

Normalized cross-correlation

You may know it as correlation coefficients

$$r_{ij} = \frac{c_{ij}}{\sqrt{c_{ii}c_{jj}}}$$

where $c_{i,j}$ are elements of the covariance matrix

Having template $T(x, y)$ and image $I(x, y)$,

$$r(x, y) = \frac{\sum_k \sum_l (T(k, l) - \bar{T}) (I(x + k, y + l) - \overline{I(x, y)})}{\sqrt{\sum_k \sum_l (T(k, l) - \bar{T})^2 \sum_k \sum_l (I(x + k, y + l) - \overline{I(x, y)})^2}}$$

Be careful about coordinate systems (sketch on blackboard)

ncc - remind the notes from statistics ...

Image intensities, though organized in a matrix form, can be re-arranged into vectors. Best visualised with plots. Remember *variance, correlation*?

Sketch about coordinate systems

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x}; \mathbf{p} + \Delta \mathbf{p})) - T(\mathbf{x})]^2$$

linearized by performing first order Taylor expansion⁵

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x}; \mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x})]^2$$

$\nabla I = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}]$ is the **gradient** image⁶ computed at $\mathbf{W}(\mathbf{x}; \mathbf{p})$. The term $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ is the **Jacobian** of the warp.

⁵Detailed explanation on the blackboard.

⁶As a vector it should have been a column wise oriented. However, for sake of clarity of equations row vector is exceptionally considered here.

Taylor series and gradient of a compound funtion

Few notes that may help in understanding of the derivation. General first order Taylor series expansion of a scalar-valued function f of more than one variable (\mathbf{x}, \mathbf{a} are vectors):

$$T(\mathbf{x}) = f(\mathbf{a}) + (\mathbf{x} - \mathbf{a})^\top Df(\mathbf{a}), \quad (1)$$

where $Df(\mathbf{a})$ is the *gradient* of f evaluated at $\mathbf{x} = \mathbf{a}$.

Gradient of a function f is a vector of partial derivatives

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots \right)^\top \quad (2)$$

Gradient of a compound funtion (chain rule). Suppose that $f : A \rightarrow R$ is a real-valued function defined on a subset A of R^n , and that f is differentiable at a point \mathbf{a} . If function g is also differntiable and $g(\mathbf{c}) = \mathbf{a}$ then for tha gradient of the compound function hold

$$D(f \circ g)(\mathbf{c}) = (Dg(\mathbf{c}))^\top \nabla f(\mathbf{a}), \quad (3)$$

where $(Dg)^\top$ denotes the transpose of the *Jacobian matrix*.

Our problem is the linearization of the multidimensional warp that affects one pixel at a position $\mathbf{x} = (x, y)^\top$, $I(W(\mathbf{x}, \mathbf{p} + \Delta \mathbf{p}))$. Taylor expansion (1) becomes

$$T(\mathbf{p}^{k+1}) = I(\mathbf{p}^k) + \Delta \mathbf{p}^\top DI(\mathbf{p}^k) \quad (4)$$

The inside function is the geometric warp hence, $g(\mathbf{p}) = W(\mathbf{x}, \mathbf{p})$. From the above it follows that the linearization is

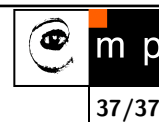
$$I(W(\mathbf{x}, \mathbf{p} + \Delta \mathbf{p})) \approx I(W(\mathbf{x}, \mathbf{p})) + \Delta \mathbf{p}^\top \frac{\partial W}{\partial \mathbf{p}}^\top \nabla I, \quad (5)$$

where

$$\nabla I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)^\top, \quad (6)$$

is the image gradient and $\frac{\partial W}{\partial \mathbf{p}}$ is the Jacobian of the warp.

End



References