Part III

Computing with a Single Camera

9 Calibration: Internal Camera Parameters from Vanishing Points and Lines

10 Resectioning: Projection Matrix from 6 Known Points

11 Exterior Orientation: Camera Rotation and Translation from 3 Known Points

covered by

[1] [H&Z] Secs: 8.6, 7.1, 22.1

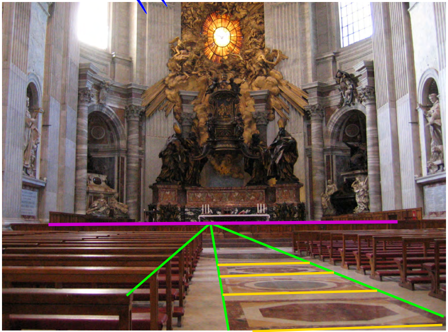
[2] Fischler, M.A. and Bolles, R.C . Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. *Communications of the ACM* 24(6):381–395, 1981

[3] [Golub & van Loan 1996, Sec. 2.5]

Obtaining Vanishing Points and Lines

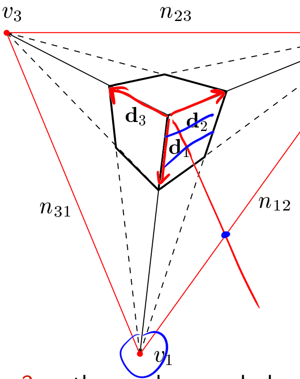
- orthogonal pairs can be collected from more images by camera rotation

- vanishing line can be obtained without vanishing points (see Slide 46)



►Camera Calibration from Vanishing Points and Lines

Problem: Given finite vanishing points and/or vanishing lines, compute $\mathbf{K}$



$$\mathbf{d}_i = \mathbf{Q}^{-1}\underline{\mathbf{v}}_i, \qquad i = 1,2,3 \qquad \text{Slide 33}$$

$$\mathbf{p}_{ij} = \mathbf{Q}^{\top}\underline{\mathbf{n}}_{ij}, \qquad i,j = 1,2,3,\ i \neq j \qquad \text{Slide 36}$$

Constraints

1. orthogonal rays $\mathbf{d}_1 \perp \mathbf{d}_2$ in space then

$$0 = \mathbf{d}_1^{\top}\mathbf{d}_2 = \underline{\mathbf{v}}_1^{\top}\mathbf{Q}^{-\top}\mathbf{Q}^{-1}\underline{\mathbf{v}}_2 = \underline{\mathbf{v}}_1^{\top}\underbrace{(\mathbf{K}\mathbf{K}^{\top})^{-1}}_{\boldsymbol{\omega}\ \text{(IAC)}}\underline{\mathbf{v}}_2$$

2. orthogonal planes $\mathbf{p}_{ij} \perp \mathbf{p}_{ik}$ in space

$$0 = \mathbf{p}_{ij}^{\top}\mathbf{p}_{ik} = \underline{\mathbf{n}}_{ij}^{\top}\mathbf{Q}\mathbf{Q}^{\top}\underline{\mathbf{n}}_{ik} = \underline{\mathbf{n}}_{ij}^{\top}\boldsymbol{\omega}^{-1}\underline{\mathbf{n}}_{ik}$$

3. orthogonal ray and plane $\mathbf{d}_k \parallel \mathbf{p}_{ij}$, $k \neq i,j$ — normal parallel to optical ray

$$\mathbf{p}_{ij} \simeq \mathbf{d}_k \quad \Rightarrow \quad \mathbf{Q}^{\top}\underline{\mathbf{n}}_{ij} = \lambda\mathbf{Q}^{-1}\underline{\mathbf{v}}_k \quad \Rightarrow \quad \underline{\mathbf{n}}_{ij} = \lambda\mathbf{Q}^{-\top}\mathbf{Q}^{-1}\underline{\mathbf{v}}_k = \lambda\boldsymbol{\omega}\,\underline{\mathbf{v}}_k, \qquad \lambda \neq 0$$

- n_{ij} may be constructed from non-orthogonal v_i and v_j, e.g. using the cross-ratio
- $\boldsymbol{\omega}$ is a symmetric, positive definite 3×3 matrix — IAC = Image of Absolute Conic

►cont'd

	condition	constraint	# constraints on ω
(2)	orthogonal v.p.	$\underline{\mathbf{v}}_i^\top \boldsymbol{\omega} \underline{\mathbf{v}}_j = 0$	1
(3)	orthogonal v.l.	$\underline{\mathbf{n}}_{ij}^\top \boldsymbol{\omega}^{-1} \underline{\mathbf{n}}_{ik} = 0$	1
(4)	v.p. orthogonal to v.l.	$\underline{\mathbf{n}}_{ij} = \lambda \boldsymbol{\omega} \underline{\mathbf{v}}_k$	2
(5)	orthogonal raster $\theta = \pi/2$	$\omega_{12} = \omega_{21} = 0$	1
(6)	unit aspect $a = 1$ when $\theta = \pi/2$	$\omega_{11} = \omega_{22}$	1
(7)	known principal point $u_0 = v_0 = 0$	$\omega_{13} = \omega_{31} = \omega_{23} = \omega_{32} = 0$	2

- these are homogeneous linear equations for the 5 parameters in $\boldsymbol{\omega}$ in the form $\mathbf{D}\mathbf{w} = \mathbf{0}$

  λ can be eliminated from (4)

  we will come to solving overdetermined homogeneous equations later → Slide 97
- we need at least 5 constraints for full $\mathbf{K}$
- we get $\mathbf{K}$ from $\boldsymbol{\omega}^{-1} = \mathbf{K}\mathbf{K}^\top$ by Choleski decomposition

  $K = chol(\omega)$

  this trick avoids solving a set of quadratic equations for the parameters in $\mathbf{K}$

In[1]:=
```
K = {{f, s, u[0]}, {0, a * f, v[0]}, {0, 0, 1}};
K // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} f & s & u[0] \\ 0 & a\,f & v[0] \\ 0 & 0 & 1 \end{pmatrix}$$

In[23]:=
```
ω = Inverse[K.Transpose[K]] * Det[K]^2;
ω // Simplify // MatrixForm
```

Out[24]//MatrixForm=

$$\begin{pmatrix} a^2 f^2 & -a\,f\,s & a\,f\,(-a\,f\,u[0] + s\,v[0]) \\ -a\,f\,s & f^2 + s^2 & a\,f\,s\,u[0] - (f^2 + s^2)\,v[0] \\ a\,f\,(-a\,f\,u[0] + s\,v[0]) & a\,f\,s\,u[0] - (f^2 + s^2)\,v[0] & a^2 f^2\,(f^2 + u[0]^2) - 2\,a\,f\,s\,u[0]\,v[0] + (f^2 + s^2)\,v[0]^2 \end{pmatrix}$$

In[29]:=
```
ω / f^2 /. {s → 0} // Simplify // MatrixForm
```

Out[29]//MatrixForm=

$$\begin{pmatrix} a^2 & 0 & -a^2 u[0] \\ 0 & 1 & -v[0] \\ -a^2 u[0] & -v[0] & a^2 (f^2 + u[0]^2) + v[0]^2 \end{pmatrix}$$

In[30]:=
```
% /. a → 1 // MatrixForm
```

Out[30]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & -u[0] \\ 0 & 1 & -v[0] \\ -u[0] & -v[0] & f^2 + u[0]^2 + v[0]^2 \end{pmatrix}$$

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & f^2 \end{bmatrix}$$

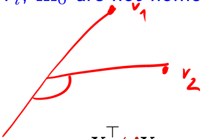
Examples

Ex 1:

Assuming known $m_0 = (u_0, v_0)$, two finite orthogonal vanishing points suffice to get f

in this formula, $\mathbf{v}_i$, $\mathbf{m}_0$ are not homogeneous!

$$(\mathbf{v}_1 - \mathbf{m}_0)^\top (\mathbf{v}_2 - \mathbf{m}_0) + f^2 = 0$$



Ex 2:

Non-orthogonal vanishing points $\mathbf{v}_i$, $\mathbf{v}_j$, known angle ϕ: $\cos\phi = \dfrac{\underline{\mathbf{v}}_i^\top \boldsymbol{\omega} \underline{\mathbf{v}}_j}{\sqrt{\underline{\mathbf{v}}_i^\top \boldsymbol{\omega} \underline{\mathbf{v}}_i}\sqrt{\underline{\mathbf{v}}_j^\top \boldsymbol{\omega} \underline{\mathbf{v}}_j}}$

- leads to polynomial equations
- e.g. assuming orthogonal raster, unit aspect (ORUA): $a = 1$, $\theta = \pi/2$

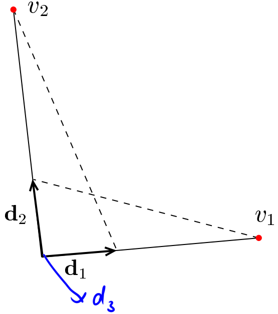
$$\boldsymbol{\omega} = \frac{1}{f^2}\begin{bmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ -u_0 & -v_0 & f^2 + u_0^2 + v_0^2 \end{bmatrix}$$

- ORUA and $u_0 = v_0 = 0$ gives

$$(f^2 + \mathbf{v}_i^\top \mathbf{v}_j)^2 = (f^2 + \|\mathbf{v}_i\|^2) \cdot (f^2 + \|\mathbf{v}_j\|^2) \cdot \cos^2\phi$$

►Camera Orientation from Vanishing Points

Problem: Given $\mathbf{K}$ and two vanishing points corresponding to two known orthogonal directions $\mathbf{d}_1$, $\mathbf{d}_2$, compute camera orientation $\mathbf{R}$ with respect to the plane.



- coordinate system choice, e.g.:

$$\mathbf{d}_1 = (1, 0, 0), \quad \mathbf{d}_2 = (0, 1, 0)$$

- we know that

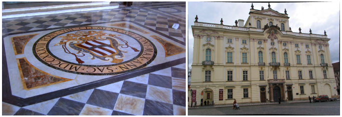
$$\mathbf{d}_i \simeq \mathbf{Q}^{-1}\underline{\mathbf{v}}_i = (\mathbf{K}\mathbf{R})^{-1}\underline{\mathbf{v}}_i = \mathbf{R}^{-1}\underbrace{\mathbf{K}^{-1}\underline{\mathbf{v}}_i}_{\underline{\mathbf{w}}_i}$$

$$\mathbf{R}\mathbf{d}_i \simeq \underline{\mathbf{w}}_i$$

- then $\underline{\mathbf{w}}_i / \|\underline{\mathbf{w}}_i\|$ is the i–th column $\mathbf{r}_i$ of $\mathbf{R}$
- the third column is orthogonal:
  $\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$

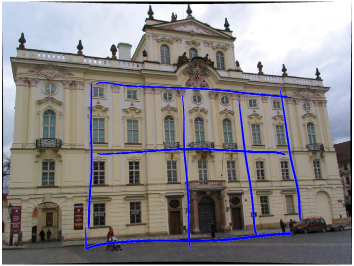
$$\mathbf{R} = \begin{bmatrix} \frac{\underline{\mathbf{w}}_1}{\|\underline{\mathbf{w}}_1\|} & \frac{\underline{\mathbf{w}}_2}{\|\underline{\mathbf{w}}_2\|} & \frac{\underline{\mathbf{w}}_1 \times \underline{\mathbf{w}}_2}{\|\underline{\mathbf{w}}_1 \times \underline{\mathbf{w}}_2\|} \end{bmatrix}$$

some suitable scenes



Application: Planar Rectification

Principle: Rotate camera parallel to the plane of interest.



$$\underline{\mathbf{m}} \simeq \mathbf{K}\mathbf{R}\,[\mathbf{I} \quad -\mathbf{C}]\,\underline{\mathbf{X}} \qquad\qquad \underline{\mathbf{m}}' \simeq \mathbf{K}\,[\mathbf{I} \quad -\mathbf{C}]\,\underline{\mathbf{X}}$$

$H = K R^{\top} K^{-1}$

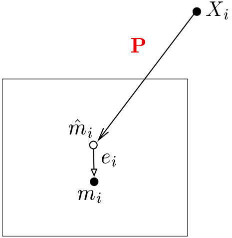
$$\underline{\mathbf{m}}' \simeq \mathbf{K}(\mathbf{K}\mathbf{R})^{-1}\,\underline{\mathbf{m}} = \mathbf{K}\mathbf{R}^{\top}\mathbf{K}^{-1}\,\underline{\mathbf{m}} = \mathbf{H}\,\underline{\mathbf{m}}$$

- $\mathbf{H}$ is the rectifying homography
- both $\mathbf{K}$ and $\mathbf{R}$ can be calibrated from two finite vanishing points
- not possible when one (or both) of them are infinite

▶Camera Resectioning

Camera calibration and orientation from a known set of $k \geq 6$ reference points and their images $\{(X_i, m_i)\}_{i=1}^{6}$.

$\hat{m}_i = P X_i$



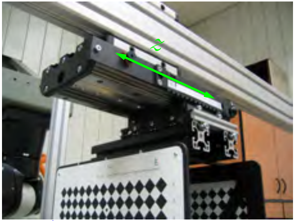
X_i is considered exact

m_i is a measurement

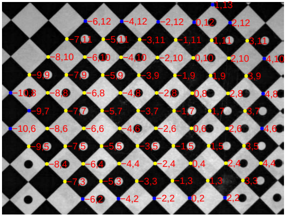
$e_i^2 = \|\mathbf{m}_i - \hat{\mathbf{m}}_i\|^2$

where $\underline{\hat{\mathbf{m}}}_i \simeq \mathbf{P}\underline{\mathbf{X}}_i$

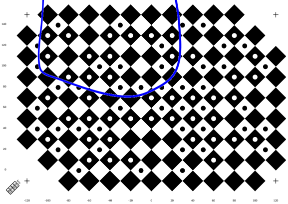
projection error



calibration target with translation stage



automatic calibration point detection



calibration chart

▶The Minimal Problem for Resectioning

Problem: Given $k = 6$ corresponding pairs $\{(X_i, m_i)\}_{i=1}^{k}$, find $\mathbf{P}$

$m_i \simeq P X_i$ finite points! easy to modify

$$\lambda_i \underline{\mathbf{m}}_i = \mathbf{P}\underline{\mathbf{X}}_i, \qquad \mathbf{P} = \begin{bmatrix} \mathbf{q}_1^\top & q_{14} \\ \mathbf{q}_2^\top & q_{24} \\ \mathbf{q}_3^\top & q_{34} \end{bmatrix} \qquad \begin{aligned} \underline{\mathbf{X}}_i &= (x_i, y_i, z_i, 1), \quad i = 1, 2, \ldots, k, \; k = 6 \\ \underline{\mathbf{m}}_i &= (u_i, v_i, 1), \quad \lambda_i \in \mathbb{R}, \; \lambda_i \neq 0 \end{aligned}$$

expanded: $\lambda_i u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad \lambda_i v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}, \quad \lambda_i = \mathbf{q}_3^\top \mathbf{X}_i + q_{34}$

eliminating λ gives: $(\mathbf{q}_3^\top \mathbf{X}_i + q_{34}) u_i = \mathbf{q}_1^\top \mathbf{X}_i + q_{14}, \quad (\mathbf{q}_3^\top \mathbf{X}_i + q_{34}) v_i = \mathbf{q}_2^\top \mathbf{X}_i + q_{24}$

Then

$$\mathbf{A}\,\mathbf{q} = \begin{bmatrix} \mathbf{X}_1^\top & 1 & \mathbf{0}^\top & 0 & -u_1 \mathbf{X}_1^\top & -u_1 \\ \mathbf{0}^\top & 0 & \mathbf{X}_1^\top & 1 & -v_1 \mathbf{X}_1^\top & -v_1 \\ \vdots & & & & & \vdots \\ \mathbf{X}_k^\top & 1 & \mathbf{0}^\top & 0 & -u_k \mathbf{X}_k^\top & -u_k \\ \mathbf{0}^\top & 0 & \mathbf{X}_k^\top & 1 & -v_k \mathbf{X}_k^\top & -v_k \end{bmatrix} \cdot \begin{bmatrix} \mathbf{q}_1 \\ q_{14} \\ \mathbf{q}_2 \\ q_{24} \\ \mathbf{q}_3 \\ q_{34} \end{bmatrix} = \mathbf{0} \qquad (8)$$

12 elements

- we need 11 indepedent parameters for $\mathbf{P}$
- $\mathbf{A} \in \mathbb{R}^{2k,12}$, $\mathbf{q} \in \mathbb{R}^{12}$
- 6 points in a general position give $\operatorname{rank} \mathbf{A} = 12$ and there is no non-trivial null space
- drop one row to get rank 11 matrix, then the basis of the null space of $\mathbf{A}$ gives $\mathbf{q}$

▶The Jack-Knife Solution for $k = 6$

- given the 6 correspondences, we have 12 equations for the 11 parameters
- can we use all the information present in data?

Jack-knife estimation

1. $n := 0$
2. for $i = 1, 2, \ldots, 2k$ do
   a. delete i-th row from $\mathbf{A}$, this gives $\mathbf{A}_i$
   b. if $\dim \operatorname{null} \mathbf{A}_i > 1$ continue with the next i
   c. $n := n + 1$
   d. compute the right null-space $\mathbf{q}_i$ of $\mathbf{A}_i$ — e.g. by 'economy-size' SVD
   e. normalize $\mathbf{q}_i$ to $\hat{\mathbf{q}}_i = \mathbf{q}_i / q_{12}$ — this assumes finite camera with $P_{3,3} = 1$
3. from all n vectors $\hat{\mathbf{q}}_i$ collected in Step 1d compute

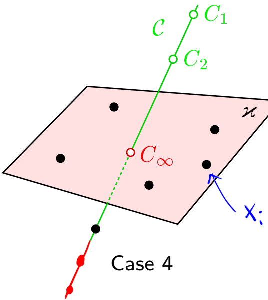
$$\mathbf{q} = \frac{1}{n} \sum_{i=1}^{n} \hat{\mathbf{q}}_i, \qquad \operatorname{var}[\mathbf{q}] = \frac{n-1}{n} \operatorname{diag} \sum_{i-1}^{n} (\hat{\mathbf{q}}_i - \mathbf{q})(\hat{\mathbf{q}}_i - \mathbf{q})^\top$$

- have a solution + an error estimate, per individual elements of $\mathbf{P}$
- at least 5 points must be in a general position — see Slide 67
- large error indicates near degeneracy
- computation not efficient with $k > 6$ points, needs $\binom{2k}{11}$ draws, e.g. $k = 7 \to 364$ draws
- one needs $k \geq 7$ for the full covariance matrix
- better error estimation method: decompose $\mathbf{P}_i$ to $\mathbf{K}_i$, $\mathbf{R}_i$, $\mathbf{C}_i$ (Slide 30), represent $\mathbf{R}_i$ with 3 parameters (e.g. Euler angles, or in Cayley representation, see Slide 144) and compute the errors for the parameters

▶Degenerate (Critical) Configurations for Resectioning

Let $\mathcal{X} = \{X_i;\, i = 1, \ldots\}$ be a set of points and $\mathbf{P}_1 \not\simeq \mathbf{P}_2$ be two regular (rank-3) cameras. Then two configurations $(\mathbf{P}_1, \mathcal{X})$ and $(\mathbf{P}_2, \mathcal{X})$ are image-equivalent if

$$\mathbf{P}_1 \underline{\mathbf{X}}_i \simeq \mathbf{P}_2 \underline{\mathbf{X}}_i \quad \text{for all} \quad X_i \in \mathcal{X}$$



Case 4

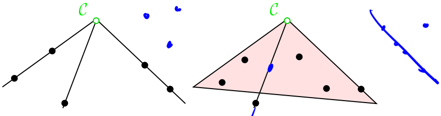
- if all calibration points $X_i \in \mathcal{X}$ lie on a plane $\varkappa$ the camera resectioning is non-unique and all image-equivalent camera centers lie on a spatial line $\mathcal{C}$ with the $C_\infty = \varkappa \cap \mathcal{C}$ excluded
- by adding points $X_i \in \mathcal{X}$ to $\mathcal{C}$ we gain nothing
- there are additional image-equivalent configurations, see next

see proof sketch in the notes or in [H&Z, Sec. 22.1.2]

Note that if $\mathbf{Q}$, $\mathbf{T}$ are suitable non-singular homographies then $\mathbf{P}_1 \simeq \mathbf{Q}\mathbf{P}_0\mathbf{T}$, where $\mathbf{P}_0$ is canonical and

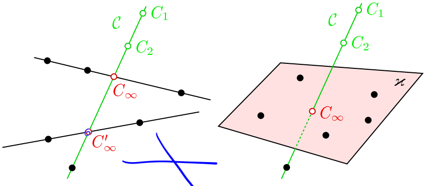
$$\mathbf{P}_0 \underbrace{\mathbf{T}\underline{\mathbf{X}}_i}_{\underline{\mathbf{Y}}_i} \simeq \mathbf{P}_2 \underbrace{\mathbf{T}\underline{\mathbf{X}}_i}_{\underline{\mathbf{Y}}_i} \quad \text{for all} \quad Y_i \in \mathcal{Y}$$

cont'd (all cases)



Case 5 Case 6

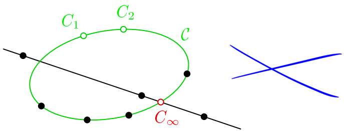
- cameras C_1, C_2 co-located at point $\mathcal{C}$
- points on three optical rays or one optical ray and one optical plane
- Case 5: we see 3 isolated point images
- Case 6: we see a line of points and an isolated point



Case 3 **Case 4**

- cameras lie on a line $\mathcal{C} \setminus \{C_\infty, C'_\infty\}$
- points lie on $\mathcal{C}$ and
  1. on two lines meeting $\mathcal{C}$ at C_∞, C'_∞
  2. or on a plane meeting $\mathcal{C}$ at C_∞
- Case 3: we see 2 lines of points

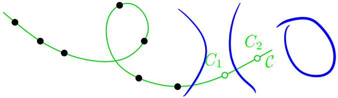
Case 2



- cameras lie on a planar conic $\mathcal{C} \setminus \{C_\infty\}$

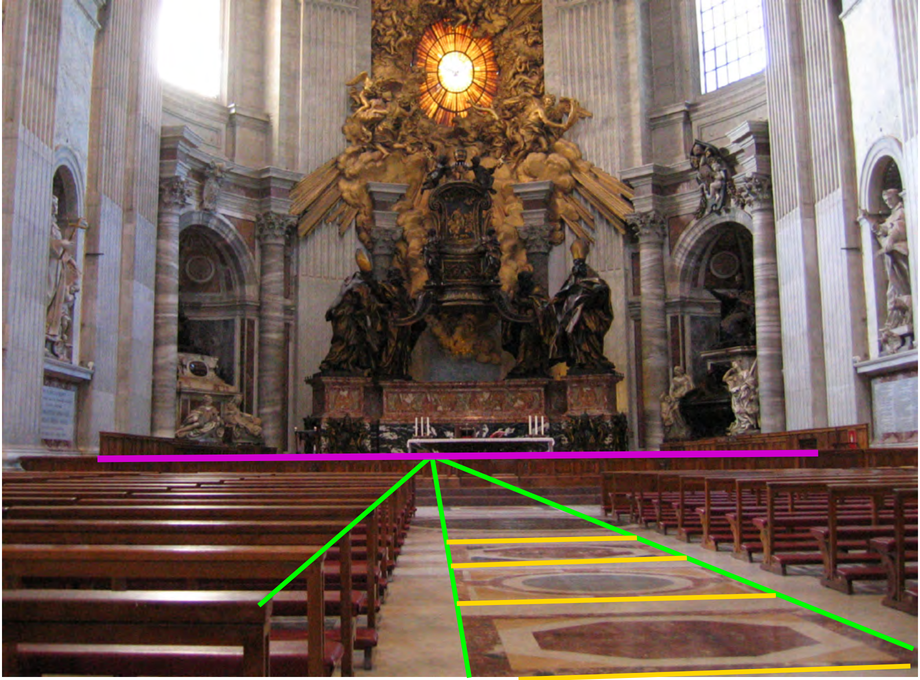
  not necessarily an ellipse
- points lie on $\mathcal{C}$ and an additional line meeting the conic at C_∞
- Case 2: we see 2 lines of points

Case 1



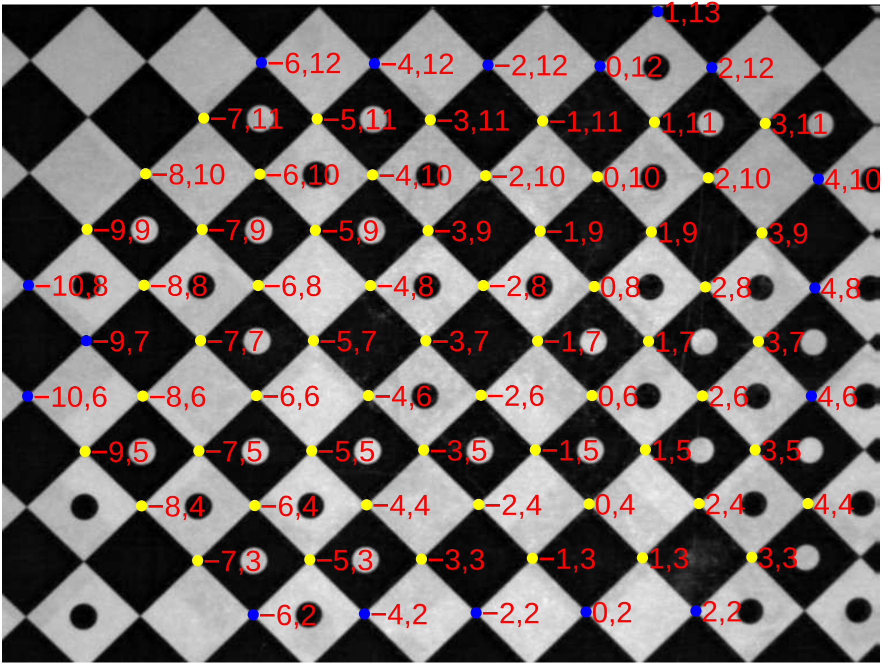
- cameras and points all lie on a twisted cubic $\mathcal{C}$
- Case 1: we see a conic

Thank You




CLEMENS·XII·PONT·MAX·ANN·SAC·MDCCXXXVII PONT·VIII


BOOKS
FILMS
TICKETS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