 1.3 (Kripke model)

A possible worlds model $\mathcal{M} = \langle \mathcal{S}, \pi \rangle$ consists of a Kripke structure $\mathcal{S}$, and a valuation of propositions $\pi : \mathcal{W} \rightarrow \mathcal{P}(\{p, q, r, \ldots\})$.

Modal logic

Remarks:

- $\mathcal{R}$ indicates which worlds are relevant for each other; $w_1 \mathcal{R} w_2$ can be read as "world w_2 is relevant for (reachable from) world w_1"
- $\mathcal{R}$ can be any binary relation from $\mathcal{W} \times \mathcal{W}$; we do not require any specific properties (yet).

Modal logic

Definition 1.4 (Semantics of modal logic)

The truth of formulae is relative to a Kripke model $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \pi \rangle$, and a world $w \in \mathcal{W}$. It can be defined through the following clauses:

- $\mathcal{M}, w \models p$ iff $p \in \pi(w)$;
- $\mathcal{M}, w \models \neg\varphi$ iff not $\mathcal{M}, w \models \varphi$;
- $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$;
- $\mathcal{M}, w \models \Box\varphi$ iff, for every $w' \in \mathcal{W}$ such that $w\mathcal{R}w'$, we have $\mathcal{M}, w' \models \varphi$.

Modal logic

Modal logic

$$run \rightarrow \Diamond stop$$

Modal logic

$$\text{run} \longrightarrow \Diamond \text{stop}$$

$$\text{stop} \longrightarrow \Box \text{stop}$$

Modal logic

$$run \longrightarrow \Diamond stop$$

$$stop \longrightarrow \Box stop$$

$$run \longrightarrow \Diamond\Box stop$$

Modal logic

Modal logic

- Note:
  - most modal logics can be translated to classical logic

    ... *but the result looks horribly ugly,*

    ... *and in most cases it is much harder to automatize anything*

Axiom in Modal logic

01

Definition 1.5 (System K)

System **K** is an extension of the propositional calculus by the axiom

Distribution axiom

$$\mathbf{K} \quad (\Box\varphi \wedge \Box(\varphi \to \psi)) \to \Box\psi$$

and the inference rule

Generalization axiom $\dfrac{\varphi}{\Box\varphi}.$

Axiom in Modal logic

Theorem 1.6 (Soundness/completeness of system K)

*System **K** is sound and complete with respect to the class of all Kripke models.*

Axiom in Modal logic

Definition 1.7 (Extending K with axioms D, T, 4, 5)

System **K** is often extended by (a subset of) the following axioms (called as below for historical reasons):

- T: $\Box\varphi \rightarrow \varphi$
- D: $\Box\varphi \rightarrow \Diamond\varphi$
- 4: $\Box\varphi \rightarrow \Box\Box\varphi$
- B: $\varphi \rightarrow \Box\Diamond\varphi$
- 5: $\Diamond\varphi \rightarrow \Box\Diamond\varphi$

Proofs

T: because $\models \varphi \Rightarrow \Box\varphi$ and due reflexivity $\forall w : (w, w) \in R$ ⊚

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Proofs

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D: $(\mathcal{M}_1 \models_w \varphi \, \forall w' : (w, w') \in R : \mathcal{M}_1 \models_{w'} \varphi)$ and due to seriality $(\mathcal{M}_1 \models_w (\exists w' : (w, w') \in R))$ we can say that $\mathcal{M}_1 \models_w \exists w'' : (w, w'') \in R : \mathcal{M}_1 \models_{w'} \varphi)$ ⊚

$$\text{D: } \Box\varphi \longrightarrow \Diamond\varphi$$

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4: provided that there is transitive relation on R we may say that $(\mathcal{M}_1 \models_w \varphi\ \forall w' : (w, w') \in R : \mathcal{M}_1 \models_{w'} \varphi) \Rightarrow (\mathcal{M}_1 \models_w \forall w' : (w, w') \in R : \mathcal{M}_1 \models_{w'} (\forall w'' : (w', w'') \in R : \mathcal{M}_1 \models_{w''} \varphi))$ ◎

$$4:\ \Box\varphi \rightarrow \Box\Box\varphi$$

Proofs

T: because $\models \varphi \Rightarrow \Box\varphi$ and due reflexivity $\forall w : (w, w) \in R$ ◎

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B: provided that there is symetric relation on R we say that $\mathcal{M}_1 \models_w \varphi \Rightarrow \forall w' : (w, w') \in R : \mathcal{M}_1 \models_{w'} \exists w'' : (w', w'') \in R : \mathcal{M}_1 \models_{w''} \varphi$ if $(\forall w, w', (w, w') \in R \Rightarrow (w', w) \in R)$ then $w = w''$ and $\mathcal{M}_1 \models_w \varphi$ ◎

$$\text{B: } \varphi \rightarrow \Box\Diamond\varphi$$

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T: because $\models \varphi \Rightarrow \Box\varphi$ and due reflexivity $\forall w : (w, w) \in R$ ◎

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5: $(\mathcal{M}_1 \models_w \exists w' : (w, w') \in R \models_{w'} \varphi) \Rightarrow (\mathcal{M}_1 \models_w \forall w'' : (w, w'') \in R : \mathcal{M}_1 \models_{w''} \exists w'(w'', w') \in R : \mathcal{M}_1 \models_{w'} \varphi)$ due to euclidean property if $(w, w') \in R \wedge (w, w'') \in R$ then $(w', w'') \in R$ ◎

$$5:\ \Diamond\varphi \rightarrow \Box\Diamond\varphi$$

Axiom in Modal logic

- T: $\Box\varphi \to \varphi$ due to reflexivity
- D: $\Box\varphi \to \Diamond\varphi$ due to seriality
- 4: $\Box\varphi \to \Box\Box\varphi$ due to transitivity
- B: $\varphi \to \Box\Diamond\varphi$ due to symetricity
- 5: $\Diamond\varphi \to \Box\Diamond\varphi$ due to euclidean property

Model of Belief

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- Knowledge is more difficult – it needs to be also true – this why the knowledge accessibility relation needs to be also reflexive.
- Therefore knowledge is a KTD45 system.

Model of Belief

- φ can be true in $\mathcal{M}$ and q ($\mathcal{M}, q \models \varphi$)
- φ can be valid in $\mathcal{M}$ ($\mathcal{M}, q \models \varphi$ for all q)
- φ can be valid ($\mathcal{M}, q \models \varphi$ for all $\mathcal{M}, q$)
- φ can be satisfiable ($\mathcal{M}, q \models \varphi$ for some $\mathcal{M}, q$)
- φ can be a theorem (it can be derived from the axioms via inference rules)

Model of Belief

- model checking (local): "given $\mathcal{M}$, q, and φ, is φ true in $\mathcal{M}, q$?"
- model checking (global): "given $\mathcal{M}$ and φ, what is the set of states in which φ is true?"

- Model checking is a technique for automatically verifying correctness properties of finite-state systems. Given a model of a system, exhaustively and automatically check whether this model meets a given specification (such as the absence of deadlocks and similar critical states that can cause the system to crash).

Model of Belief

- model checking (local): "given $\mathcal{M}$, q, and φ, is φ true in $\mathcal{M}, q$?"
- model checking (global): "given $\mathcal{M}$ and φ, what is the set of states in which φ is true?"
- satisfiability: "given φ, is φ true in at least one model and state?"
- validity: "given φ, is φ true in all models and their states?"
- theorem proving: "given φ, is it possible to prove (derive) φ?"

Model of Belief

Modal logic is a generic framework.

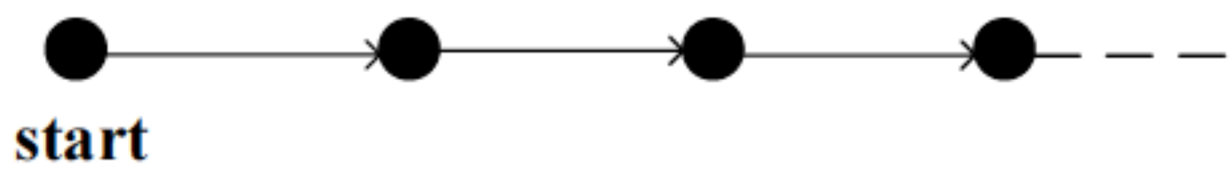
Various modal logics:

- knowledge $\rightsquigarrow$ epistemic logic,
- beliefs $\rightsquigarrow$ doxastic logic,
- obligations $\rightsquigarrow$ deontic logic,
- actions $\rightsquigarrow$ dynamic logic,
- time $\rightsquigarrow$ temporal logic,
- ability $\rightsquigarrow$ strategic logic,
- and combinations of the above

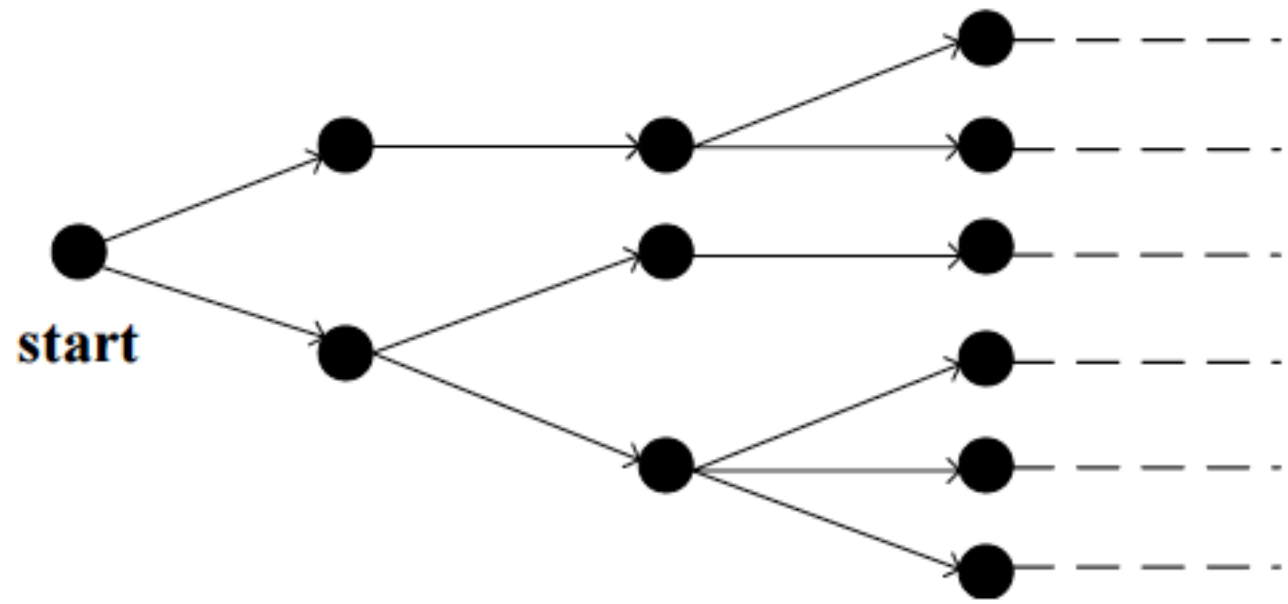
Model of Time

- Modeling time as an instance of modal logic where the accessibility relation represents the relationship between the past, current and future time moments.

- Time:
  - linear



  - branching



Typical Temporal Operators

01

$\mathcal{X}\varphi$	φ is true in the next moment in time
$\mathcal{G}\varphi$	φ is true in all future moments
$\mathcal{F}\varphi$	φ is true in some future moment
$\varphi\mathcal{U}\psi$	φ is true until the moment when ψ becomes true

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$$\mathcal{G}((\neg\text{passport} \vee \neg\text{ticket}) \rightarrow \mathcal{X}\neg\text{board\_flight})$$

$$\text{send}(\text{msg}, \text{rcvr}) \rightarrow \mathcal{F}\text{receive}(\text{msg}, \text{rcvr})$$

Safety Property

- *something bad will not happen*
- *something good will always hold*

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Usually: $\mathcal{G}\neg\ldots$

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and so on ...

Usually: $\mathcal{F}$....

Fairness Property

01

- *Useful when scheduling processes, responding to messages, etc.*
- *Good for specifying interaction properties of the environment*

- Typical examples:

$$\mathcal{G}(\text{rocketLondon} \rightarrow \mathcal{F}\text{rocketParis})$$

- Strong Fairness:
if something is attempted/requested, then it will be successful

- Typical examples:

$$\mathcal{G}(\text{attempt} \rightarrow \mathcal{F}\text{success})$$

$$\mathcal{G}\mathcal{F}\text{attempt} \rightarrow \mathcal{G}\mathcal{F}\text{success}$$

Linear Temporal Logic - LTL

- Reasoning about a particular computation of a system where time is linear - just one possible future path is included.

Definition 3.4 (Models of LTL)

A model of LTL is a sequence of time moments. We call such models paths, and denote them by λ.

Evaluation of atomic propositions at particular time moments is also needed.

Notation:

- $\lambda[i]$: ith time moment
- $\lambda[i \ldots j]$: all time moments between i and j
- $\lambda[i \ldots \infty]$: all timepoints from i on

Linear Temporal Logic - LTL

Definition 3.5 (Semantics of LTL)

$\lambda \models p$	iff p is true at moment $\lambda[0]$;
$\lambda \models \mathcal{X}\varphi$	iff $\lambda[1..\infty] \models \varphi$;
$\lambda \models \mathcal{F}\varphi$	iff $\lambda[i..\infty] \models \varphi$ for some $i \geq 0$;
$\lambda \models \mathcal{G}\varphi$	iff $\lambda[i..\infty] \models \varphi$ for all $i \geq 0$;
$\lambda \models \varphi\mathcal{U}\psi$	iff $\lambda[i..\infty] \models \psi$ for some $i \geq 0$, and $\lambda[j..\infty] \models \varphi$ for all $0 \leq j \leq i$.

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Note that:

$$\mathcal{G}\varphi \equiv \neg\mathcal{F}\neg\varphi$$

$$\mathcal{F}\varphi \equiv \neg\mathcal{G}\neg\varphi$$

$$\mathcal{F}\varphi \equiv \top\,\mathcal{U}\varphi$$

Computational Tree Logic - CTL

- Reasoning about possible computations of a system. Time is branching -- we want all alternative paths included.

- Path quantifiers: **A** (for all paths), **E** (there is a path);
- Temporal operators: $\mathcal{X}$ (nexttime), $\mathcal{F}$ (sometime), $\mathcal{G}$ (always) and $\mathcal{U}$ (until);

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- Temporal operators: $\mathcal{X}$ (nexttime), $\mathcal{F}$ (sometime), $\mathcal{G}$ (always) and $\mathcal{U}$ (until);

- Vanilla CTL: every temporal operator must be immediately preceded by exactly one path quantier
- CTL*: no syntactic restrictions
- Reasoning in Vanilla CTL can be automatized.

Computational Tree Logic - CTL

Definition 3.8 (Semantics of CTL*: state formulae)

$M, q \models \mathbf{E}\varphi$ iff there is a path λ, starting from q, such that $M, \lambda \models \varphi$;

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Definition 3.9 (Semantics of CTL*: path formulae)

Exactly like for LTL!

Computational Tree Logic - CTL

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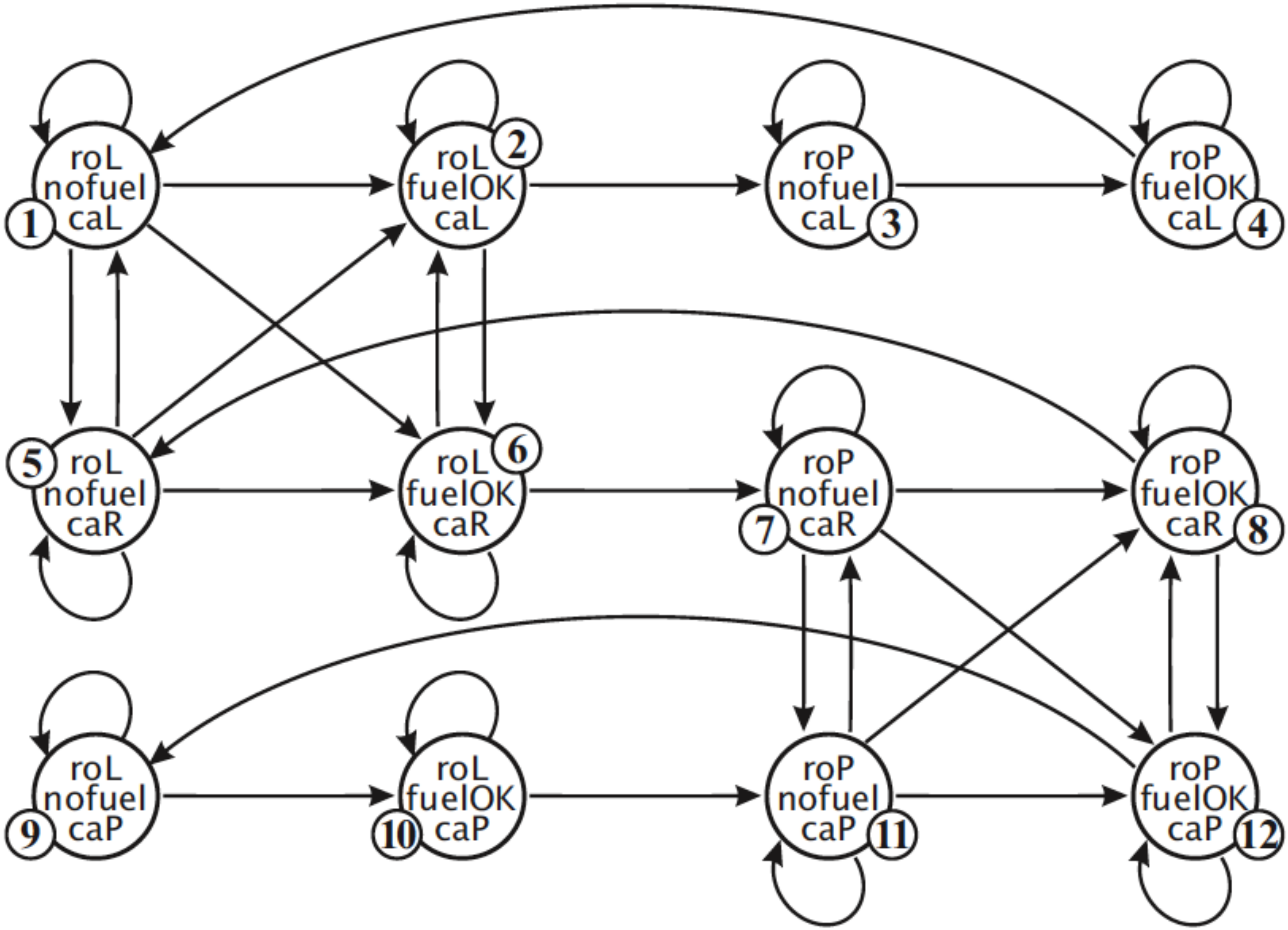

Dynamic Logic

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As usual, $\langle\alpha\rangle\varphi \equiv \neg[\alpha]\neg\varphi$.

Dynamic Logic

3rd **idea:** Programs/actions can be combined (sequentially, nondeterministically, iteratively), e.g.:

$$[\alpha;\beta]\varphi$$

would mean "after every execution of α and then β, formula φ holds".

Dynamic Logic

Definition 3.1 (Labelled Transition System)

A labelled transition system is a pair

$$\langle St, \{\xrightarrow{\alpha} : \alpha \in \mathbf{L}\}\rangle$$

where St is a non-empty set of states and $\mathbf{L}$ is a non-empty set of labels and for each $\alpha \in \mathbf{L}$: $\xrightarrow{\alpha} \subseteq St \times St$.

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Definition 3.2 (Dynamic Logic: Models)

A model of propositional dynamic logic is given by a labelled transition systems and an evaluation of propositions.

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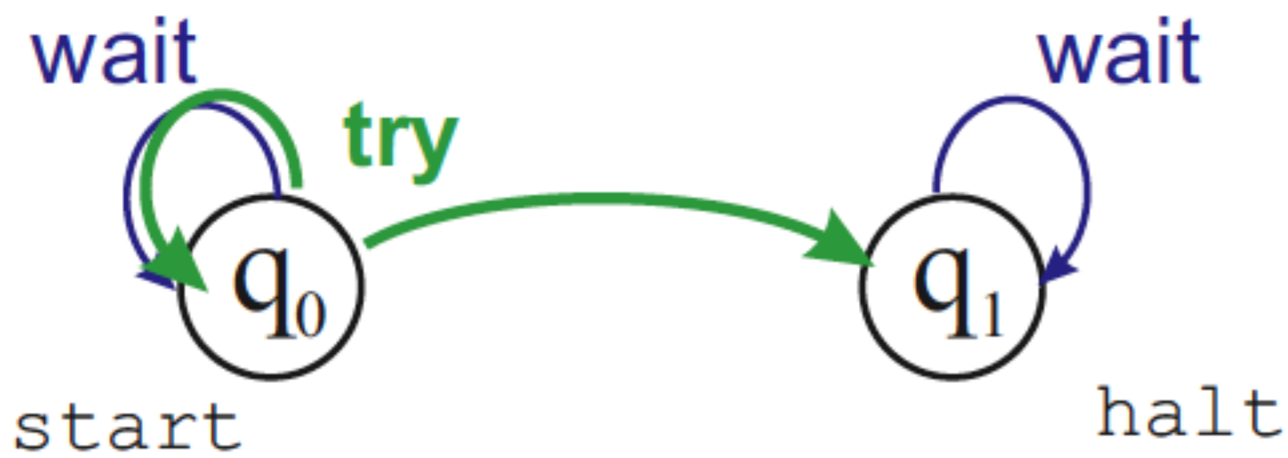
A model of propositional dynamic logic is given by a labelled transition systems and an evaluation of propositions.

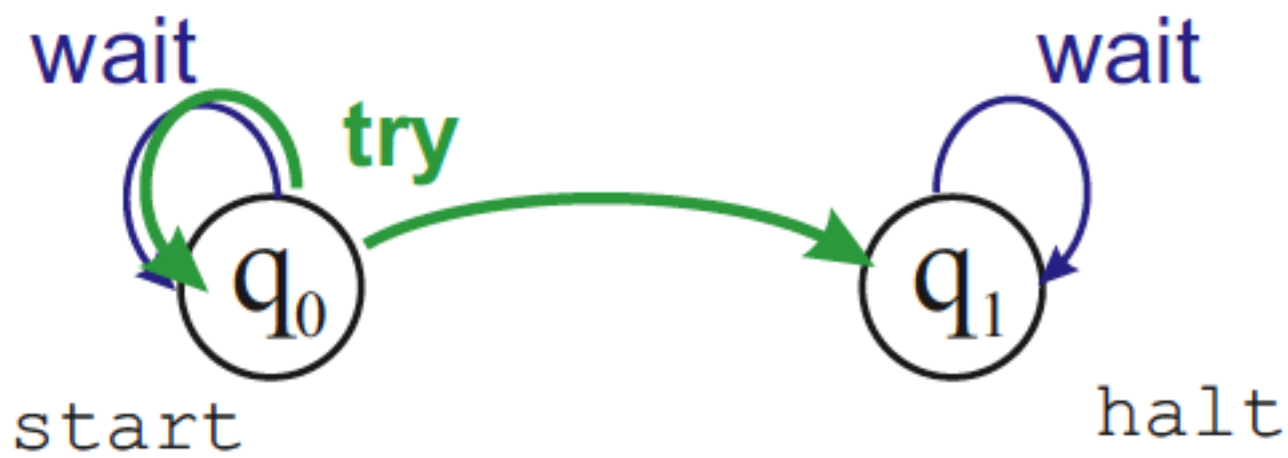
Definition 3.3 (Semantics of DL)

$\mathcal{M}, s \models [\alpha]\varphi$ iff for every t such that $s \xrightarrow{\alpha} t$, we have $\mathcal{M}, t \models \varphi$.

Dynamic Logic

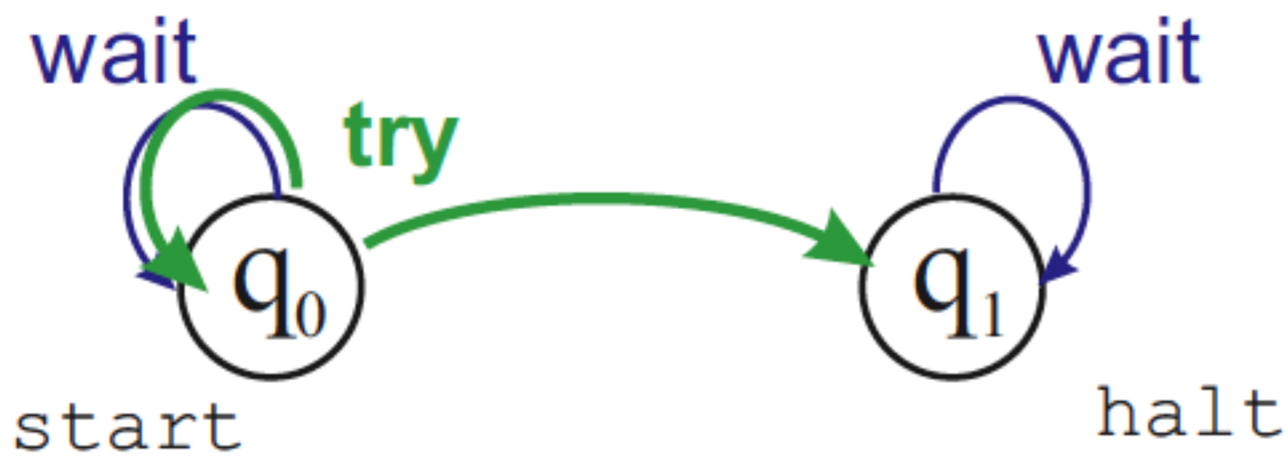


Dynamic Logic



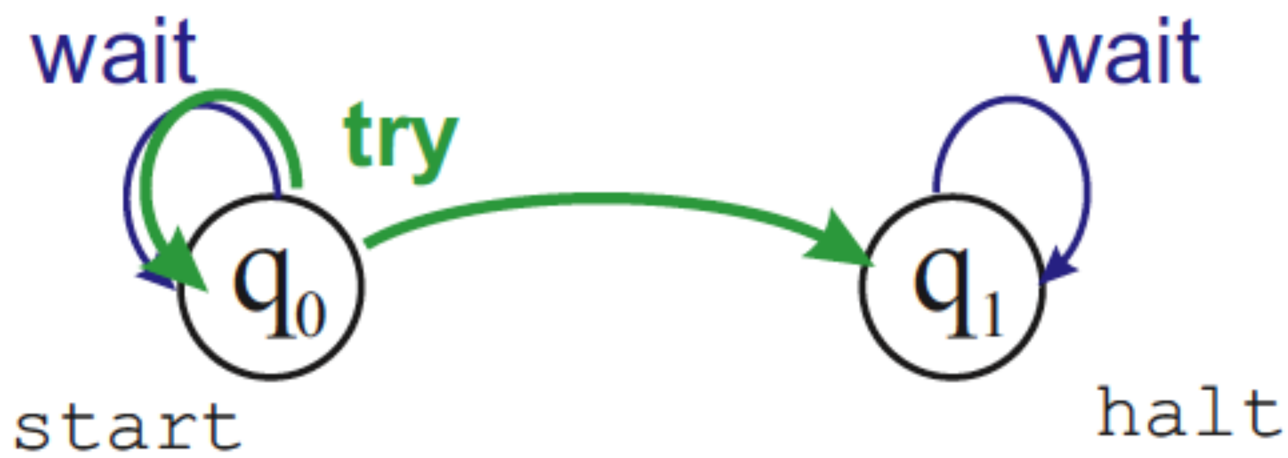
$$\text{start} \rightarrow \langle try \rangle \text{halt}$$

Dynamic Logic



$$\text{start} \longrightarrow \langle \mathit{try} \rangle \text{halt}$$
$$\text{start} \longrightarrow \neg [\mathit{try}] \text{halt}$$

Dynamic Logic



$$\text{start} \rightarrow \langle try \rangle \text{halt}$$

$$\text{start} \rightarrow \neg [try] \text{halt}$$

$$\text{start} \rightarrow \langle try \rangle [wait] \text{halt}$$

Concluding Remarks

- Practical Importance of Temporal and Dynamic Logics:
  - Automatic verication in principle possible (model checking).
  - Can be used for automated planning.
  - Executable specications can be used for programming.

- Note:

  When we combine time and actions with knowledge (beliefs, desires, intentions, obligations...), we finally obtain a fairly realistic model of MAS.

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