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AE4M33RZN, Fuzzy logic: Examples in fuzzy DL

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Plan of the lecture

Fuzzy DL examples

Concrete data types

Ex: Jim revisited

We will use the Lukasiewicz logic in the following examples ($\sqcap = \underset{L}{\sqcap}$, ...).

$$\langle jim : \text{Male} \,|\, 0.9 \rangle \quad (1)$$

$$\langle jim : \text{Female} \,|\, 0.2 \rangle \quad (2)$$

$$\langle \text{Male} \sqcap \text{Female} \sqsubseteq \bot \,|\, 1 \rangle \quad (3)$$

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We will use the Lukasiewicz logic in the following examples ($\sqcap = \sqcap_{\text{L}}$, ...).

$$\langle jim : \text{Male} \mid 0.9 \rangle \tag{1}$$
$$\langle jim : \text{Female} \mid 0.2 \rangle \tag{2}$$
$$\langle \text{Male} \sqcap \text{Female} \sqsubseteq \bot \mid 1 \rangle \tag{3}$$

The interpretation domain is $\Delta^{\mathcal{I}_1} = \Delta^{\mathcal{I}_2} = \{j\}, jim^{\mathcal{I}_1} = jim^{\mathcal{I}_2} = j$.

$\text{Male}^{\mathcal{I}_1} = \{(j; 0.9)\}$ $\qquad$ $\text{Male}^{\mathcal{I}_2} = \{(j; 0.9)\}$

$\text{Female}^{\mathcal{I}_1} = \{(j; 0)\}$ $\qquad$ $\text{Female}^{\mathcal{I}_2} = \{(j; 0.2)\}$

Ex: Jim revisited (check your knowledge)

Let's check the interpretation against the definitions...

$\mathscr{I} \models \tau$	$\tau_{(1)}$	$\tau_{(2)}$	$\tau_{(3)}$
$\mathscr{I}_1$	?	?	?
$\mathscr{I}_2$	?	?	?

Ex: Jim revisited (check your knowledge)

Let's check the interpretation against the definitions...

$\mathscr{I} \models \tau$	$\tau_{(1)}$	$\tau_{(2)}$	$\tau_{(3)}$
$\mathscr{I}_1$	yes	no	yes
$\mathscr{I}_2$	yse	yes	no

Ex: Jim revisited (in fuzzyDL)

Let's change the weights and encode the example in fuzzyDL:

```
(instance jim Male 0.4)
(instance jim Female 0.2)

(l-implies (and Male Female) *bottom* 0.9)

(min-instance? jim Male)
(max-instance? jim Male)
(min-instance? jim Female)
(max-instance? jim Female)
```

Let $\langle jim : \text{Male} \,|\, \alpha \rangle$ and $\langle jim : \text{Female} \,|\, \beta \rangle$, what are the bounds on α and β? fuzzyDL shows that $0.4 \leqslant \alpha \leqslant 0.9$ and $0.2 \leqslant \beta \leqslant 0.7$. Why?

Ex: Smokers

Recall the motivational example from the first lecture:

$$\langle \text{symmetric}(\text{friend}) \rangle \tag{4}$$

$$\langle (anna, bill) : \text{friend} \,|\, 1 \rangle \tag{5}$$

$$\langle (bill, cloe) : \text{friend} \,|\, 1 \rangle \tag{6}$$

$$\langle (cloe, dirk) : \text{friend} \,|\, 1 \rangle \tag{7}$$

$$\langle anna : \text{Smoker} \,|\, 1 \rangle \tag{8}$$

$$\langle \exists\, \text{friend} \cdot \text{Smoker} \sqsubseteq \text{Smoker} \,|\, 0.7 \rangle \tag{9}$$

What are the bounds on $\langle i : \text{Smoker} \rangle$ for $i \in \{anna, bill, cloe, dirk\}$?

Ex: Smokers

What changes if we add

$$\langle dirk : \neg \text{Smoker} \,|\, 0.7 \rangle \tag{10}$$

(11)

What are the bounds on $\langle i : \neg \text{Smoker} \rangle$ for $i \in \{anna, bill, cloe, dirk\}$?

Ex: Smokers (in fuzzyDL)

```
(implies (some friendOf Smoker) Smoker 0.7)

(symmetric friendOf)
(related anna bill friendOf)
(related bill cloe friendOf)
(related cloe dirk friendOf)

(instance anna Smoker)
(instance dirk (not Smoker) 0.7)

(min-instance? anna Smoker)
(min-instance? bill Smoker)
(min-instance? cloe Smoker)
(min-instance? dirk Smoker)

(max-instance? anna Smoker)
(max-instance? bill Smoker)
(max-instance? cloe Smoker)
(max-instance? dirk Smoker)
```

Concrete data types

The domain $\Delta^{\mathcal{I}}$ is an unordered set. This is good for modelling cathegorical data: e.g. colors, people, ...

General idea: Extended interpretation

But we also need to include real numbers $\mathbb{R}$. The *fuzzy description logic with concrete datatypes* $\mathcal{SHIF}(\mathcal{D})$ uses "abstract objects" and "concrete objects":

$$\Delta^{\mathcal{I}} = \Delta_a^{\mathcal{I}} \cup \mathbb{R}$$

Concrete data types

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All non-concrete notions are called *abstract*.

Concrete data types: New concepts

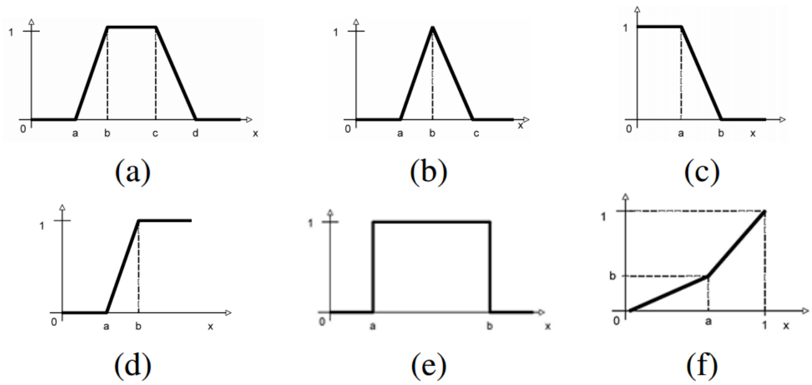

Fig. 1. (a) Trapezoidal function; (b) Triangular function; (c) L-function; (d) R-function; (e) Crisp interval; (f) Linear function.

Ex: Age of parents

```
(related adam bob parent) (related adam eve parent)

(define-fuzzy-concept around23 triangular(0,100, 18,23,26))
(define-fuzzy-concept moreTh17 right-shoulder(0,100, 13,21))
(instance bob (some age around23) 0.9)
(instance eve (some age moreTh17))

(define-fuzzy-concept young left-shoulder(0,100, 17,25))
(define-concept YoungPerson (some age young))

(min-instance? eve YoungPerson) (max-instance? eve YoungPerson)
(min-instance? bob YoungPerson) (max-instance? bob YoungPerson)
(min-instance? adam (all  parent YoungPerson))
(max-instance? adam (all  parent YoungPerson))
(min-instance? adam (some parent YoungPerson))
(max-instance? adam (some parent YoungPerson))
```

Ex: Age of parents

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Ex: Age of parents

1. What are the bounds on α from $\langle \mathit{eve} : \mathrm{YoungPerson} \mid \alpha \rangle$?

   Start by drawing the concept around23, then construct an interpretation. How much freedom do you have when constructing the interpretation?

2. Let fuzzyDL reasoner give you both bounds on $\langle i : \mathrm{YoungPerson} \mid \beta_i \rangle$ for $i \in \{\mathit{eve}, \mathit{bob}\}$.

   How do you infer the bounds on $\langle \mathit{adam} : \mathrm{YoungPerson} \mid \gamma \rangle$?

Ex: Car dealing

1. The buyer wants a **passenger** that costs **less than €26000**.
2. If there is an **alarm system** in the car, **then** he is satisfied with paying no more than **€22300**, but he can go up to **€22750** with a lesser degree of satisfaction.
3. The **driver insurance**, **air conditioning** and the **black color** are important factors.
4. Preferably the price is no more than **€22000**, but he can go to **€24000** to a lesser degree of satisfaction.

Ex: Car dealing

1. The seller wants to sell no less than **€22000**.
2. Preferably the buyer buys the **insurance plus** package.
3. If the **color is black**, then it is highly possible the car has an **air-conditioning**.

This can be formalized in fuzzy description logic.

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This can be formalized in fuzzy description logic.
We have the background knowledge:

$$\langle \mathsf{Sedan} \sqsubseteq \mathsf{PassengerCar} \,|\, 1 \rangle$$

$$\langle \mathsf{InsurancePlus} = \mathsf{DriverInsurance} \sqcap \mathsf{TheftInsurance} \,|\, 1 \rangle$$

Ex: Car dealing

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3. $B_2 = \text{DriverInsurance}, B_3 = \text{AirCondition}, B_4 = \exists \text{color} \cdot \text{Black}$
4. $B_5 = \exists \text{price} \cdot l.sh.(22000, 24000)$

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3. $S_2 = (0.5 \ (\exists \text{color} \cdot \text{Black}) \mapsto \text{AirCondition})$

Ex: Car dealing

We know that S and B are hard constraints and $B_{1..5}$ and $S_{1..2}$ are soft preferences. All the concepts can be “summed up”:

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$$\text{Buy} = B \sqcap (0.1B_1 + 0.2B_2 + 0.1B_3 + 0.4B_4 + 0.2B_5)$$

and

$$\text{Sell} = S \sqcap (0.6S_1 + 0.4S_2)$$

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and

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A good choice of $\sqcap$ can make B a hard constraint.

Ex: Car dealing

Optimal match

$$bsb(K, \text{Buy} \sqcap \text{Sell})$$

Finds the optimal match between a seller and a buyer. (Finds an ideal, imaginary car that maximizes satisfaction of both parties.)

Particular car

$$bdb(K, \langle auditTT : \text{Buy} \sqcap \text{Sell} \rangle)$$

Finds the degree of satisfaction for a particuklar car *audiTT*.

Where to find more examples?

- **Simple examples** are bundled with fuzzyDL installation (`/opt/fuzzydl/` on the `heartofgold` server).
- **Advanced examples** can be found on the fuzzyDL web site: `http://gaia.isti.cnr.it/~straccia/software/fuzzyDL/fuzzyDL.html`

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