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AdaBoost

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Presentation

Outline:

- AdaBoost algorithm
  - Why is of interest?
  - How it works?
  - Why it works?
- AdaBoost variants

What is AdaBoost?

AdaBoost is an algorithm for constructing a "strong" classifier as linear combination

$$f(x) = \sum_{t=1}^{T} \alpha_t h_t(x)$$

of "simple" "weak" classifiers $h_t(x)$.

Terminology

- $h_t(x)$ ... "weak" or basis classifier, hypothesis, "feature"
- $H(x) = sign(f(x))$ ... ''strong" or final classifier/hypothesis

Interesting properties

- AB is a linear classifier with all its desirable properties.
- AB output converges to the logarithm of likelihood ratio.
- AB has good generalisation properties.
- AB is a feature selector with a principled strategy (minimisation of upper bound on empirical error).
- AB close to sequential decision making (it produces a sequence of gradually more complex classifiers).

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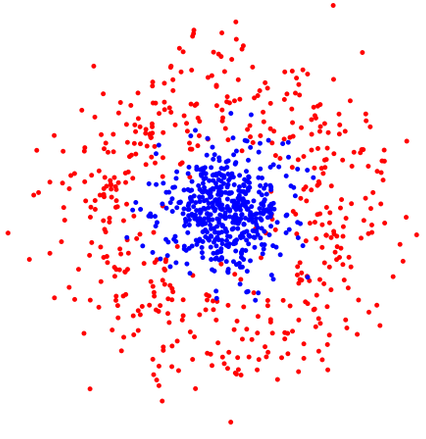

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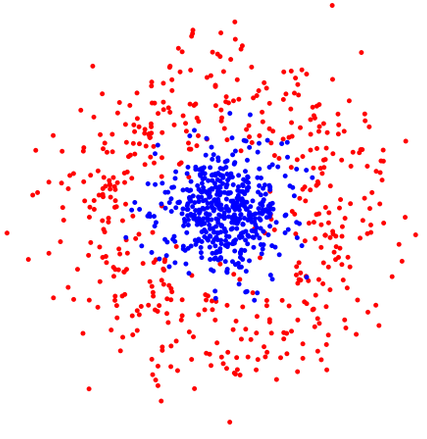

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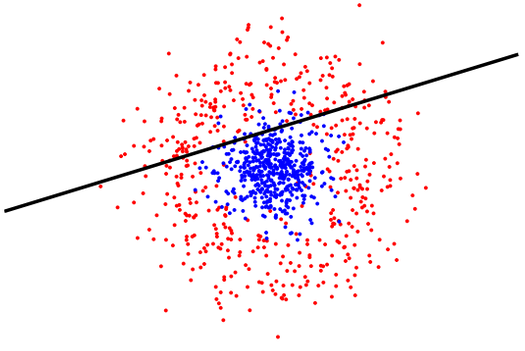
$t = 1$



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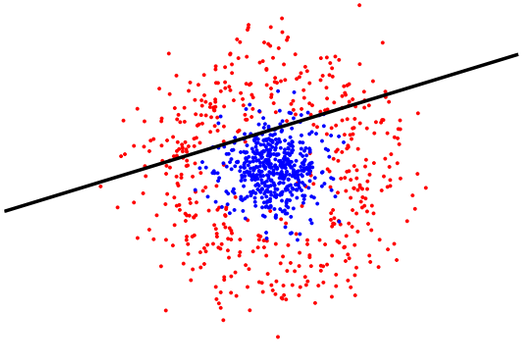

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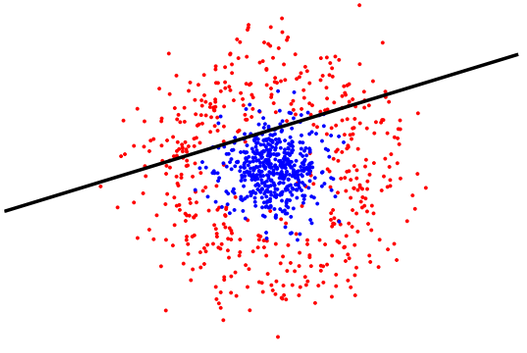

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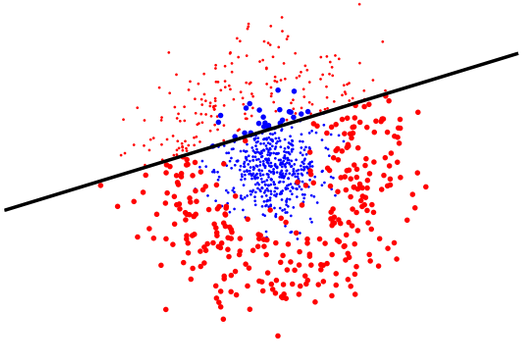

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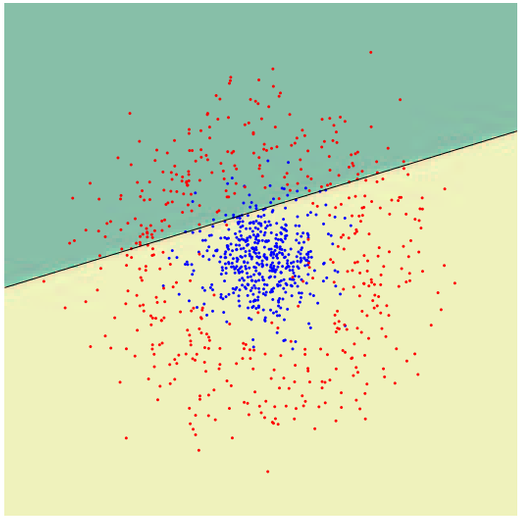

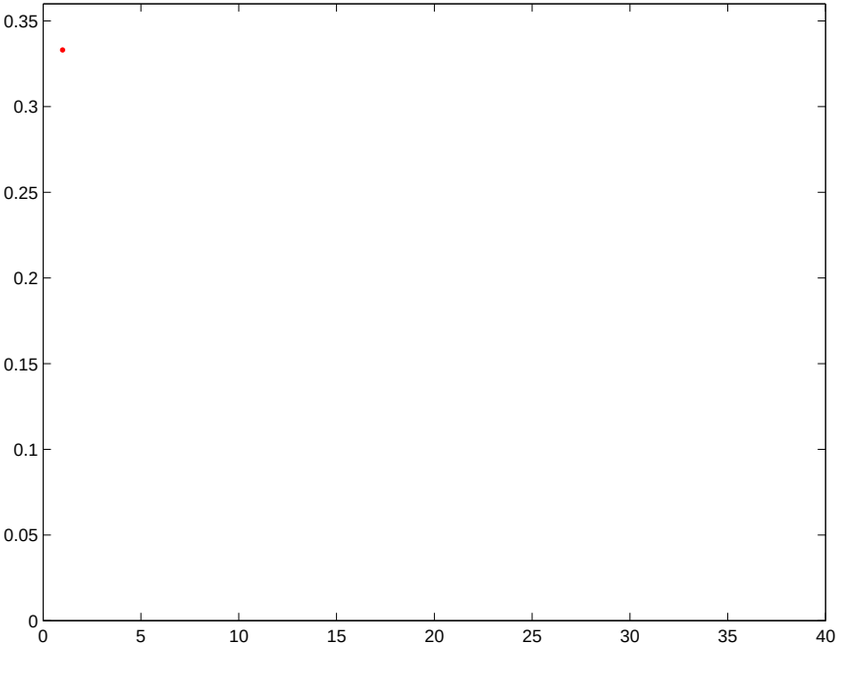

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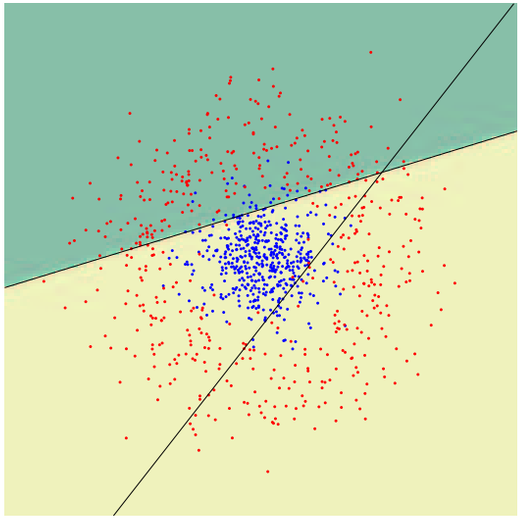

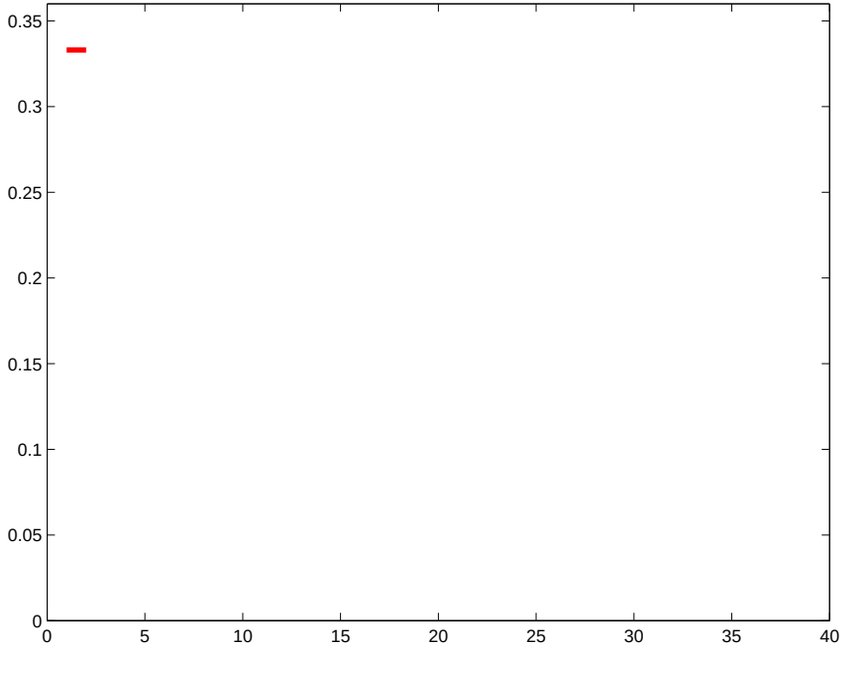

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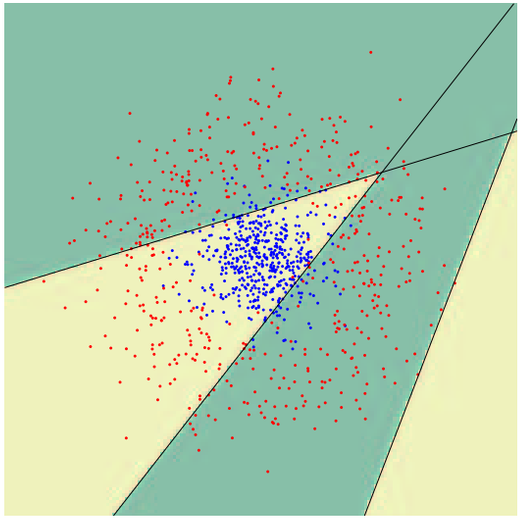
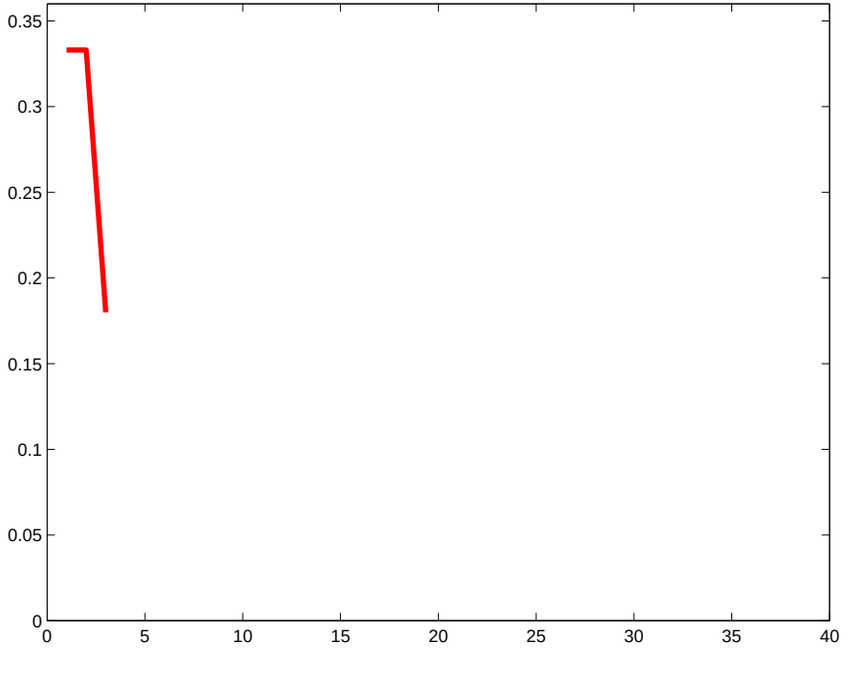
$t = 3$





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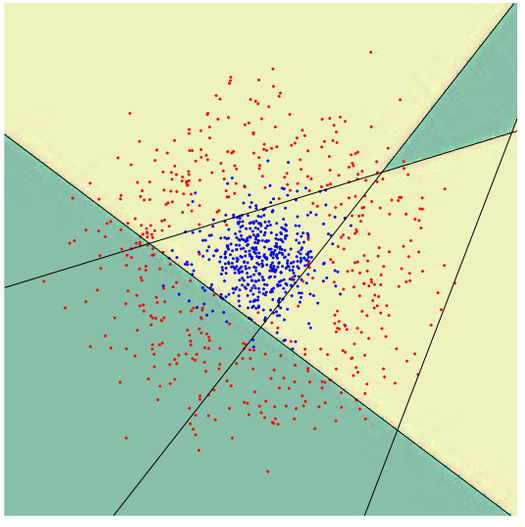
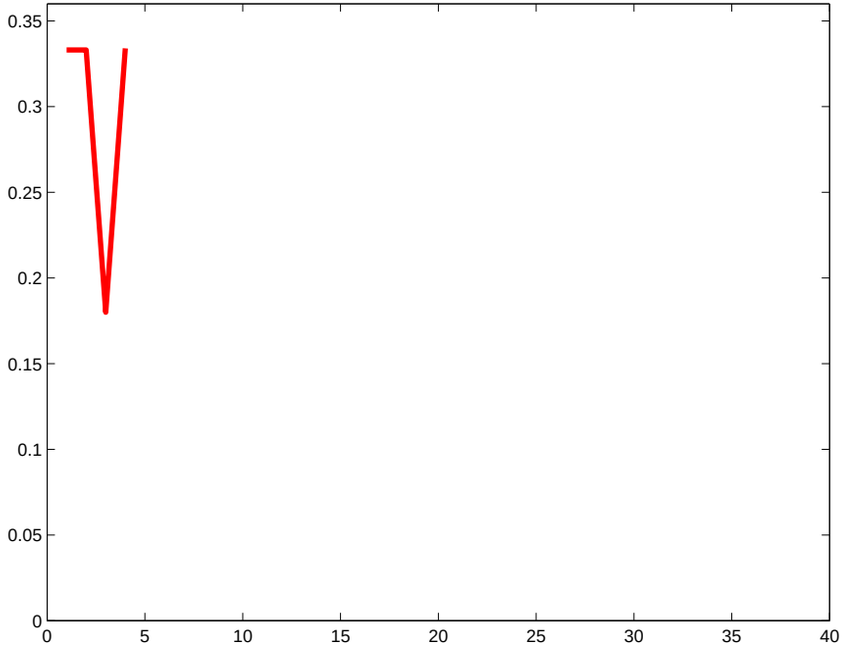
$t = 4$





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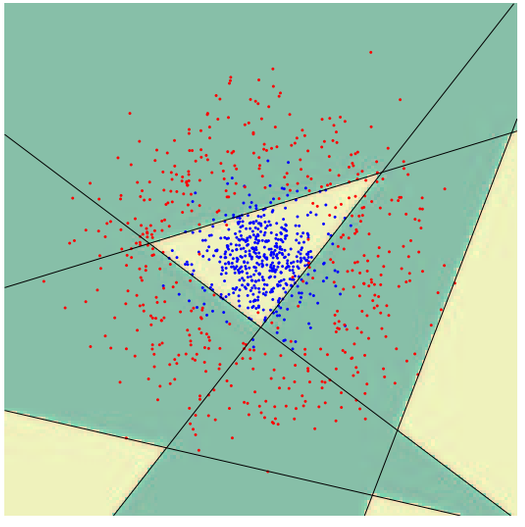
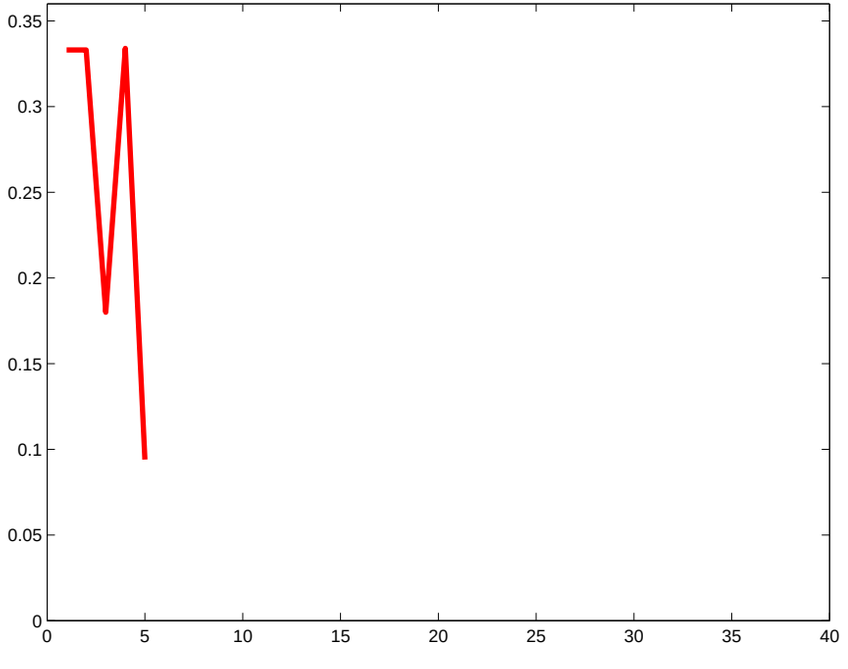
$t = 5$





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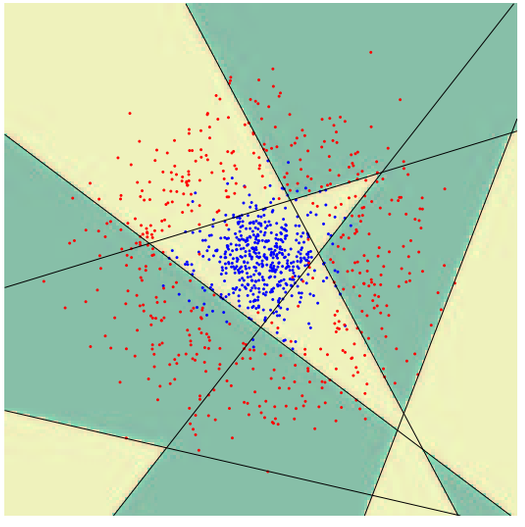

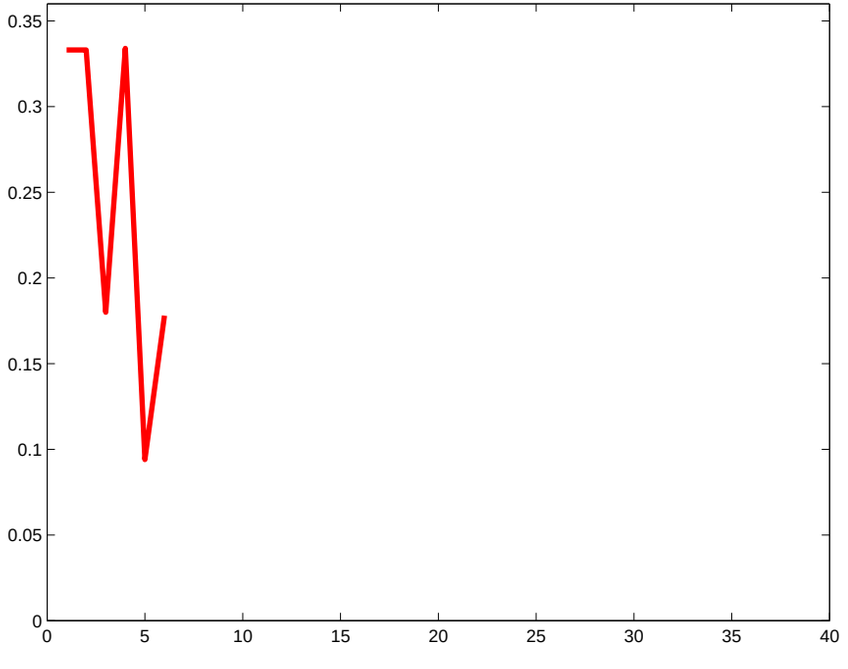

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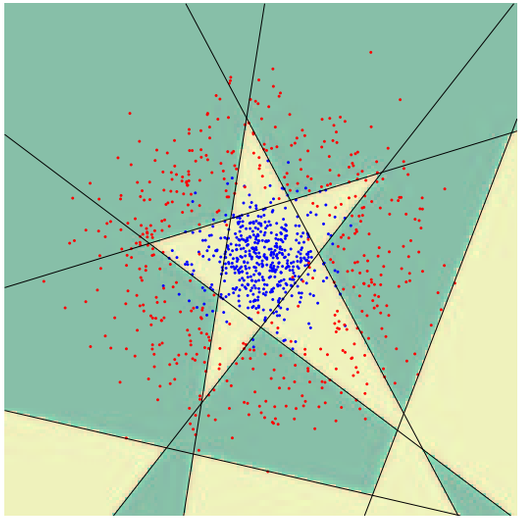

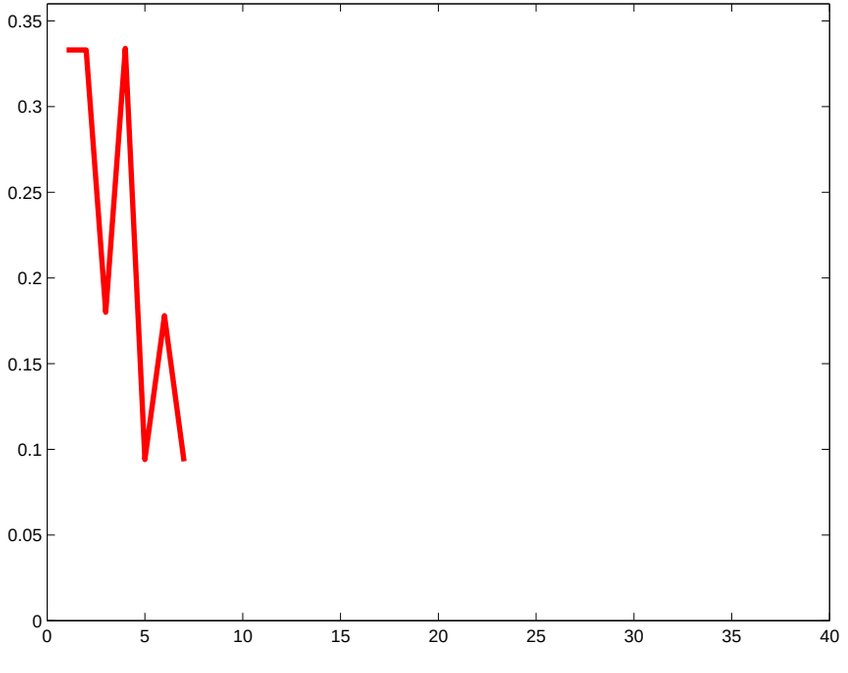

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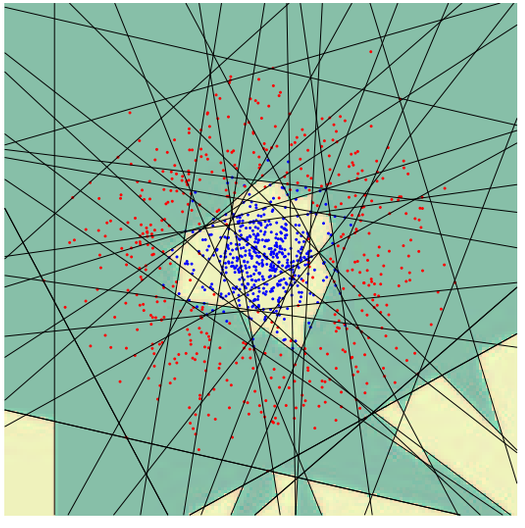
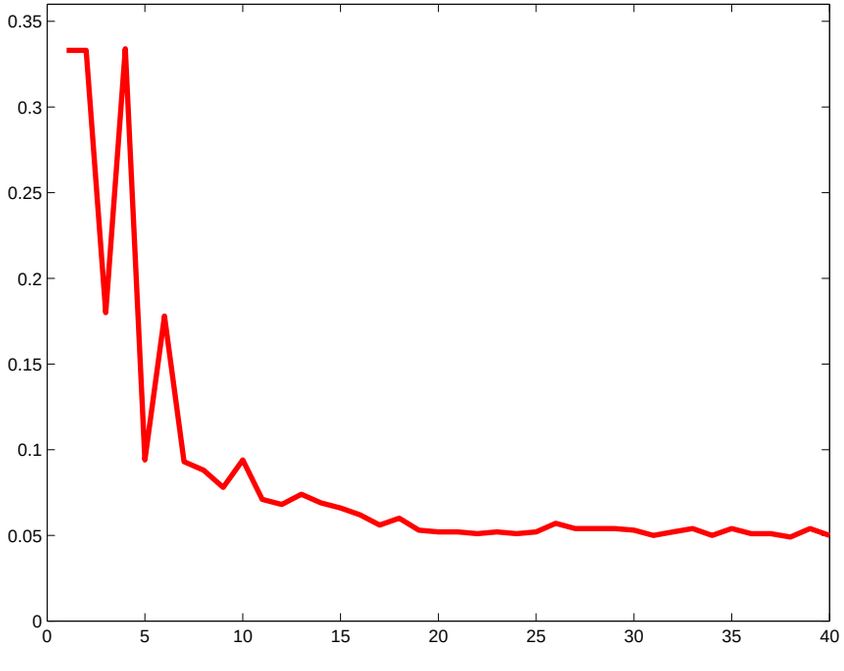
$t = 40$



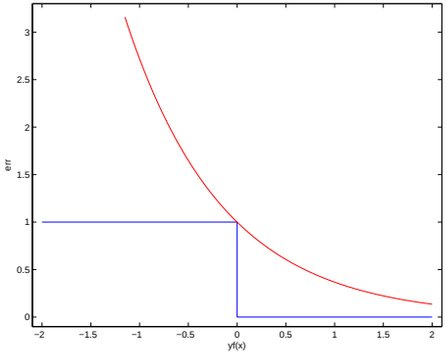


Reweighting

Effect on the training set

$$D_{t+1}(i) = \frac{D_t(i) exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

$$exp(-\alpha_t y_i h_t(x_i)) \begin{cases} < 1, & y_i = h_t(x_i) \\ > 1, & y_i \neq h_t(x_i) \end{cases}$$



$\Rightarrow$ Increase (decrease) weight of wrongly (correctly) classified examples

$\Rightarrow$ The weight is the upper bound on the error of a given example.

$\Rightarrow$ All information about previously selected "features" is captured in D_t.

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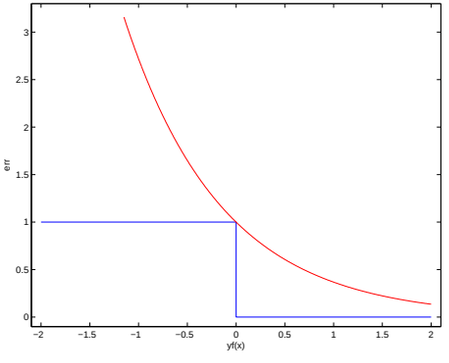

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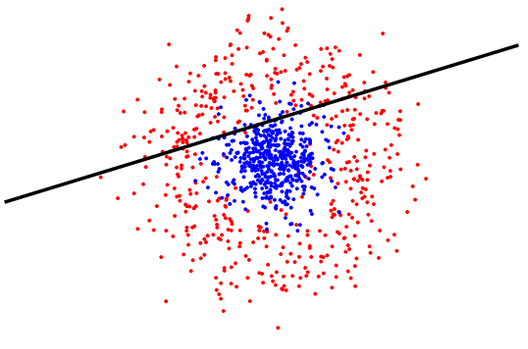

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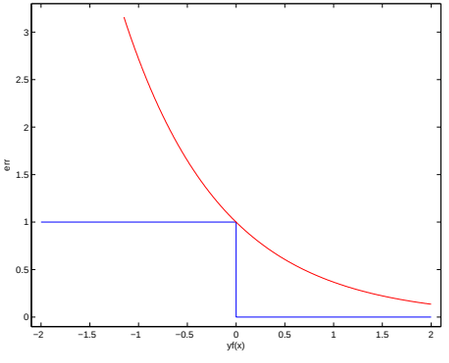

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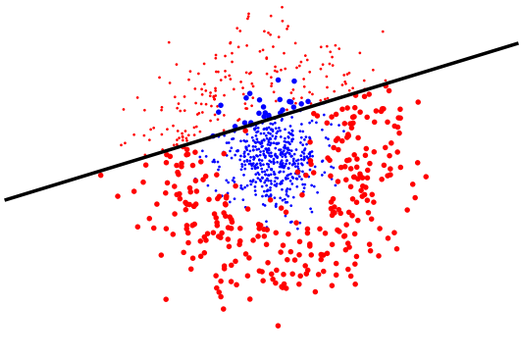

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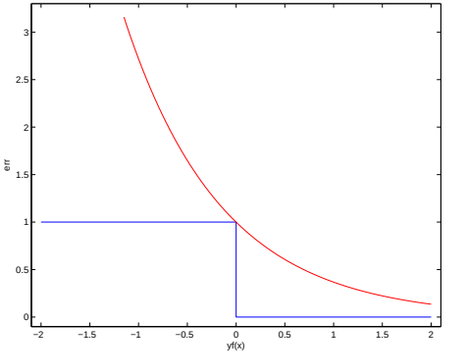

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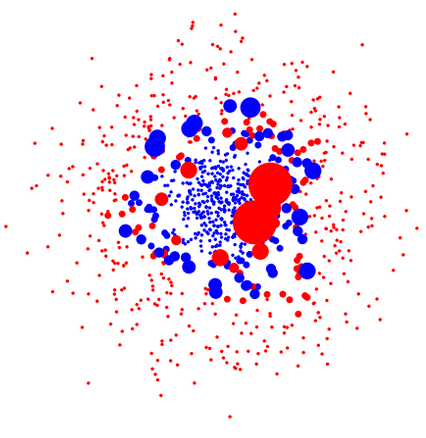


Upper Bound Theorem

Theorem: The following upper bound holds on the training error of H

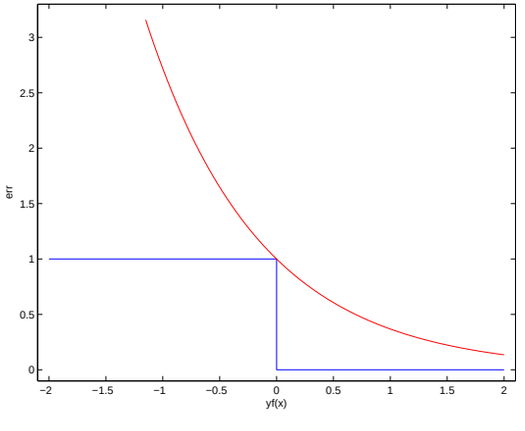
$$\frac{1}{m}|\{i : H(x_i) \neq y_i\}| \leq \prod_{t=1}^{T} Z_t$$

Proof: By unravelling the update rule

$$\begin{aligned} D_{T+1}(i) &= \frac{exp(-\sum_t \alpha_t y_i h_t(x_i))}{m \prod_t Z_t} \\ &= \frac{exp(-y_i f(x_i))}{m \prod_t Z_t}. \end{aligned}$$

If $H(x_i) \neq y_i$ then $y_i f(x_i) \leq 0$ implying that $exp(-y_i f(x_i)) > 1$, thus

$$\begin{aligned} [\![H(x_i) \neq y_i]\!] &\leq exp(-y_i f(x_i)) \\ \frac{1}{m}\sum_i [\![H(x_i) \neq y_i]\!] &\leq \frac{1}{m}\sum_i exp(-y_i f(x_i)) \\ &= \sum_i (\prod_t Z_t) D_{T+1}(i) = \prod_t Z_t \end{aligned}$$



Consequences of the Theorem

- Instead of minimising the training error, its upper bound can be minimised.
- This can be done by minimising Z_t in each training round by:
  - Choosing optimal h_t, and
  - Finding optimal α_t.
- Can be proved to maximise margin.

Choosing α_t

We attempt to minimise $Z_t = \sum_i D_t(i) exp(-\alpha_t y_i h_t(x_i))$:

$$\begin{aligned}
\frac{dZ}{d\alpha} = -\sum_{i=1}^{m} D(i) y_i h(x_i) e^{-y_i \alpha_i h(x_i)} &= 0 \\
-\sum_{i:y_i = h(x_i)} D(i) e^{-\alpha} + \sum_{i:y_i \neq h(x_i)} D(i) e^{\alpha} &= 0 \\
-e^{-\alpha}(1-\epsilon) + e^{\alpha}\epsilon &= 0 \\
\alpha &= \frac{1}{2} \log \frac{1-\epsilon}{\epsilon}
\end{aligned}$$

$\Rightarrow$ The minimisator of the upper bound is $\alpha_t = \frac{1}{2} \log \frac{1-\epsilon_t}{\epsilon_t}$.

Choosing h_t

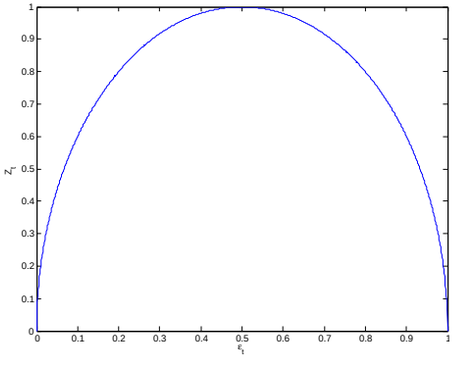
Weak classifier examples

- Decision tree, Perceptron – $\mathcal{H}$ *infinite*
- Selecting the best one from given *finite* set $\mathcal{H}$

Justification of the weighted error minimisation

Having $\alpha_t = \frac{1}{2}\log\frac{1-\epsilon_t}{\epsilon_t}$

$$
\begin{aligned}
Z_t &= \sum_{i=1}^{m} D_t(i) e^{-y_i \alpha_i h_t(x_i)} \\
&= \sum_{i:y_i=h_t(x_i)} D_t(i) e^{-\alpha_t} + \sum_{i:y_i\neq h_t(x_i)} D_t(i) e^{\alpha_t} \\
&= (1-\epsilon_t) e^{-\alpha_t} + \epsilon_t e^{\alpha_t} \\
&= 2\sqrt{\epsilon_t(1-\epsilon_t)}
\end{aligned}
$$



$\Rightarrow$ Z_t is minimised by selecting h_t with minimal weighted error ϵ_t.

The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$

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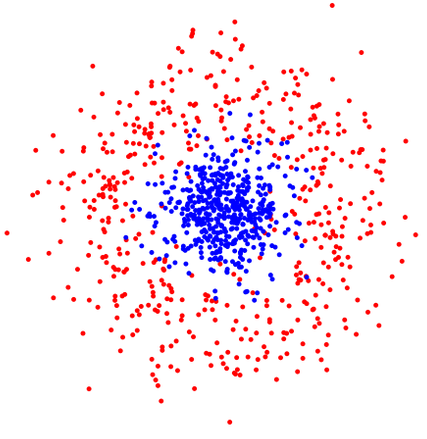
Initialise weights $D_1(i) = 1/m$.



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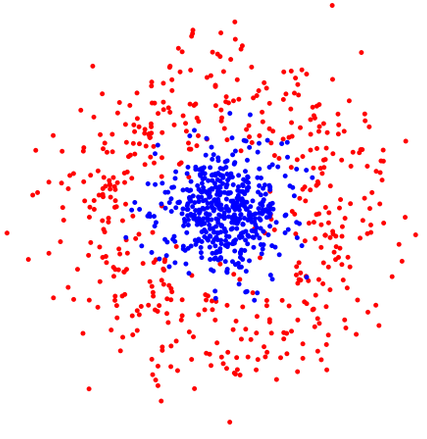

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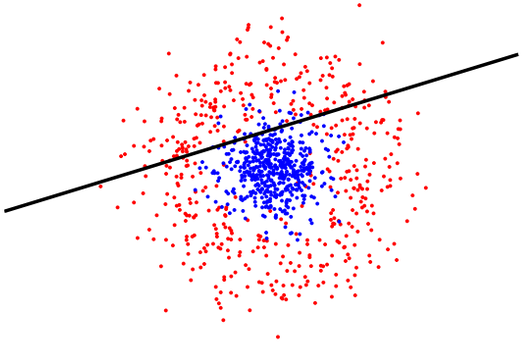
For $t = 1, ..., T$:

- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i)[\![y_i \neq h_j(x_i)]\!]$

$t = 1$



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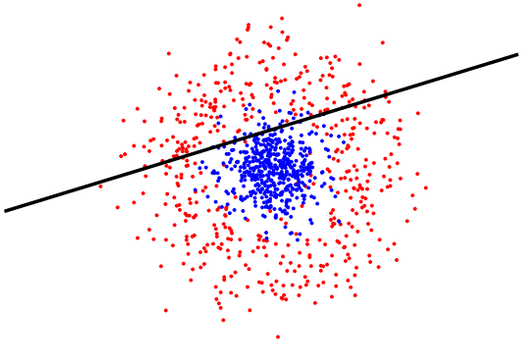
- If $\epsilon_t \geq 1/2$ then stop

$t = 1$



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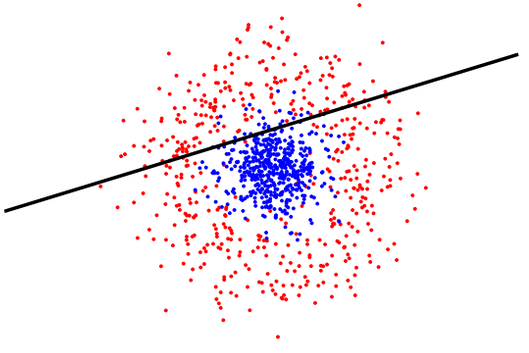
$t = 1$



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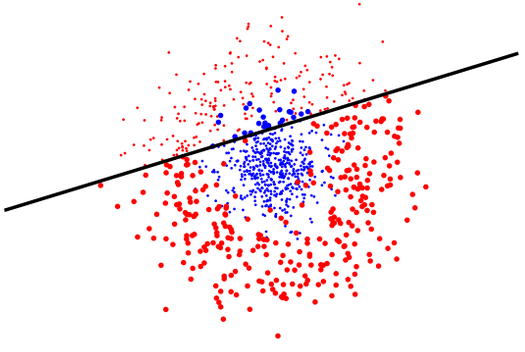
- Update

$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

$t = 1$



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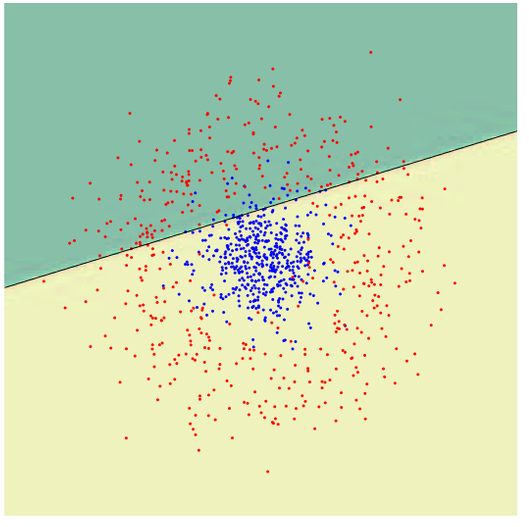
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2} \log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

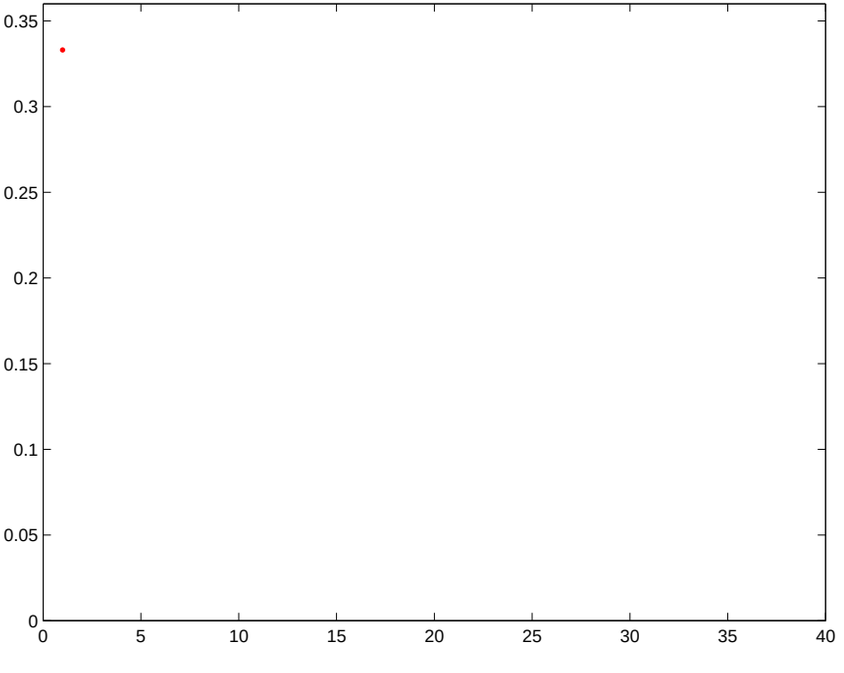
$$D_{t+1}(i) = \frac{D_t(i) exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 1$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$.
For $t = 1, ..., T$:

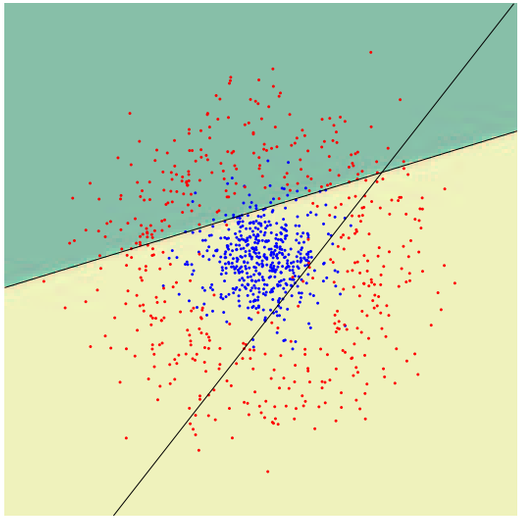
- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i)[\![y_i \neq h_j(x_i)]\!]$
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

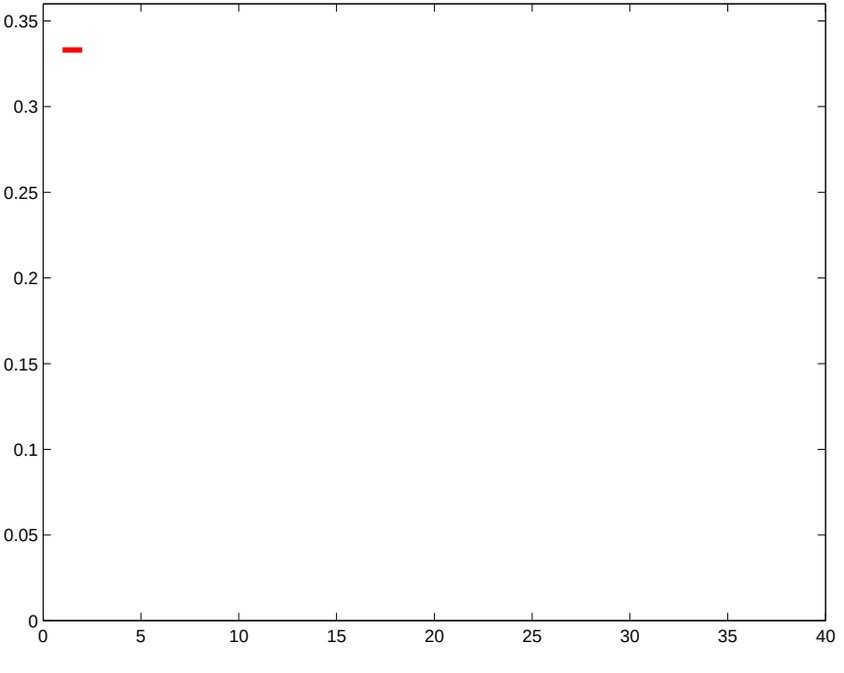
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 2$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$.
For $t = 1, ..., T$:

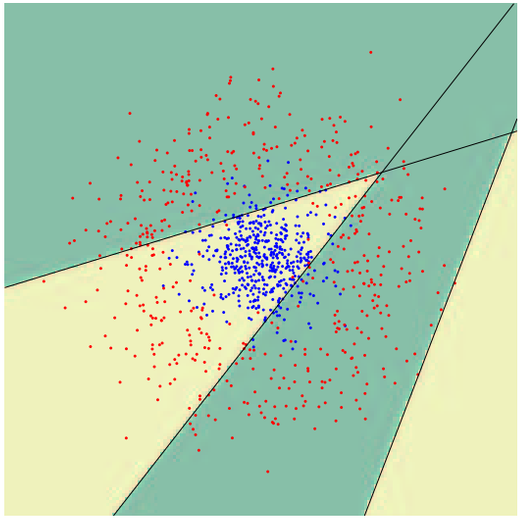
- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i)[\![y_i \neq h_j(x_i)]\!]$
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

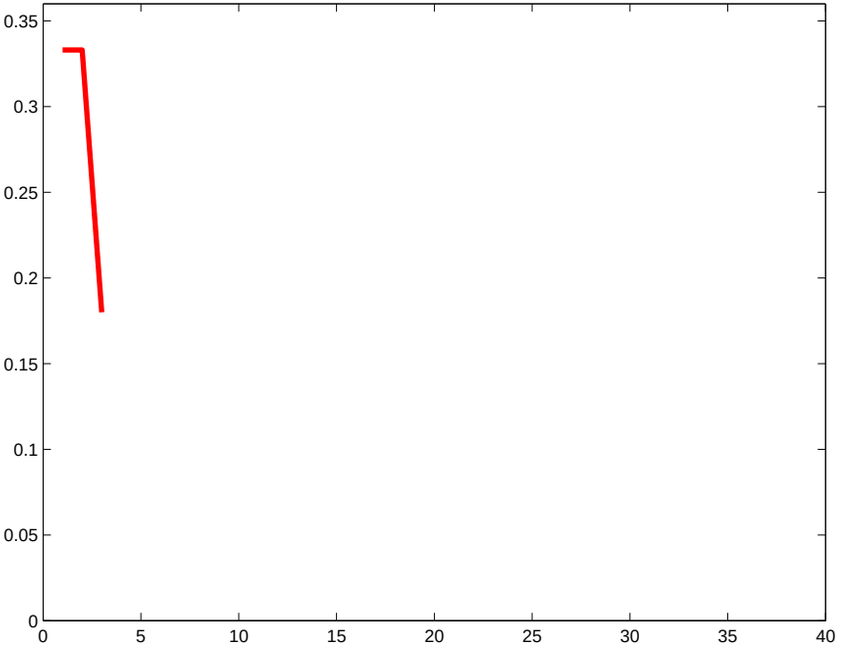
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 3$





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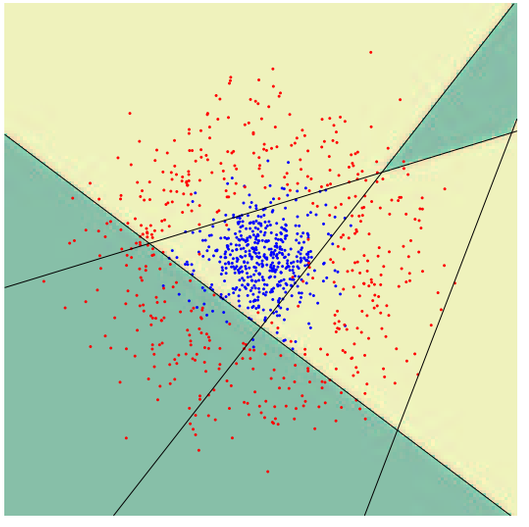
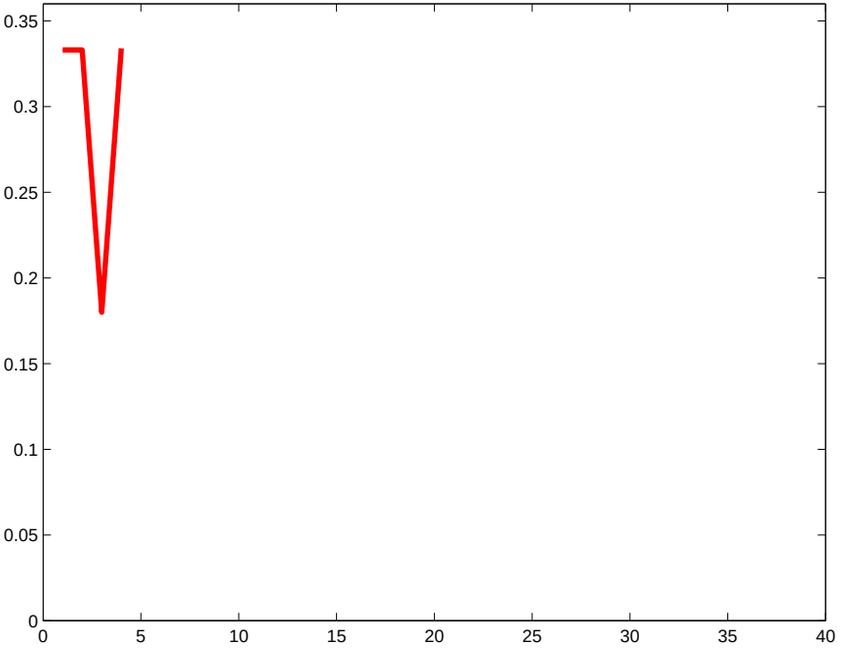
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 4$





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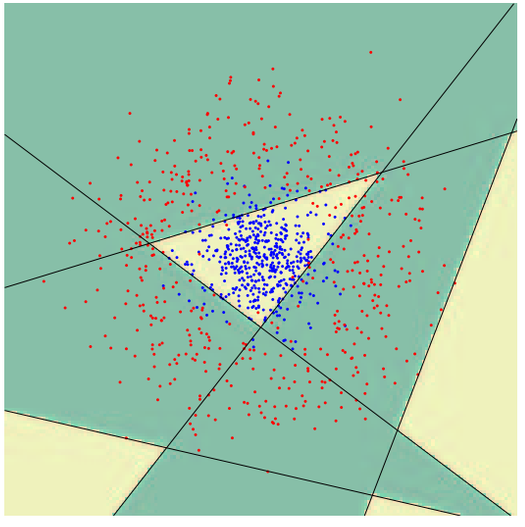
- Update

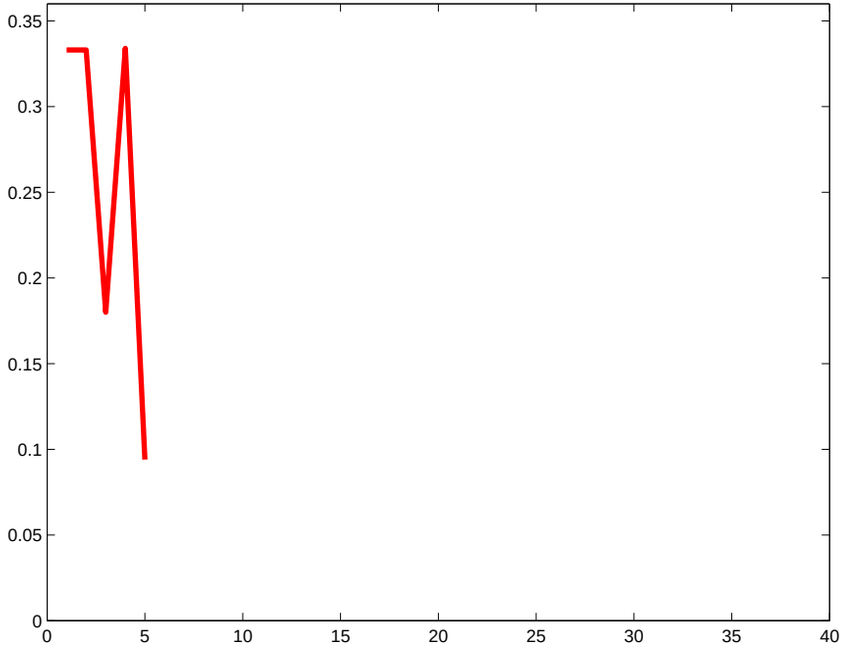
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 5$





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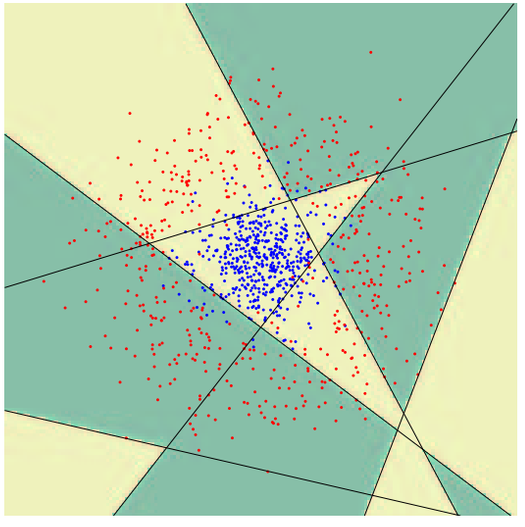
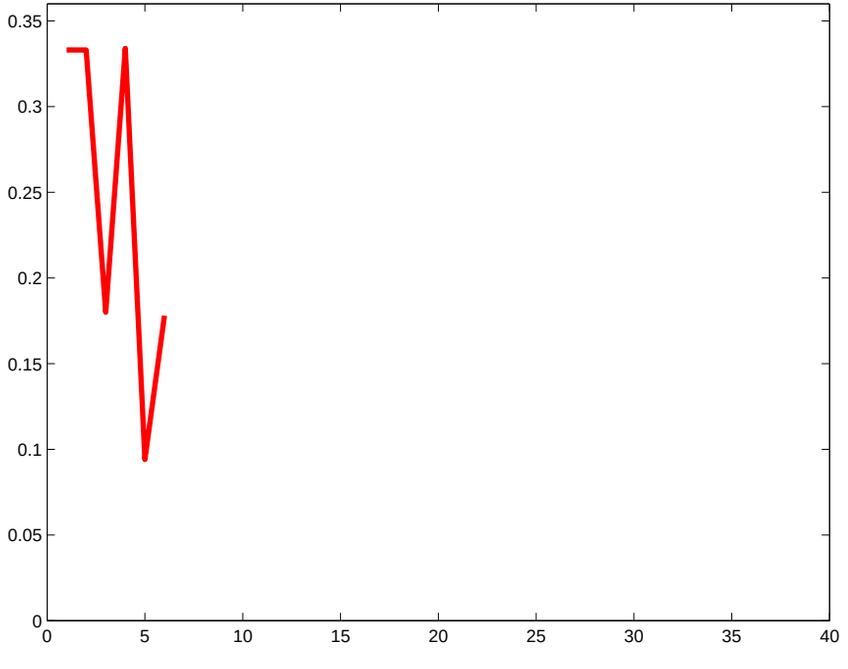
$t = 6$





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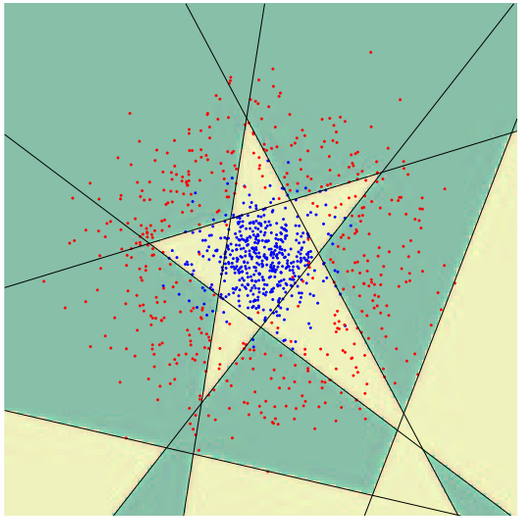
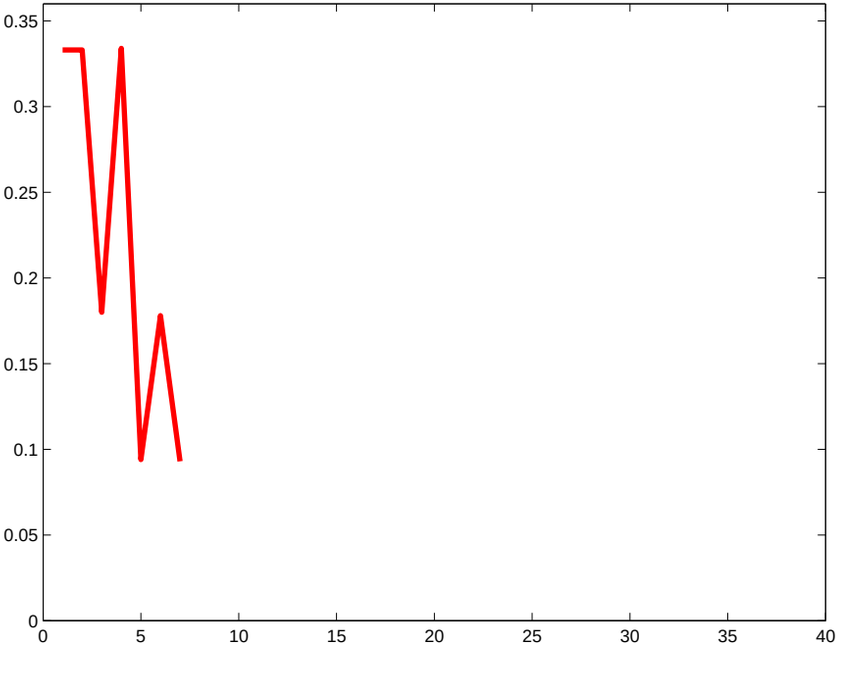
Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 7$





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Output the final classifier:

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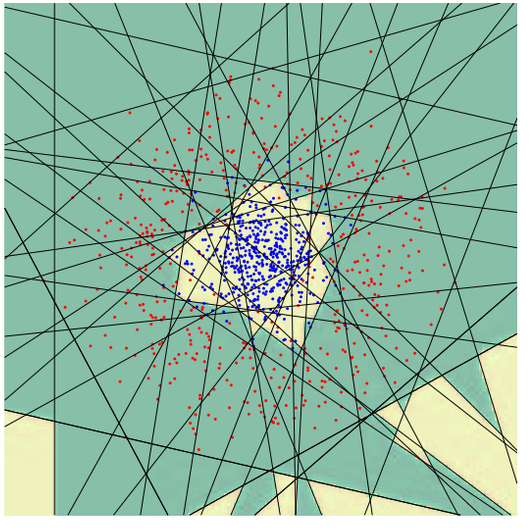
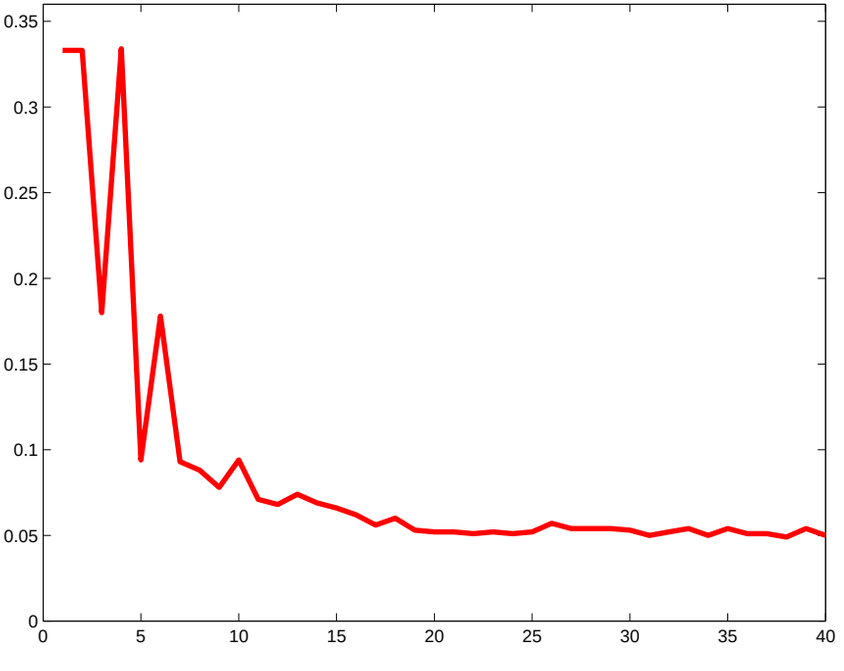
$t = 40$





AdaBoost variants

Freund & Schapire 1995

- Discrete ($h : \mathcal{X} \to \{0, 1\}$)
- Multiclass AdaBoost.M1 ($h : \mathcal{X} \to \{0, 1, ..., k\}$)
- Multiclass AdaBoost.M2 ($h : \mathcal{X} \to [0, 1]^k$)
- Real valued AdaBoost.R ($Y = [0, 1]$, $h : \mathcal{X} \to [0, 1]$)

Schapire & Singer 1997

- Confidence rated prediction ($h : \mathcal{X} \to R$, two-class)
- Multilabel AdaBoost.MR, AdaBoost.MH (different formulation of minimised loss)

... Many other modifications since then (WaldBoost, cascaded AB, online AB, ...)

Pros and cons of AdaBoost

Advantages

- Very simple to implement
- General learning scheme - can be used for various learning tasks
- Feature selection on very large sets of features
- Fairly good generalisation

Disadvantages

- Suboptimal solution (greedy learning)
- Can overfit in presence of noise (has been addressed recently)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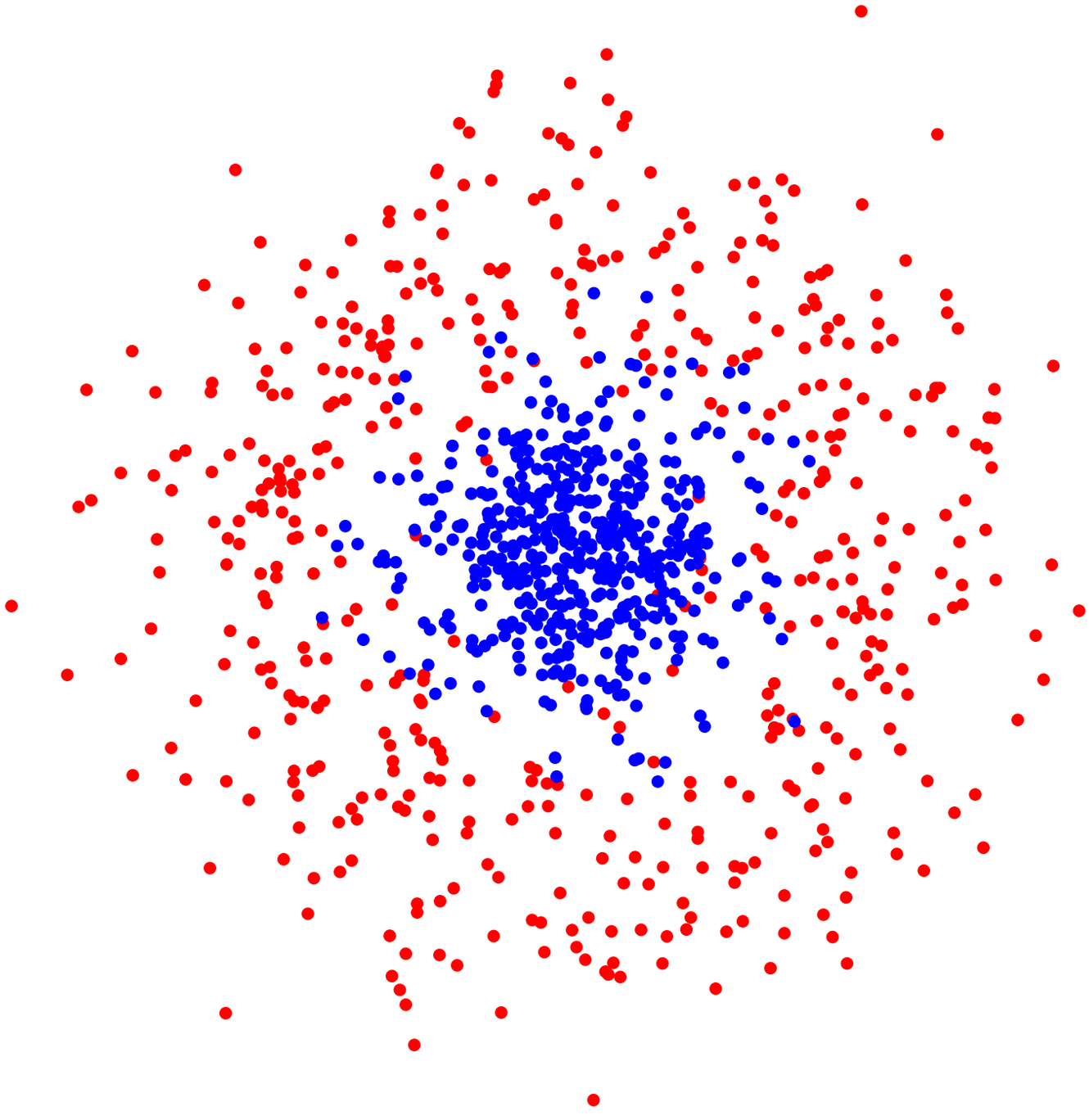

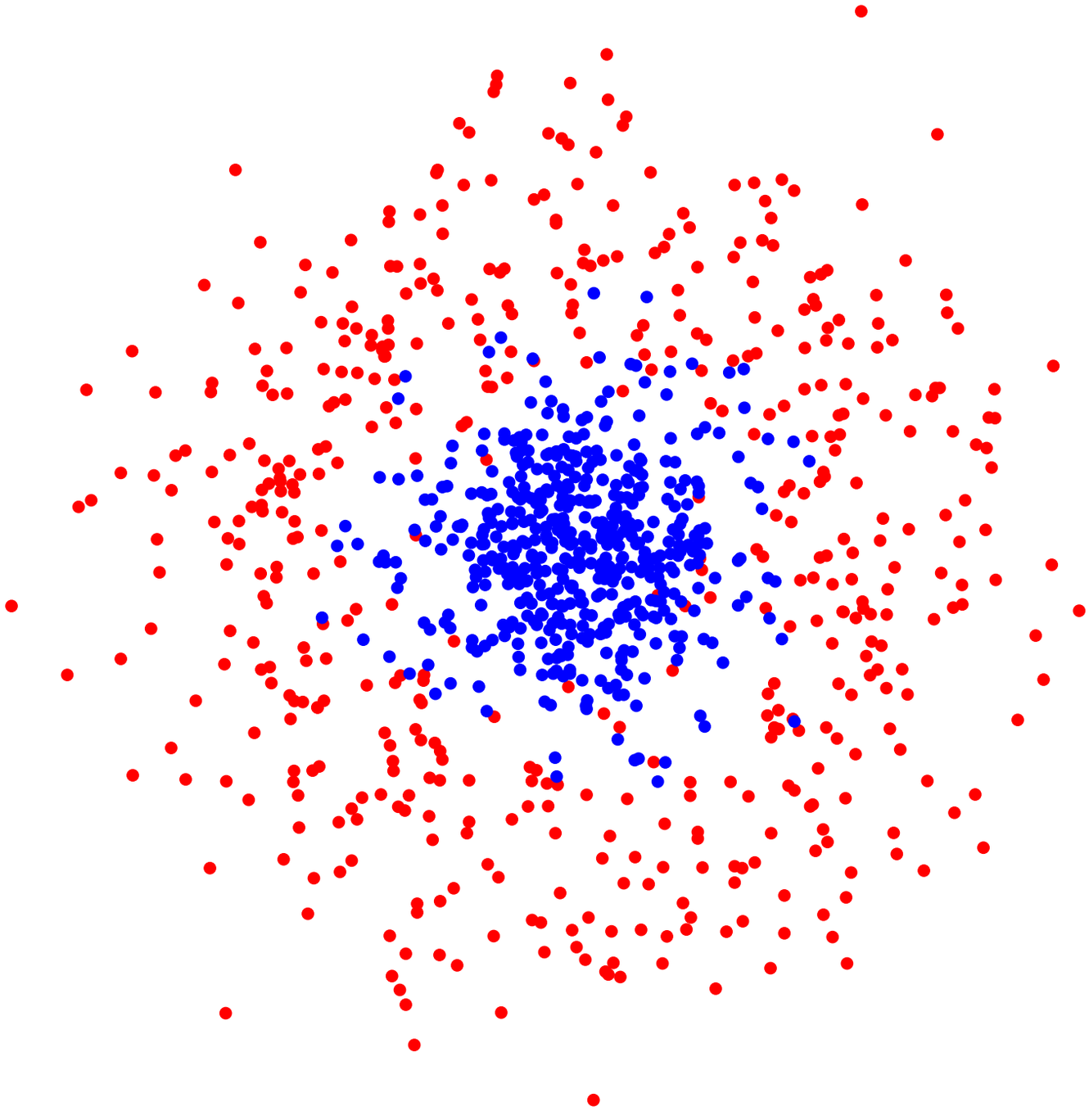

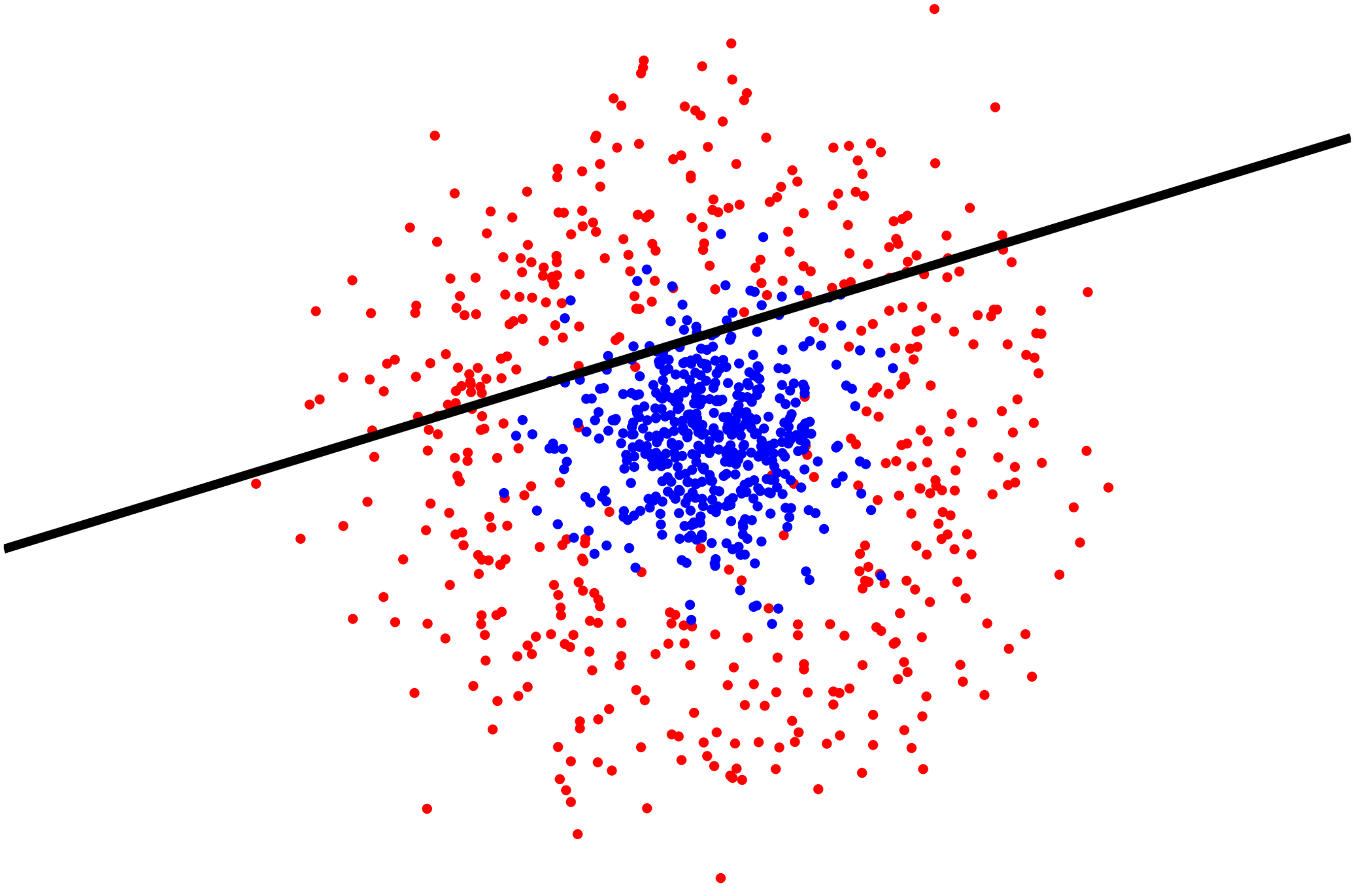

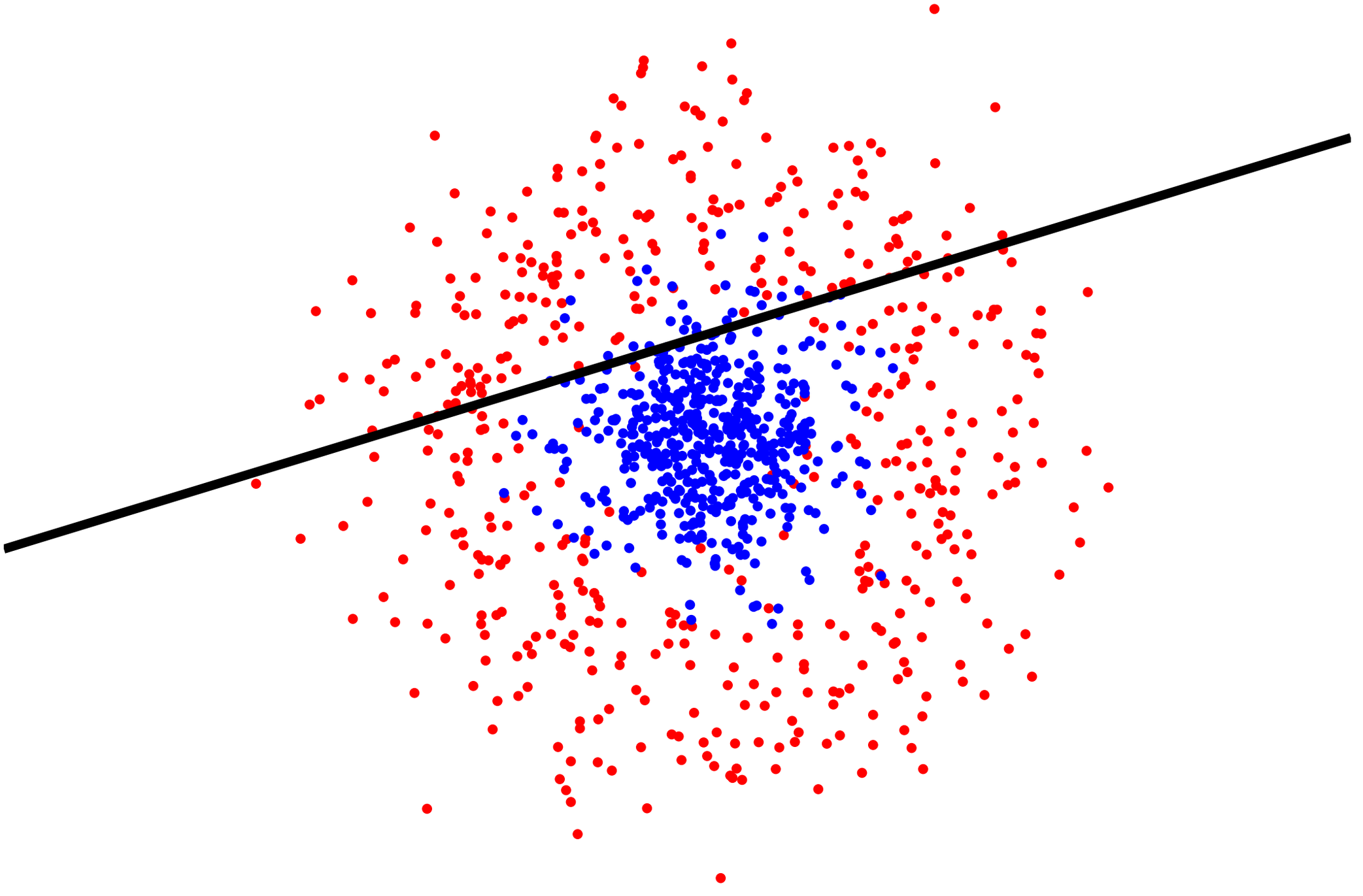

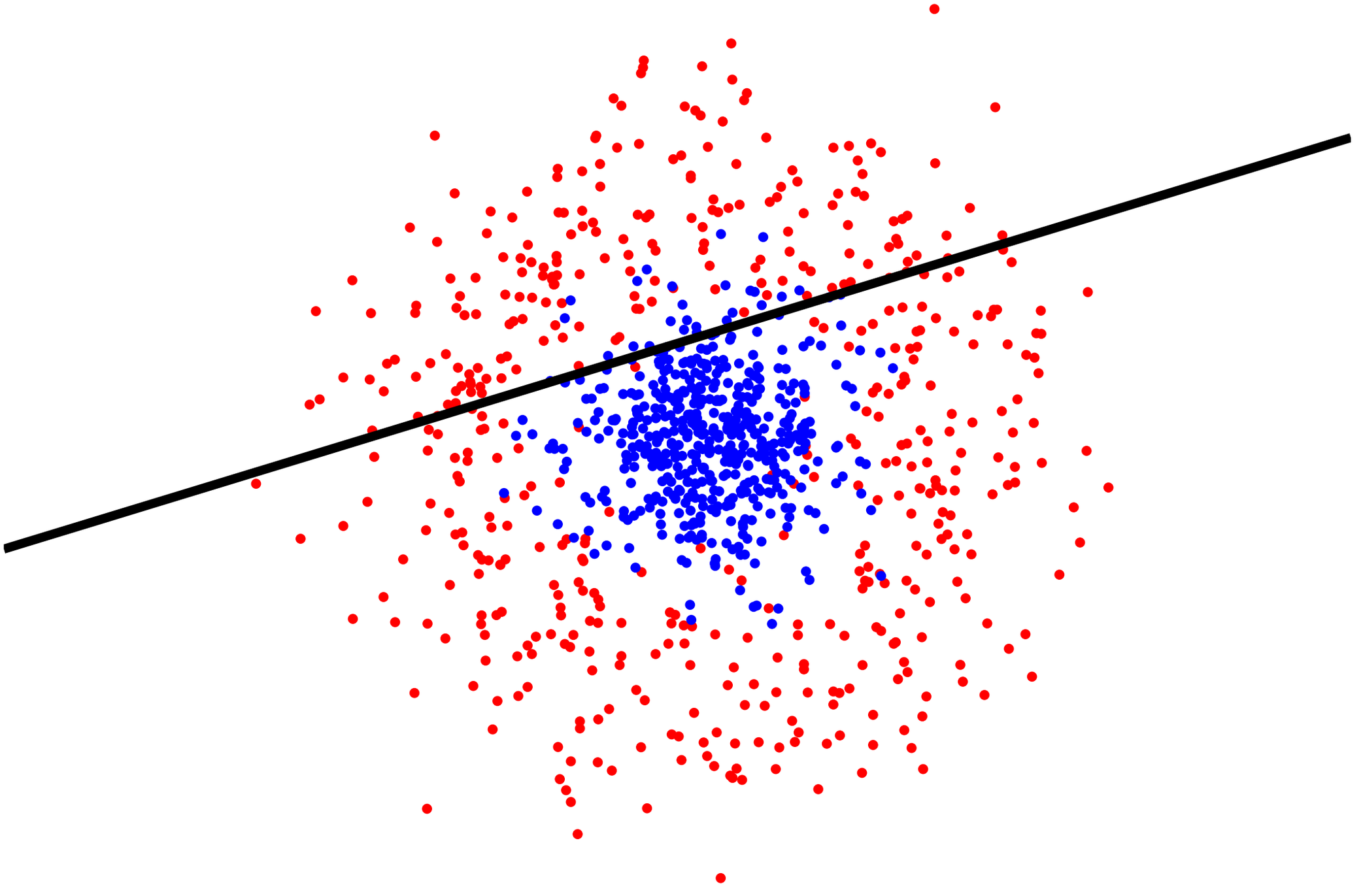

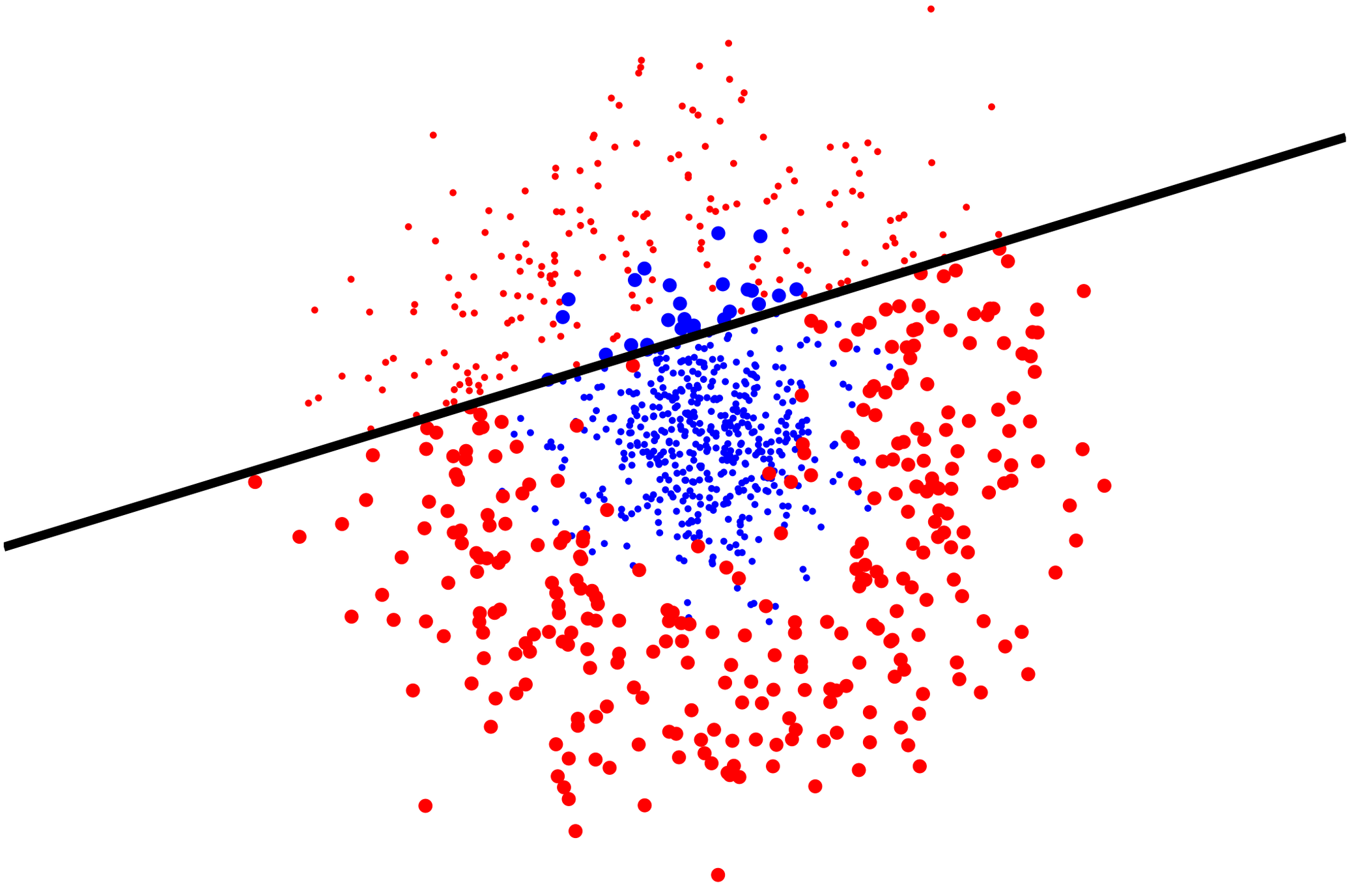

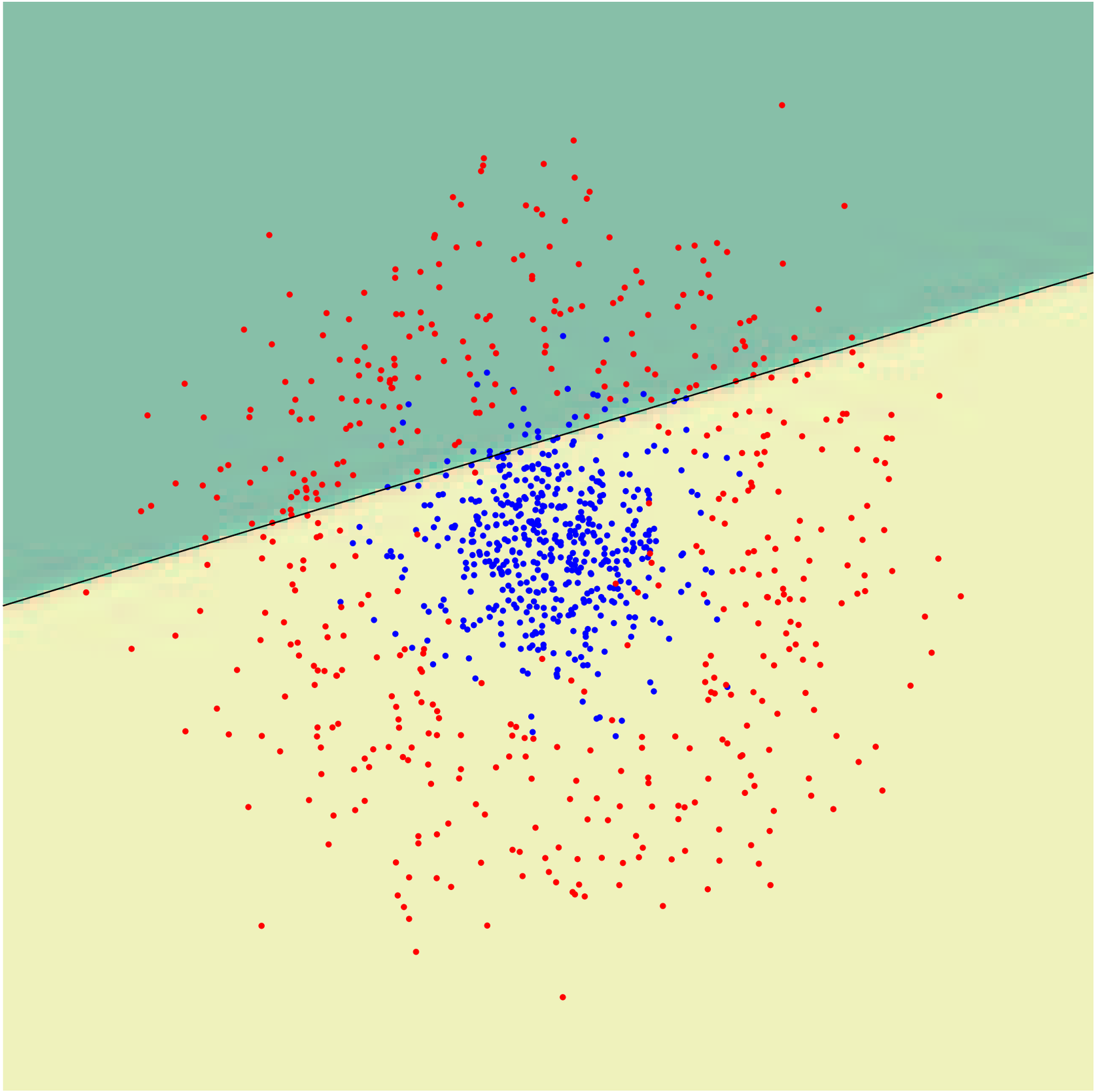

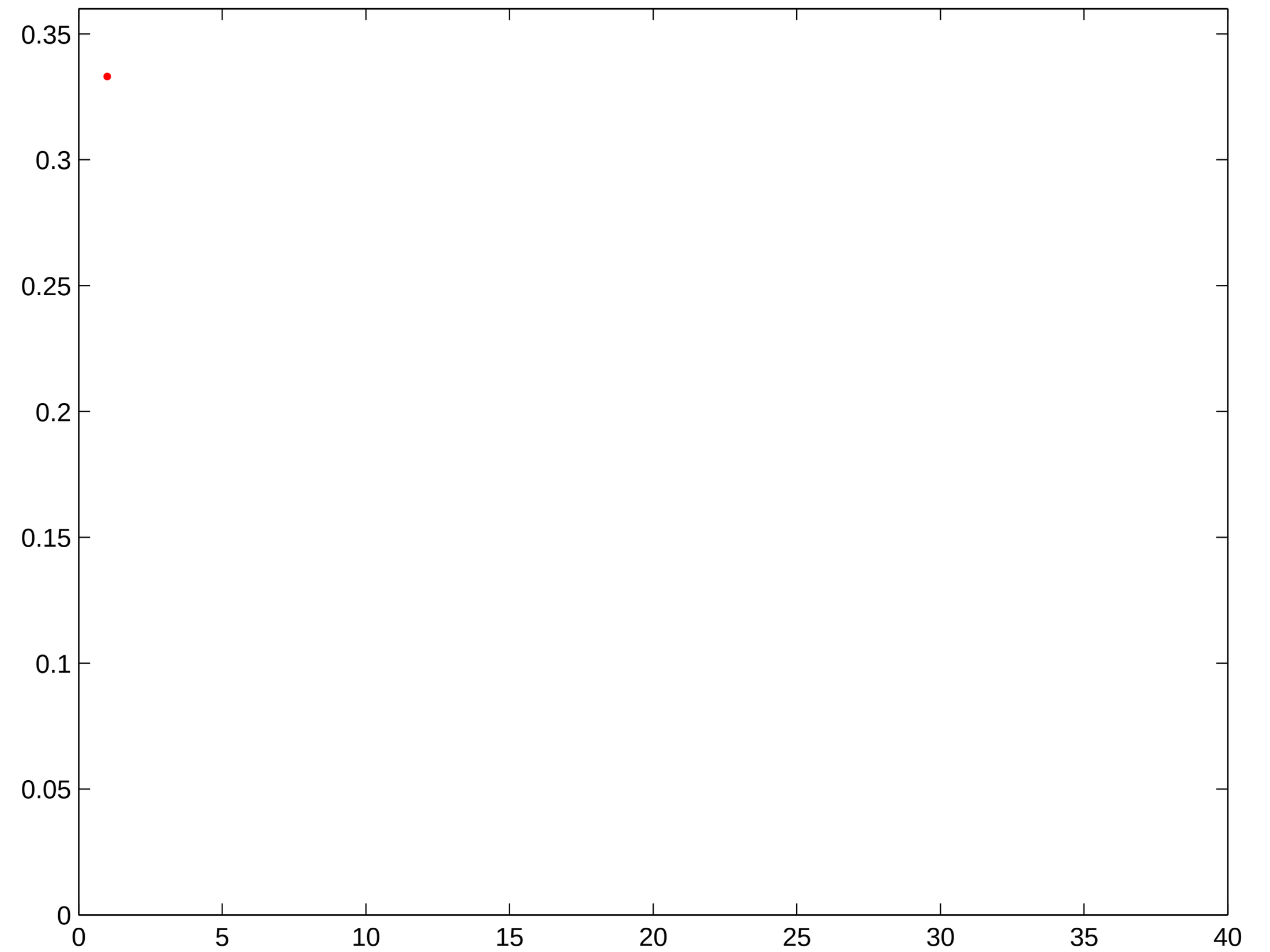
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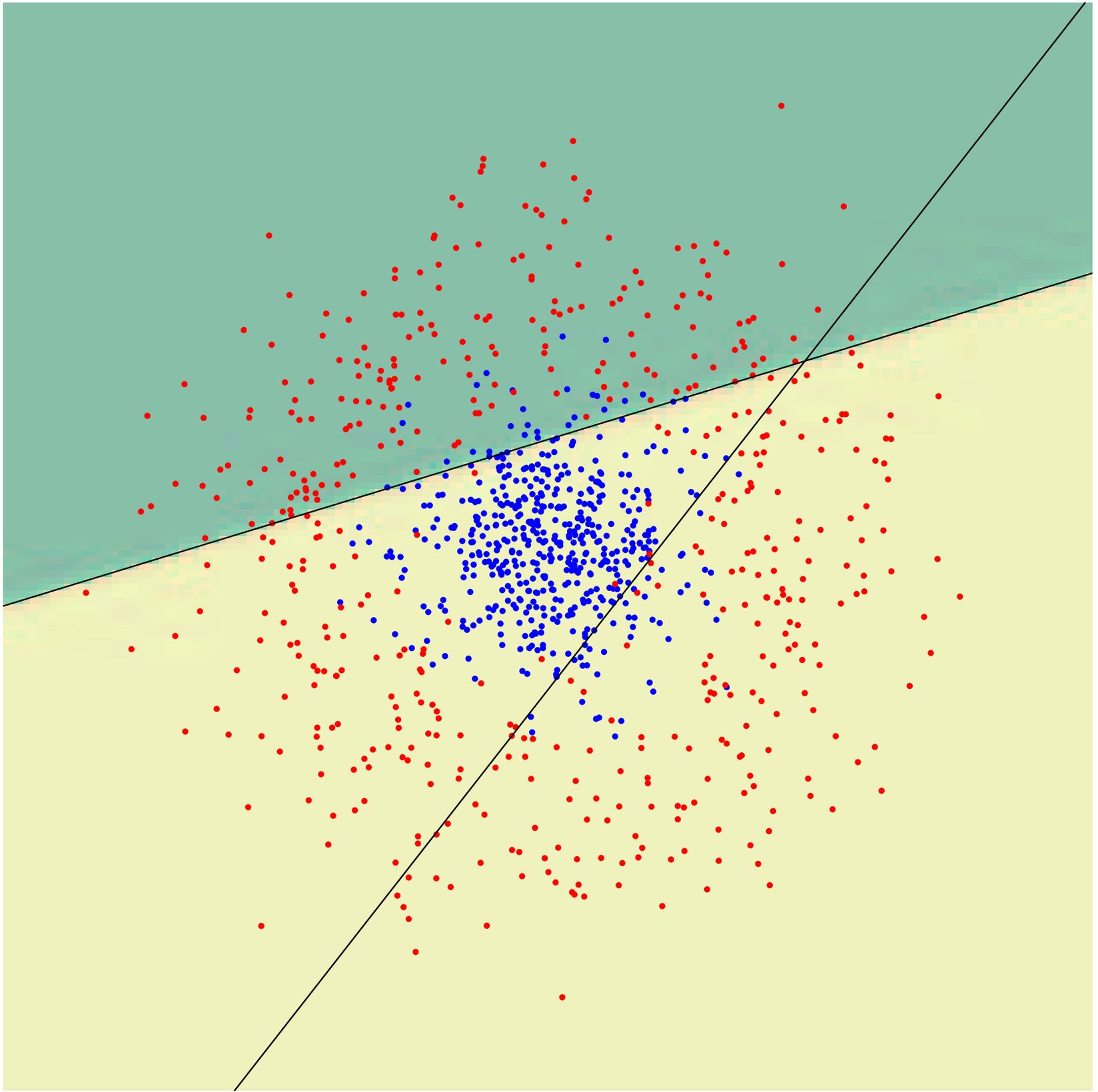

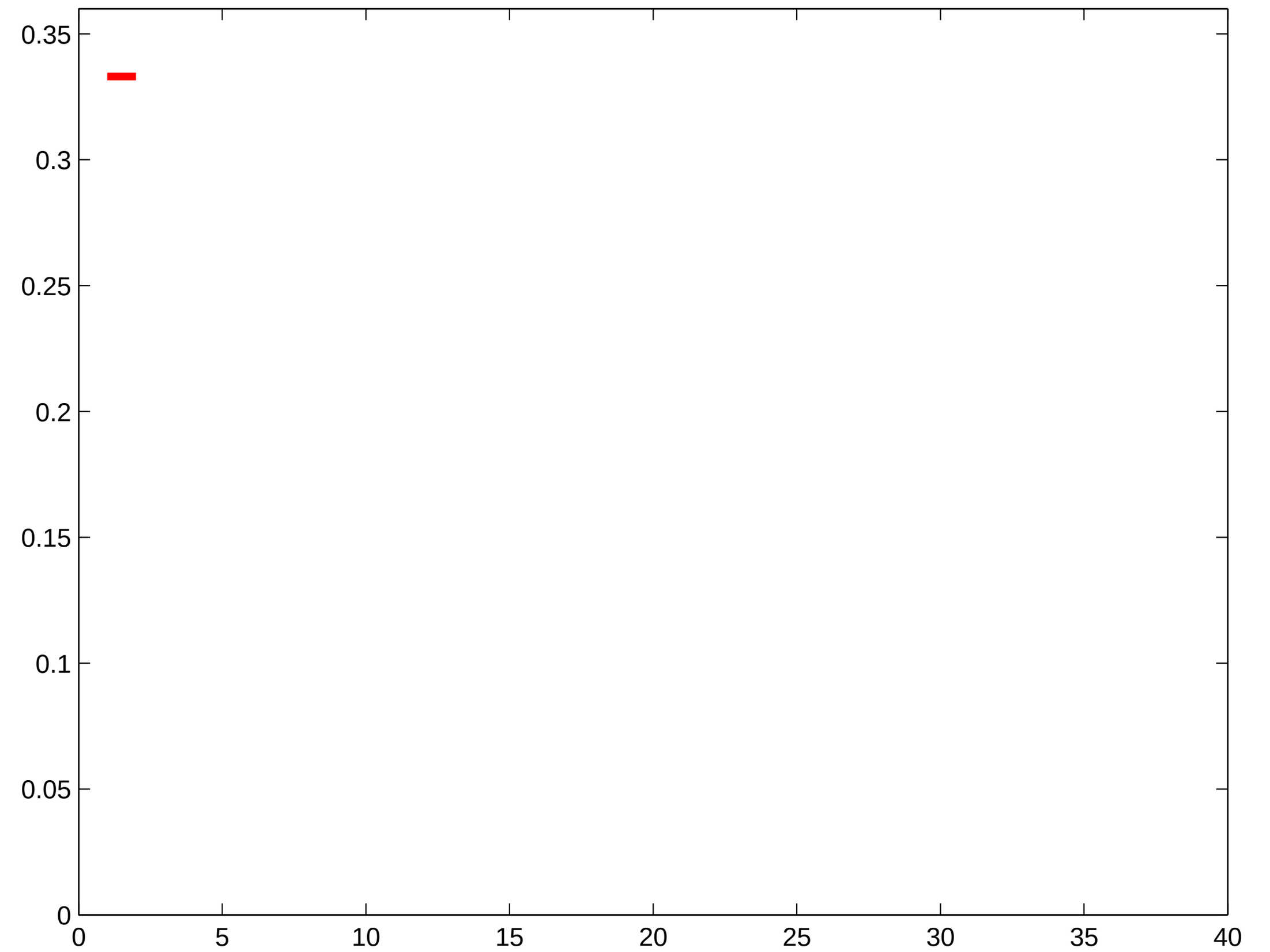
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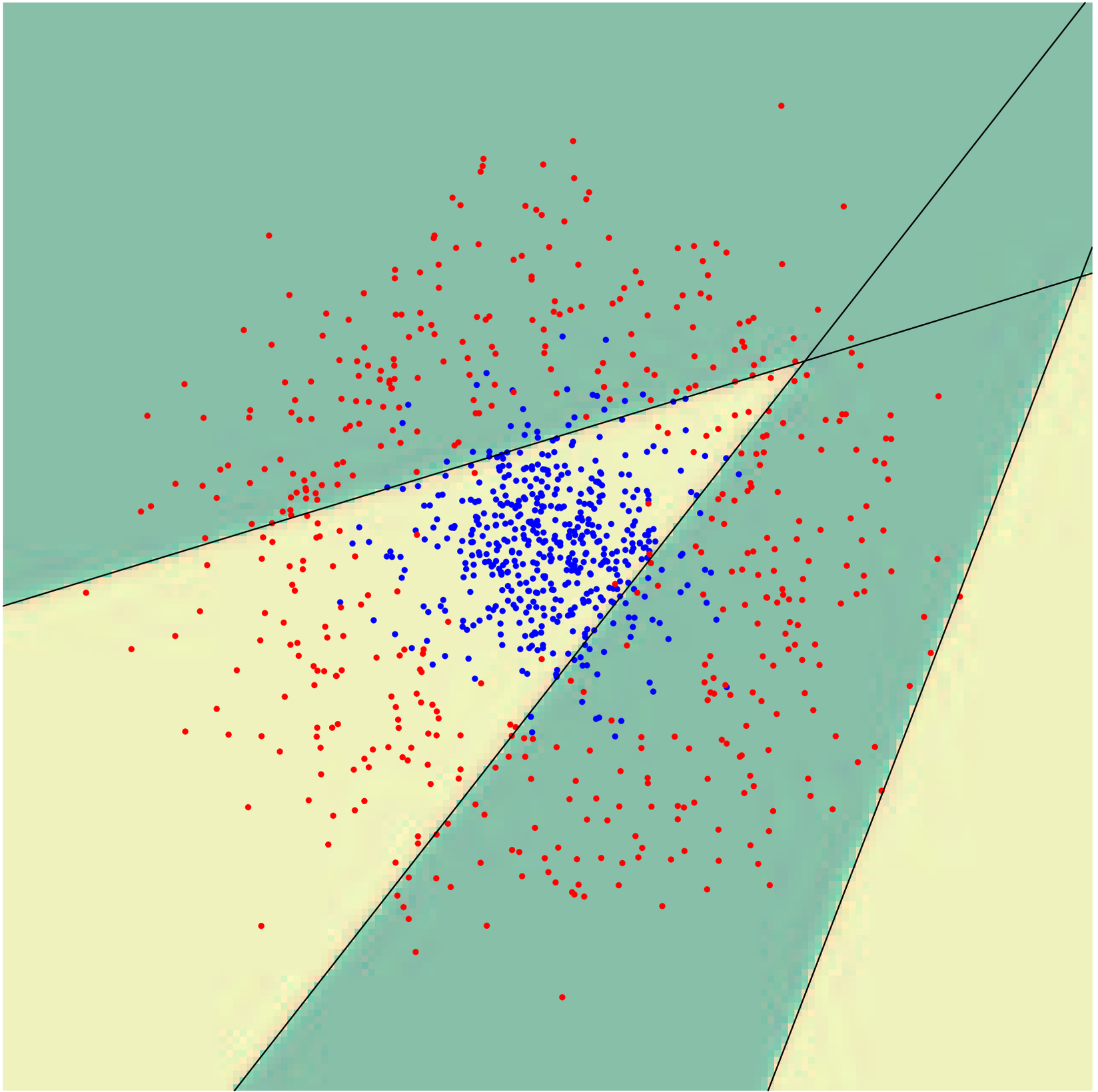

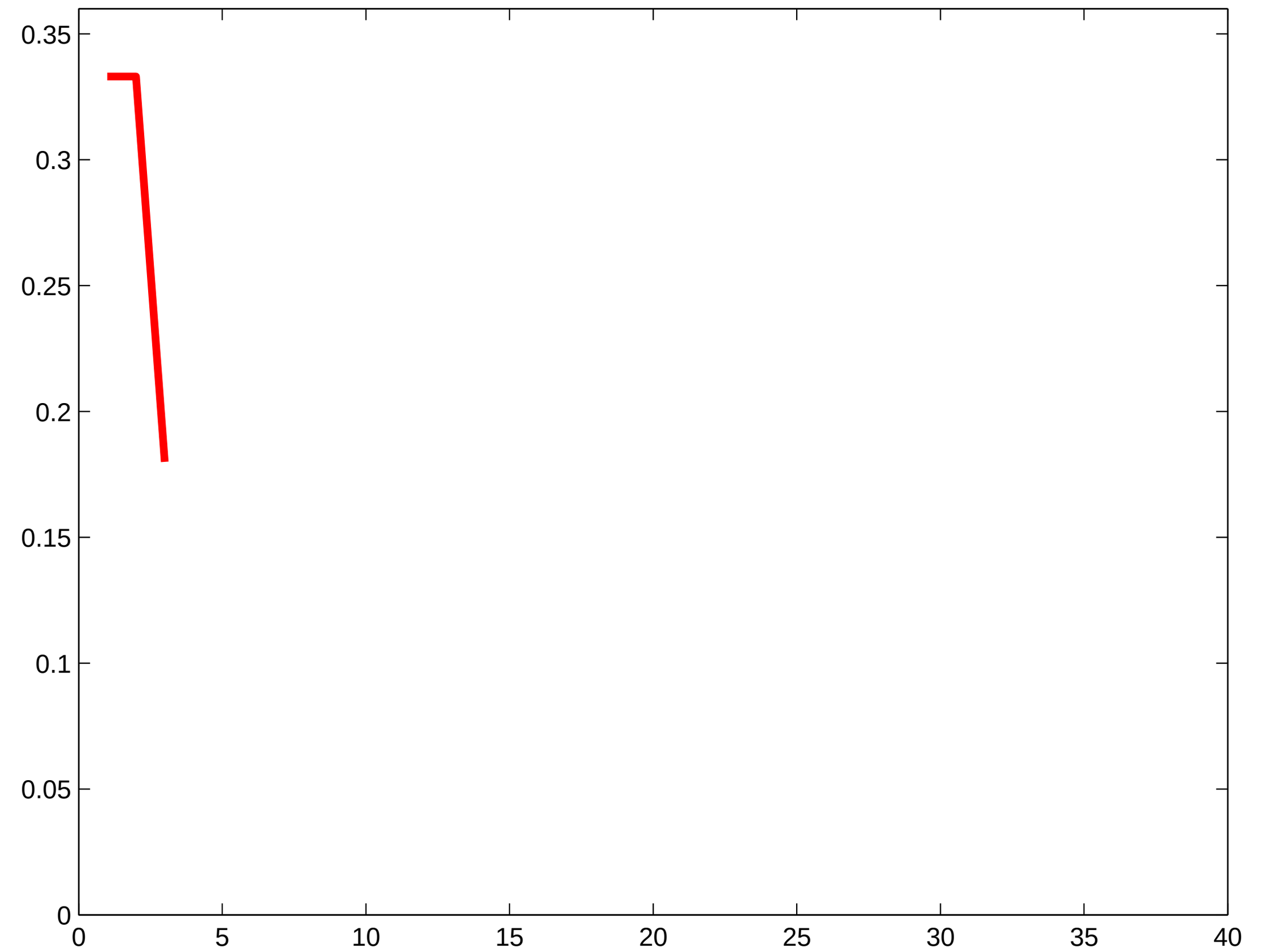
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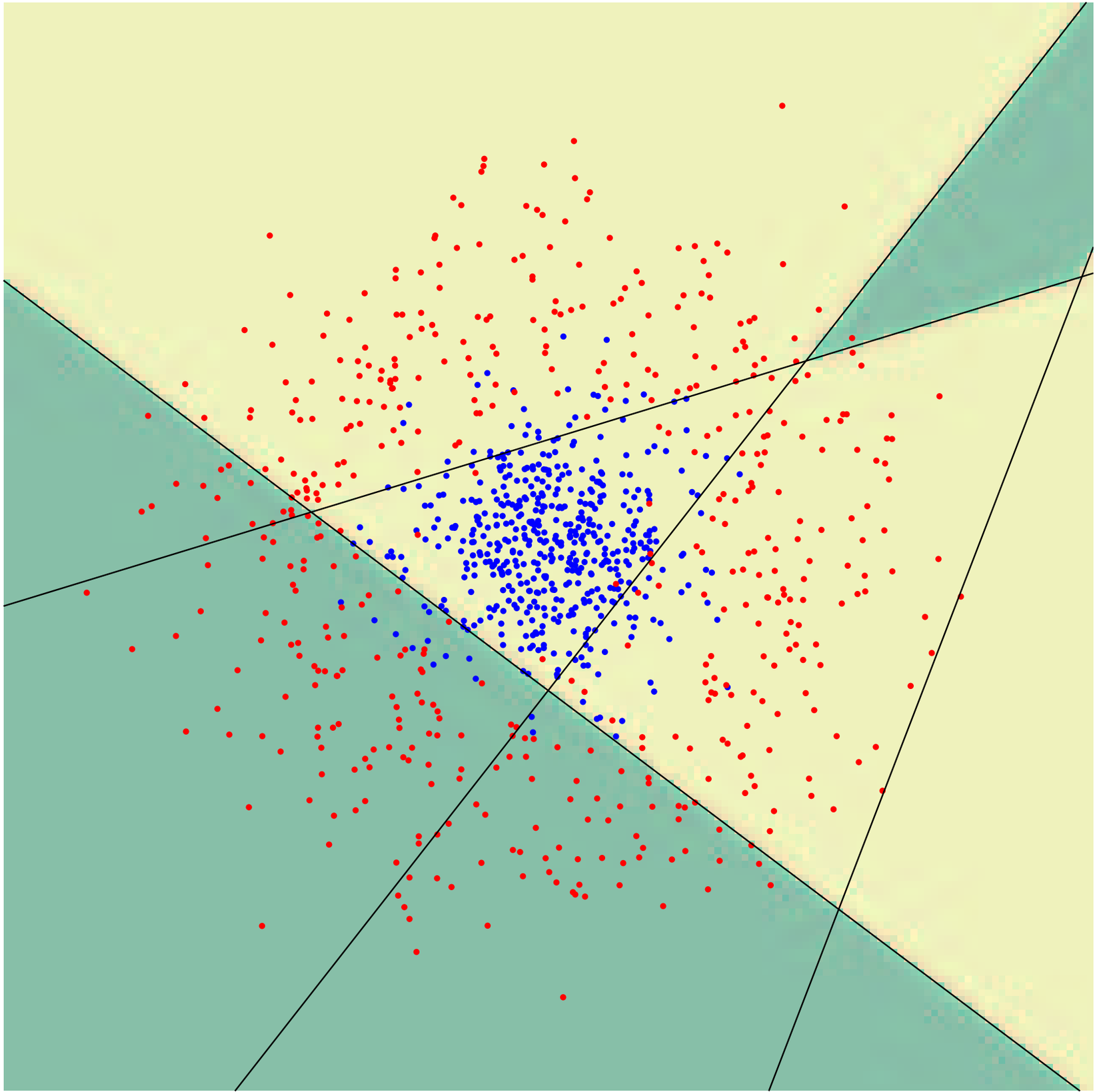

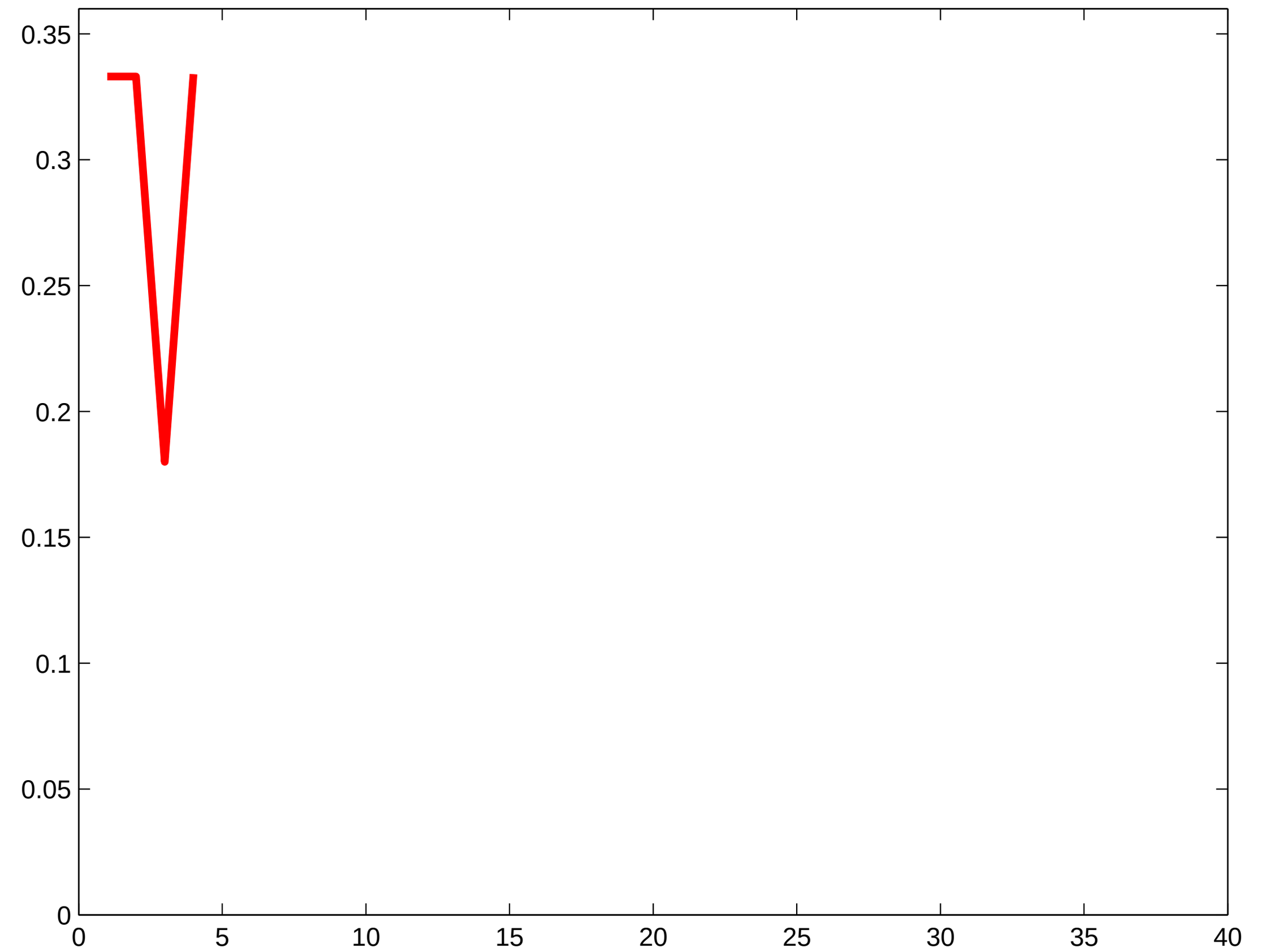
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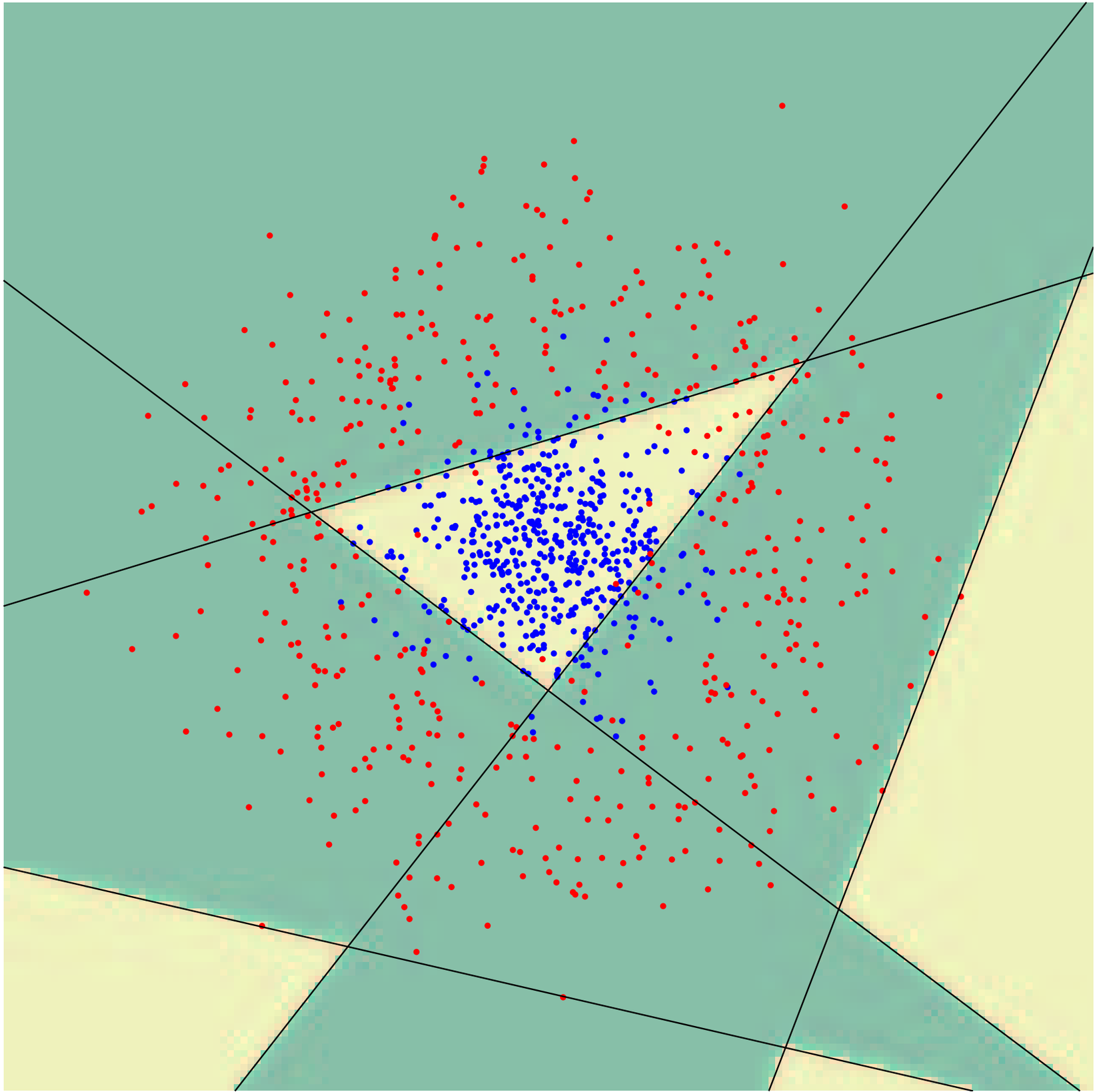

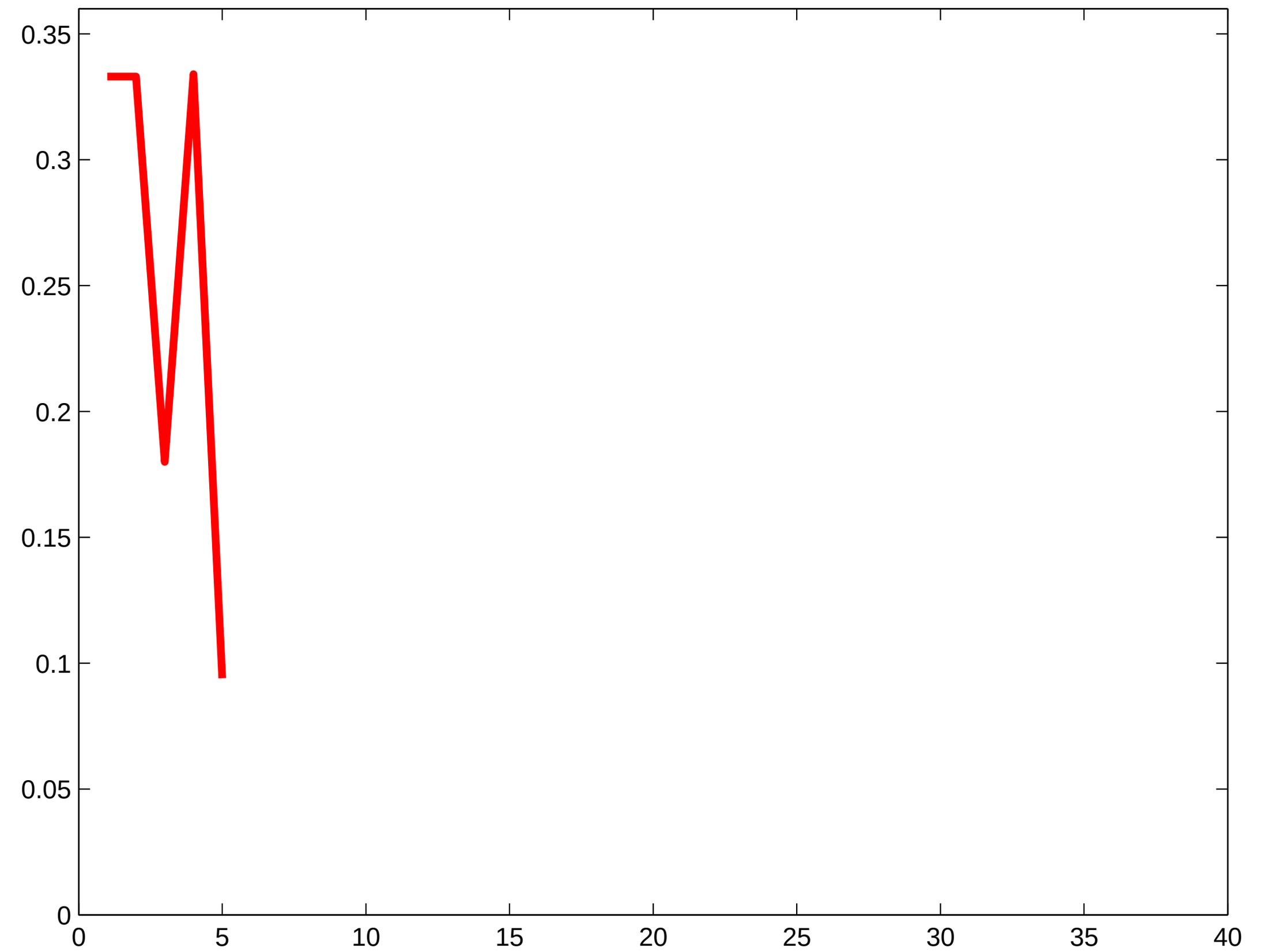
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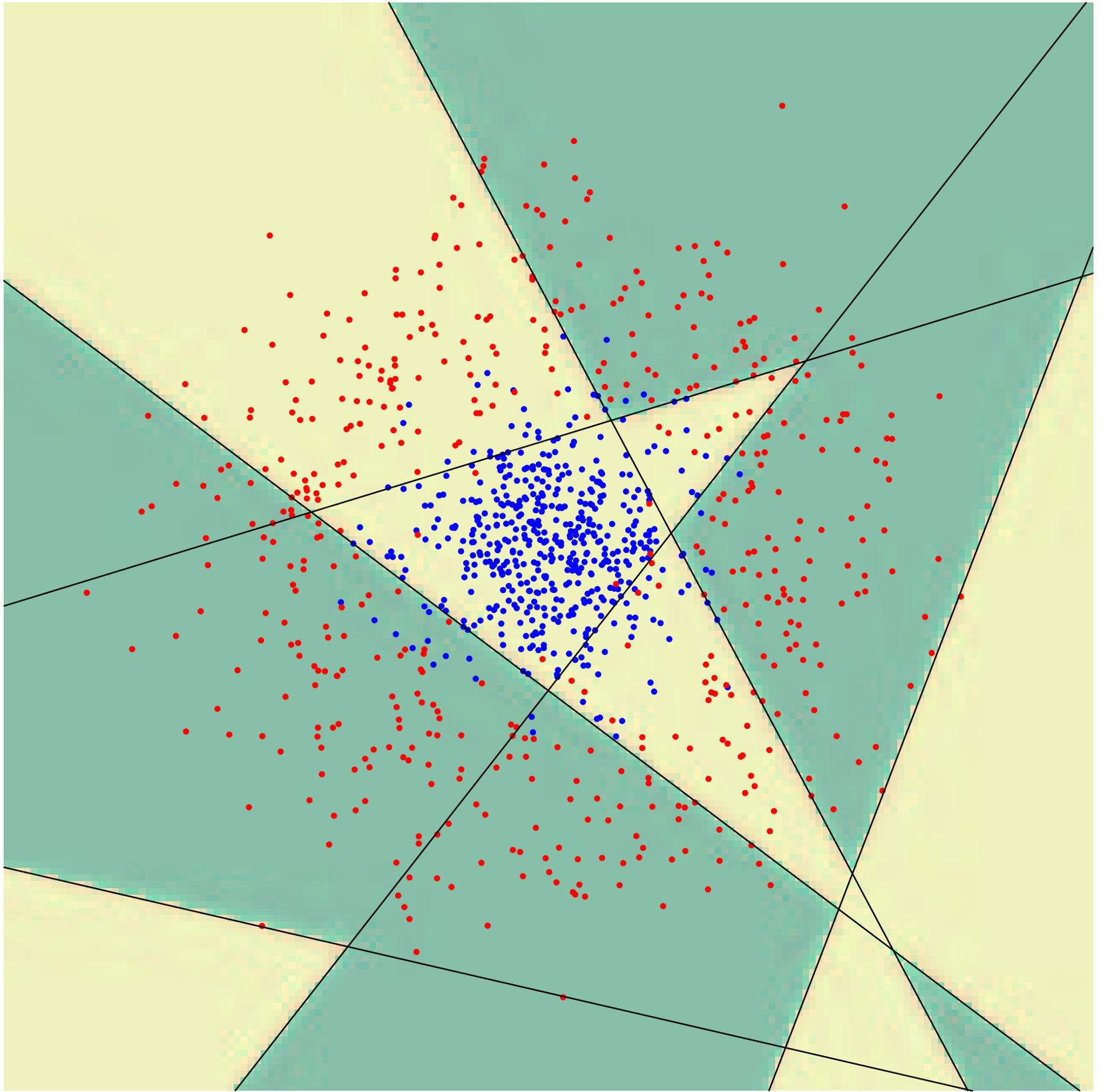

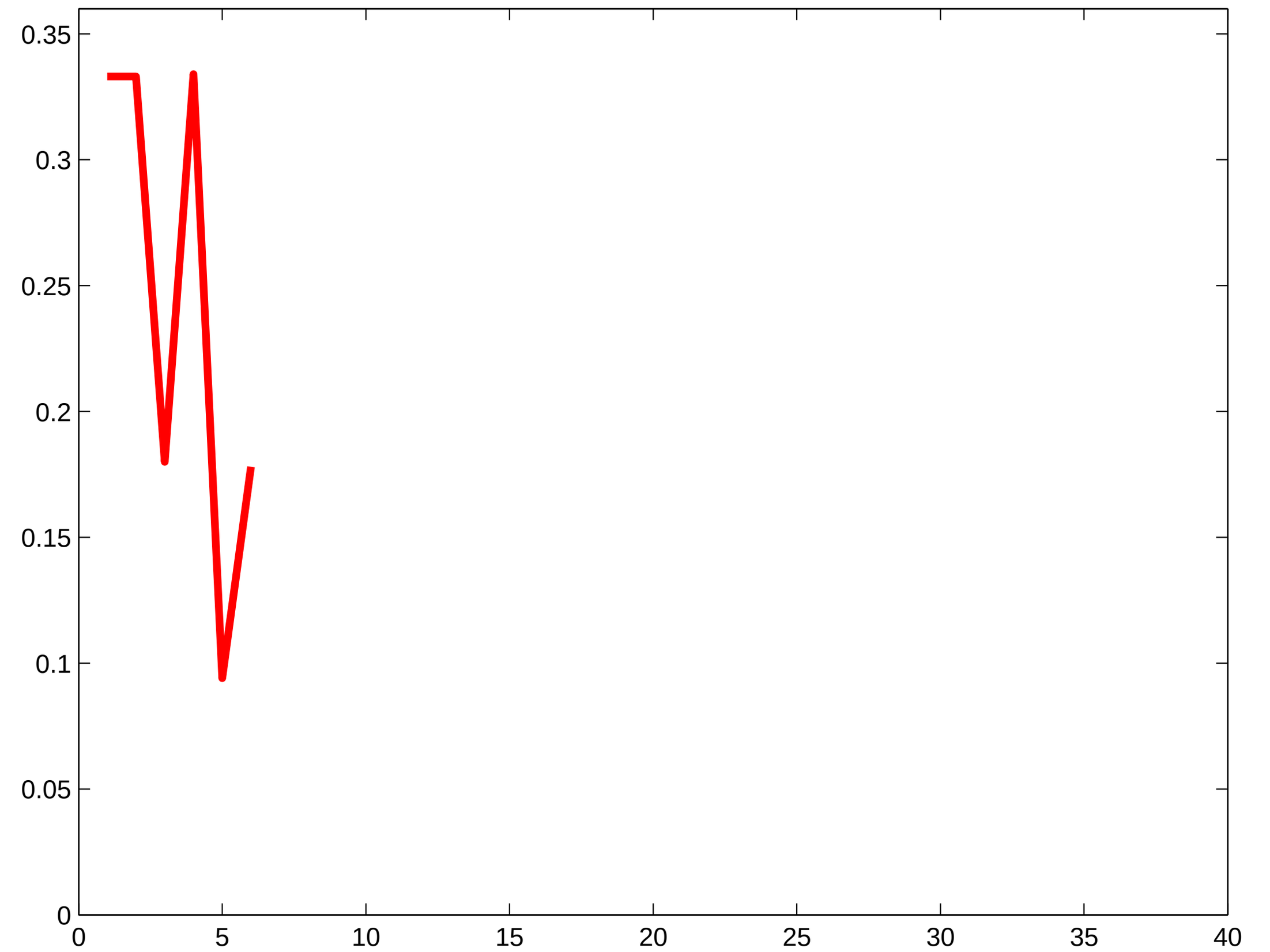
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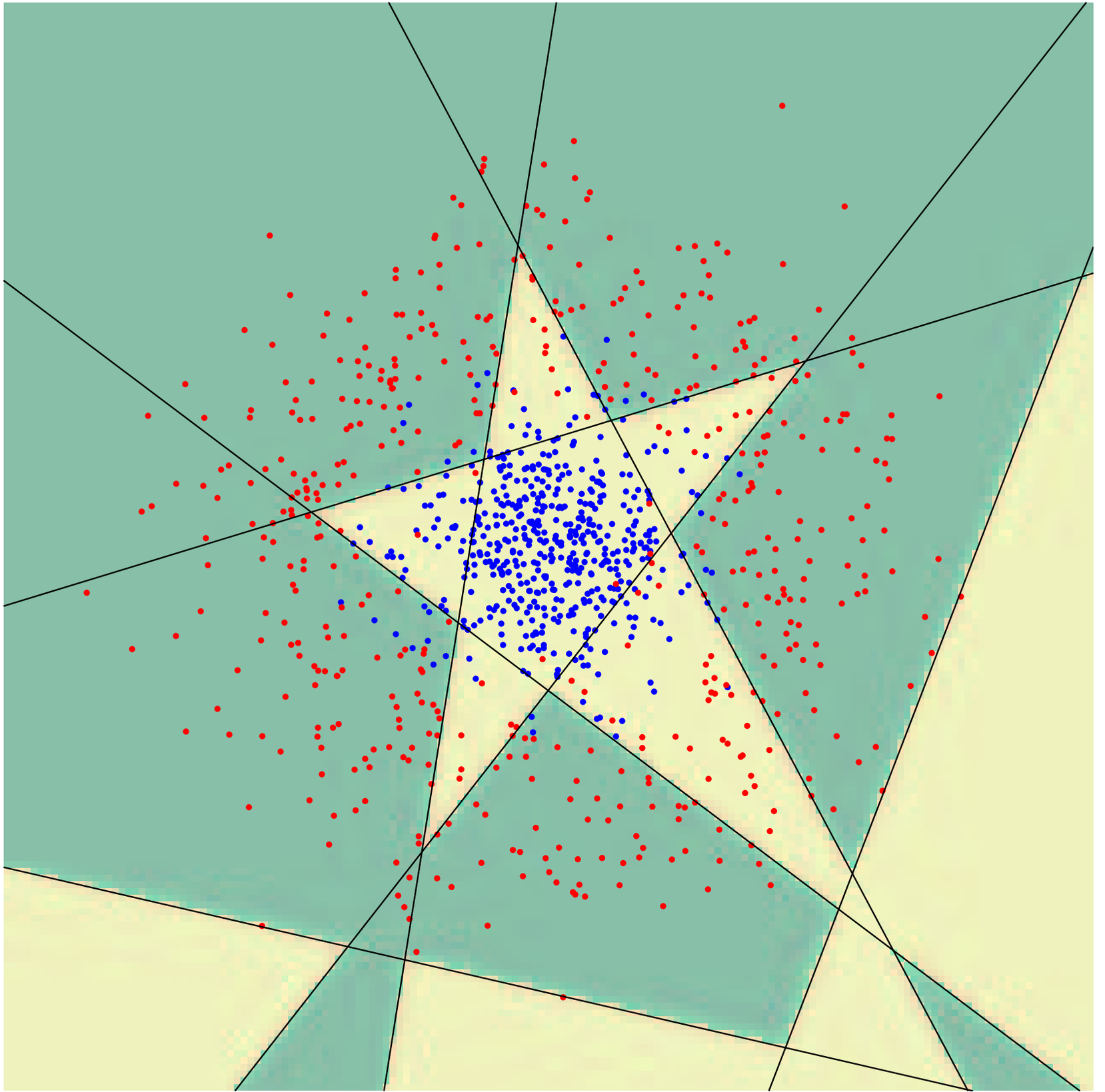

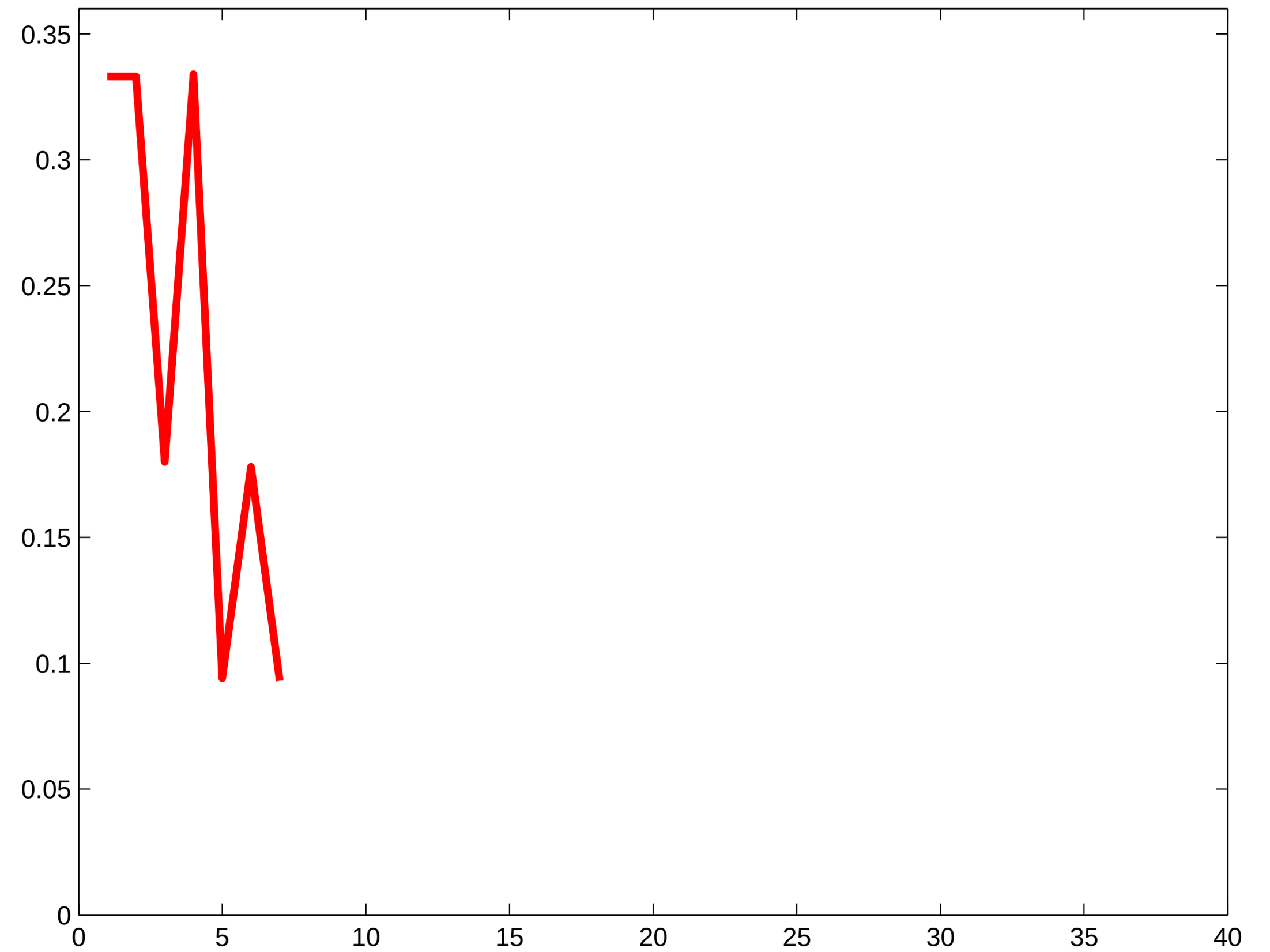
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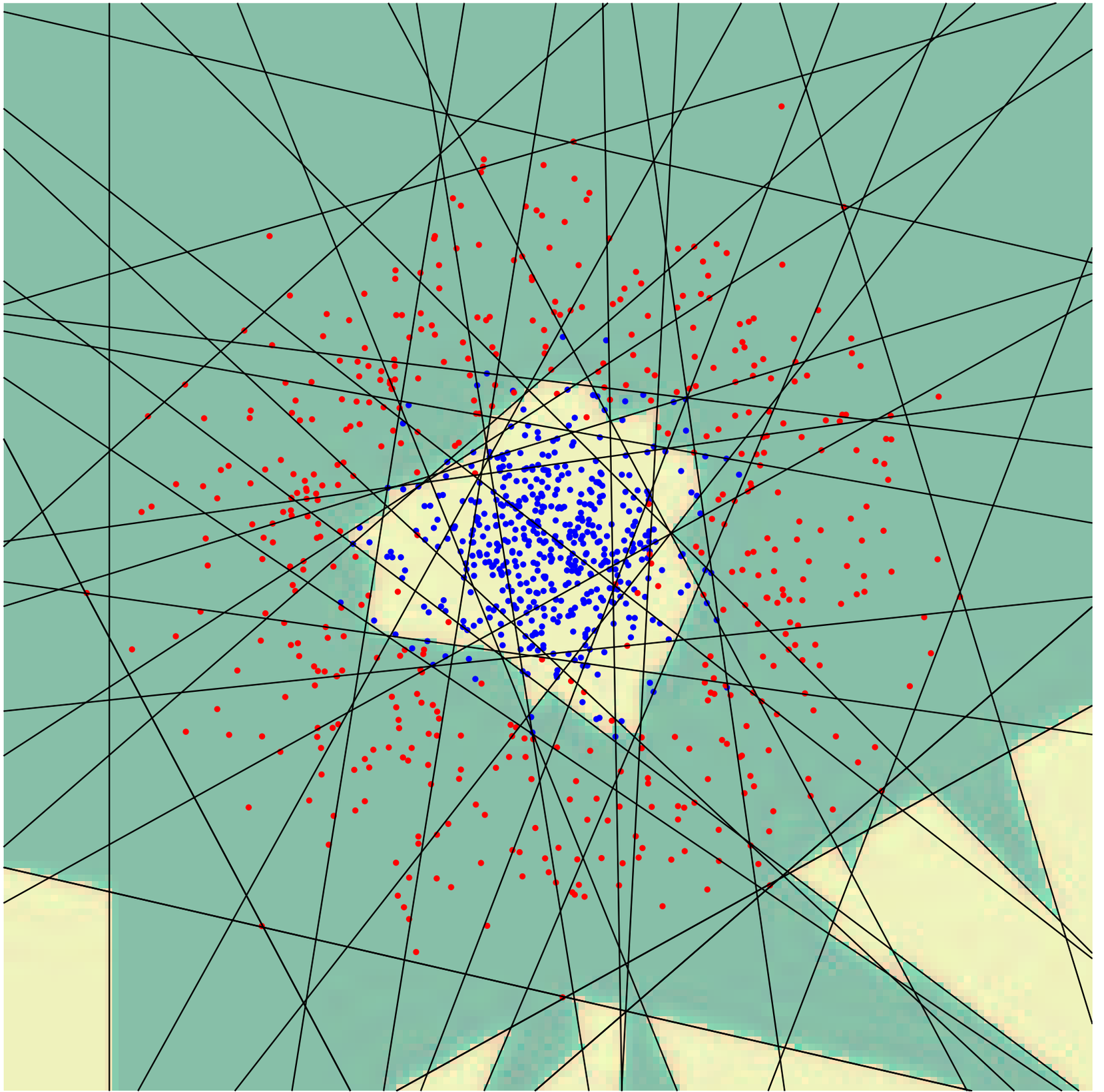

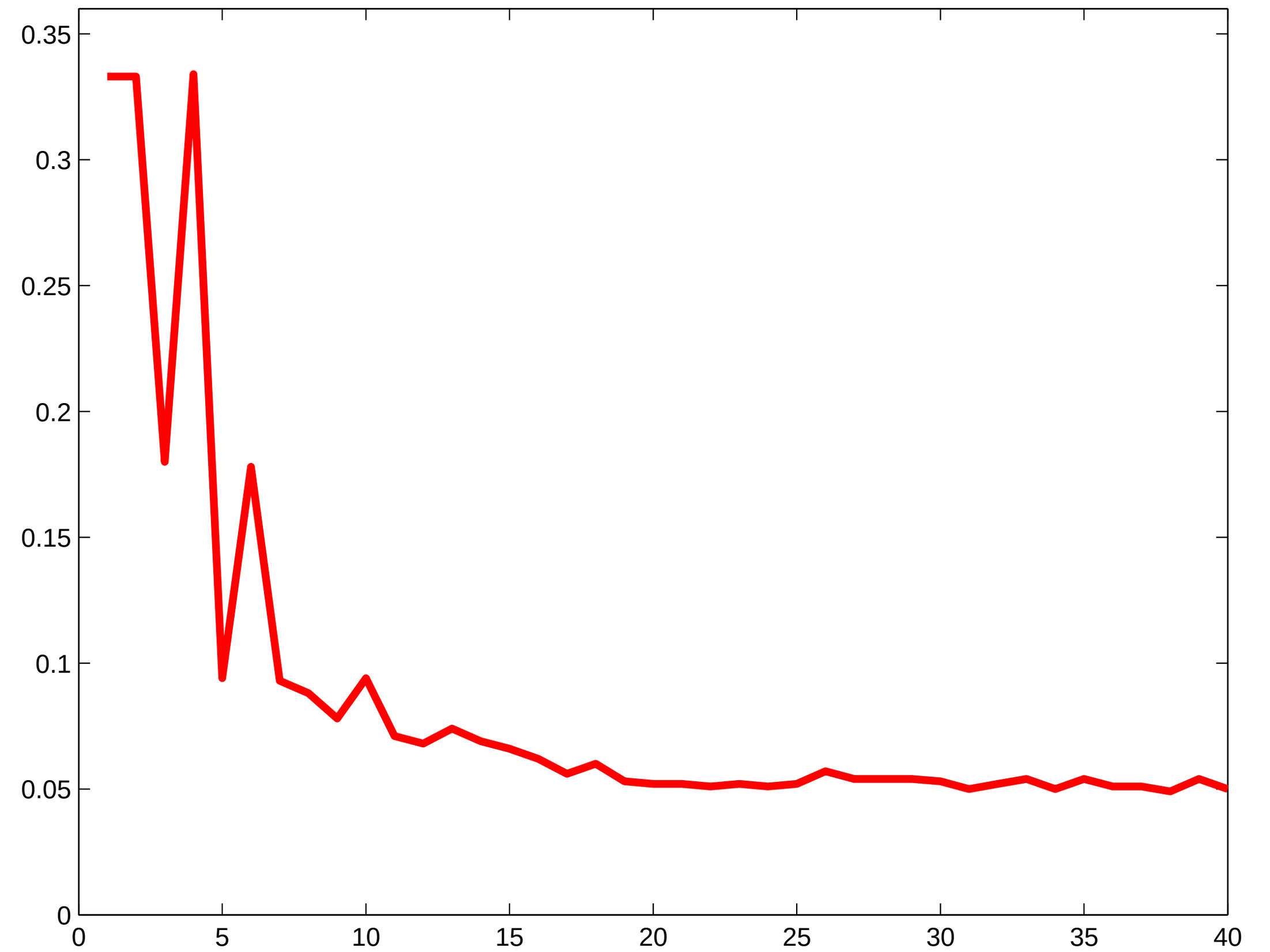
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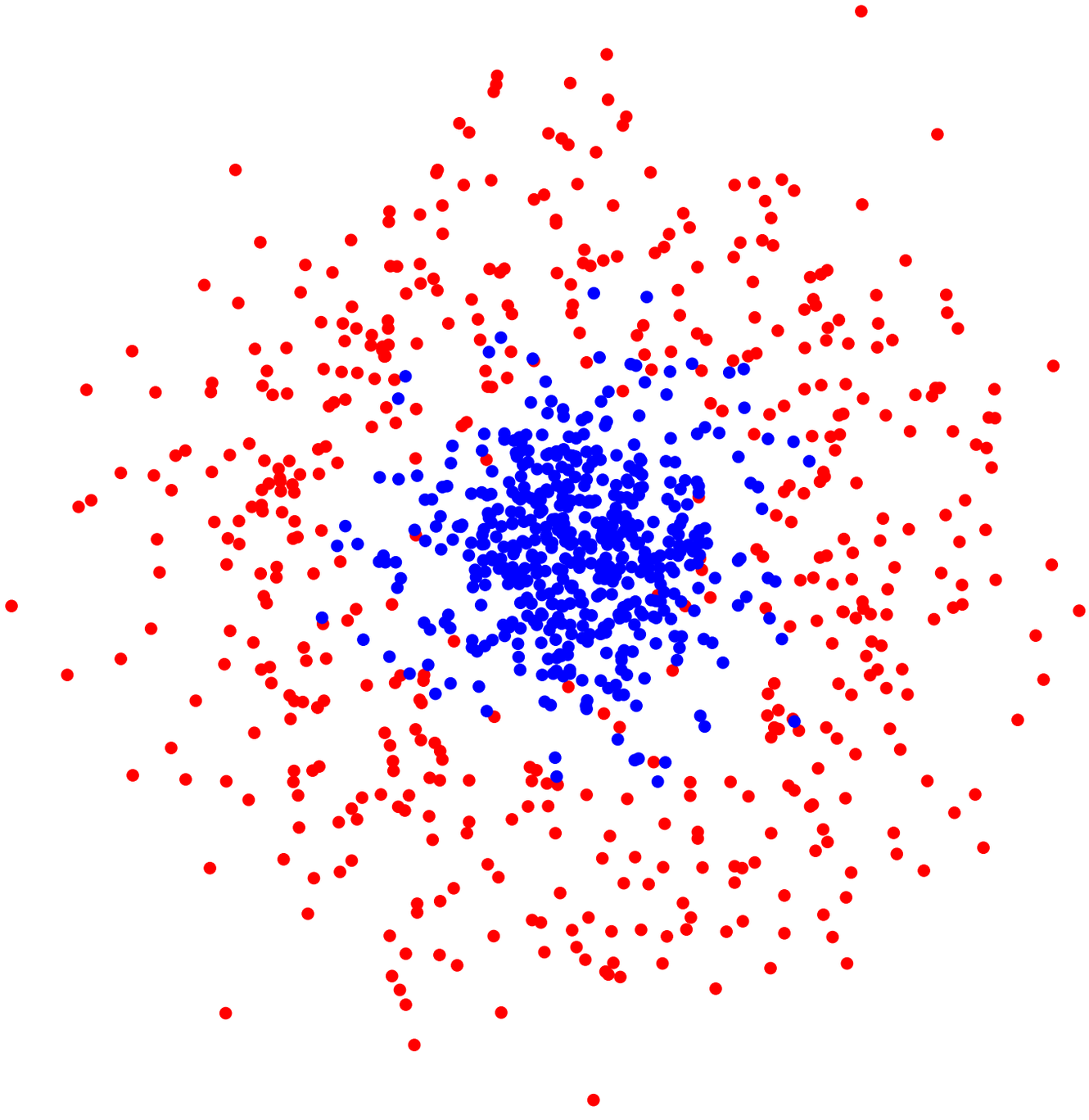

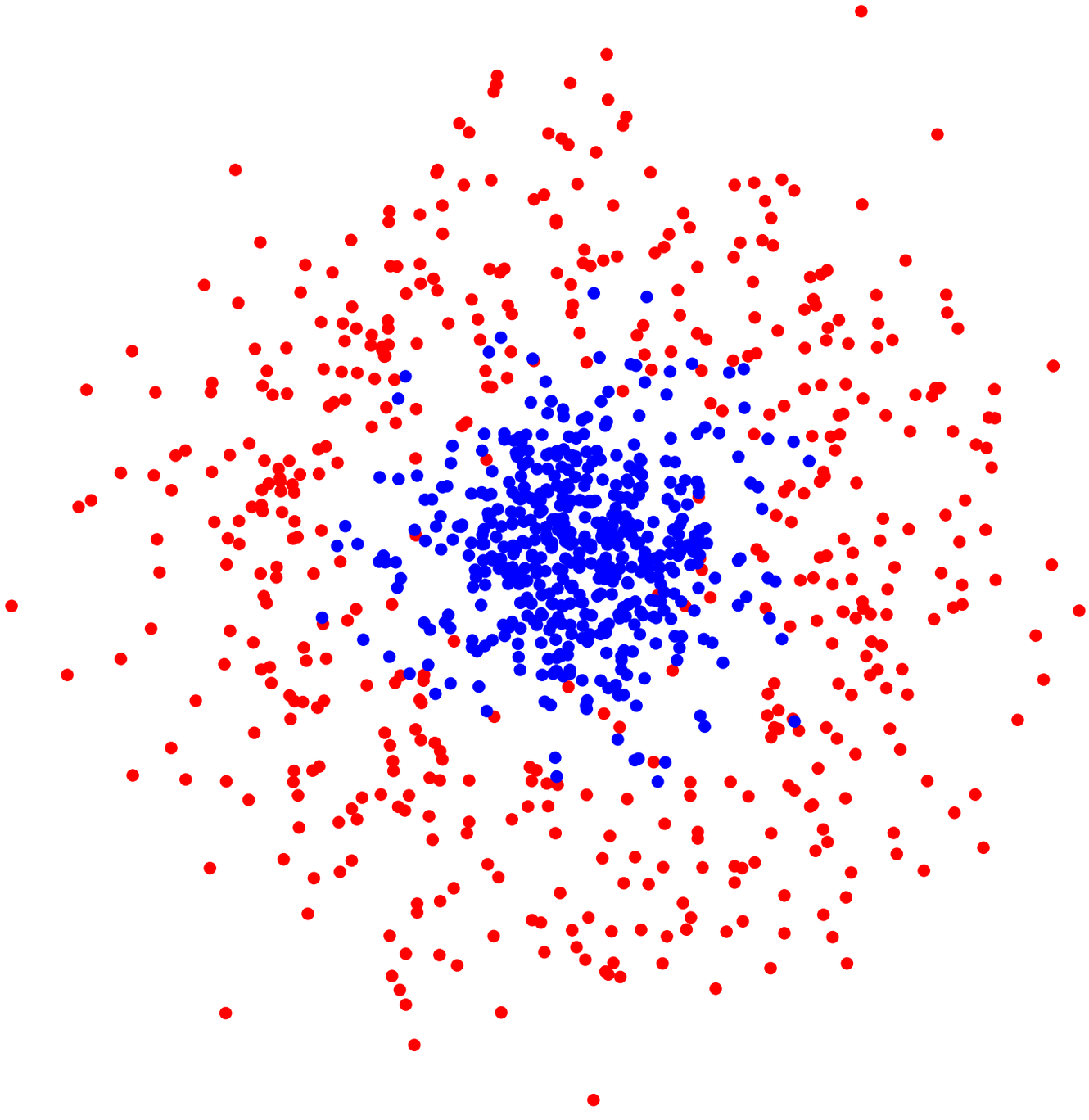

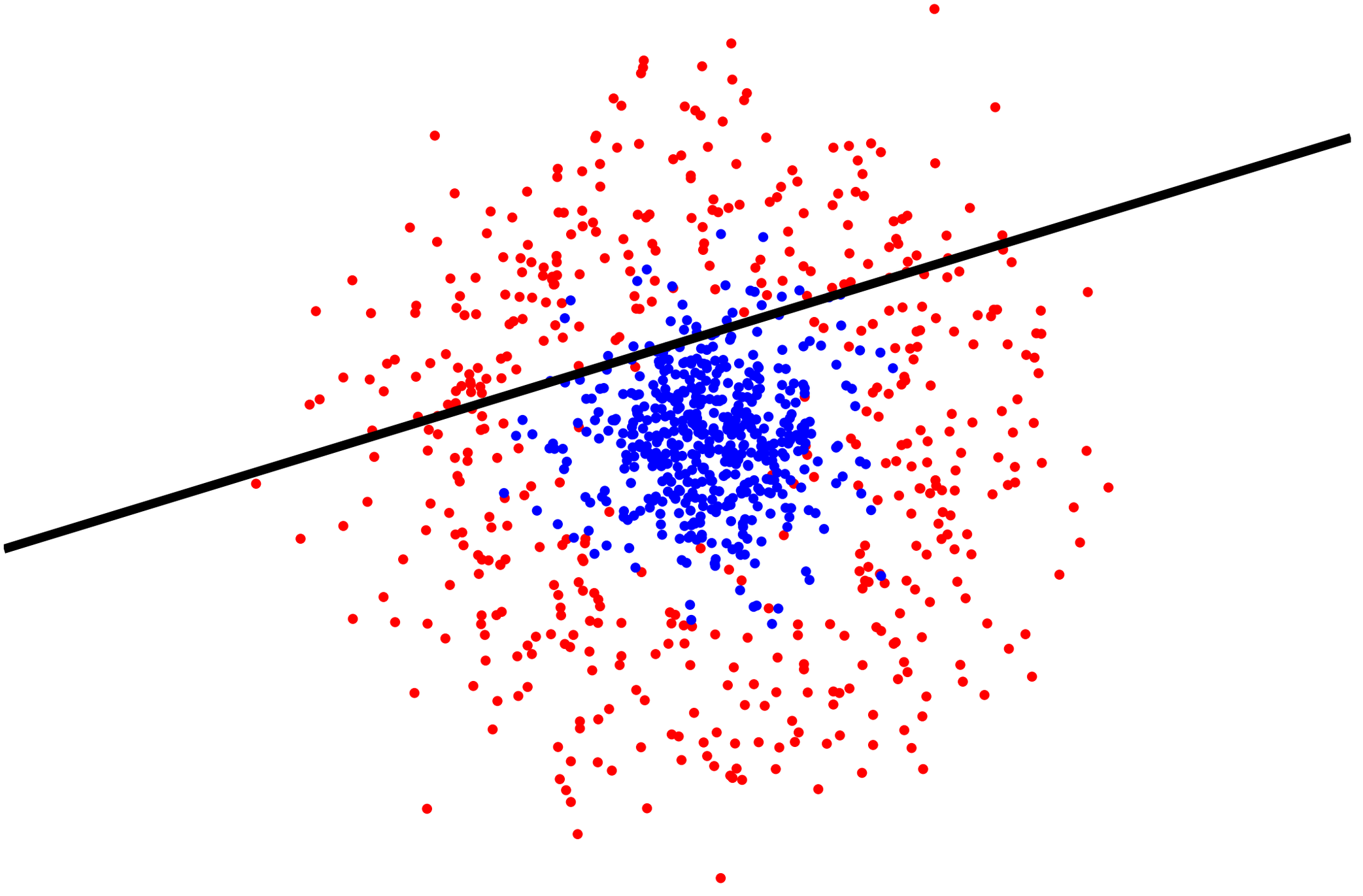

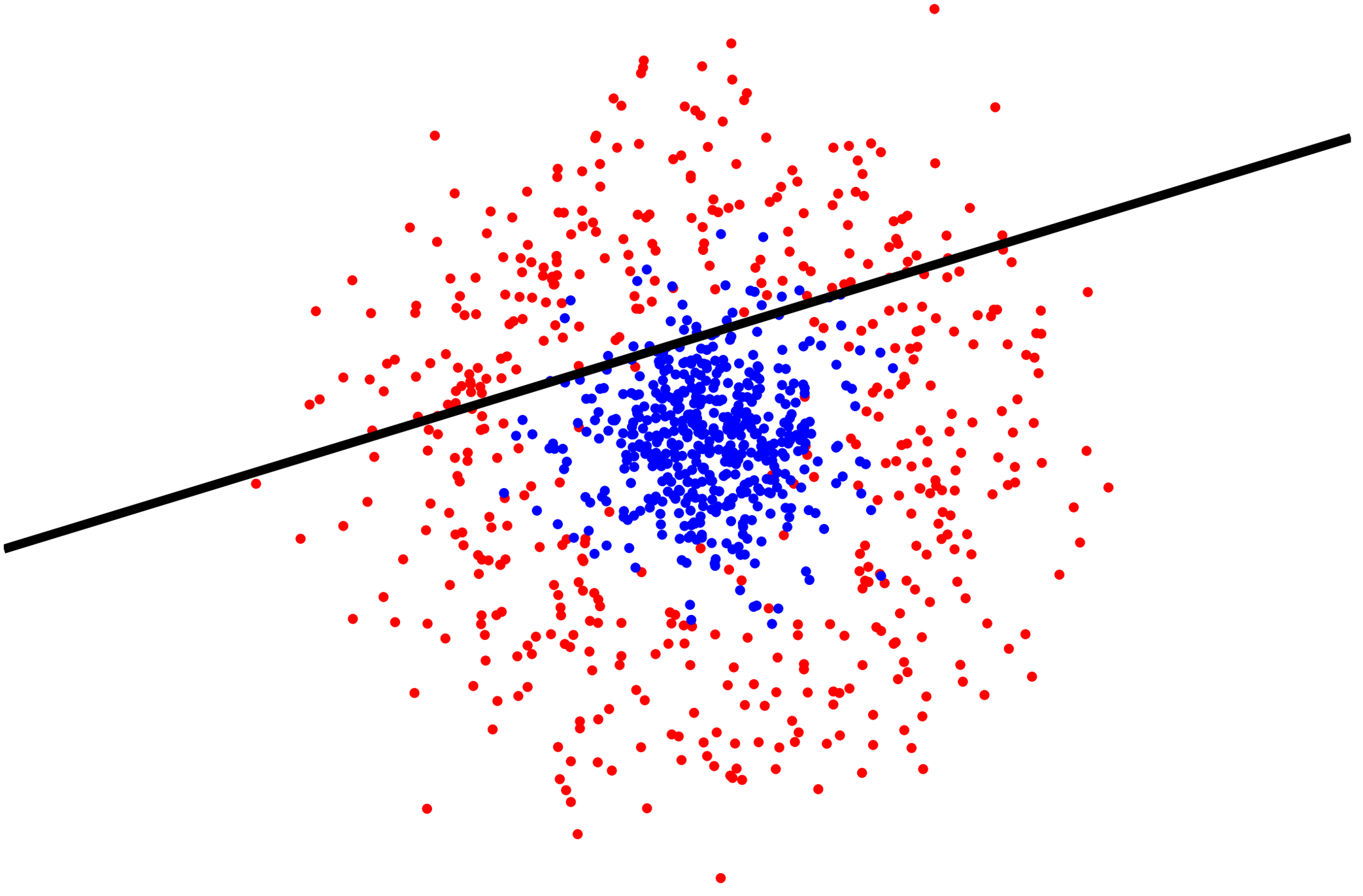

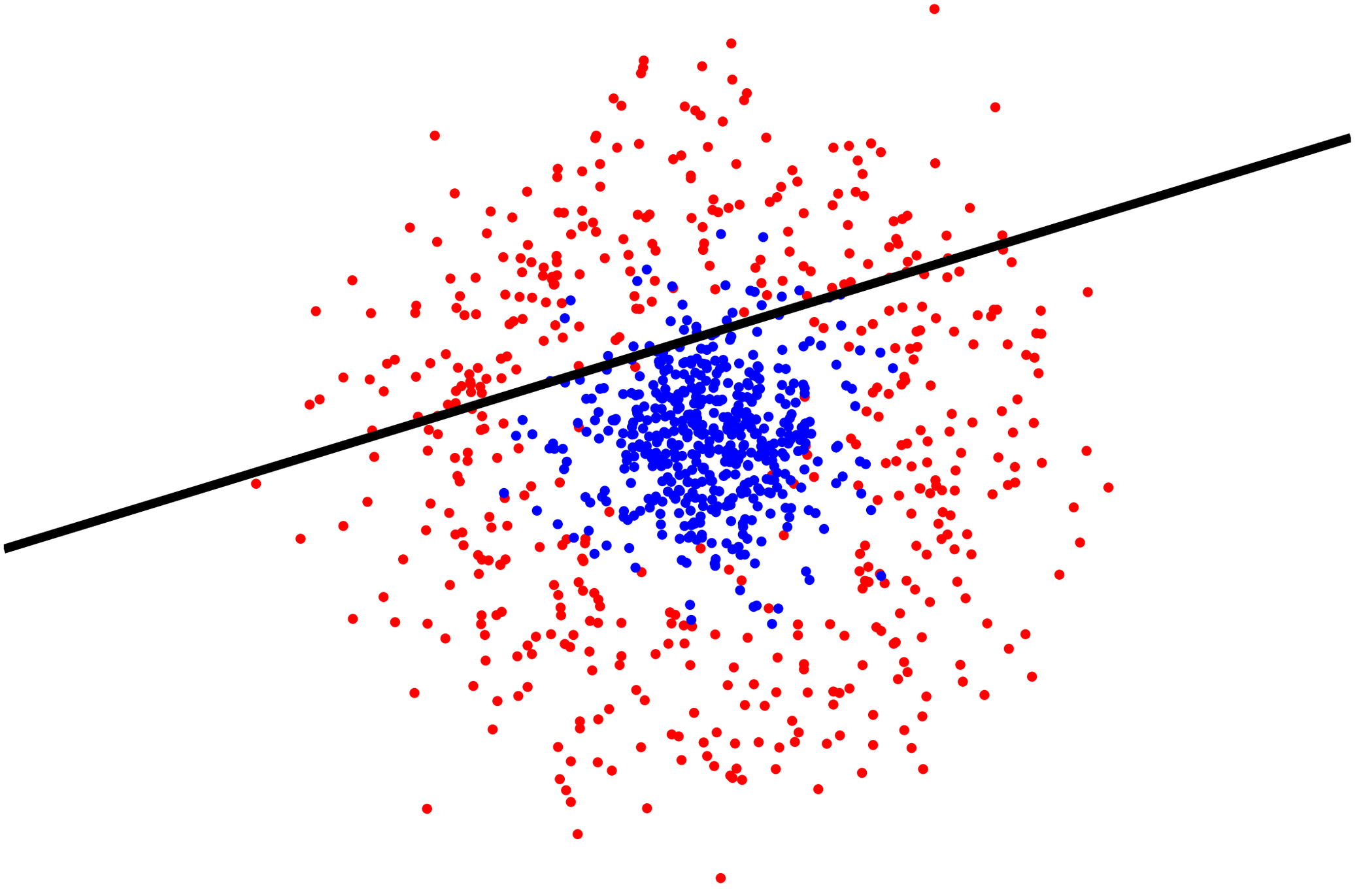

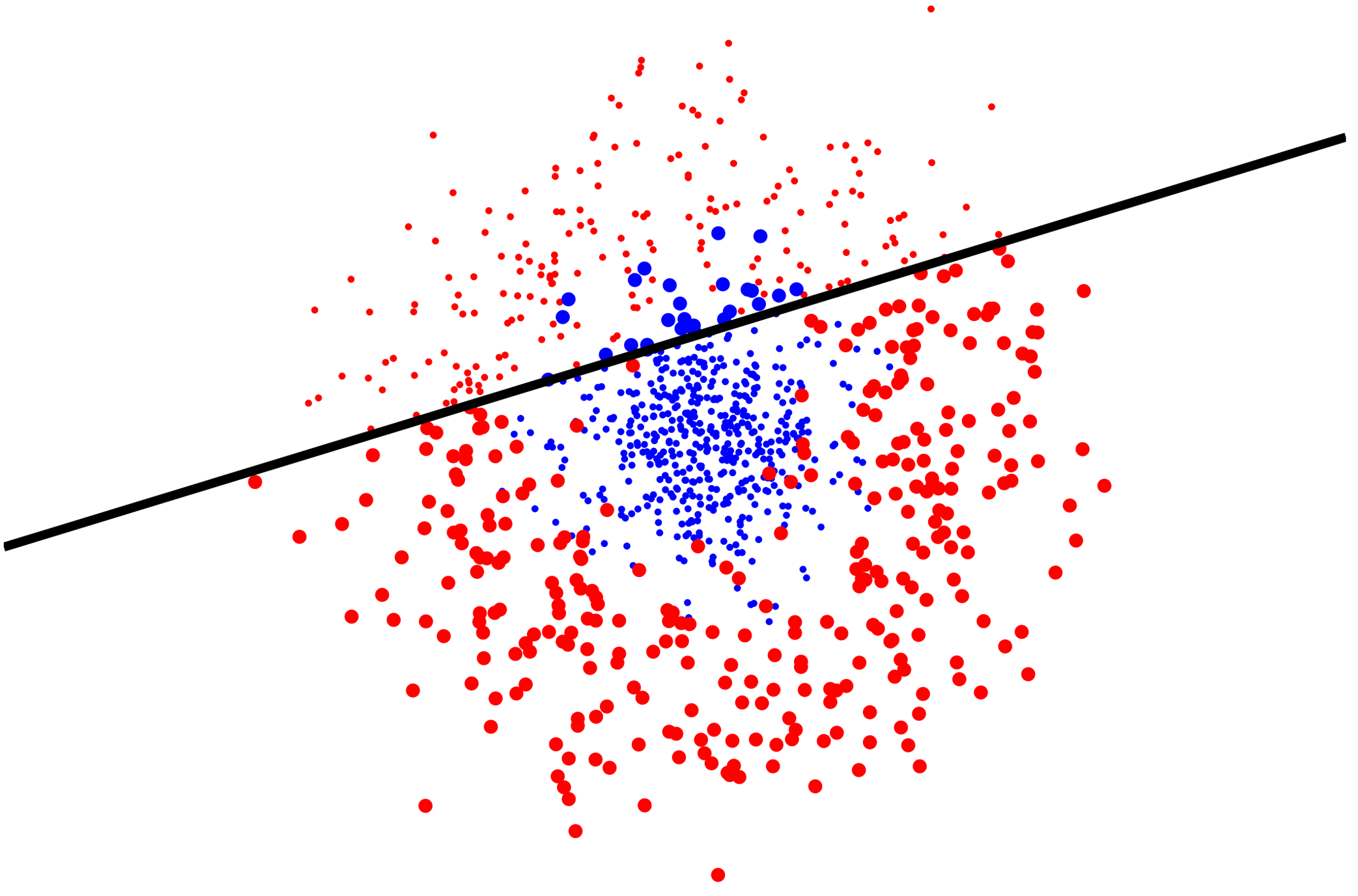

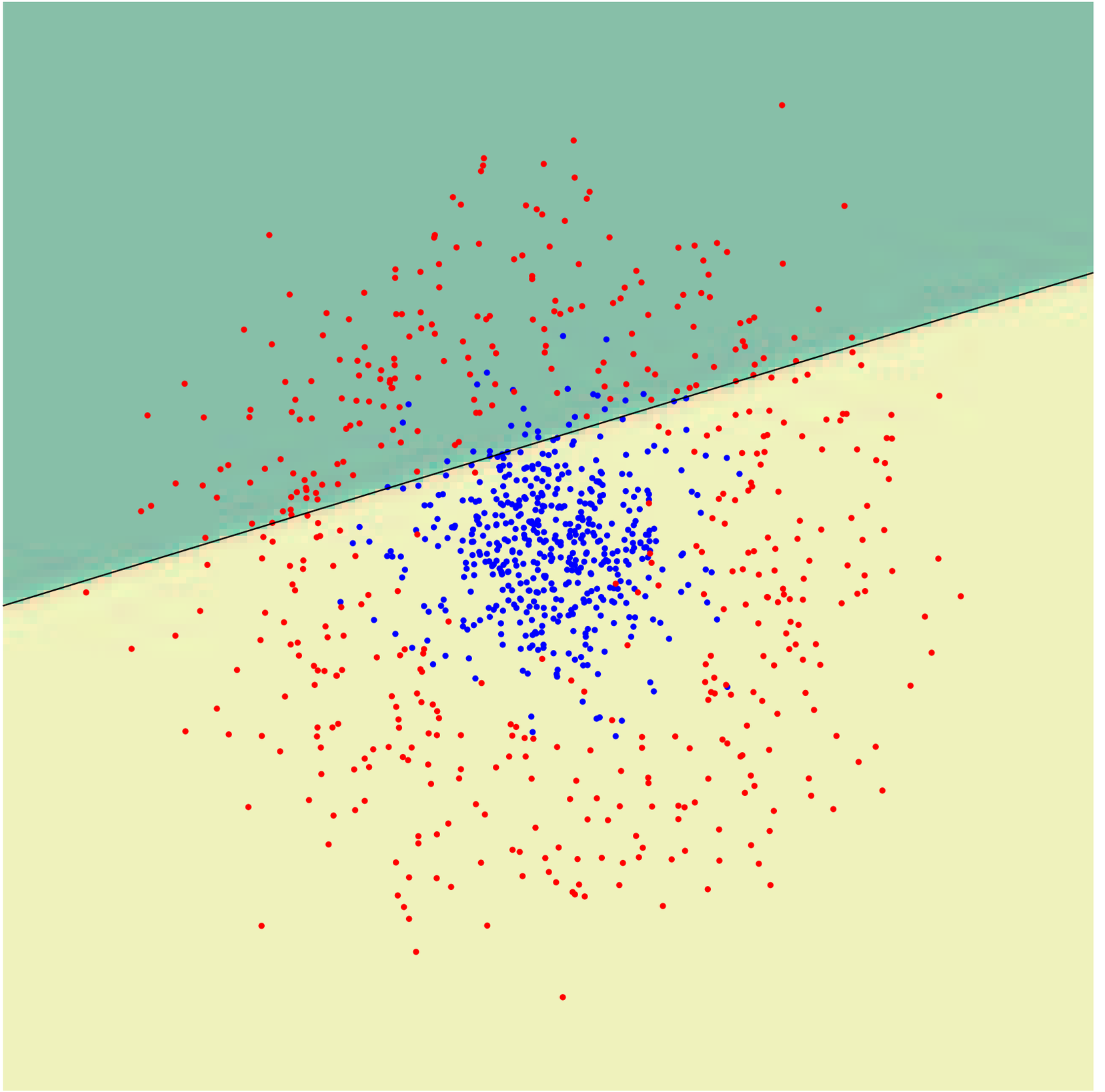

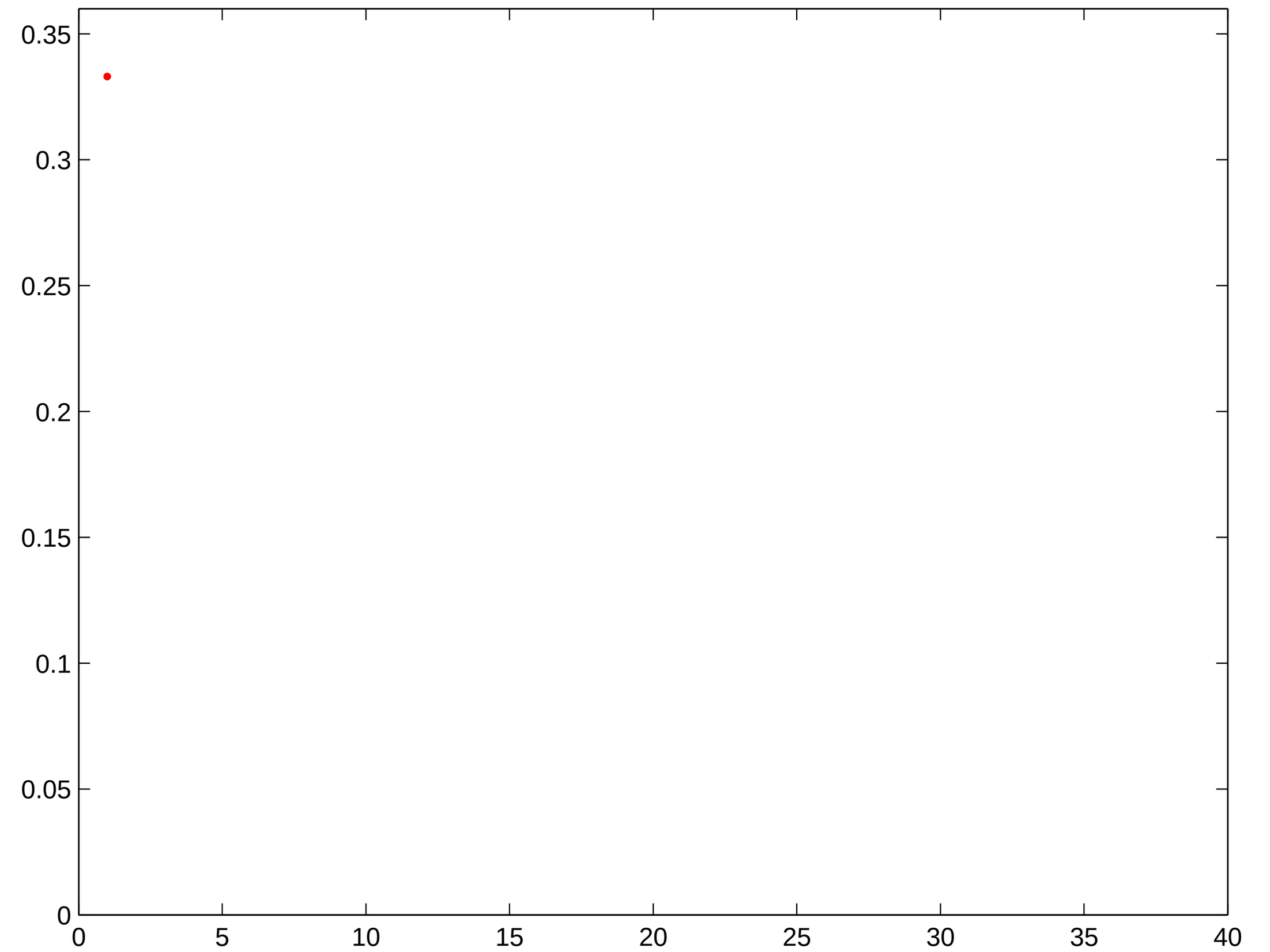
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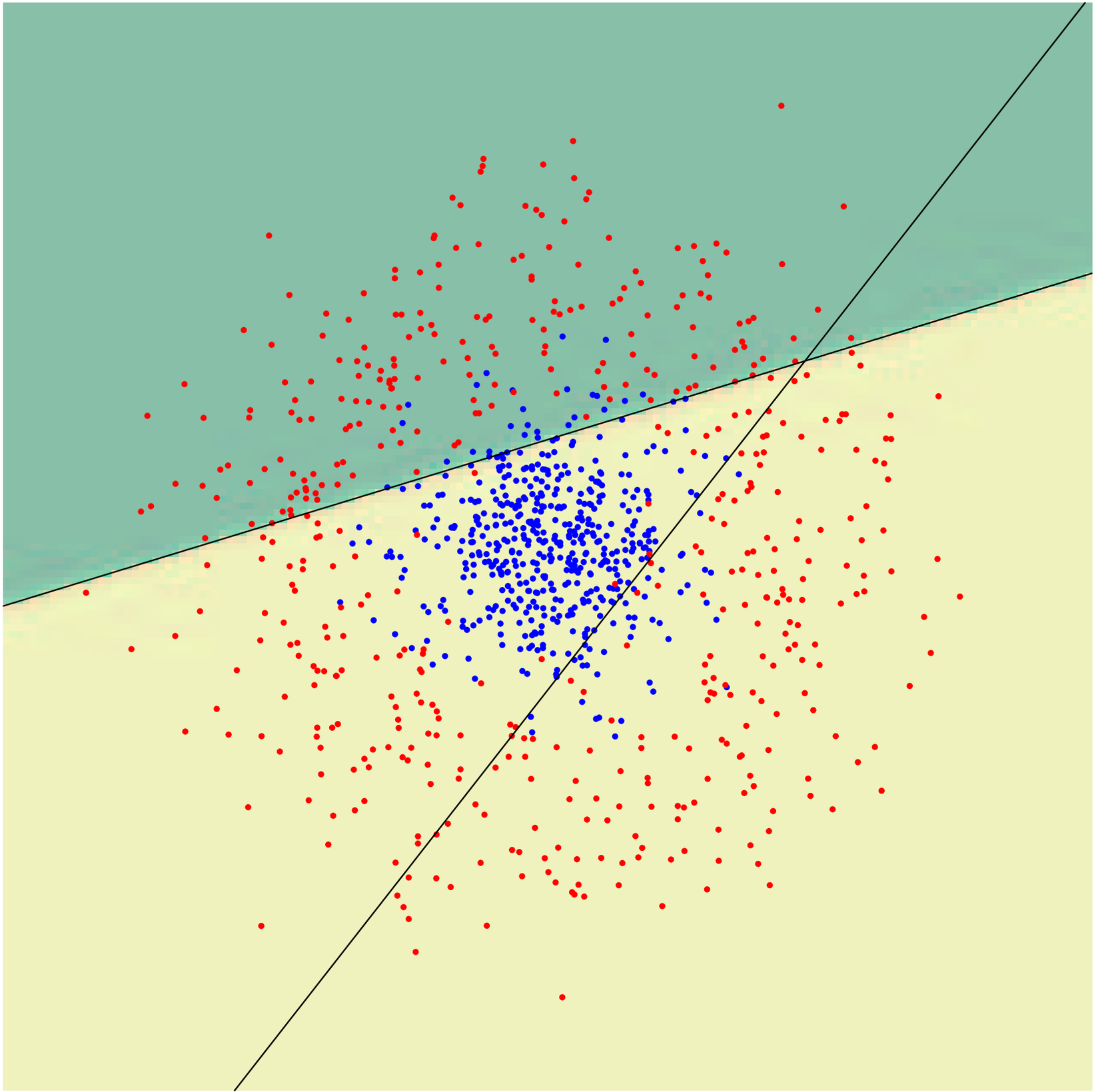

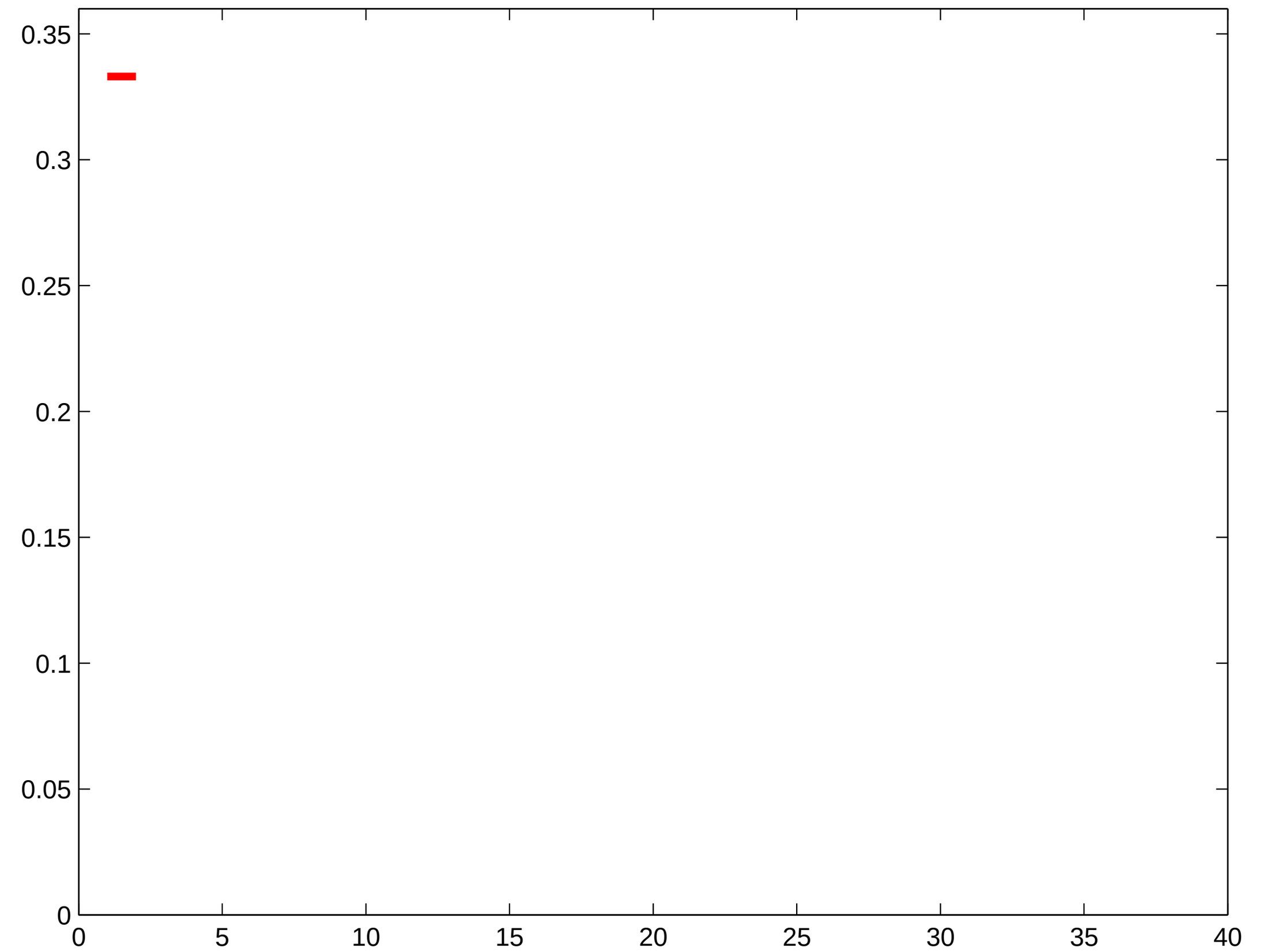
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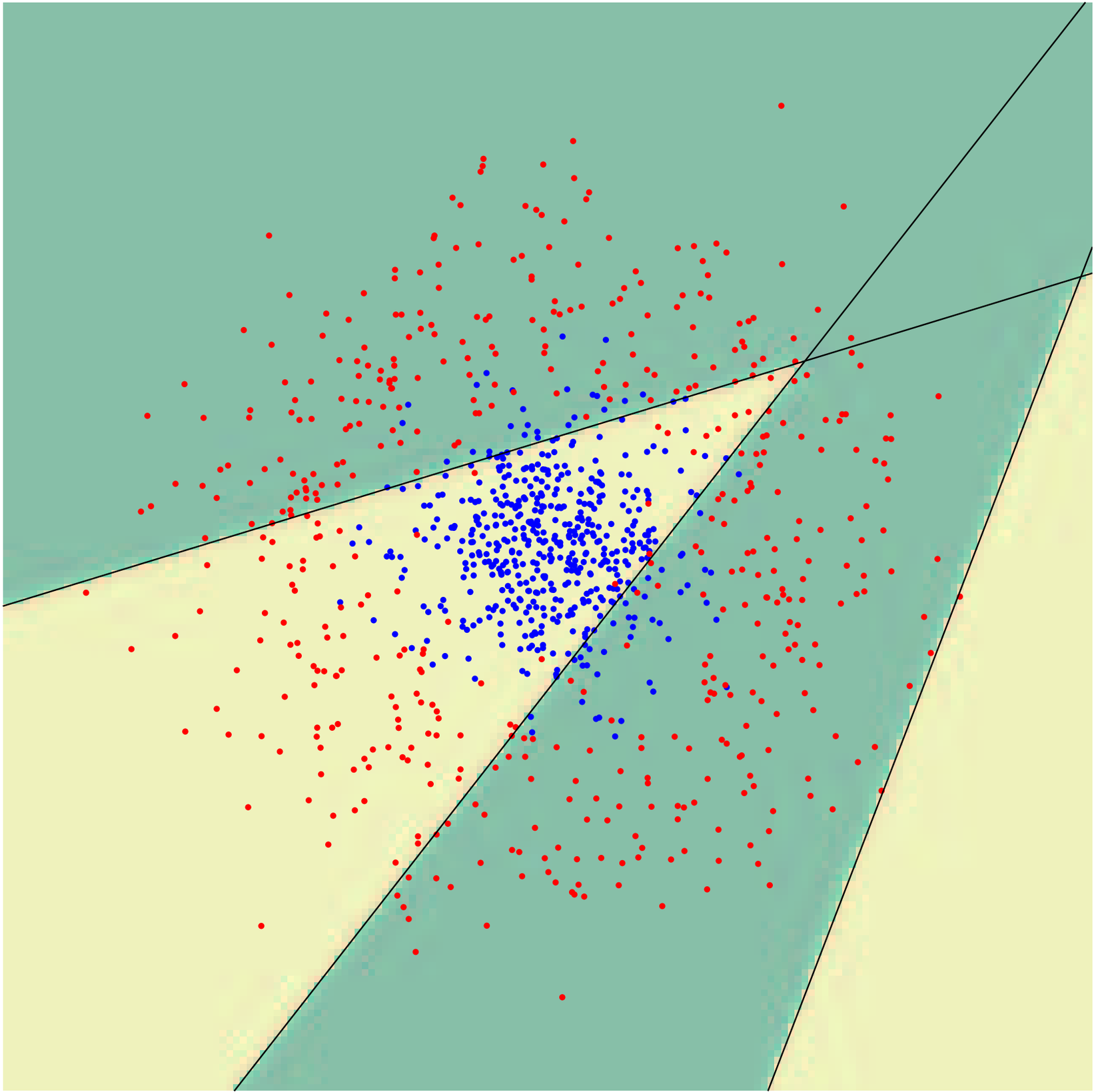

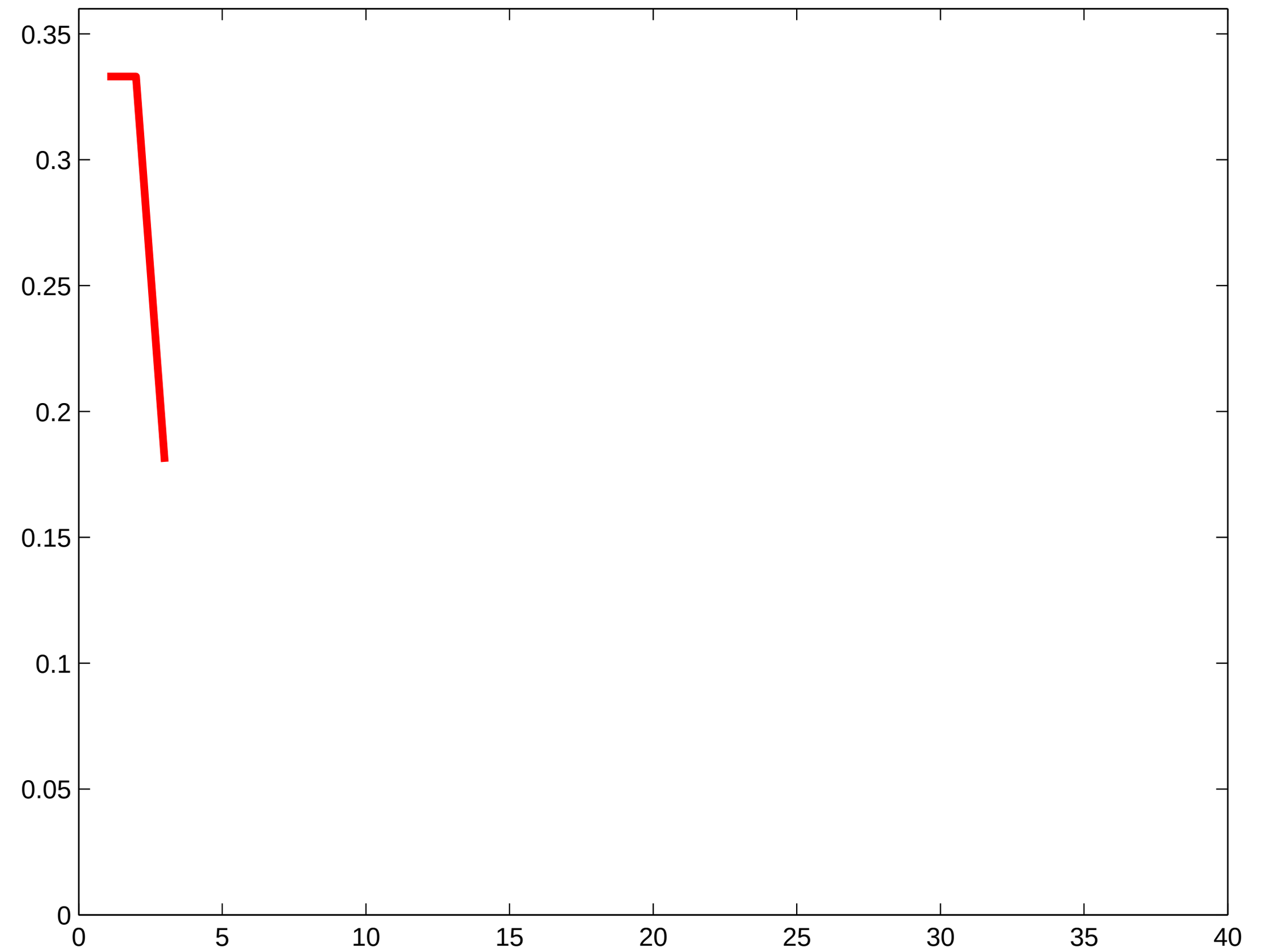
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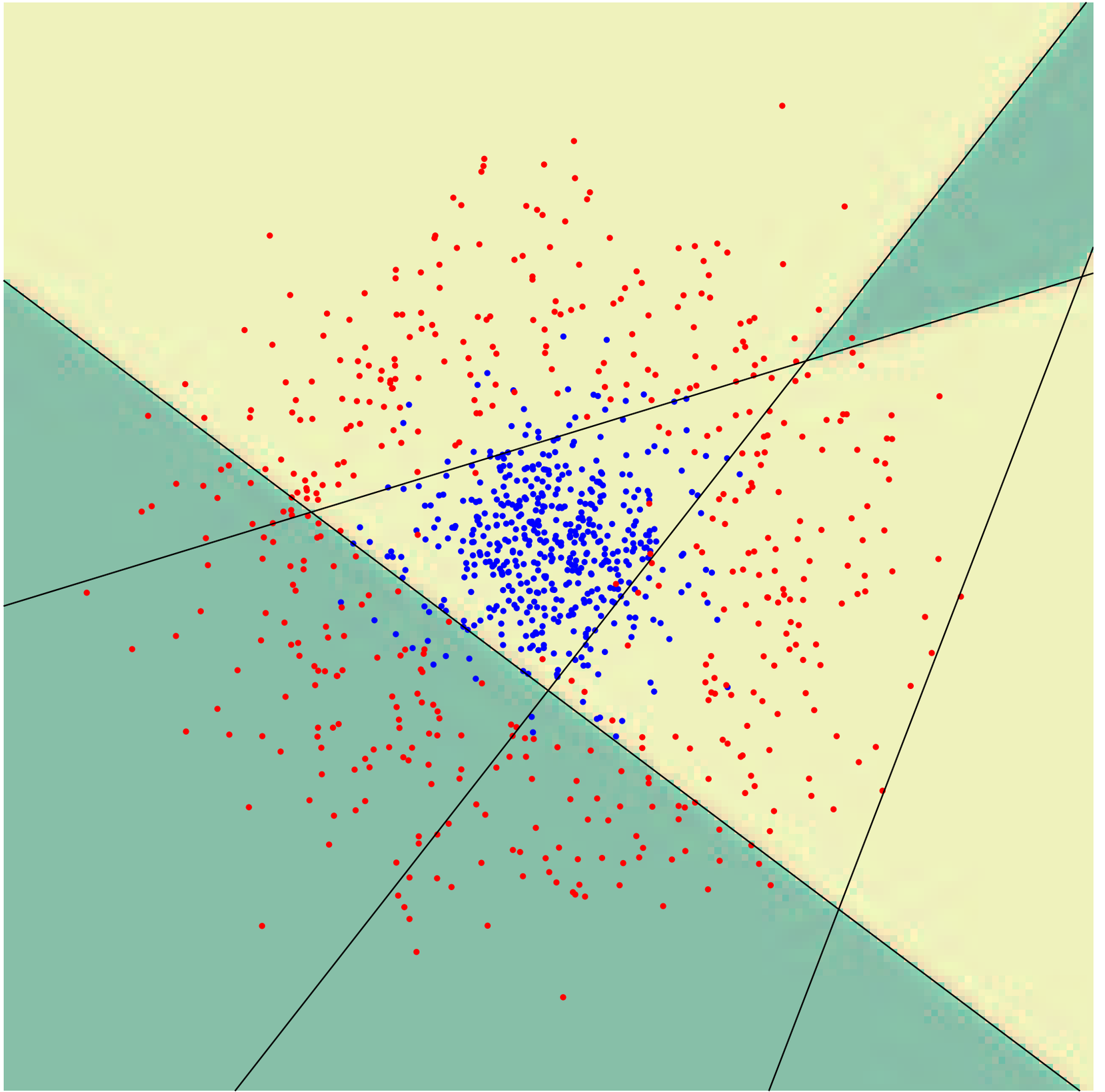

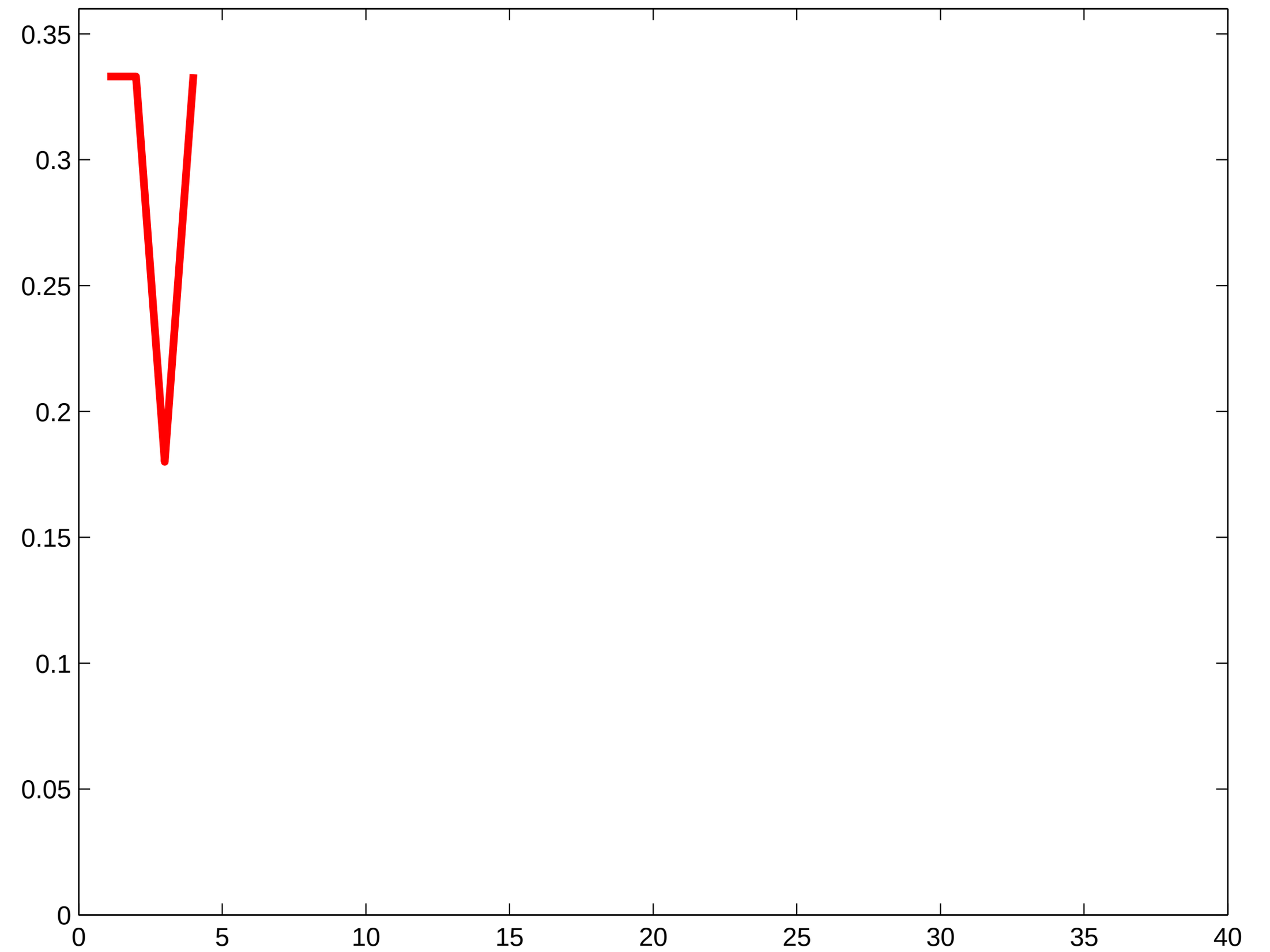
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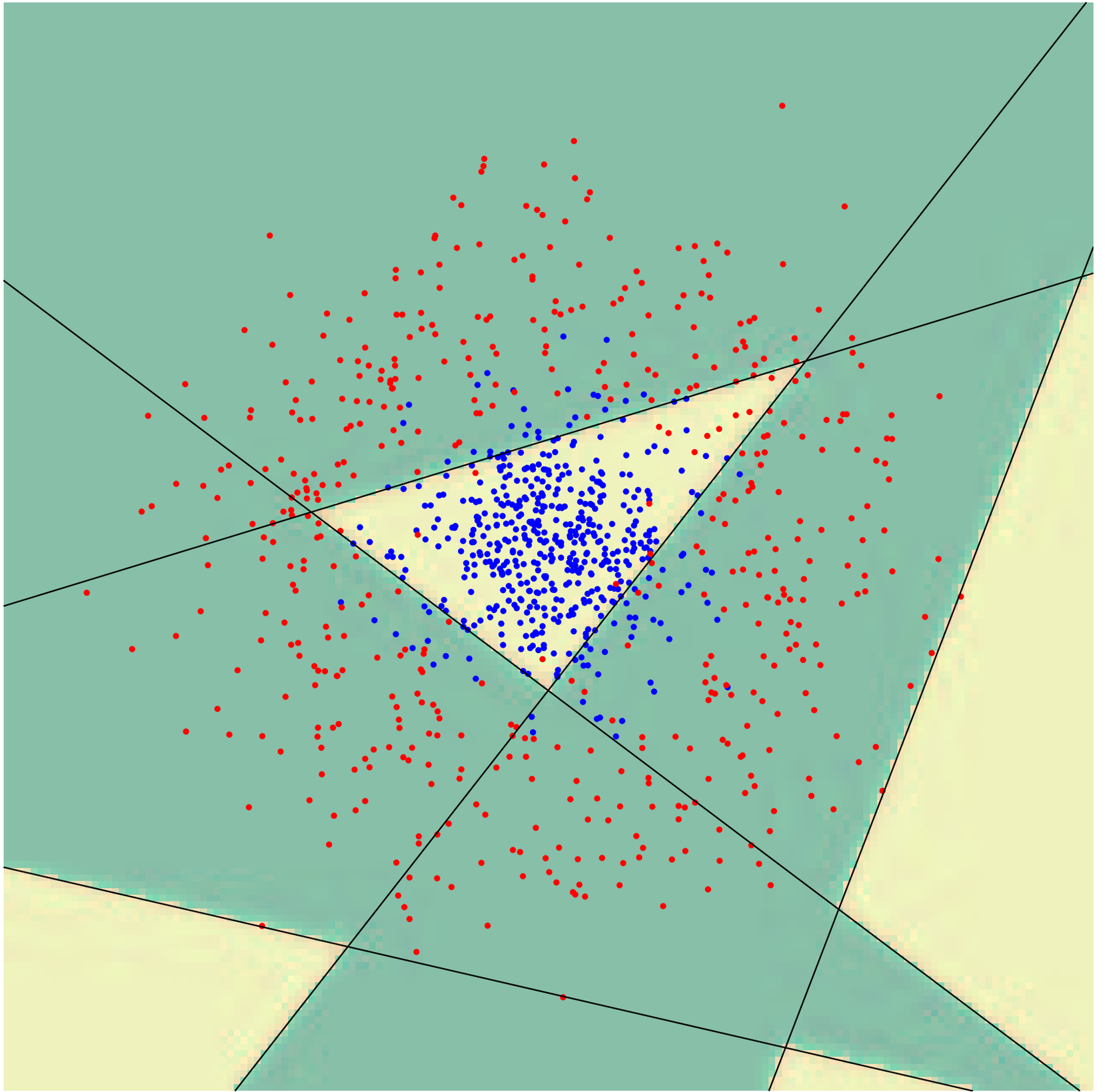

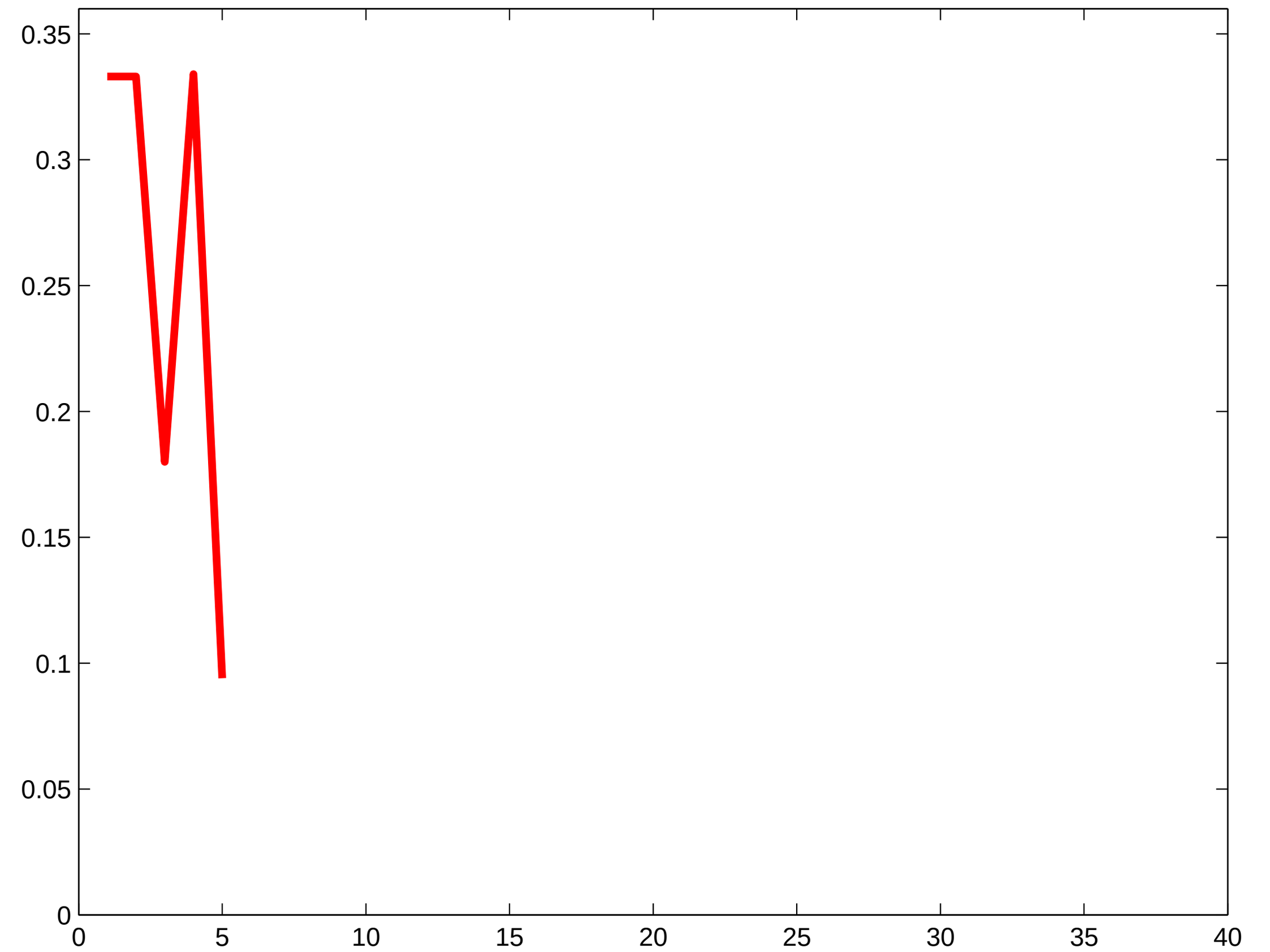
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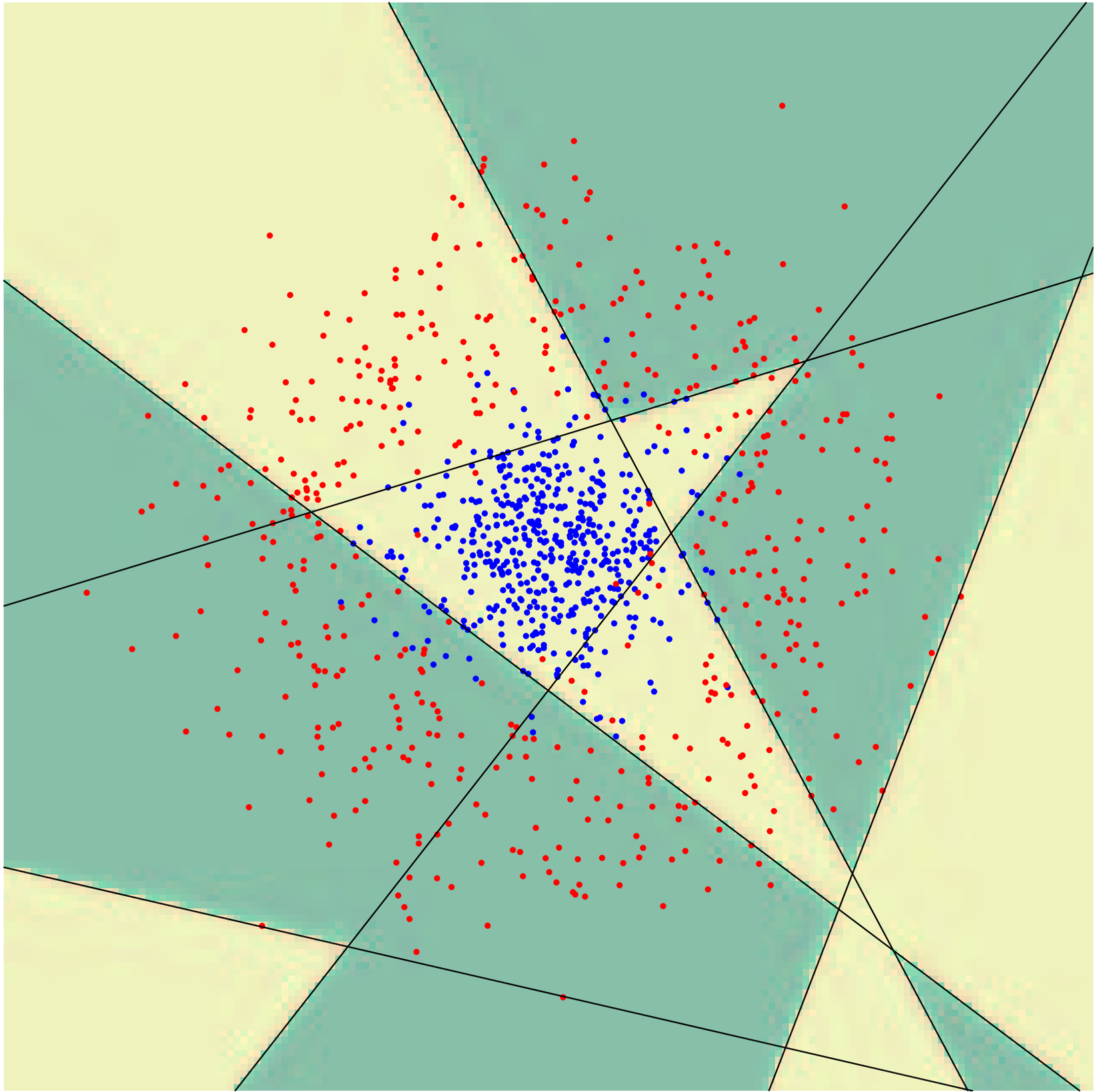

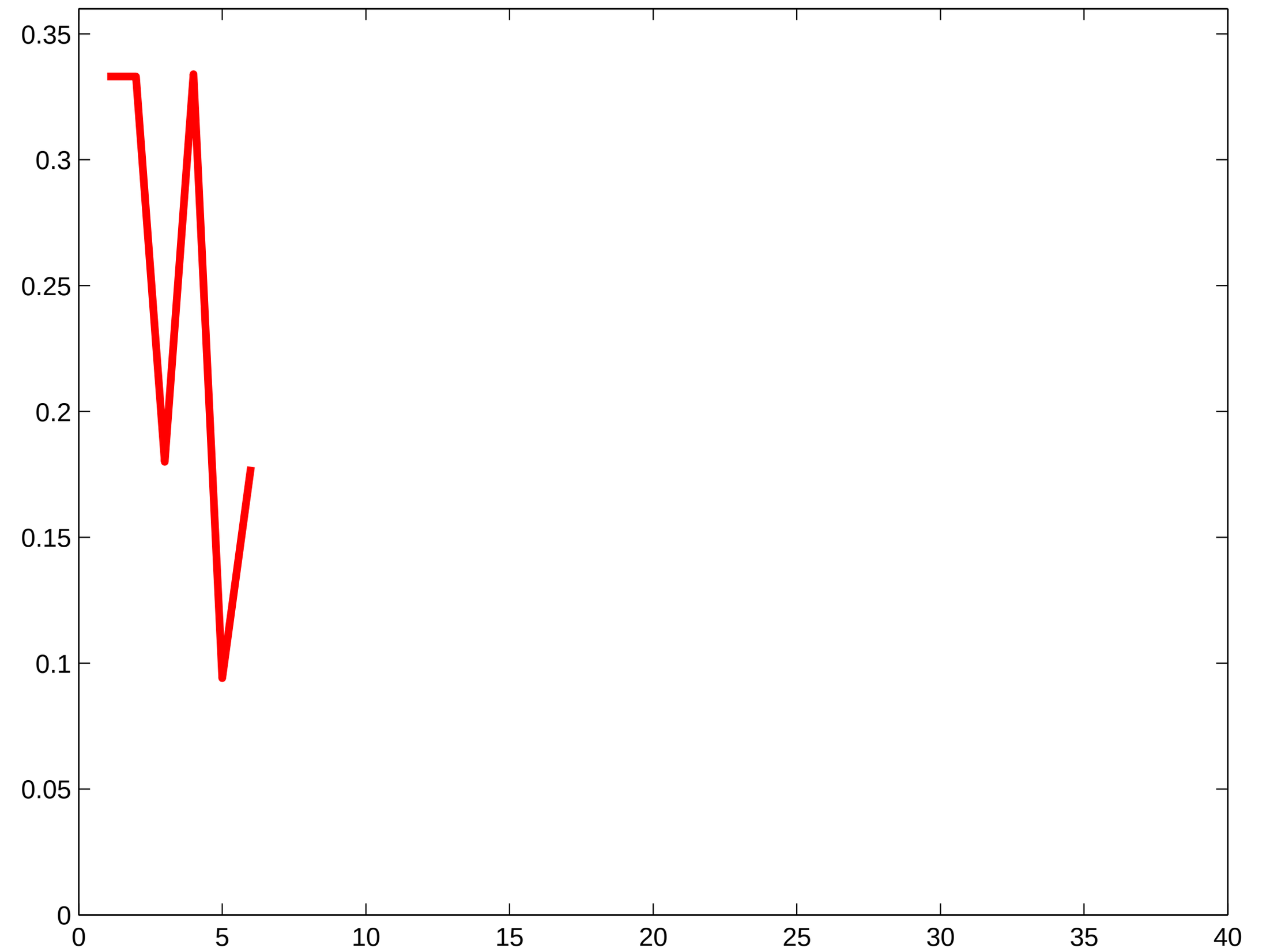
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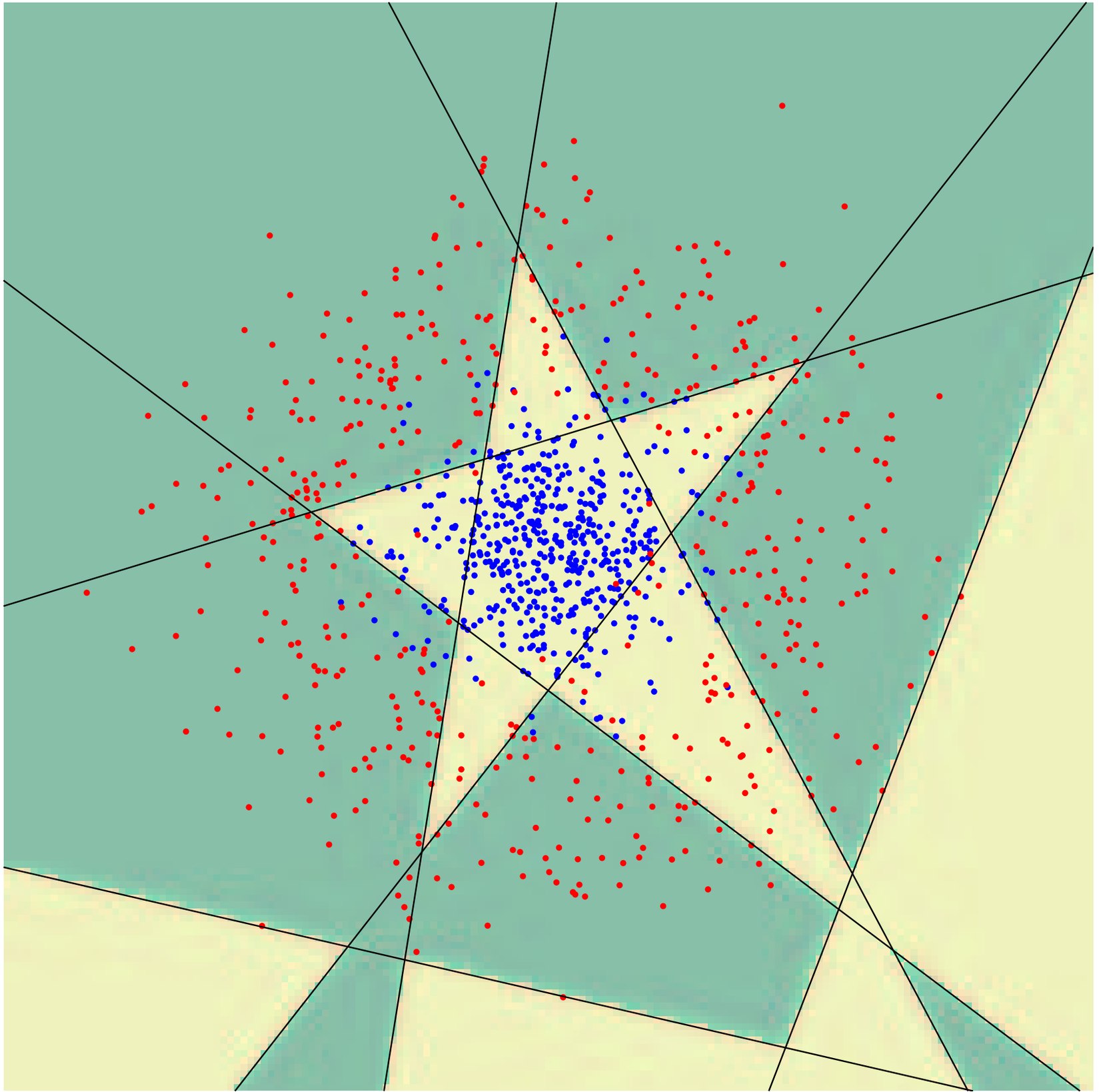

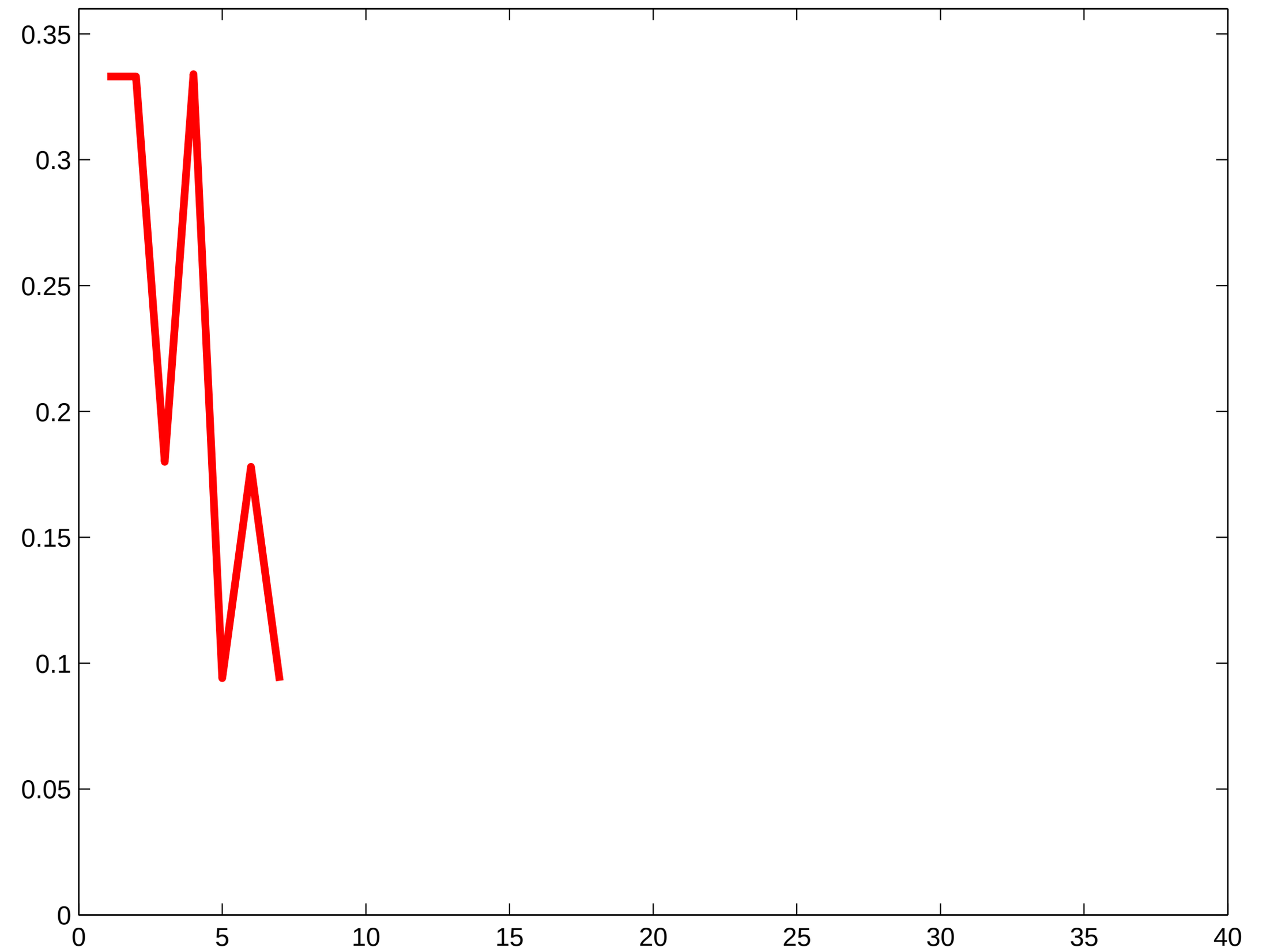
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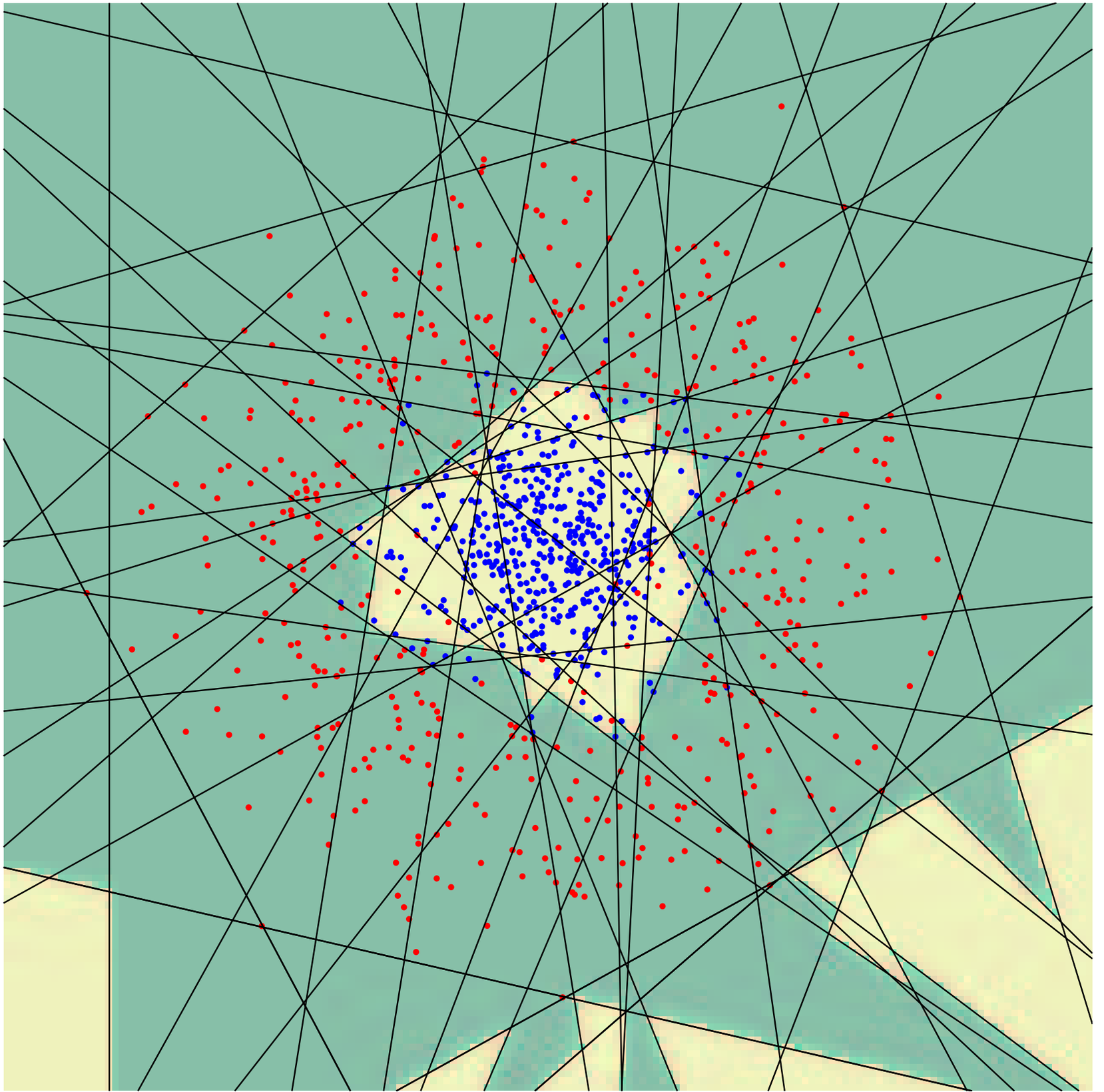

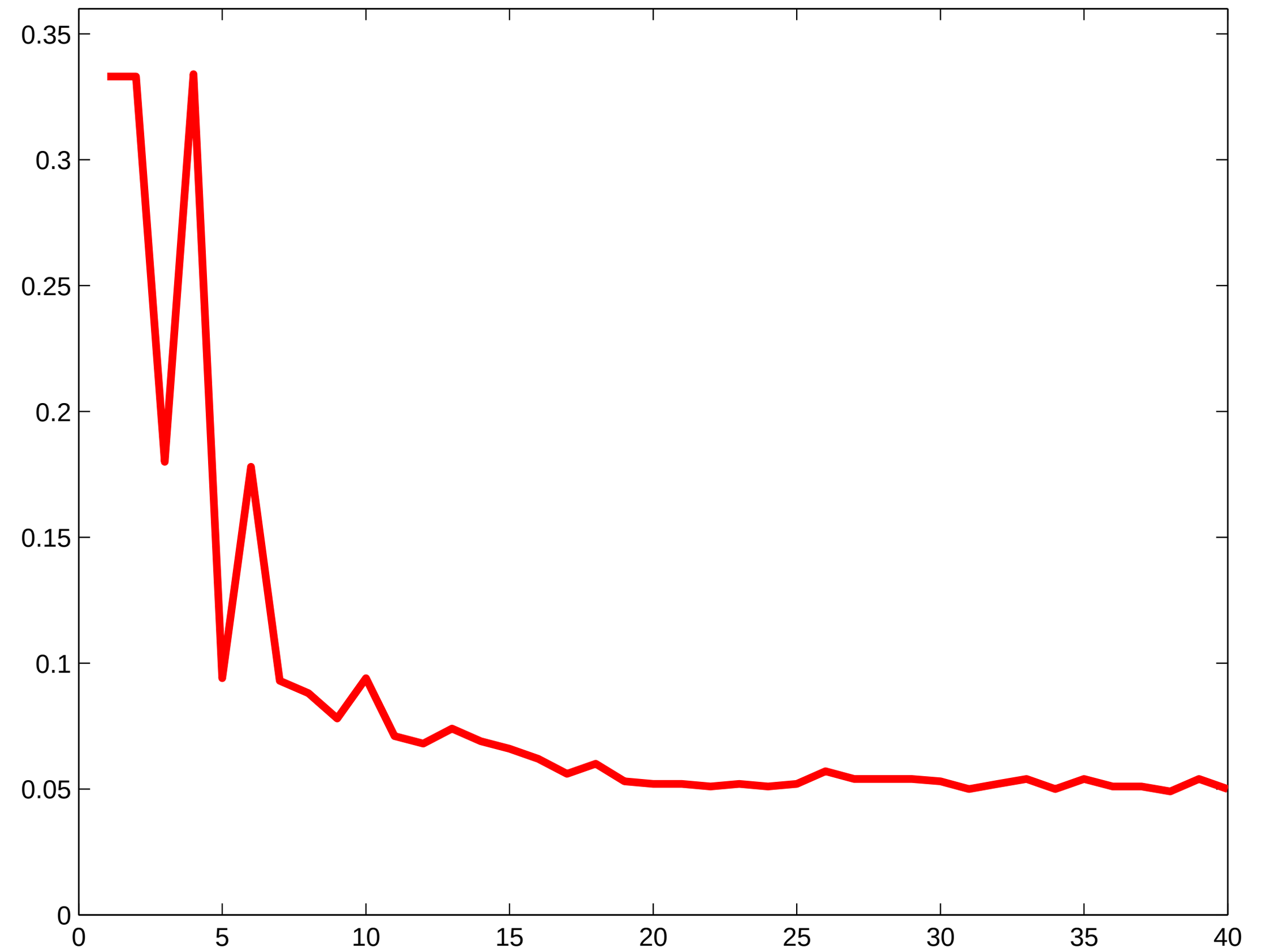
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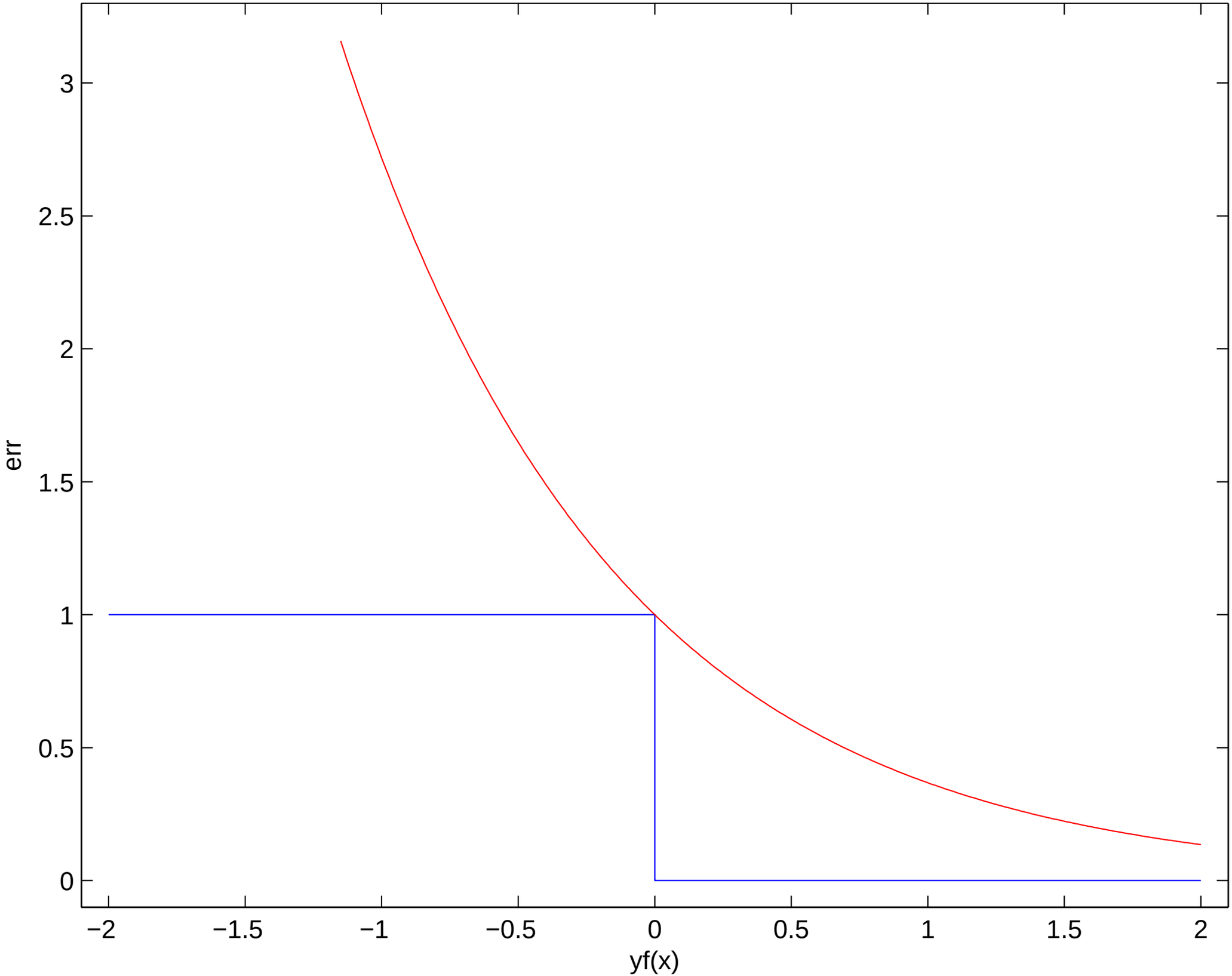
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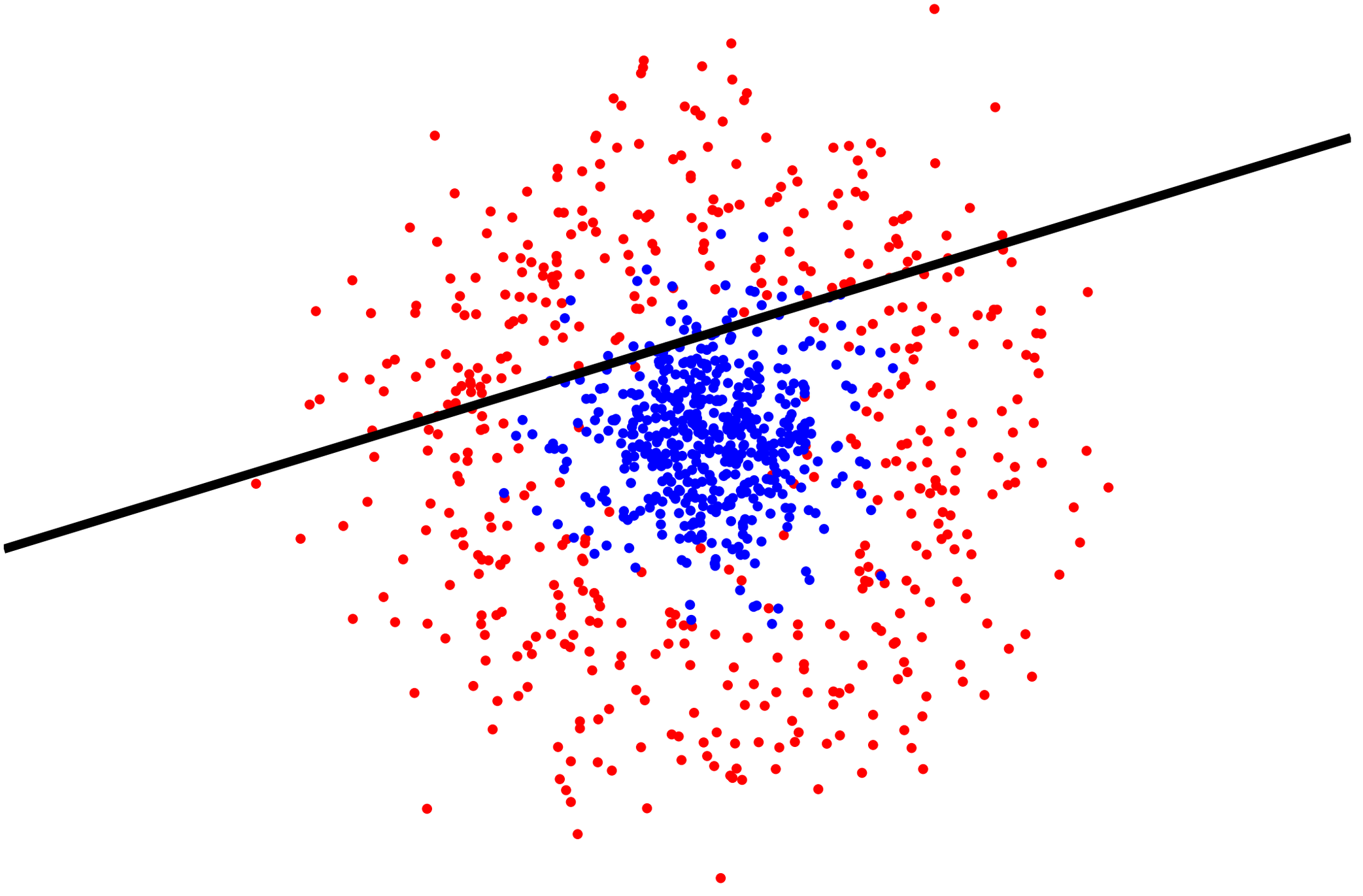

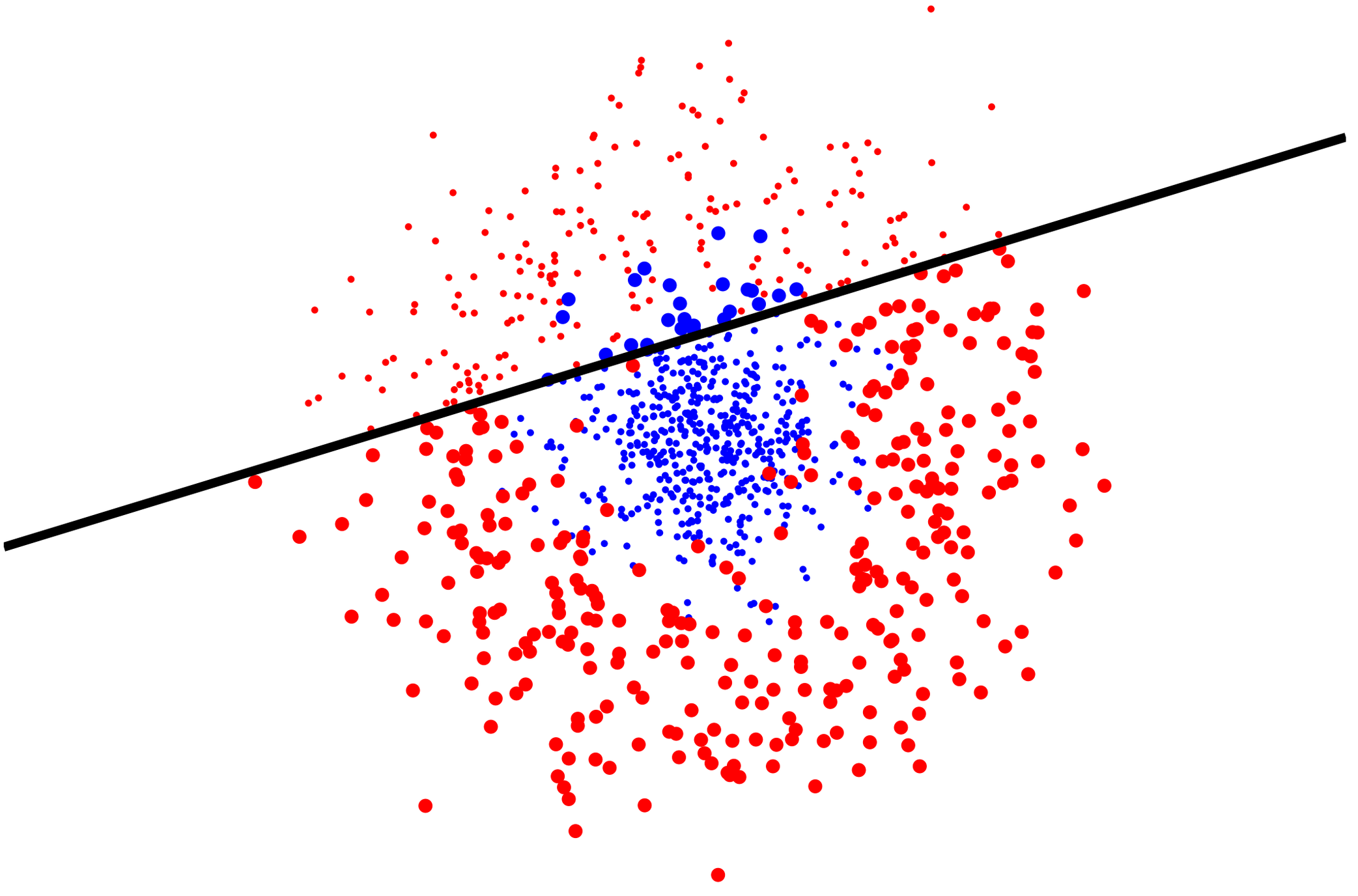

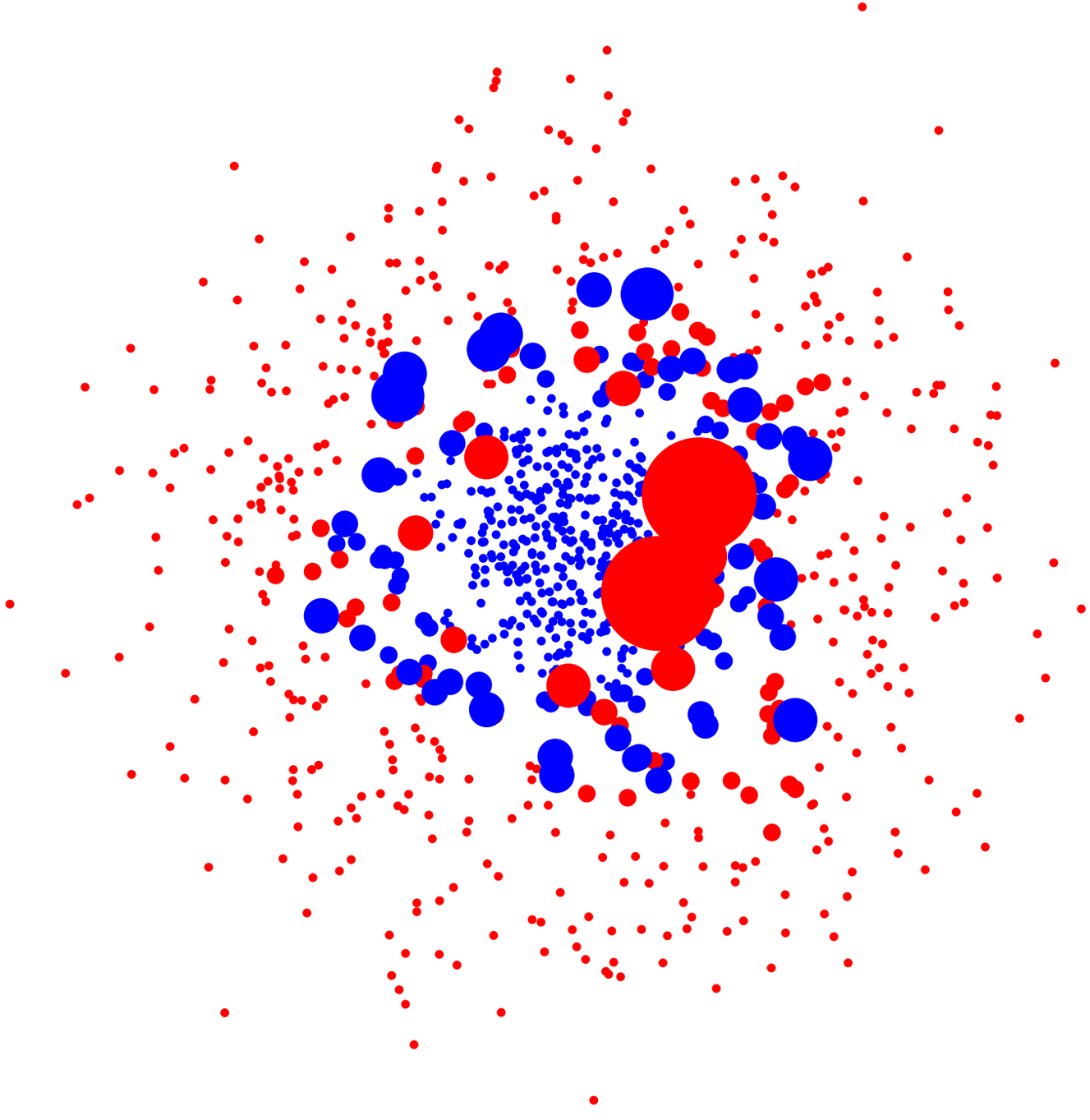

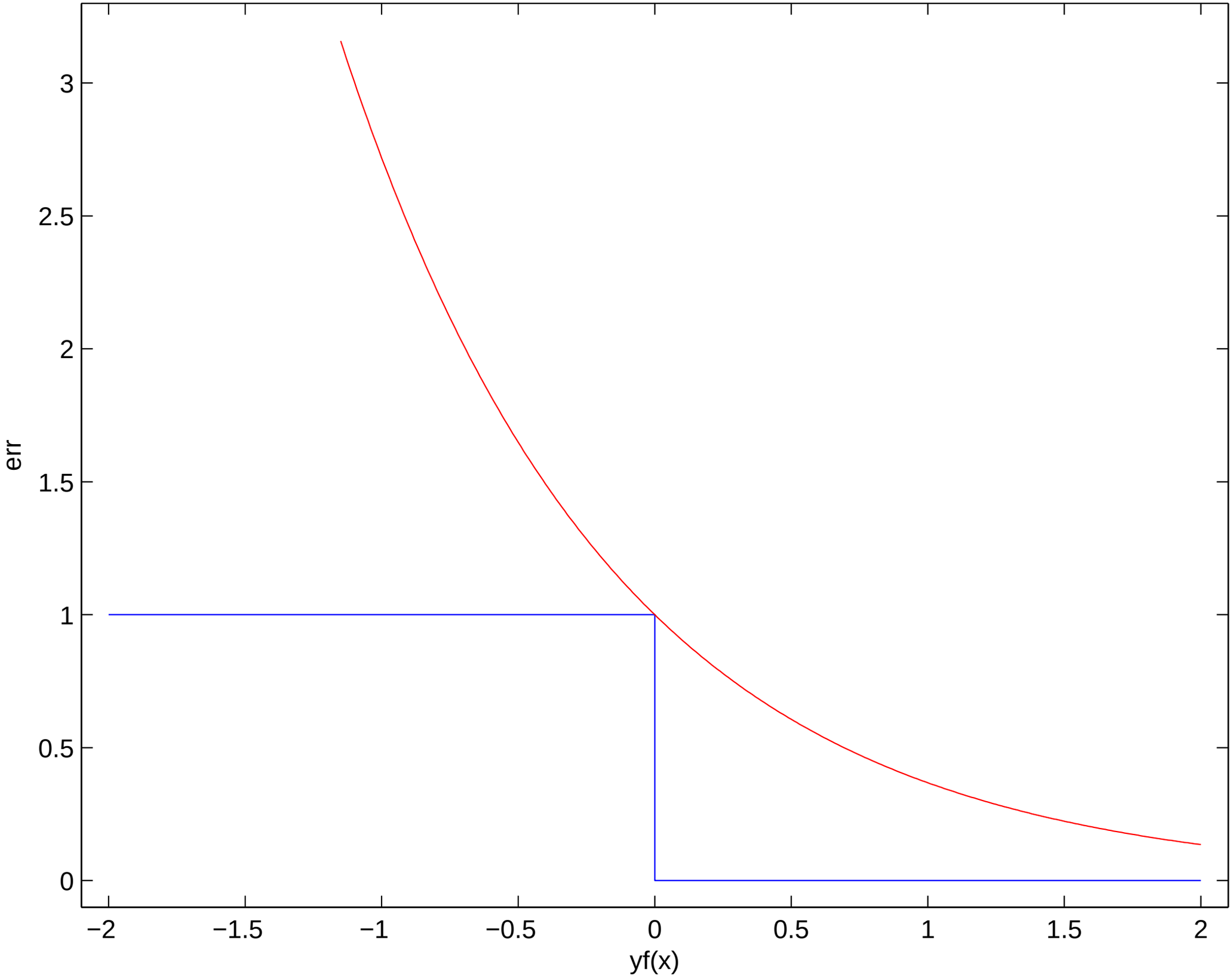
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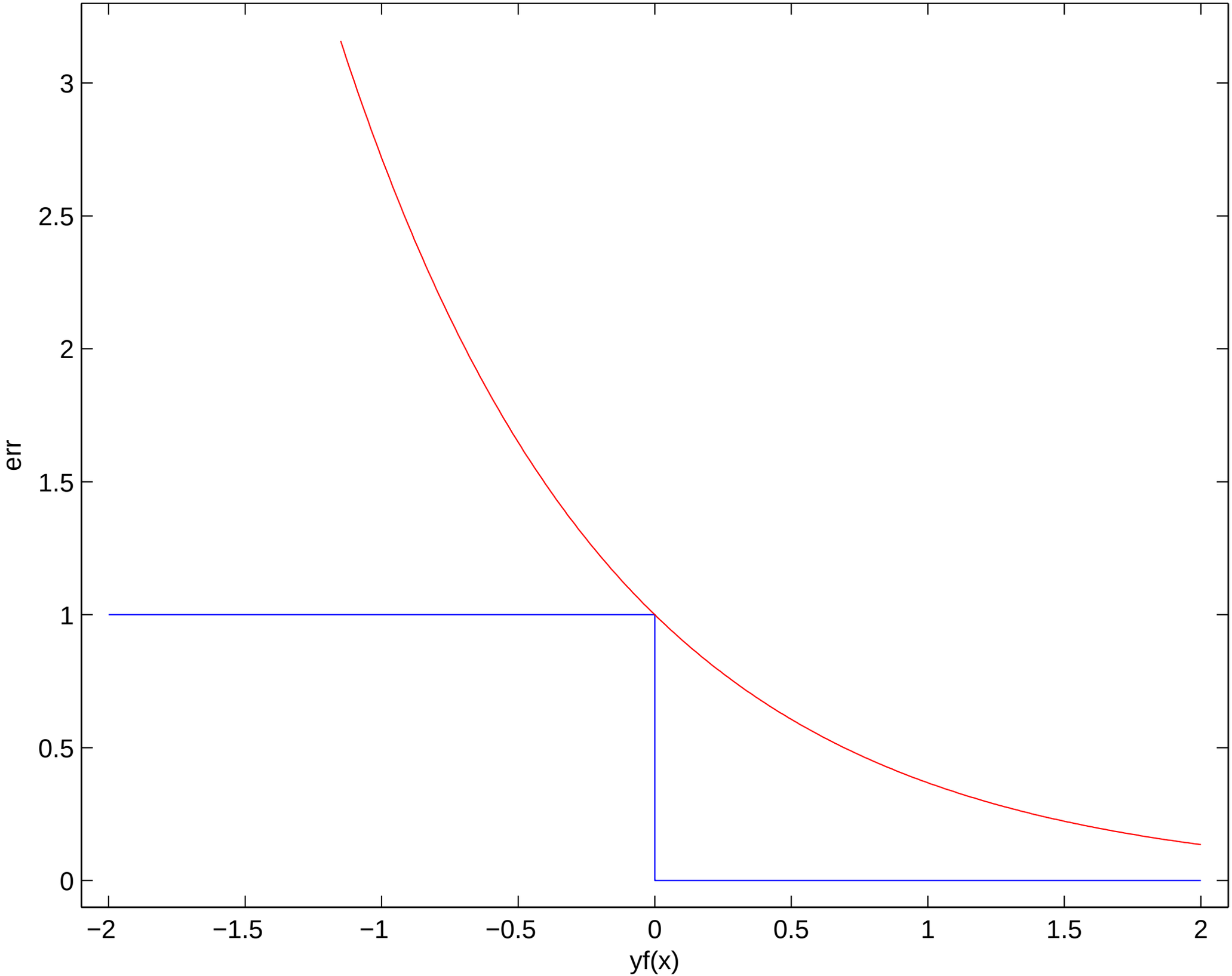
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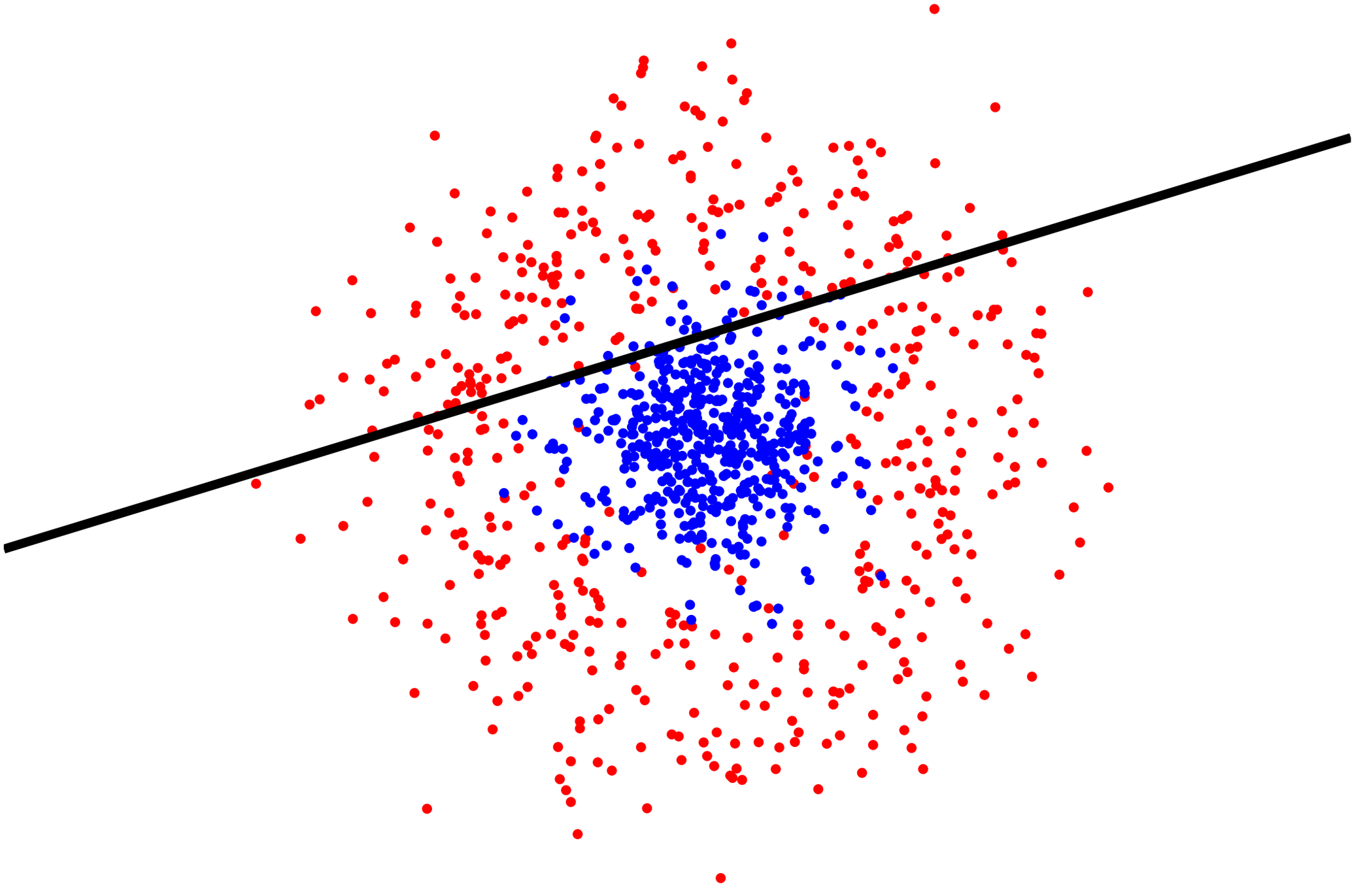

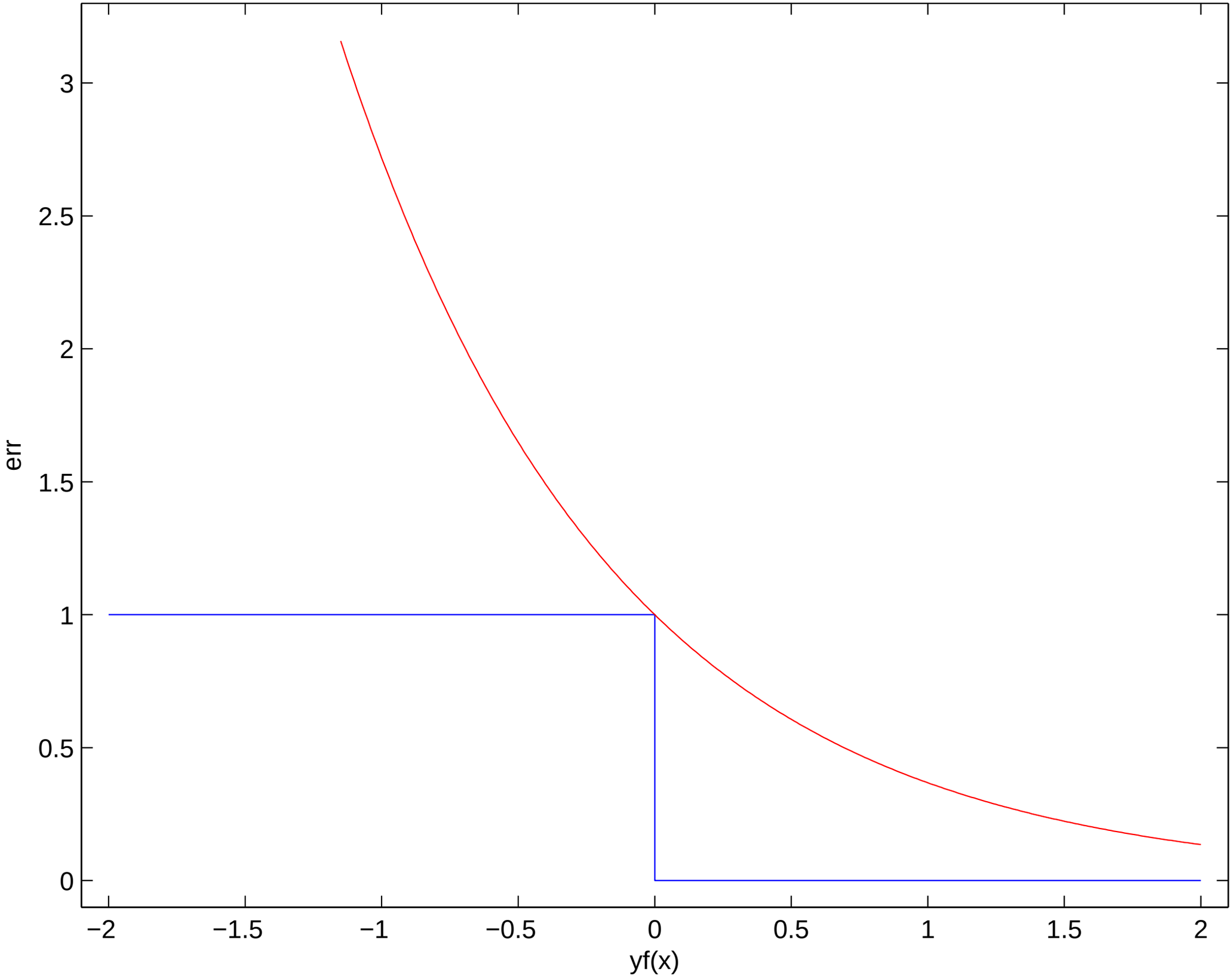
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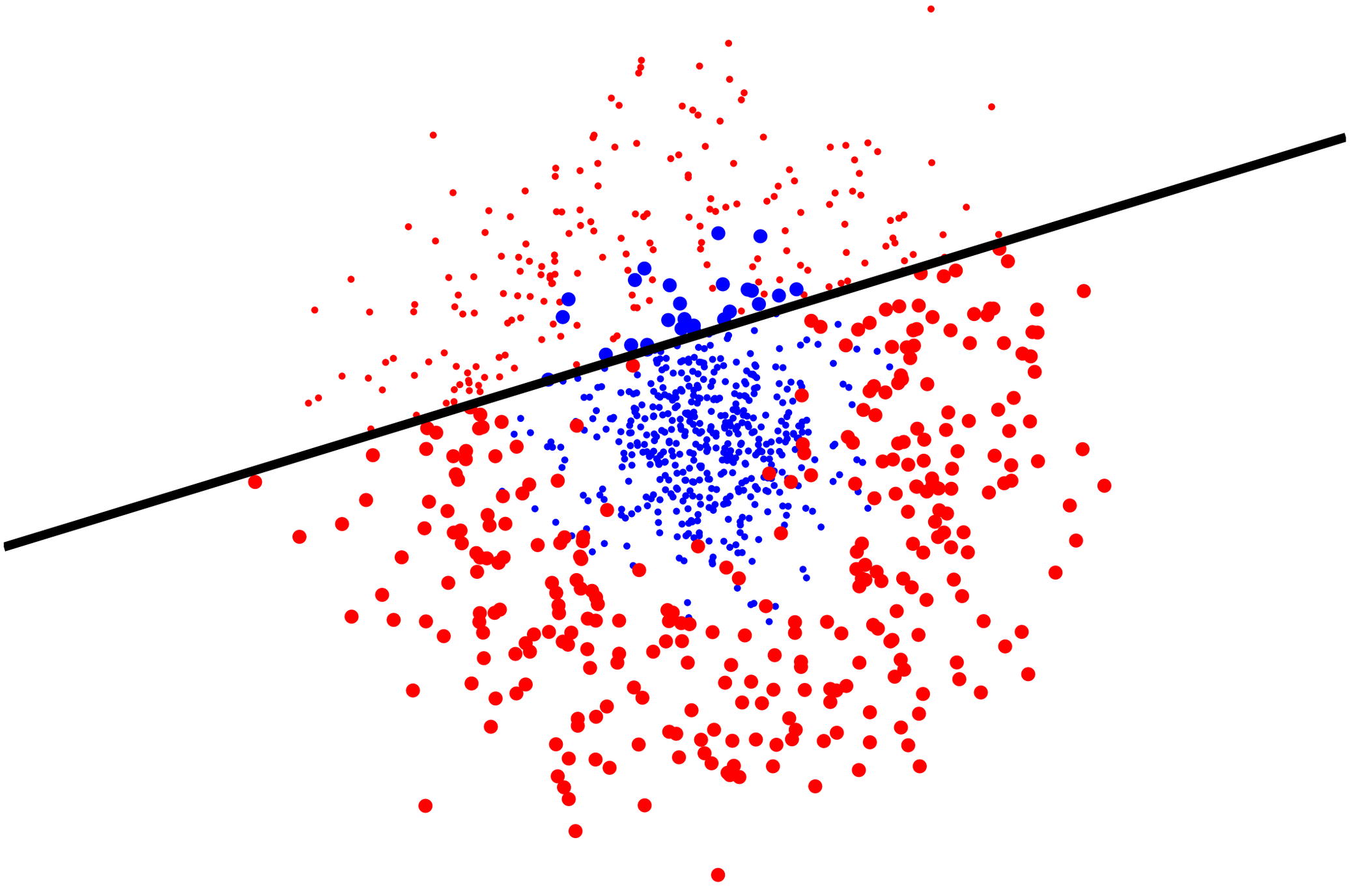

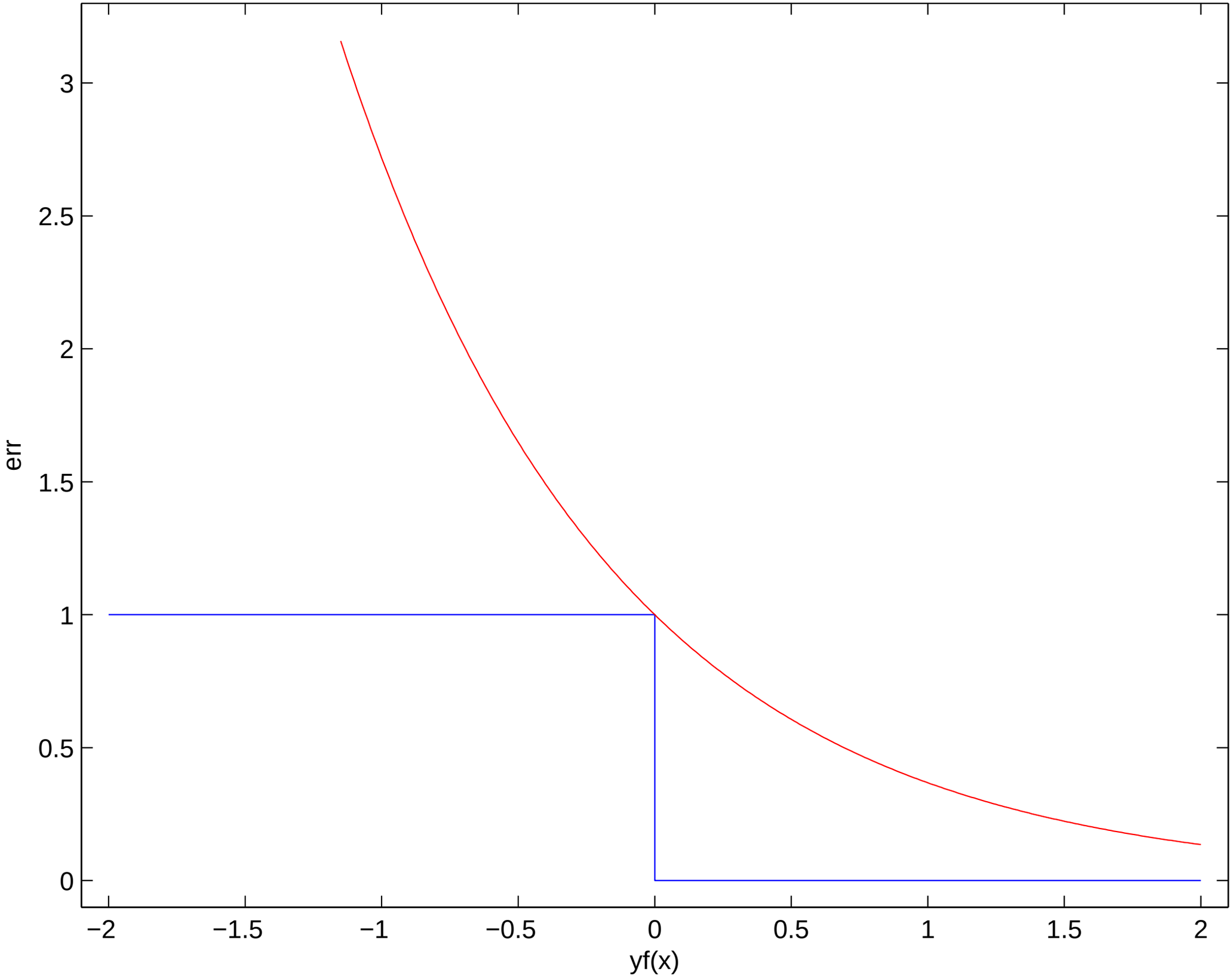
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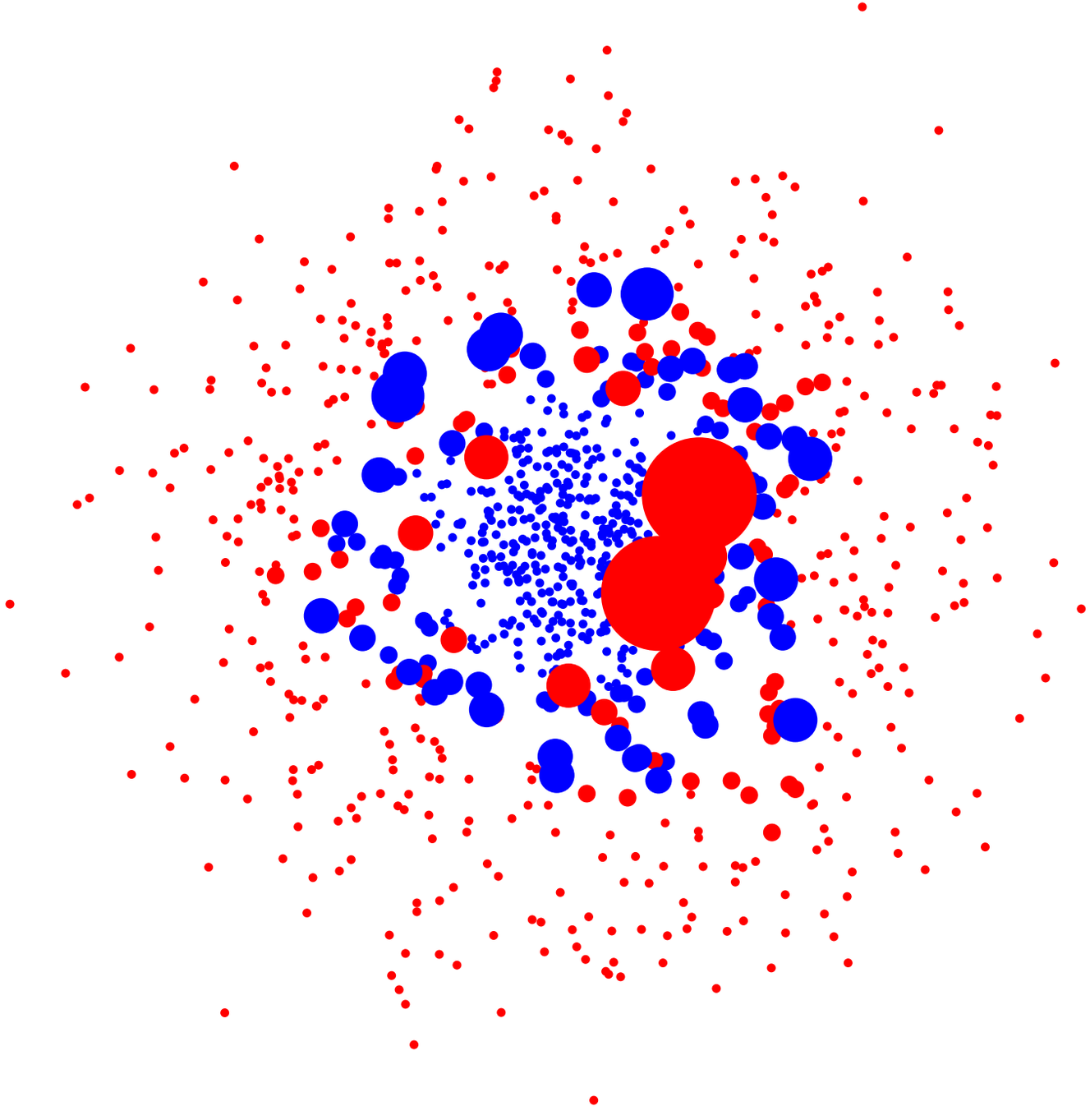

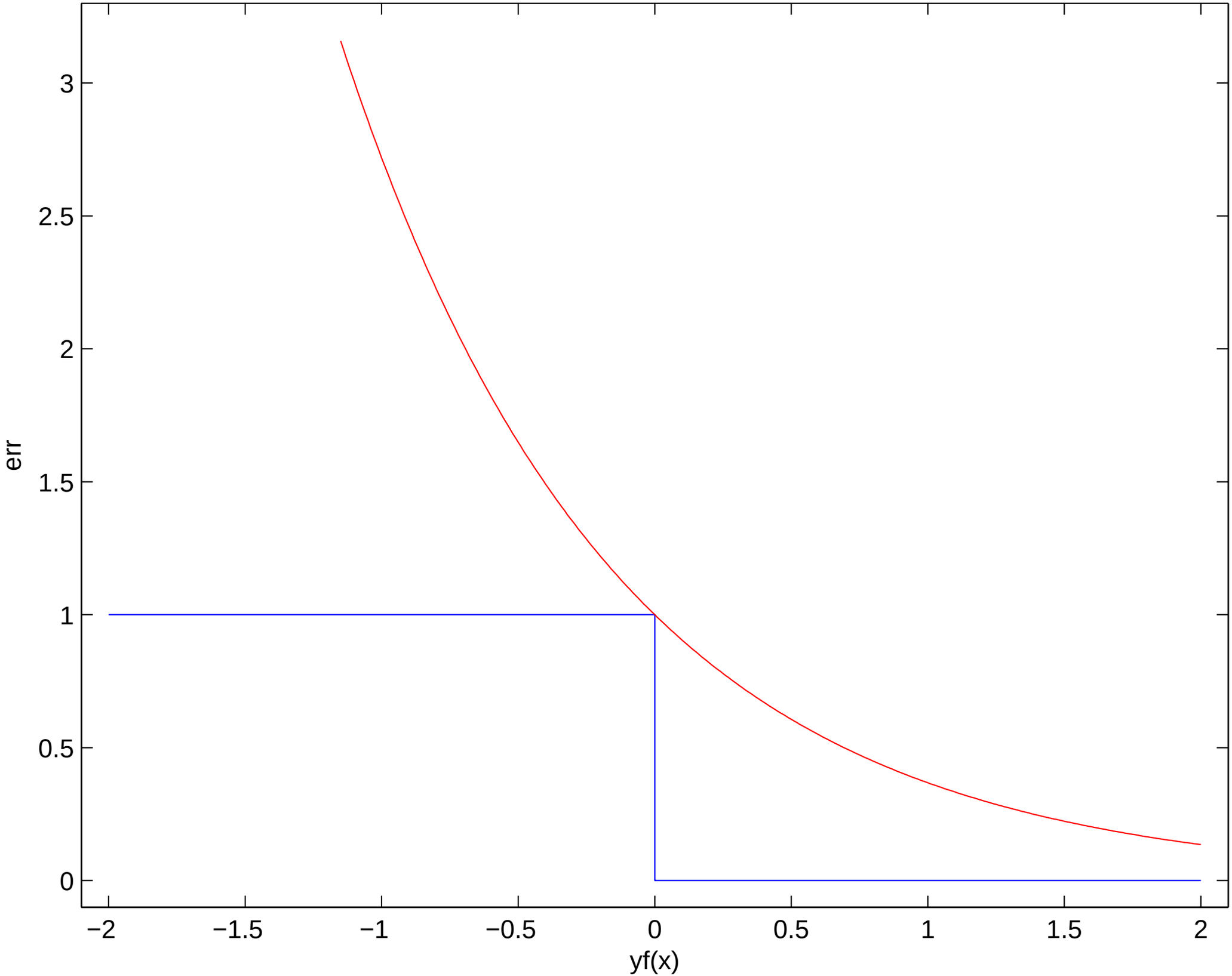
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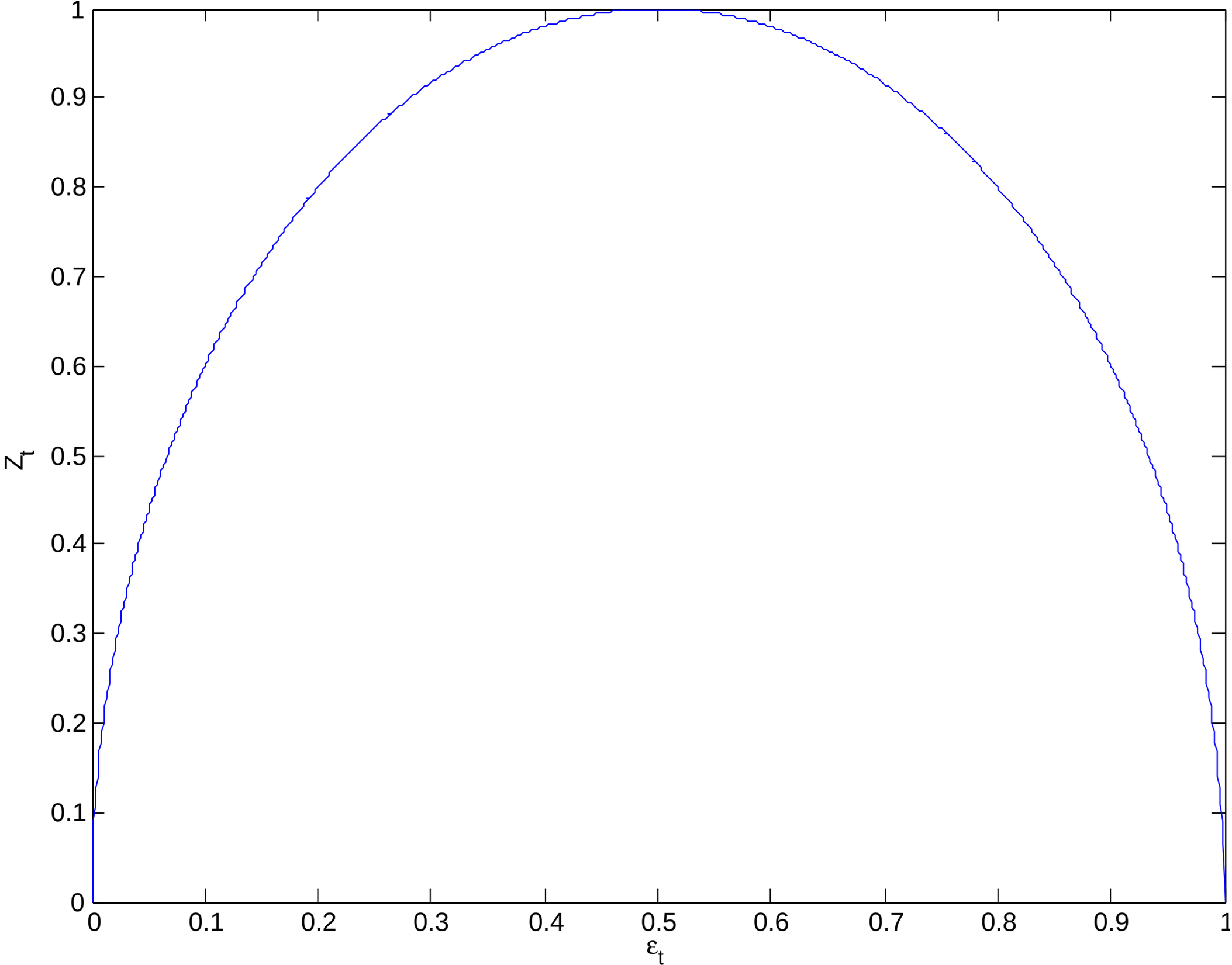
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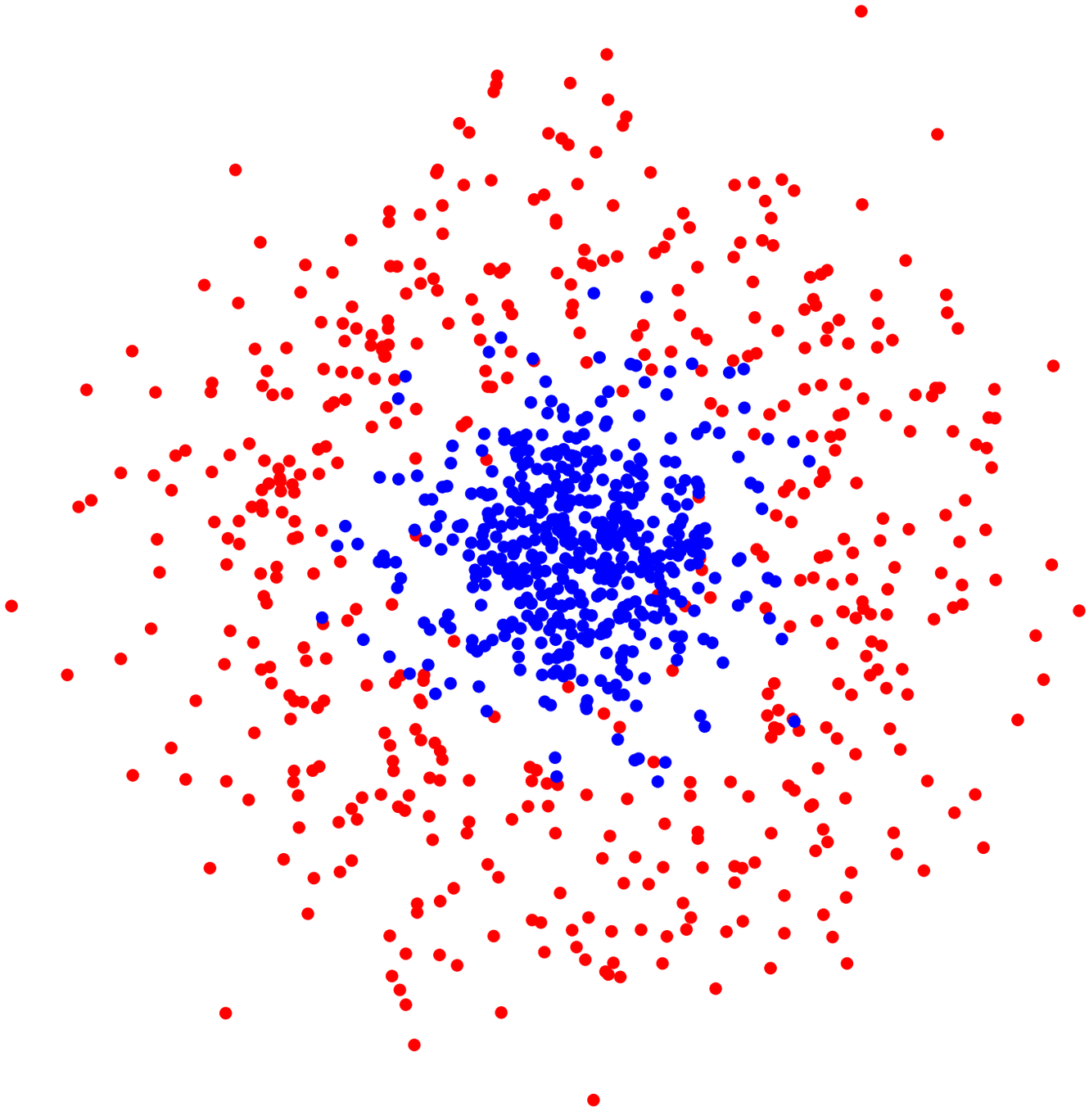

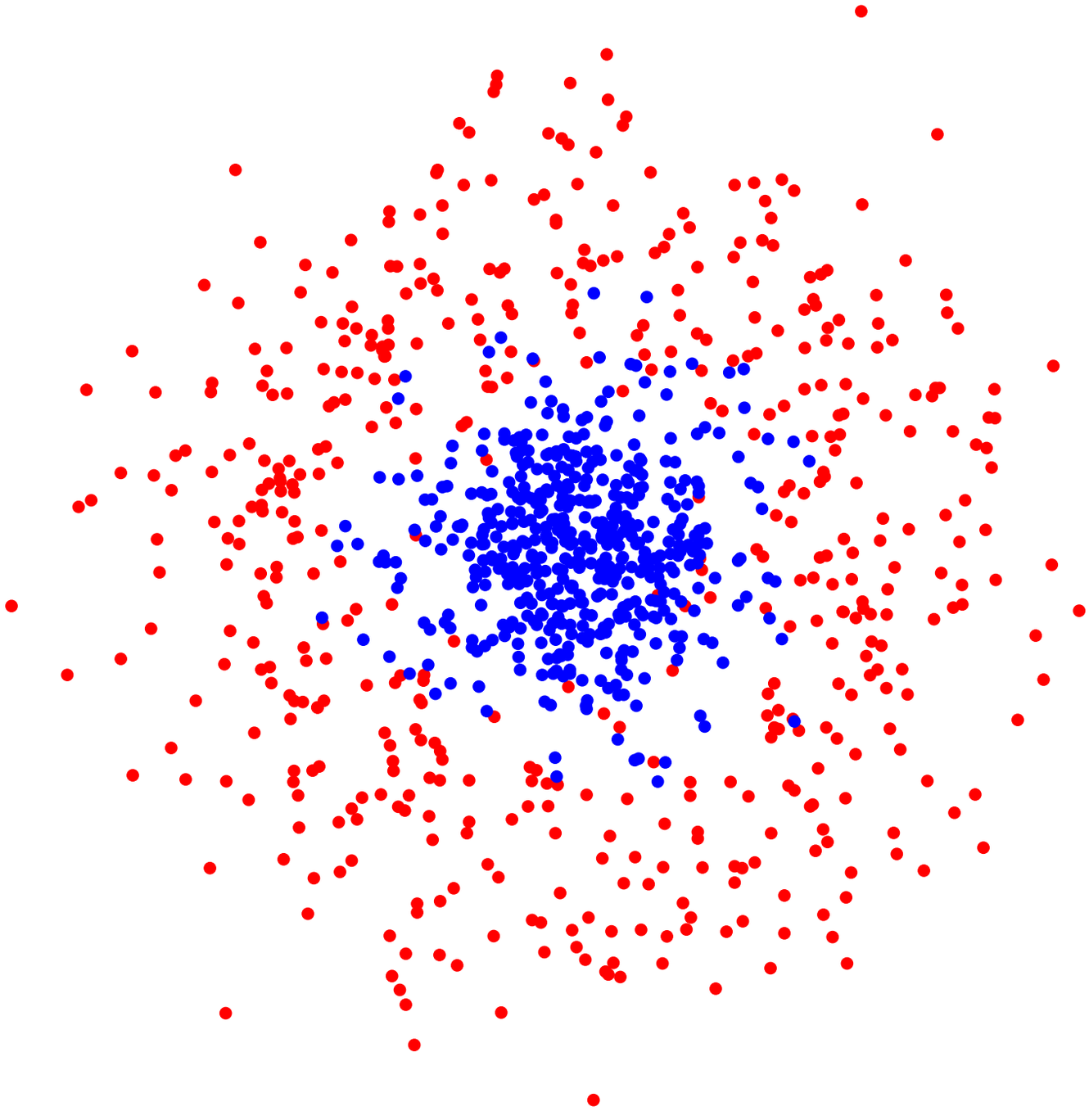

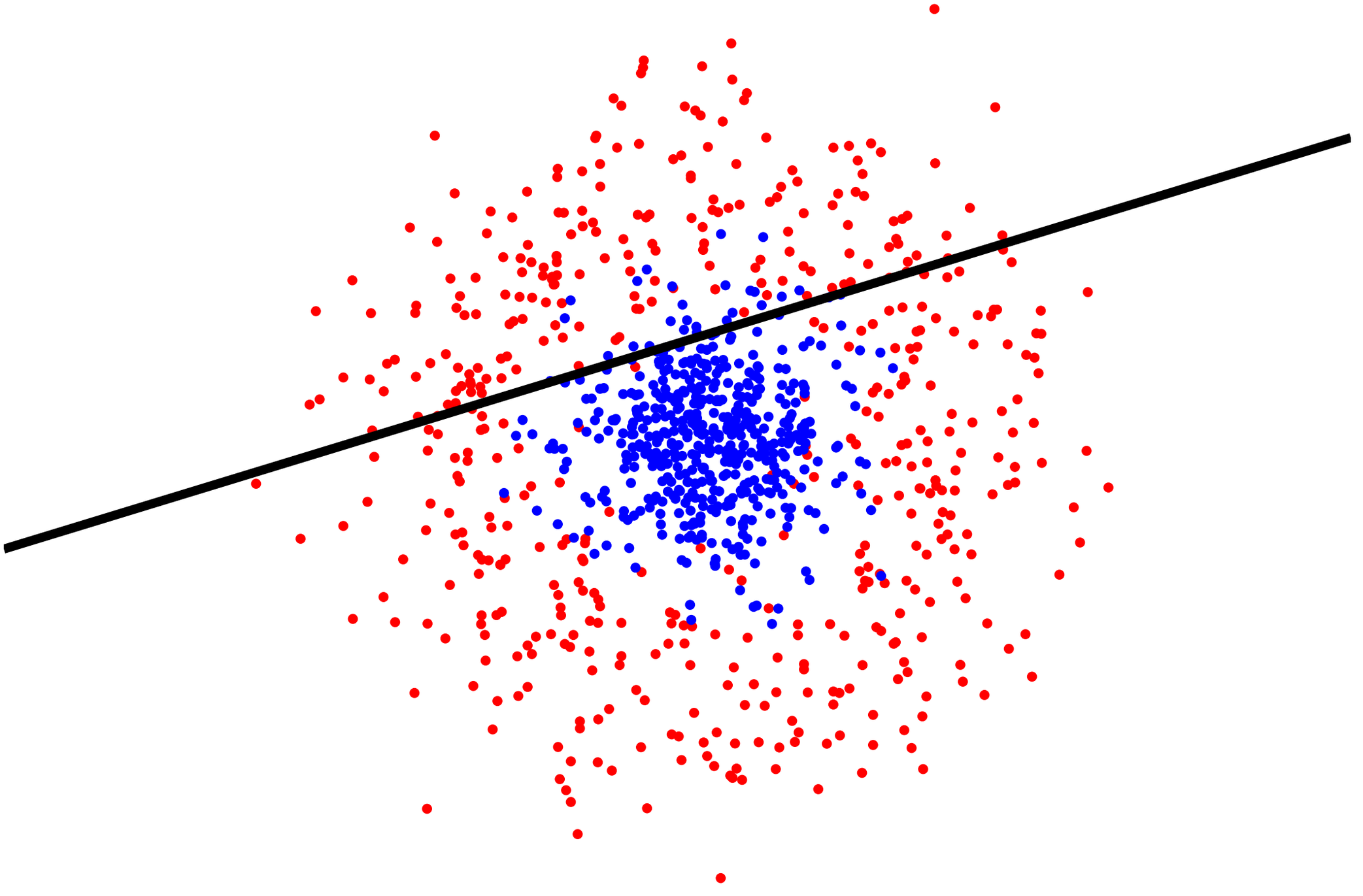

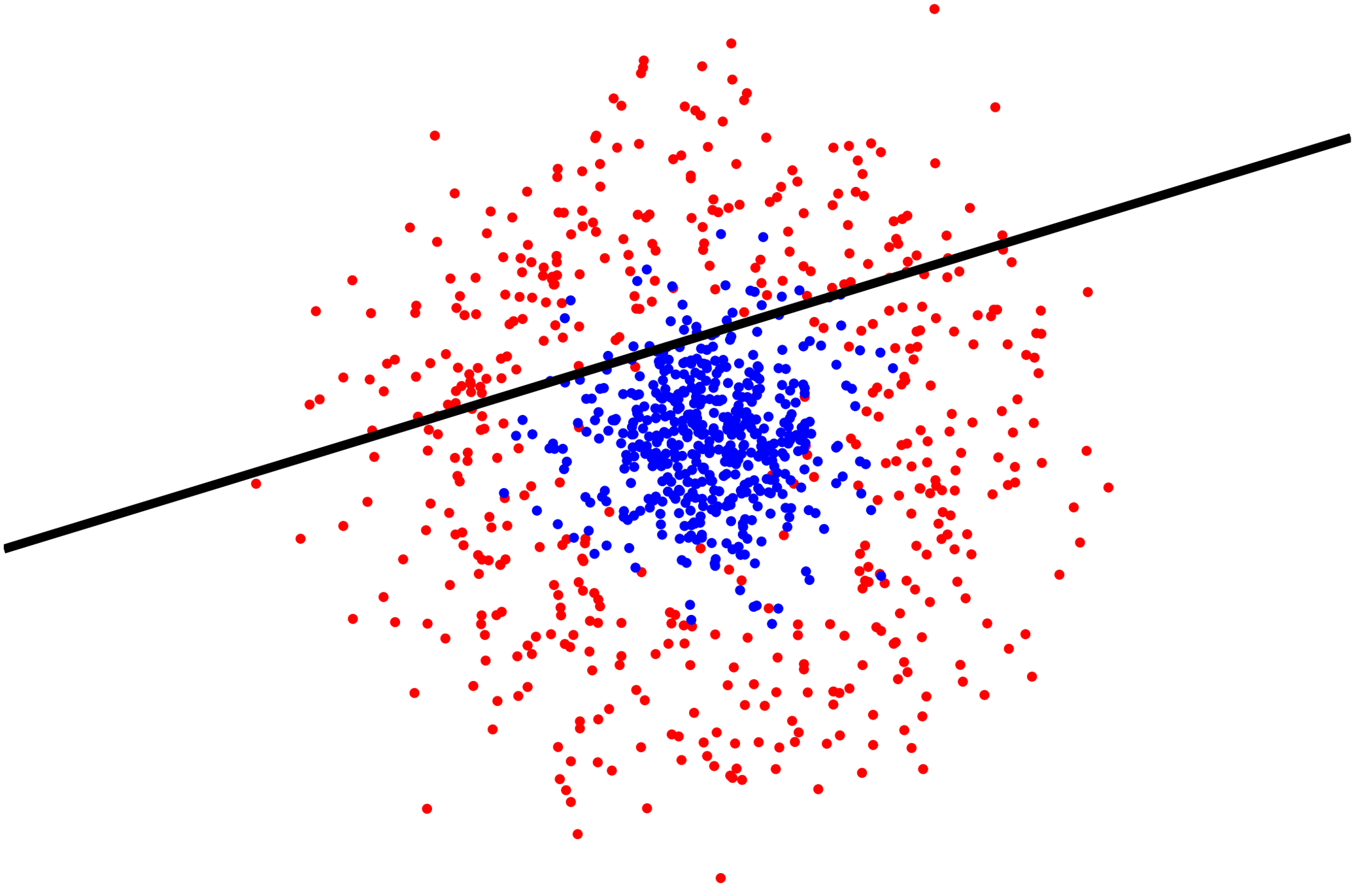

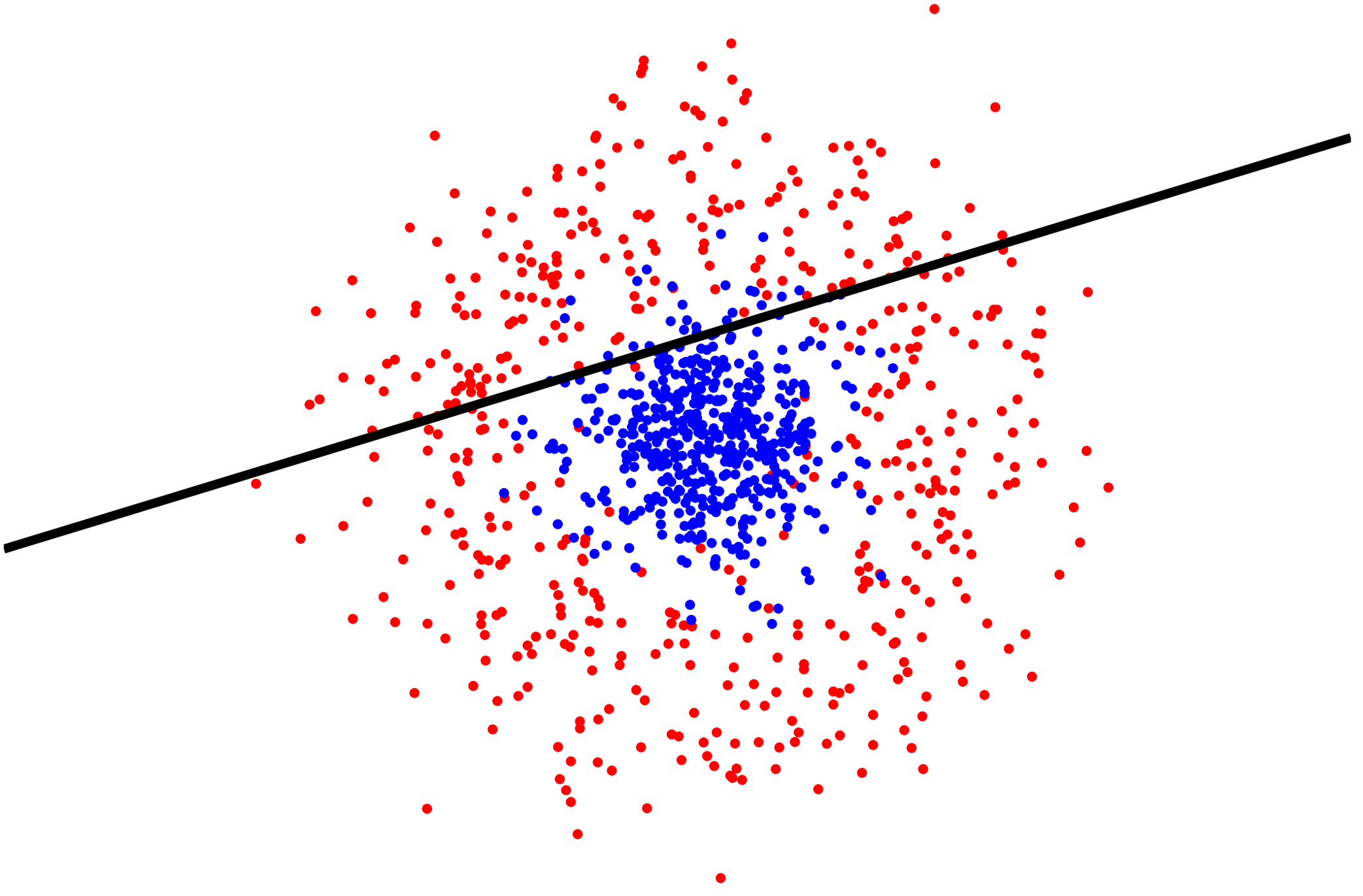

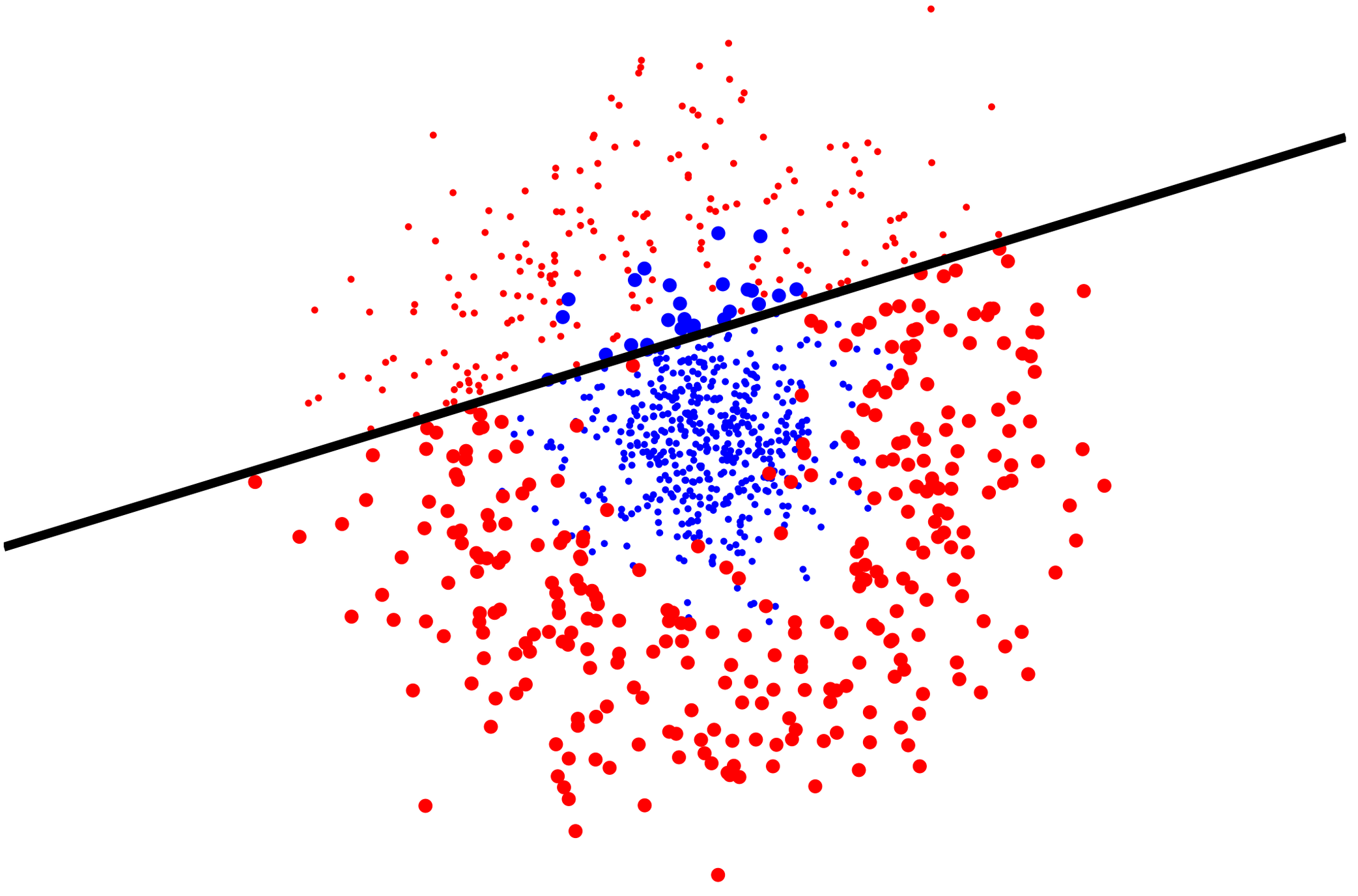

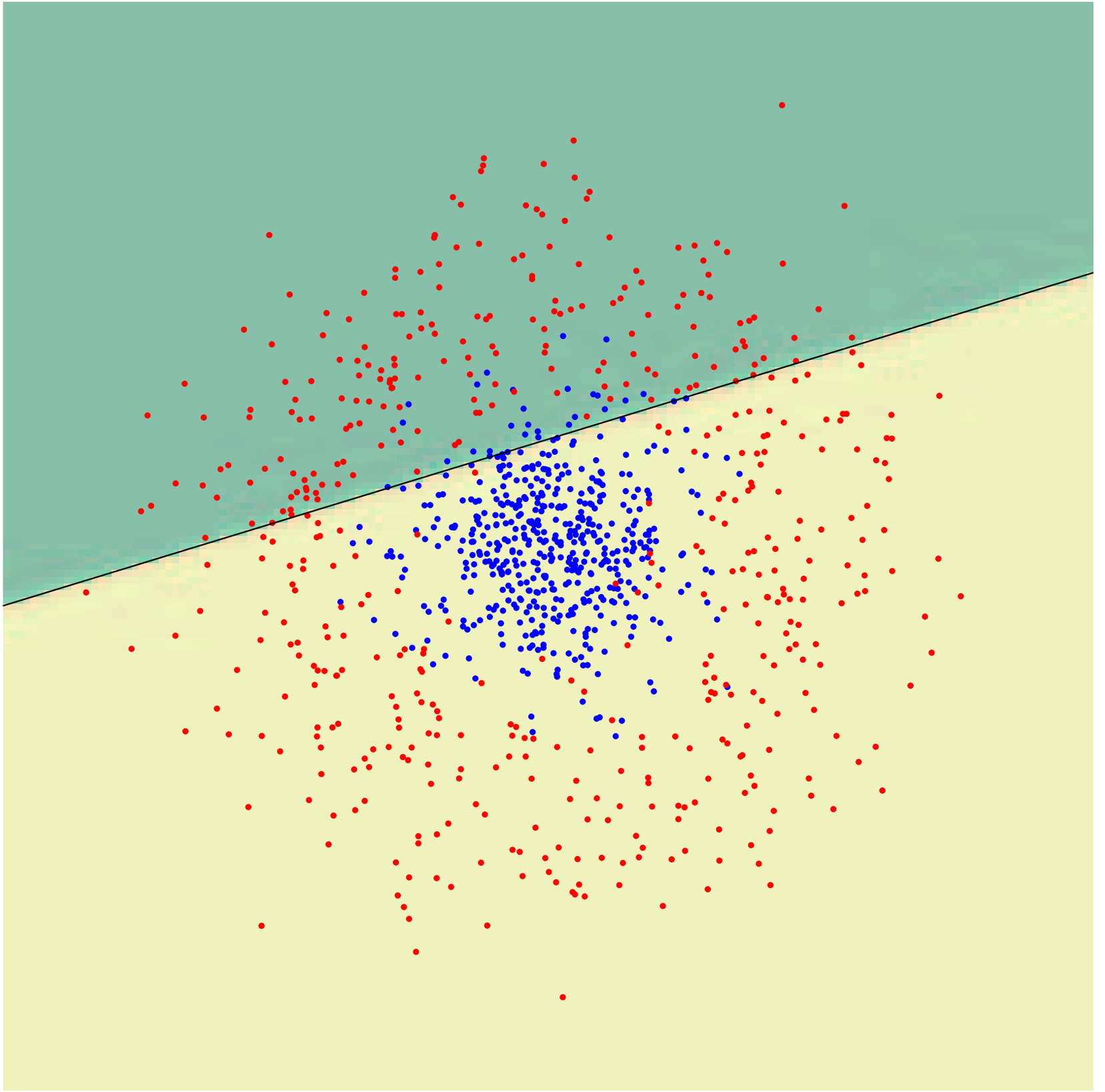

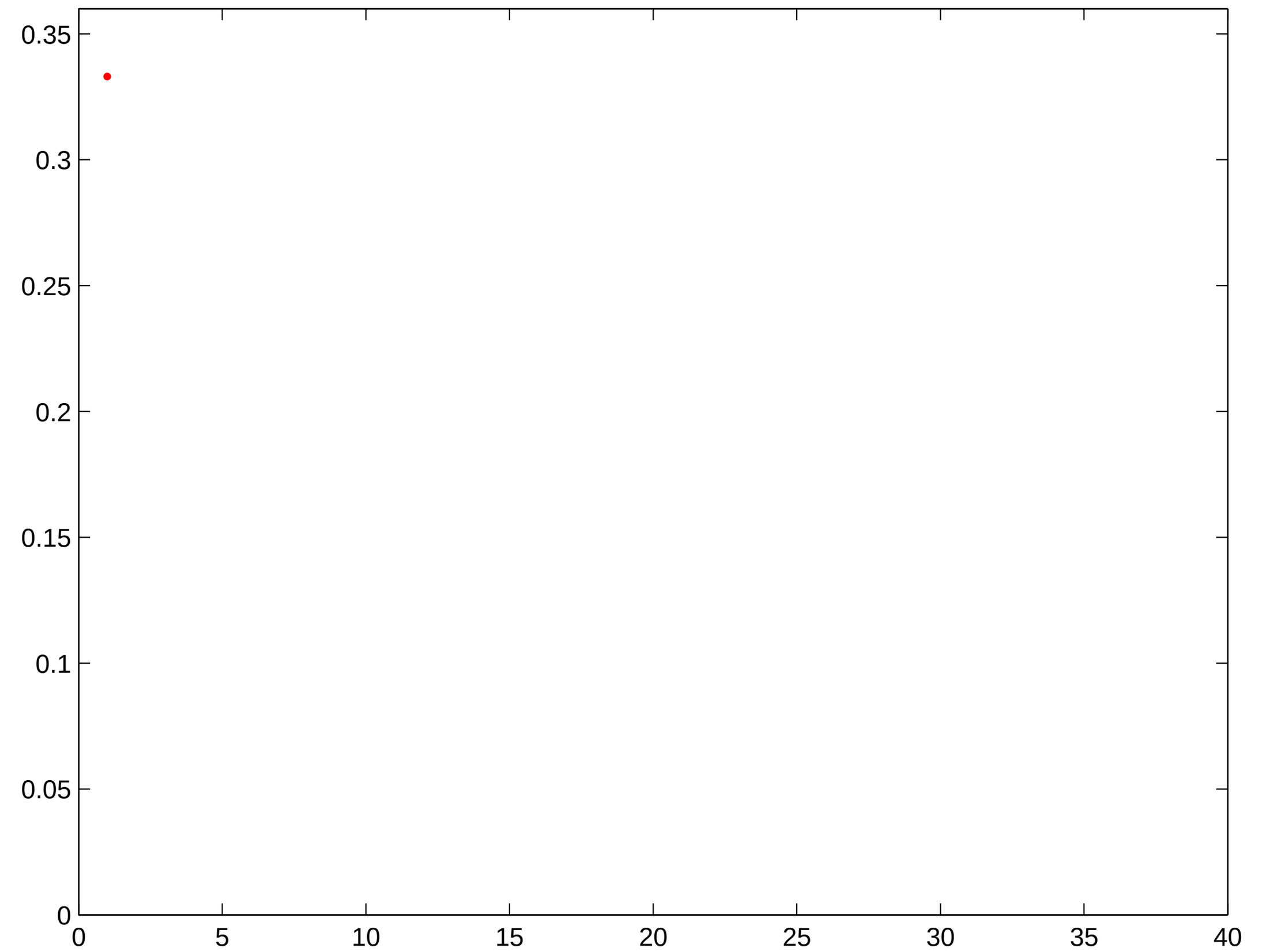
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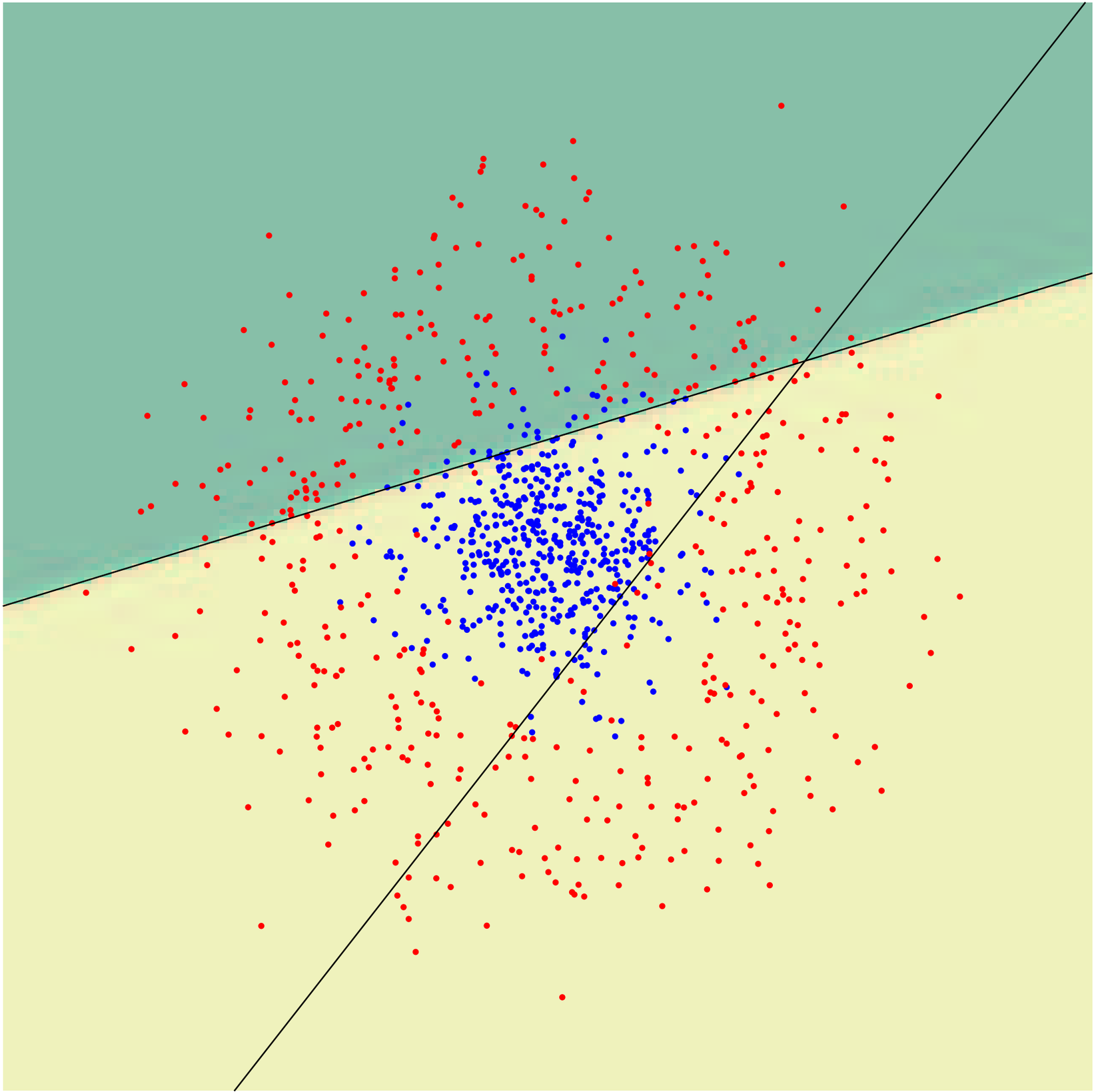

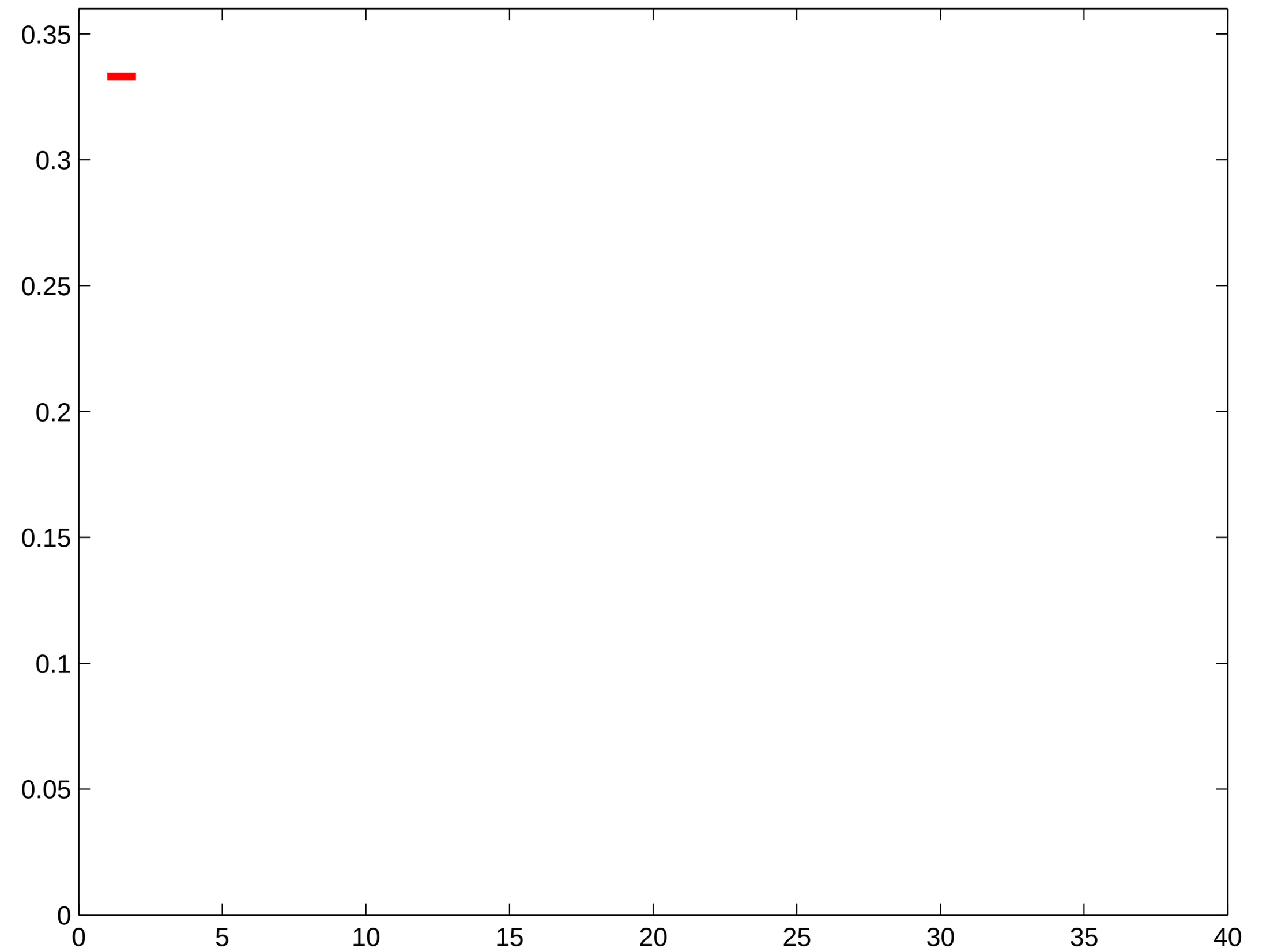
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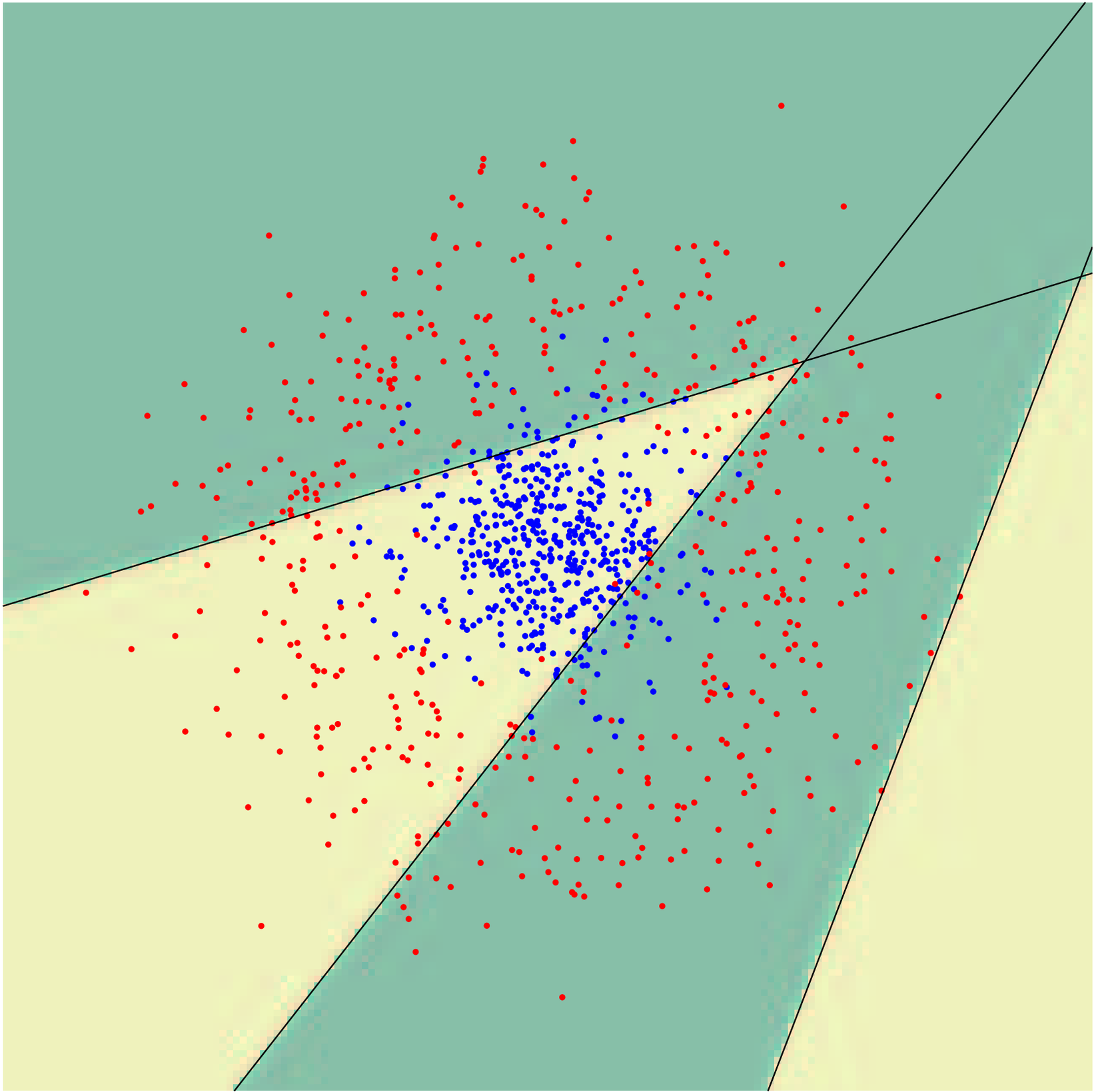

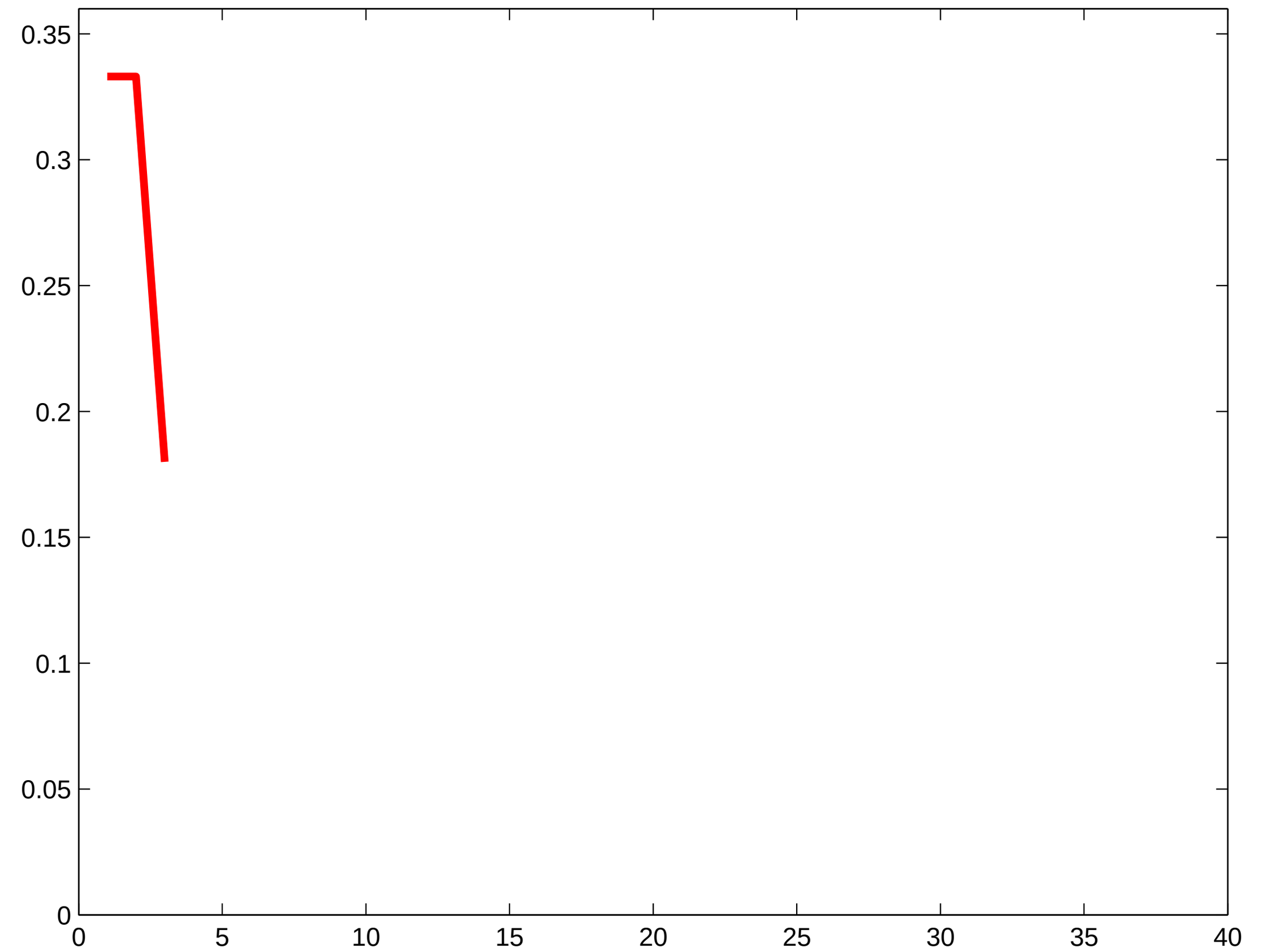
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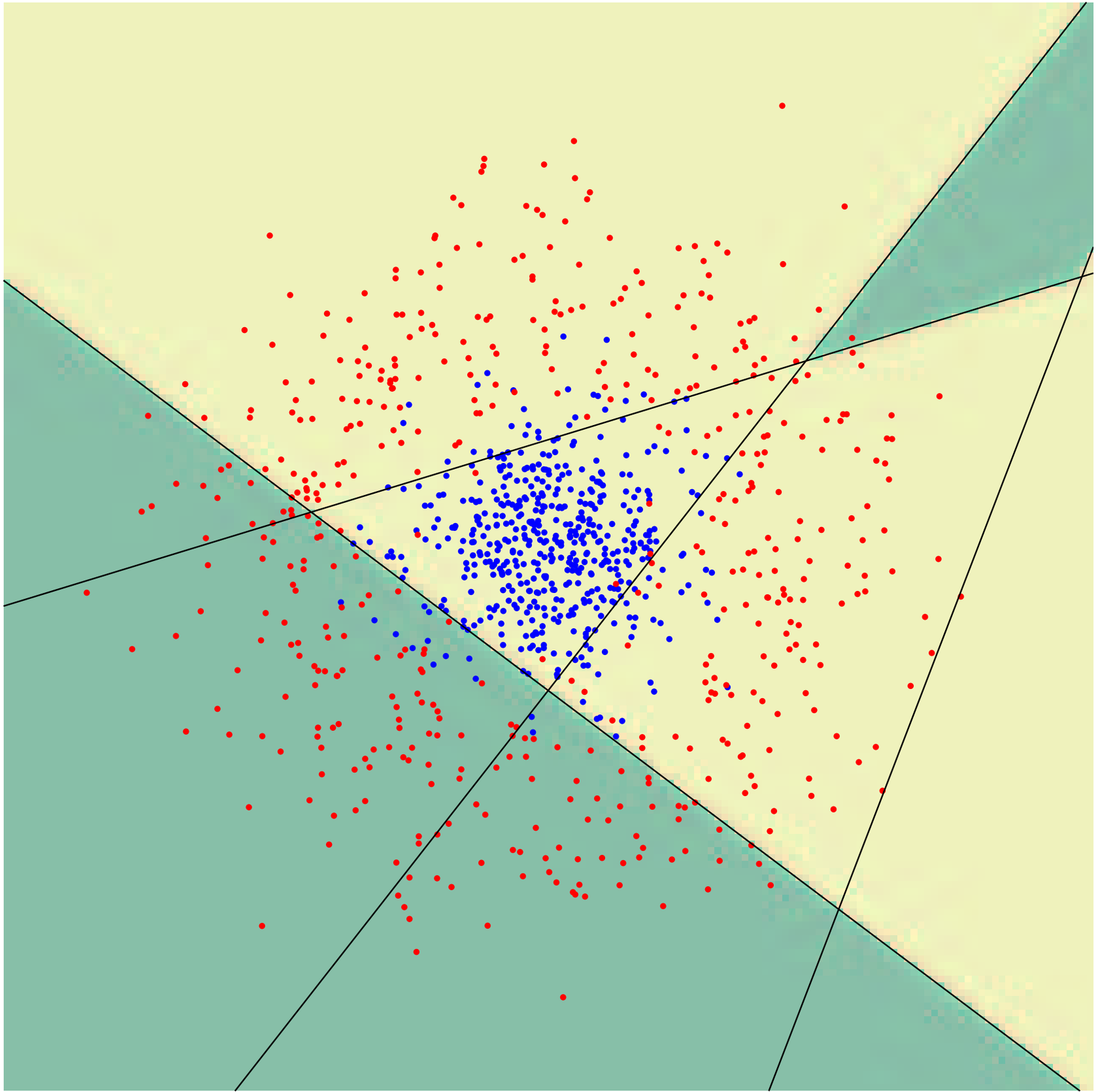

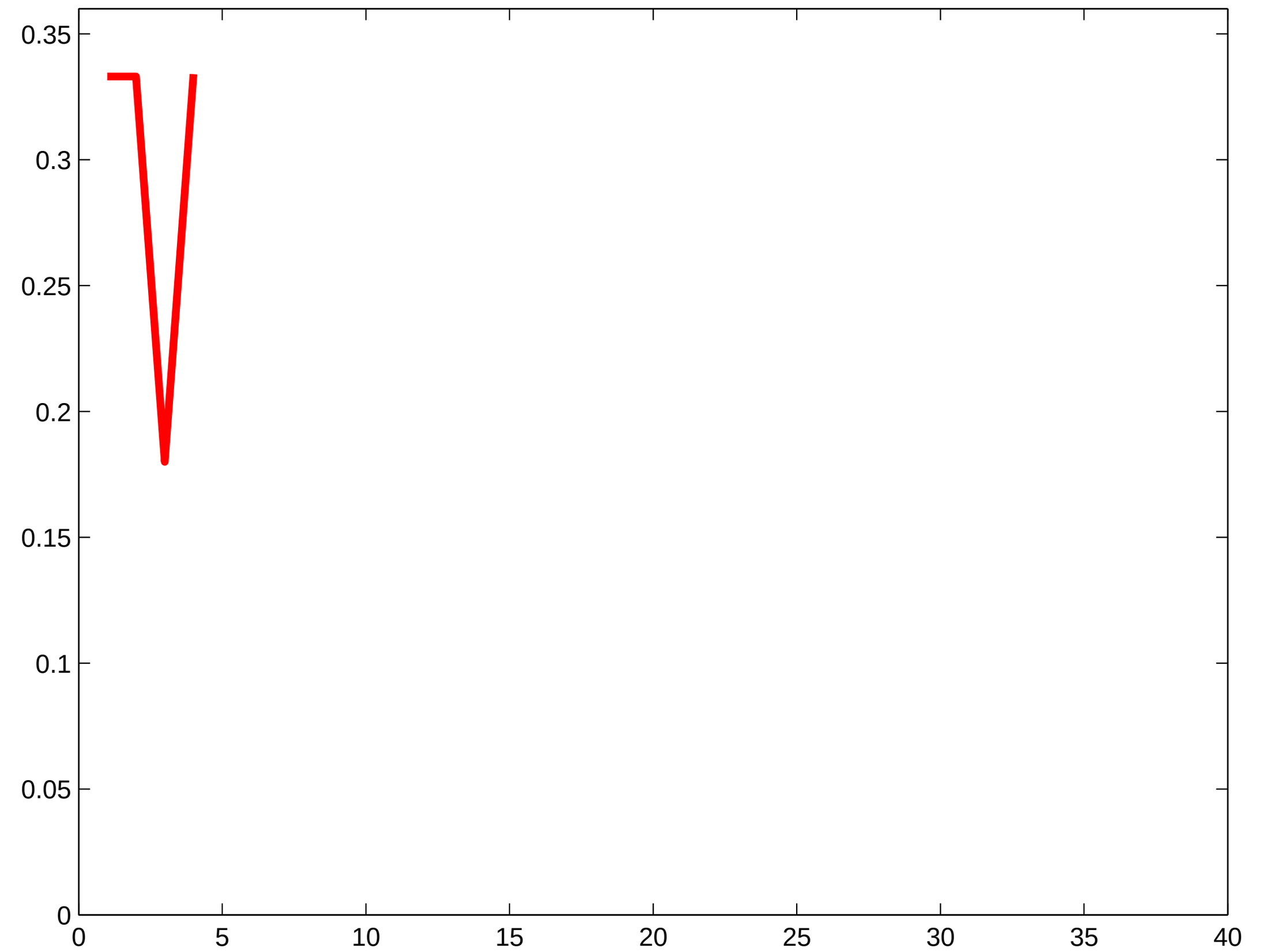
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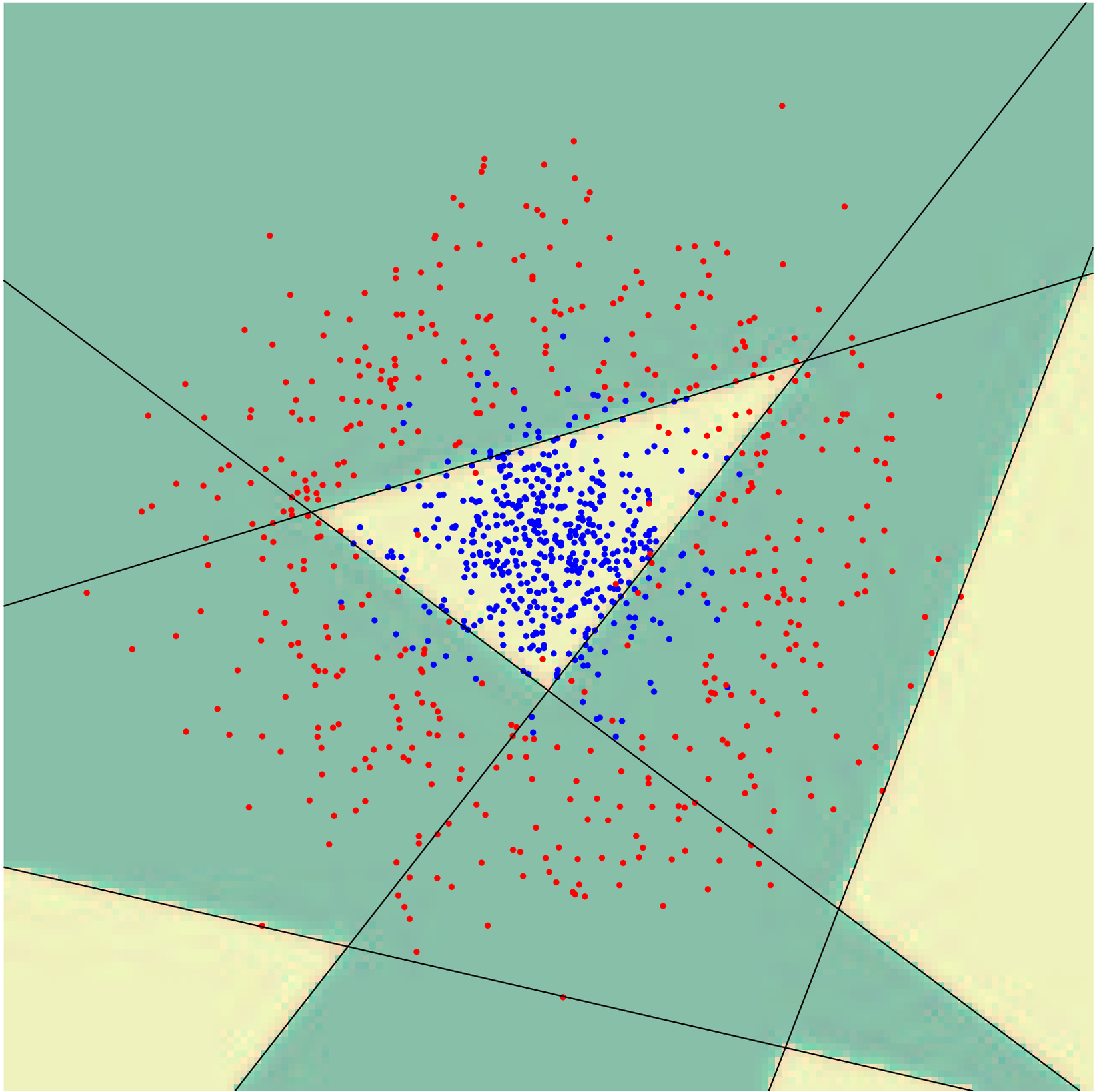

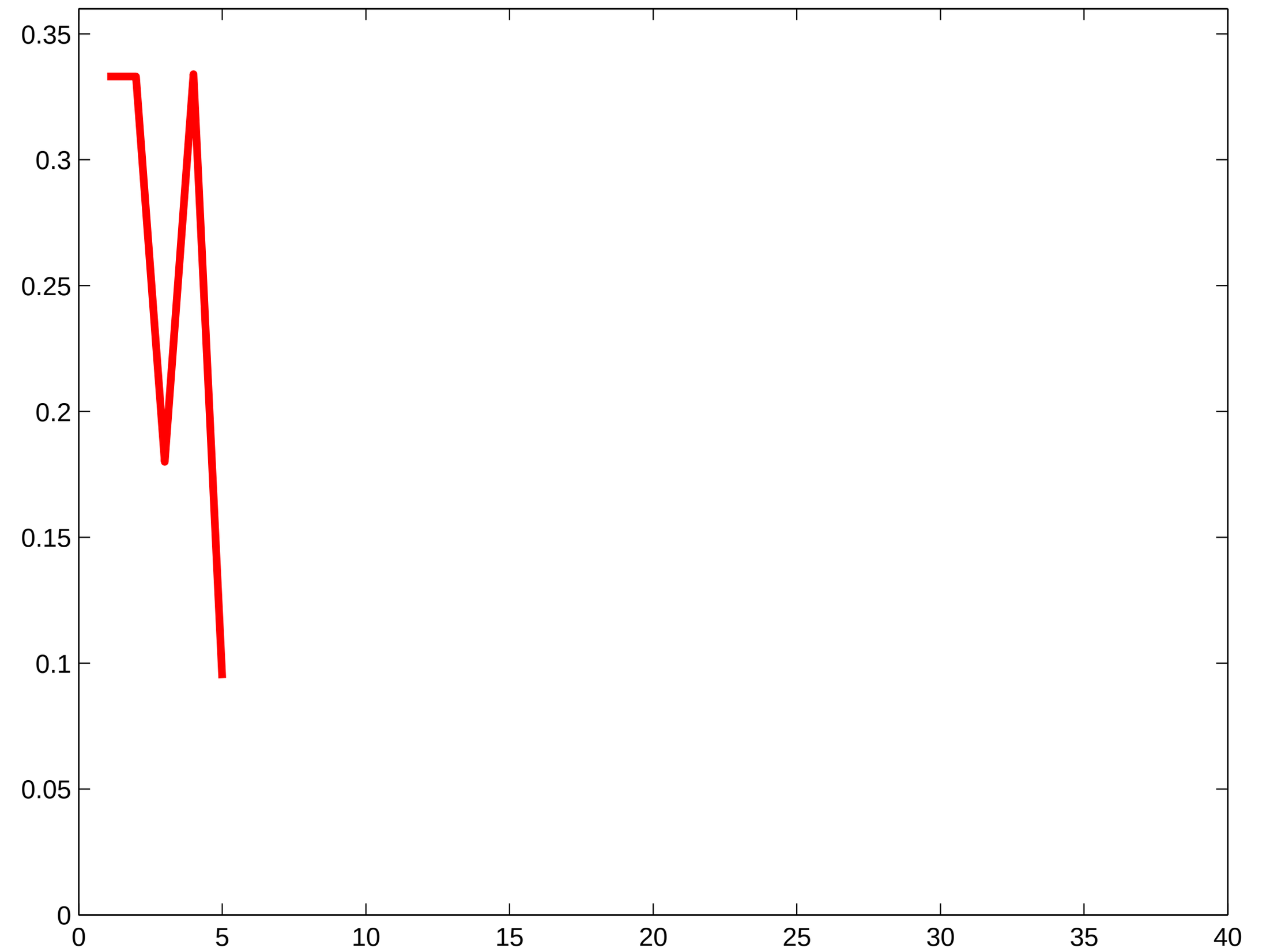
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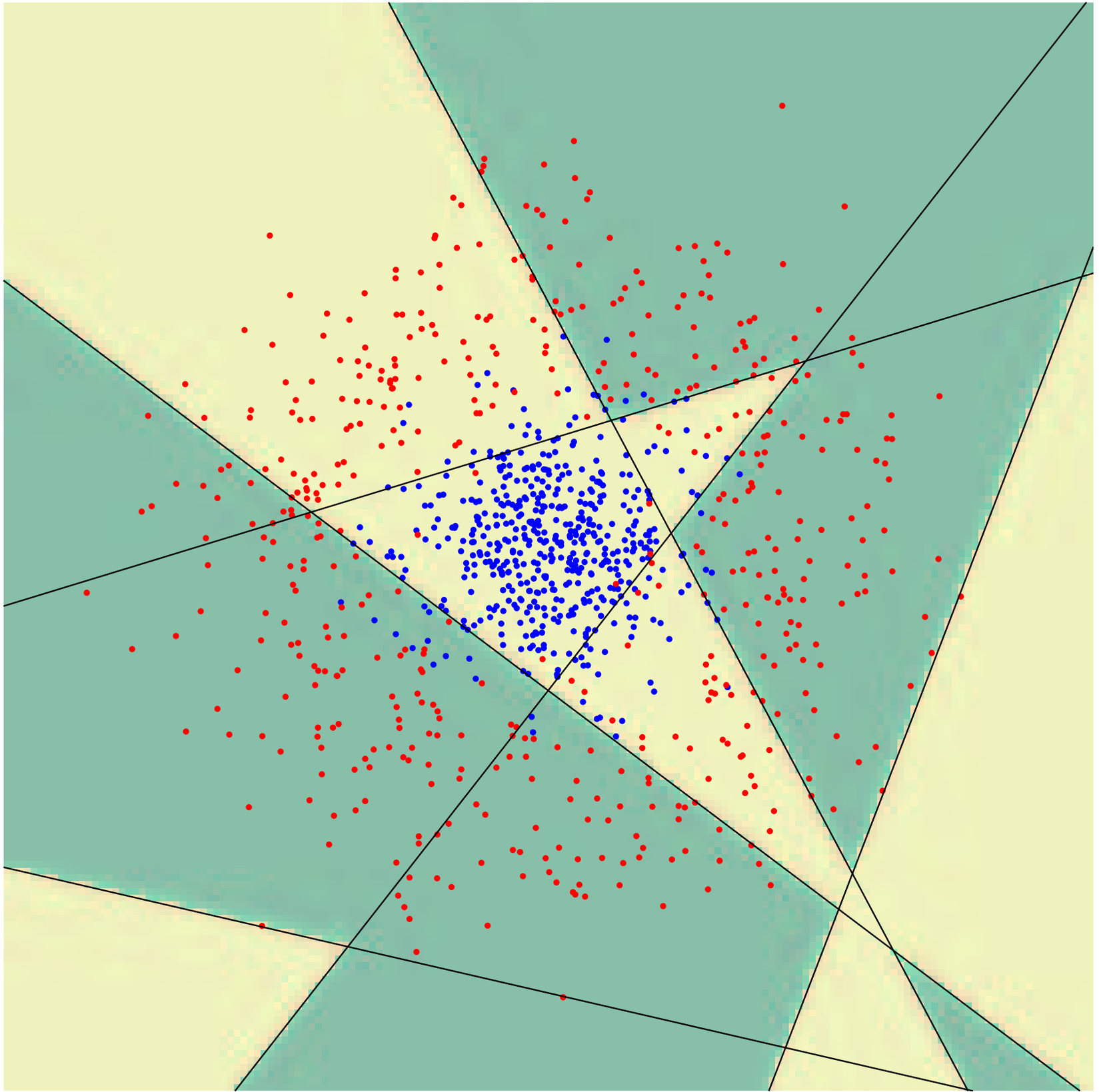

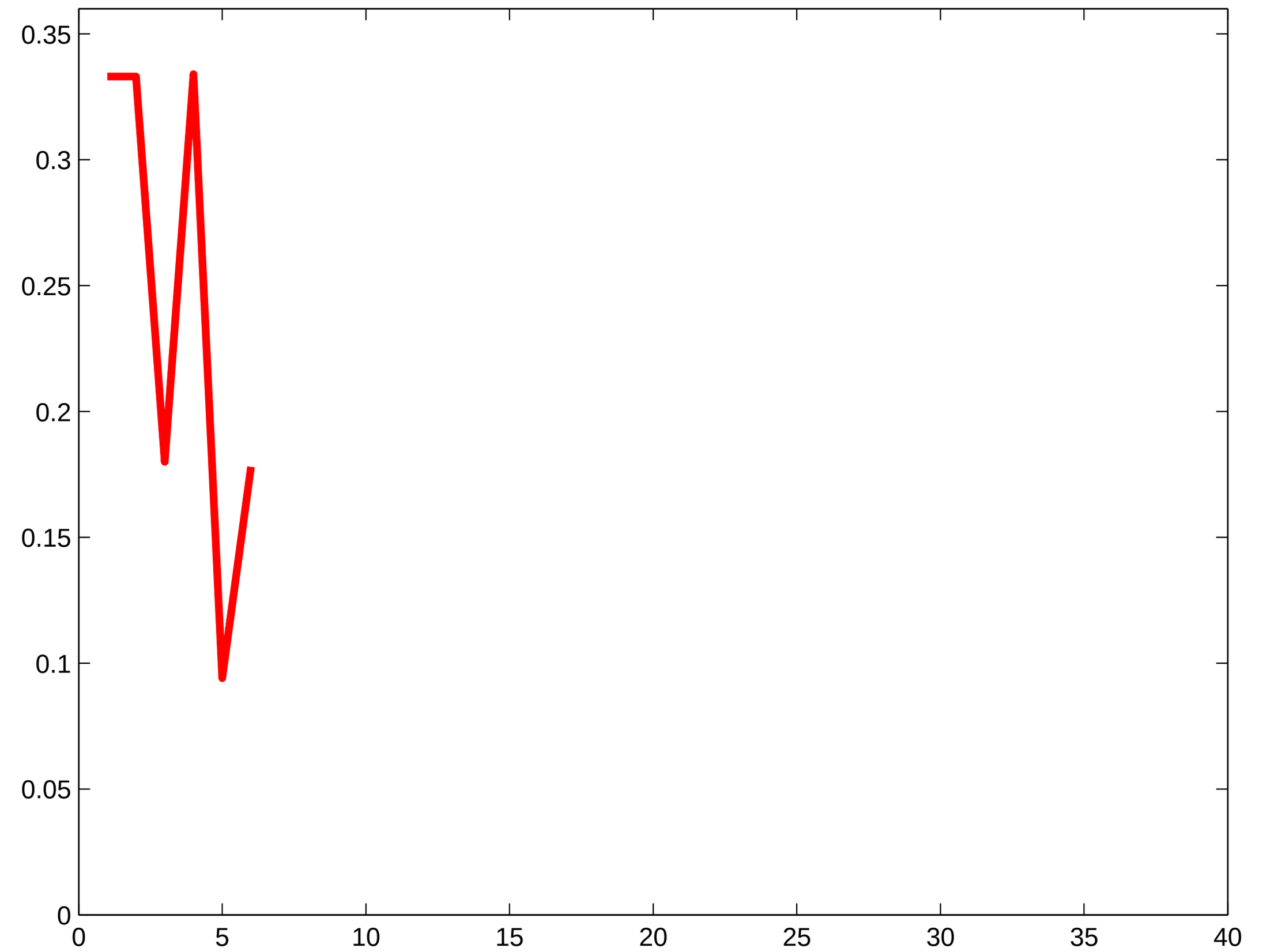
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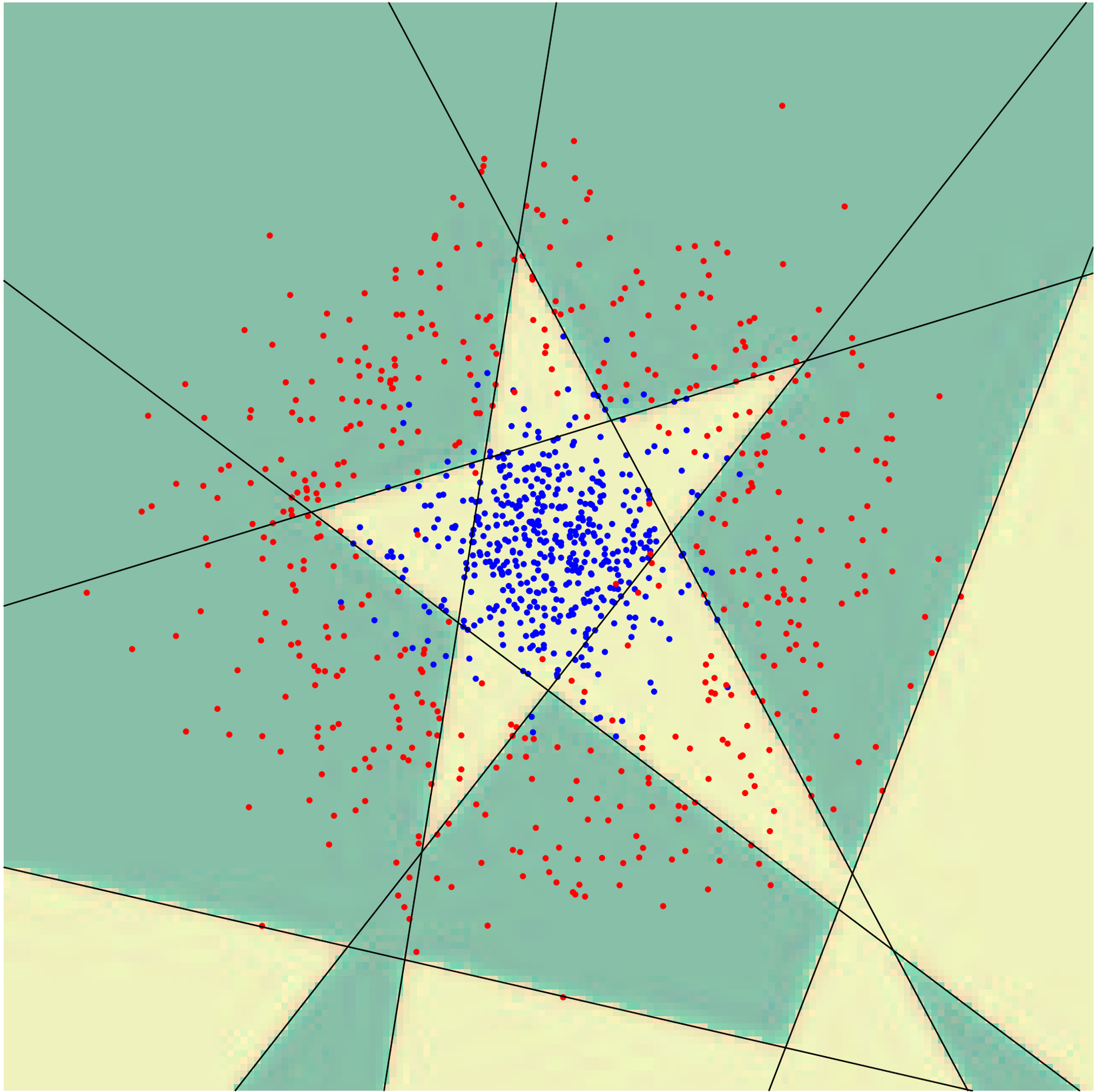

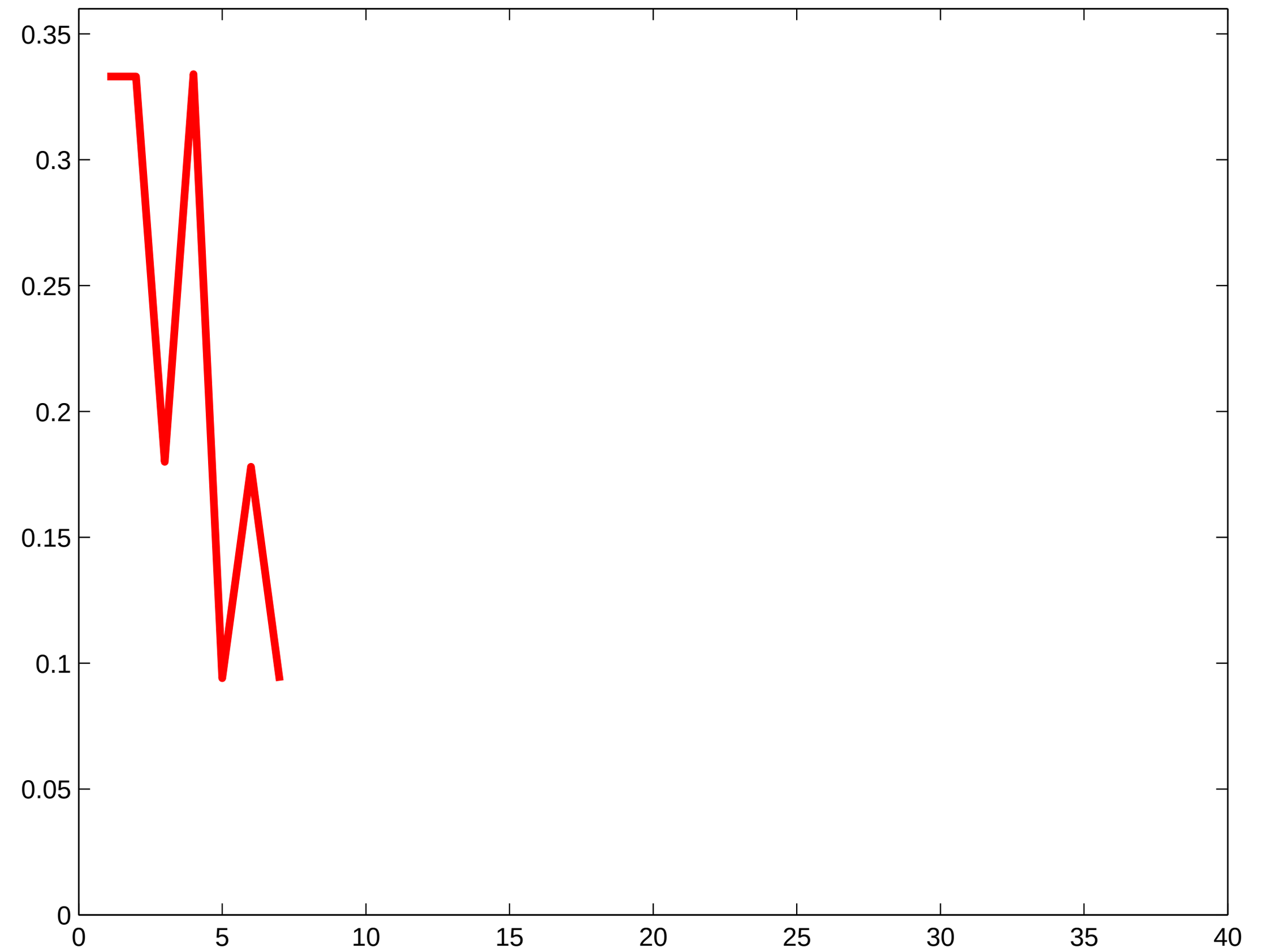
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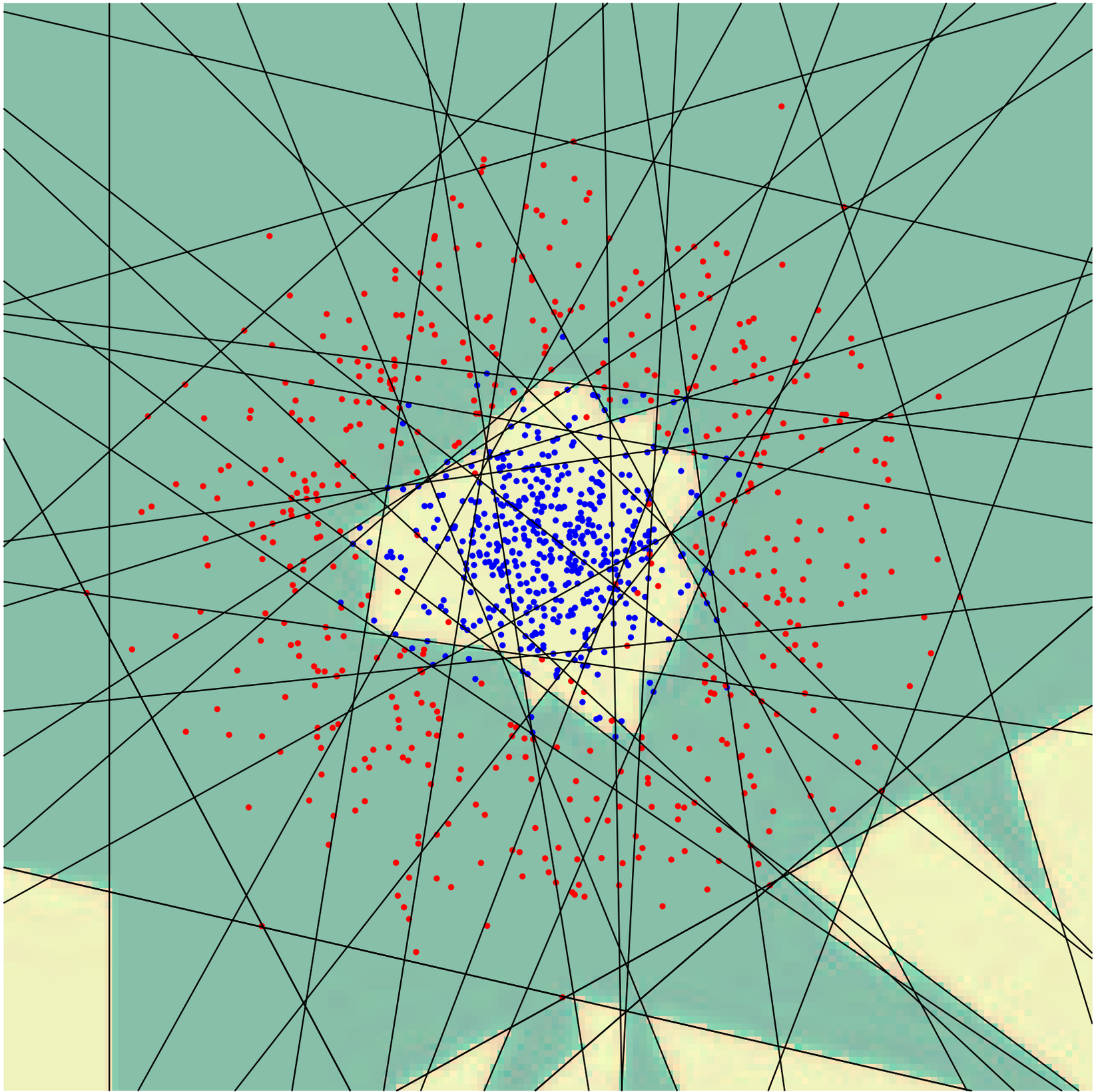

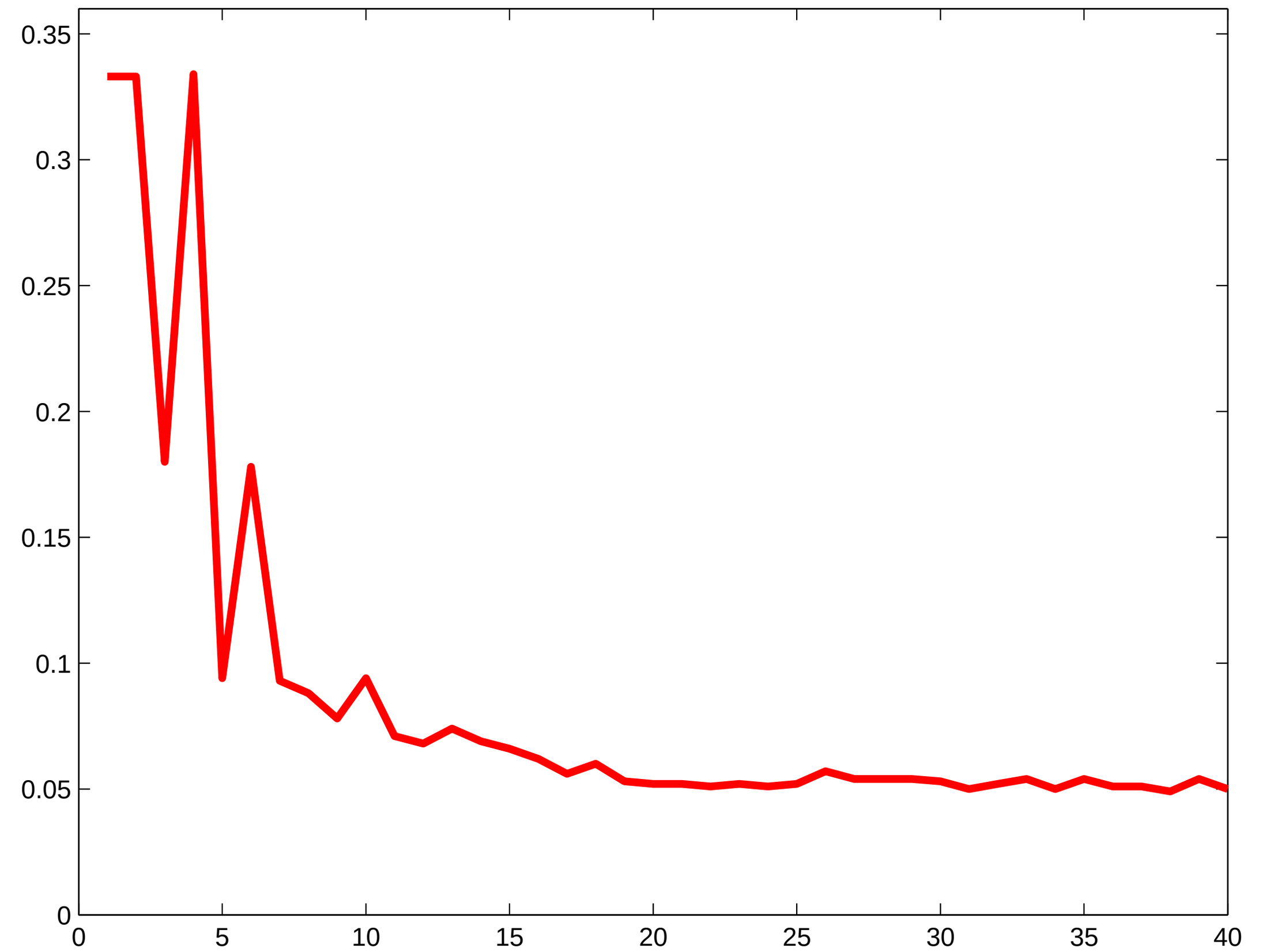
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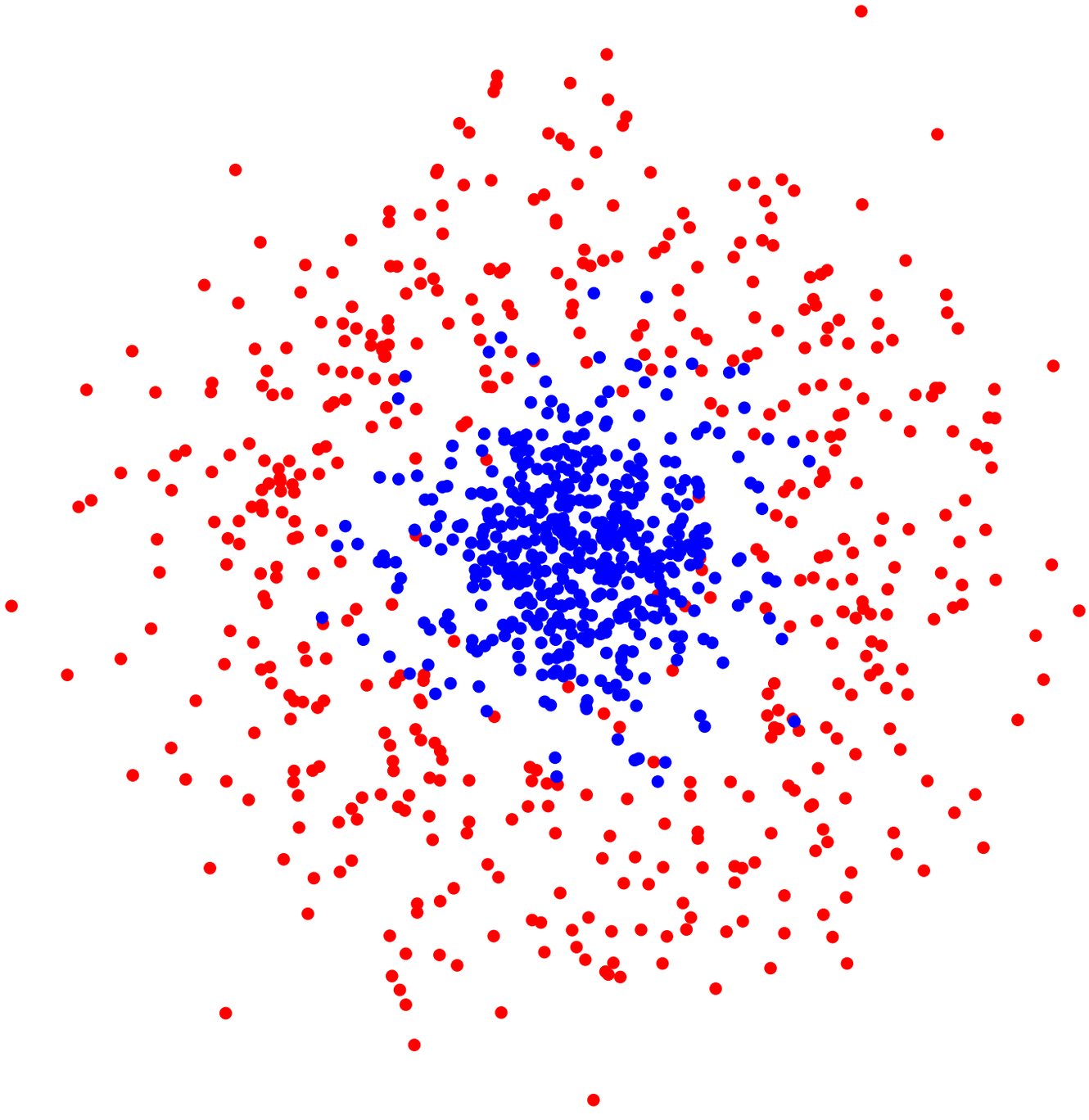

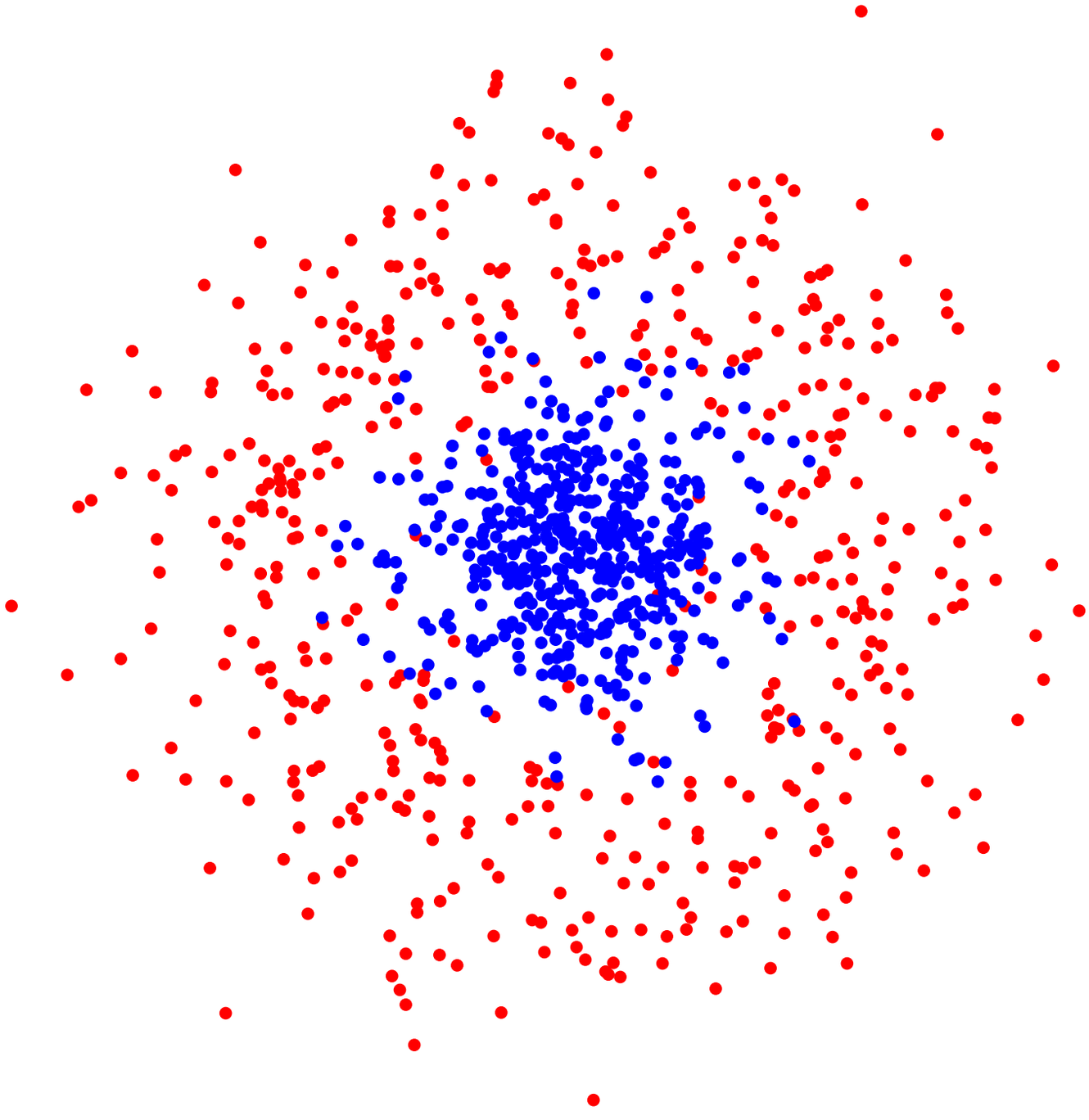

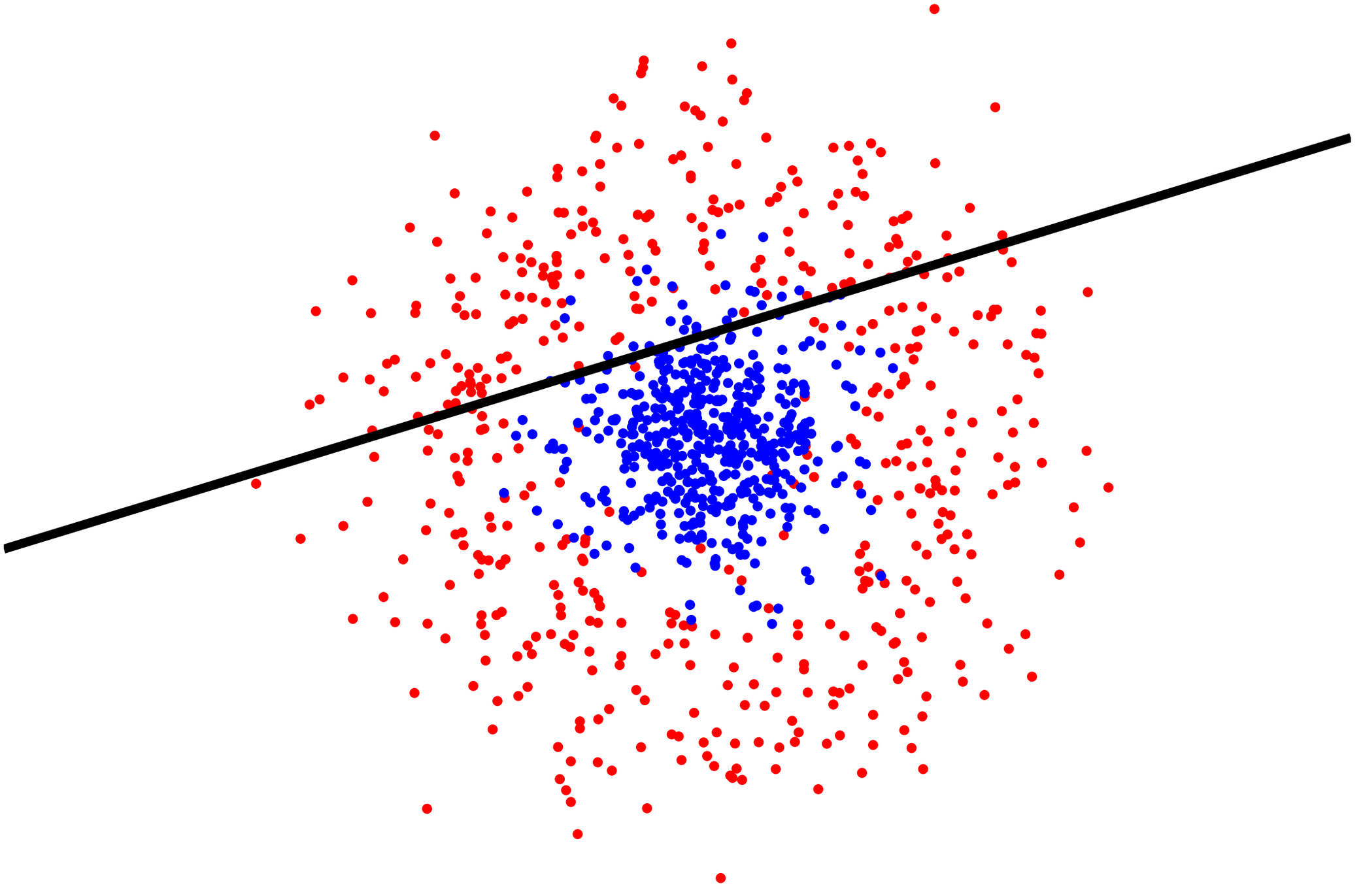

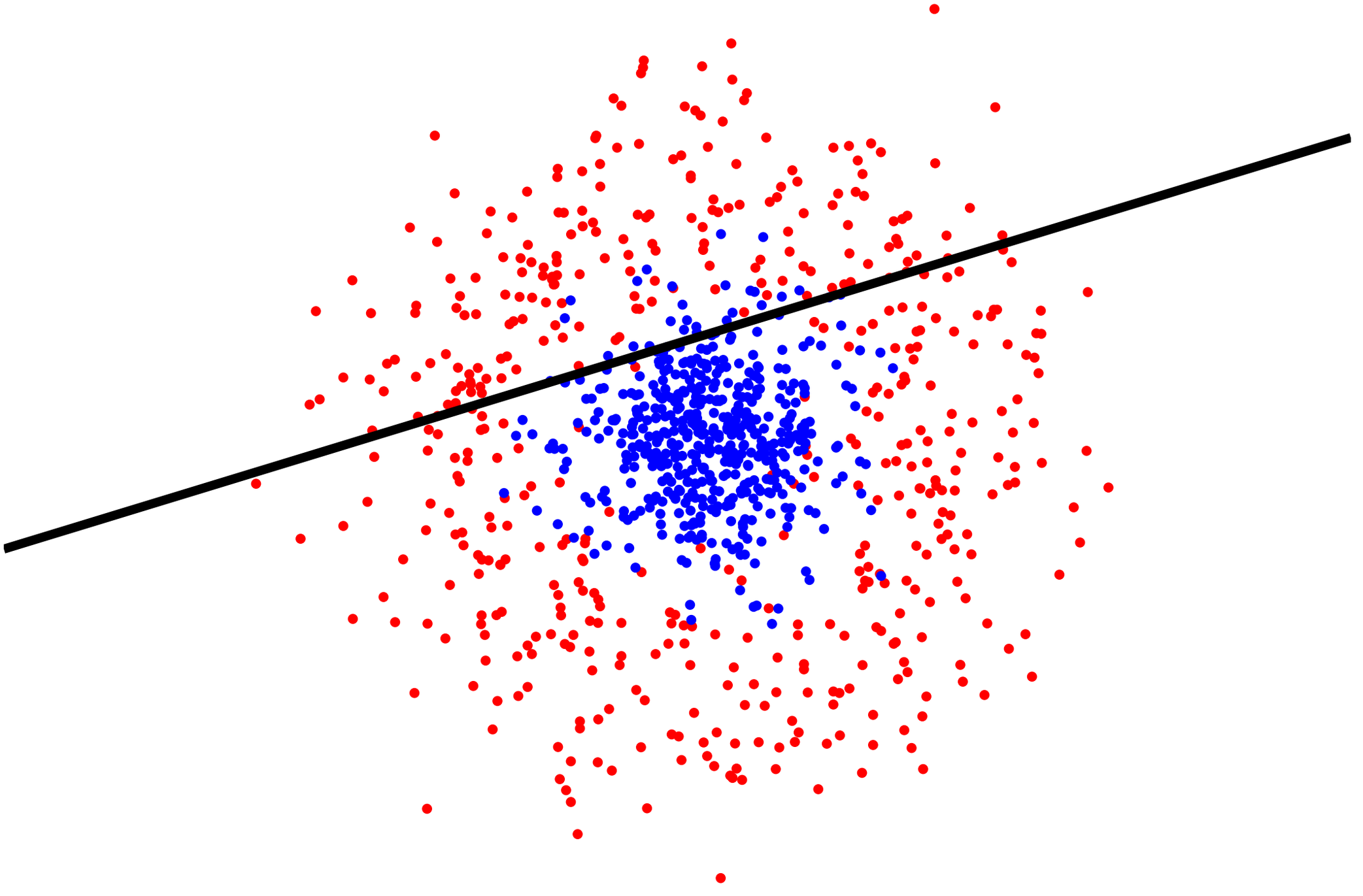

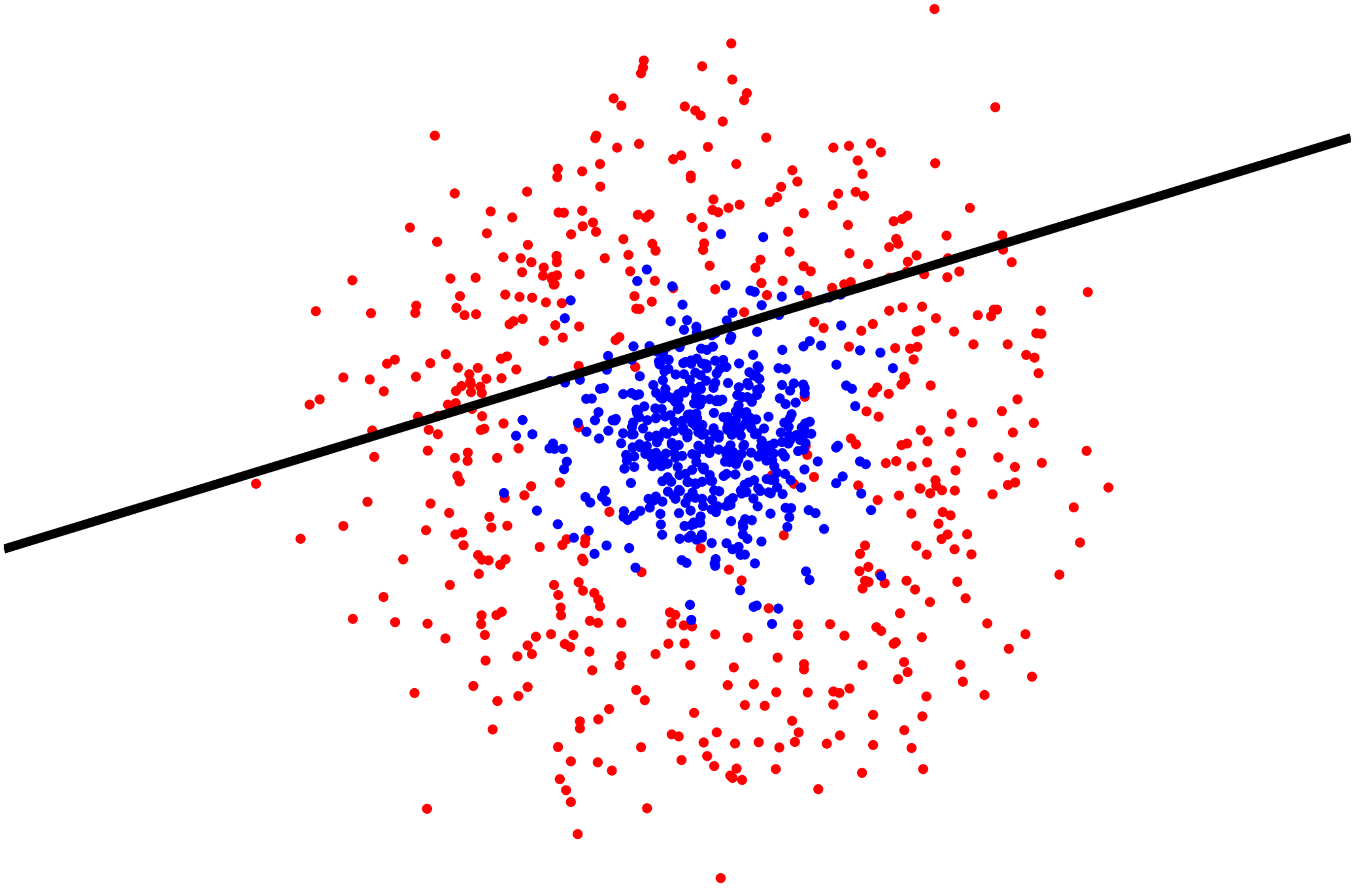

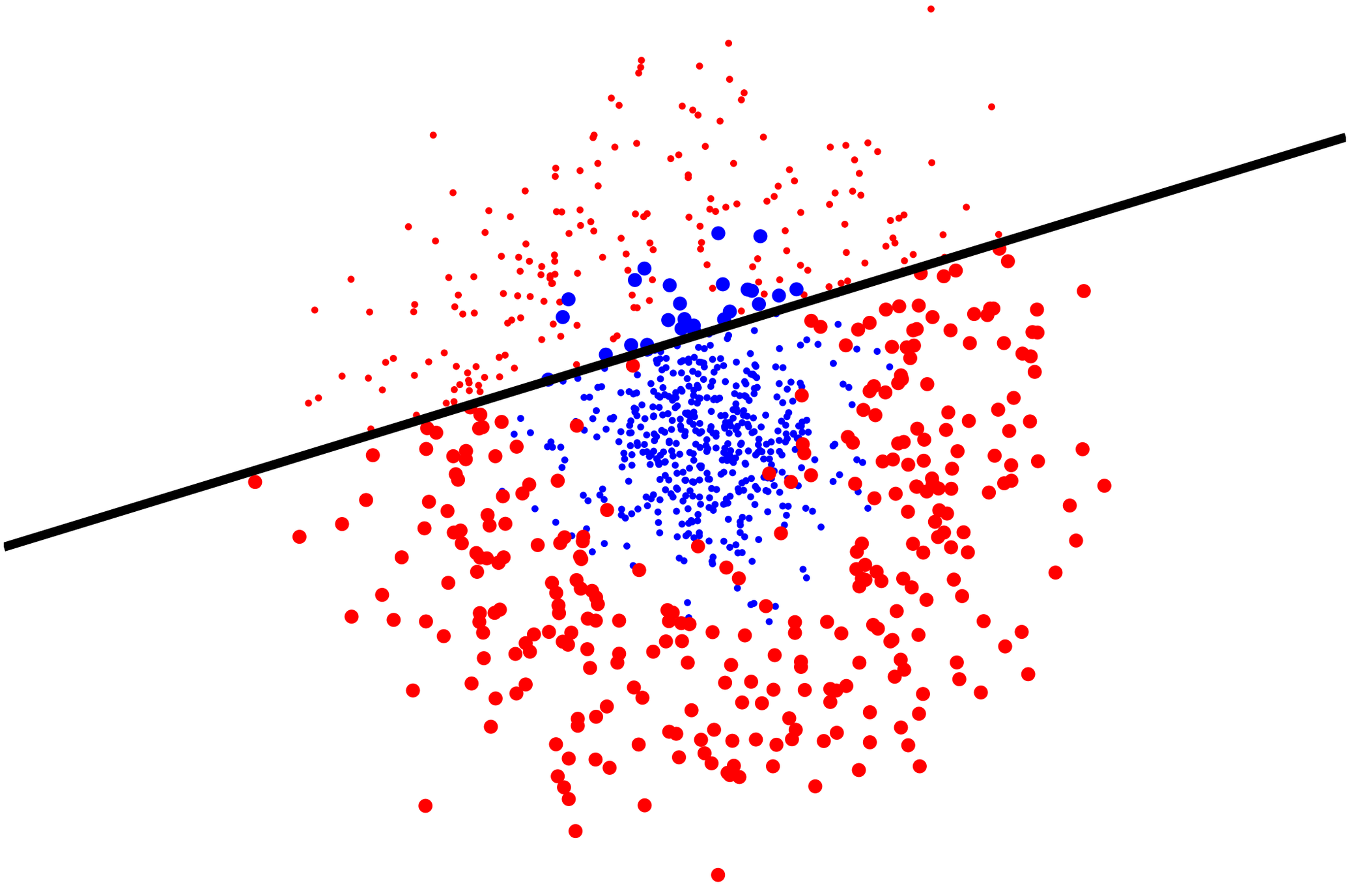

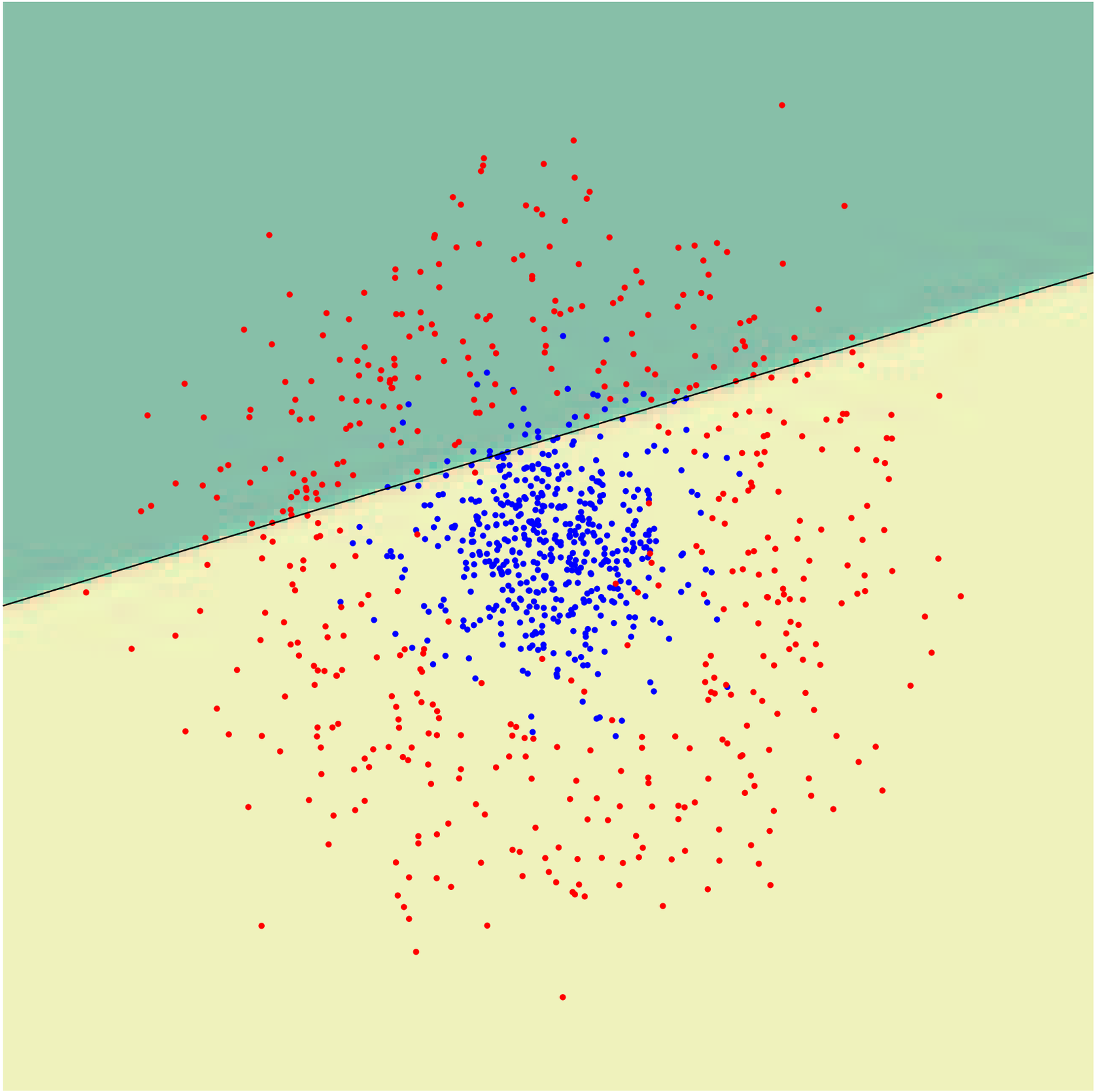

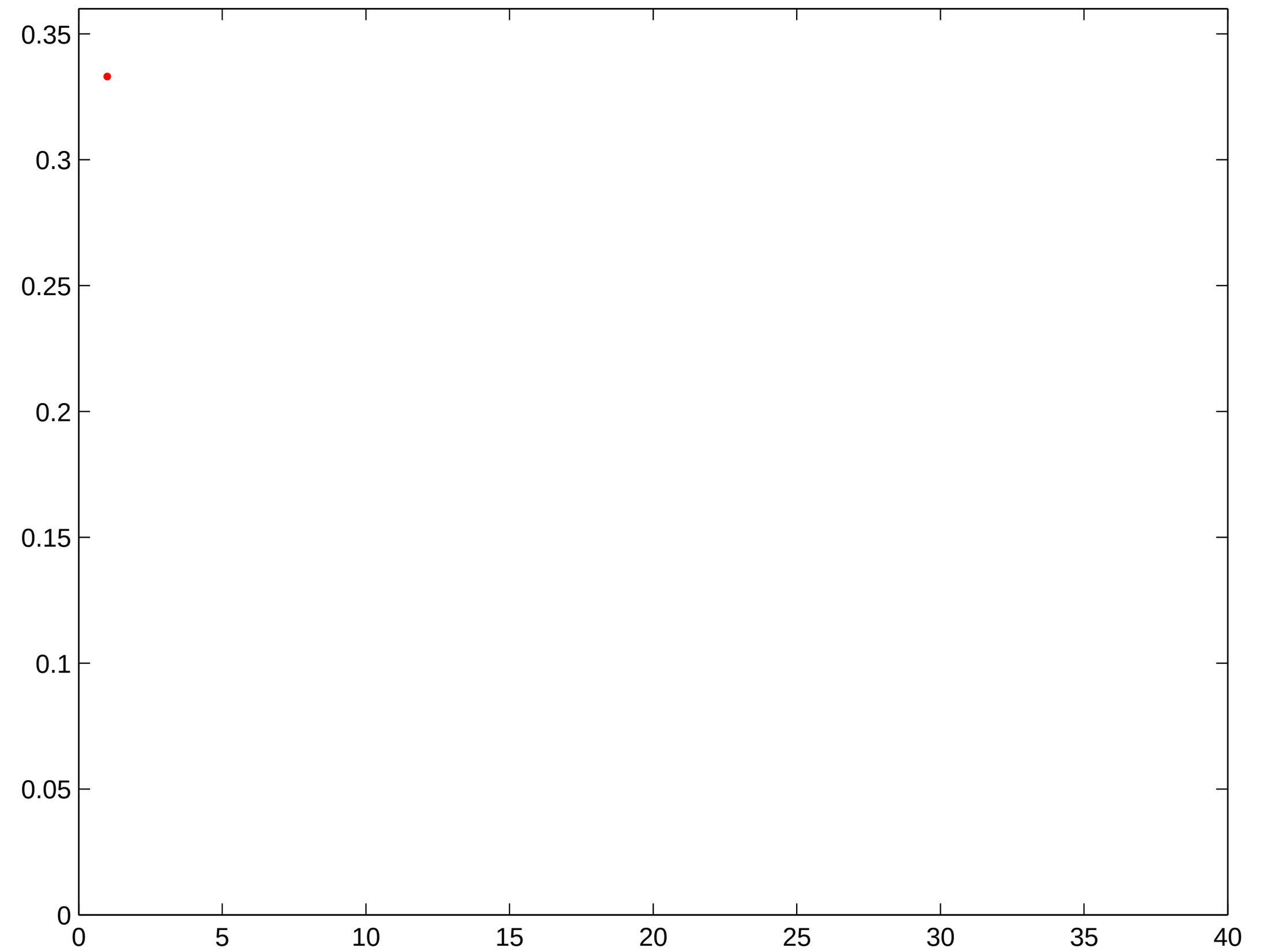
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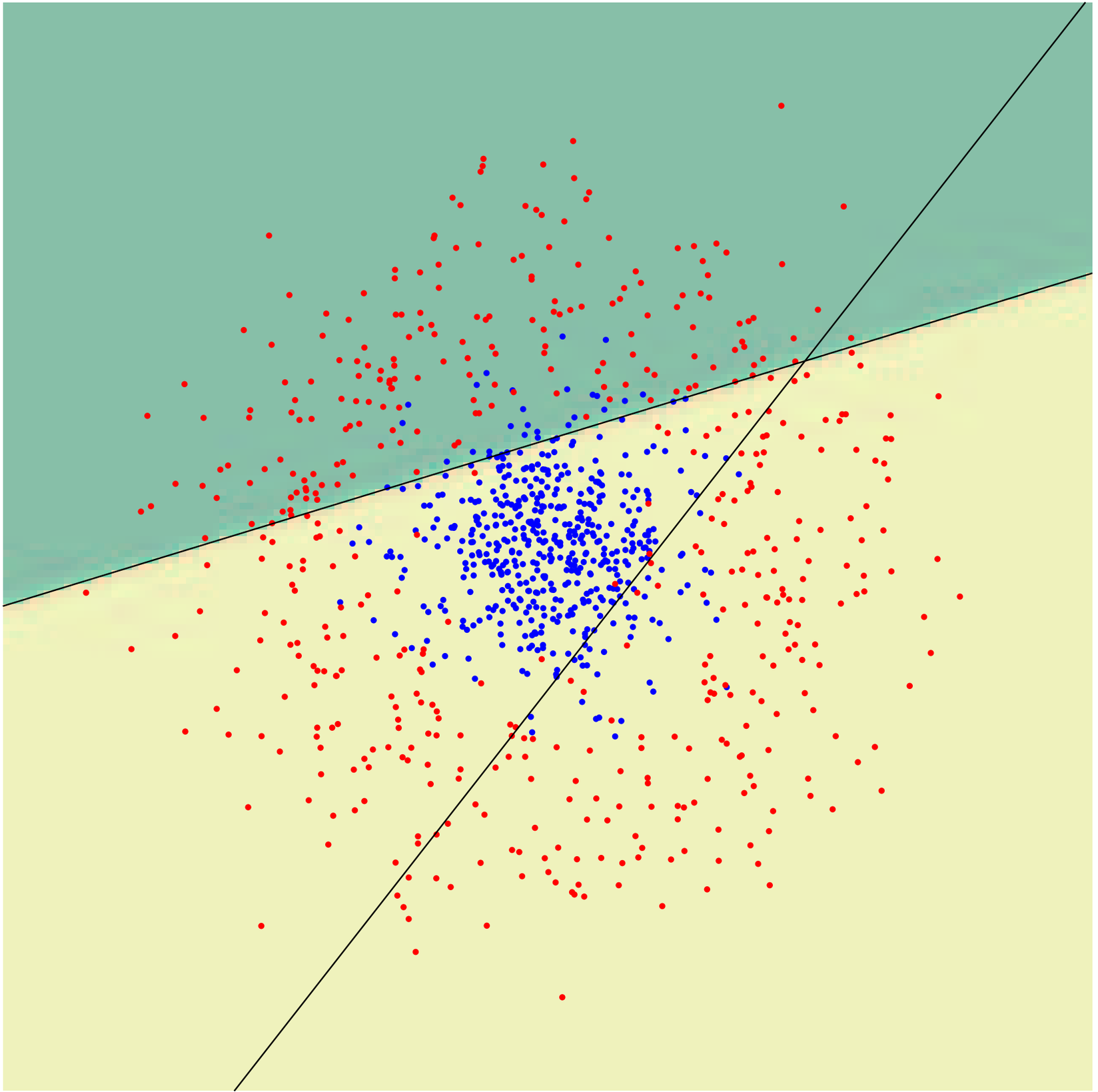

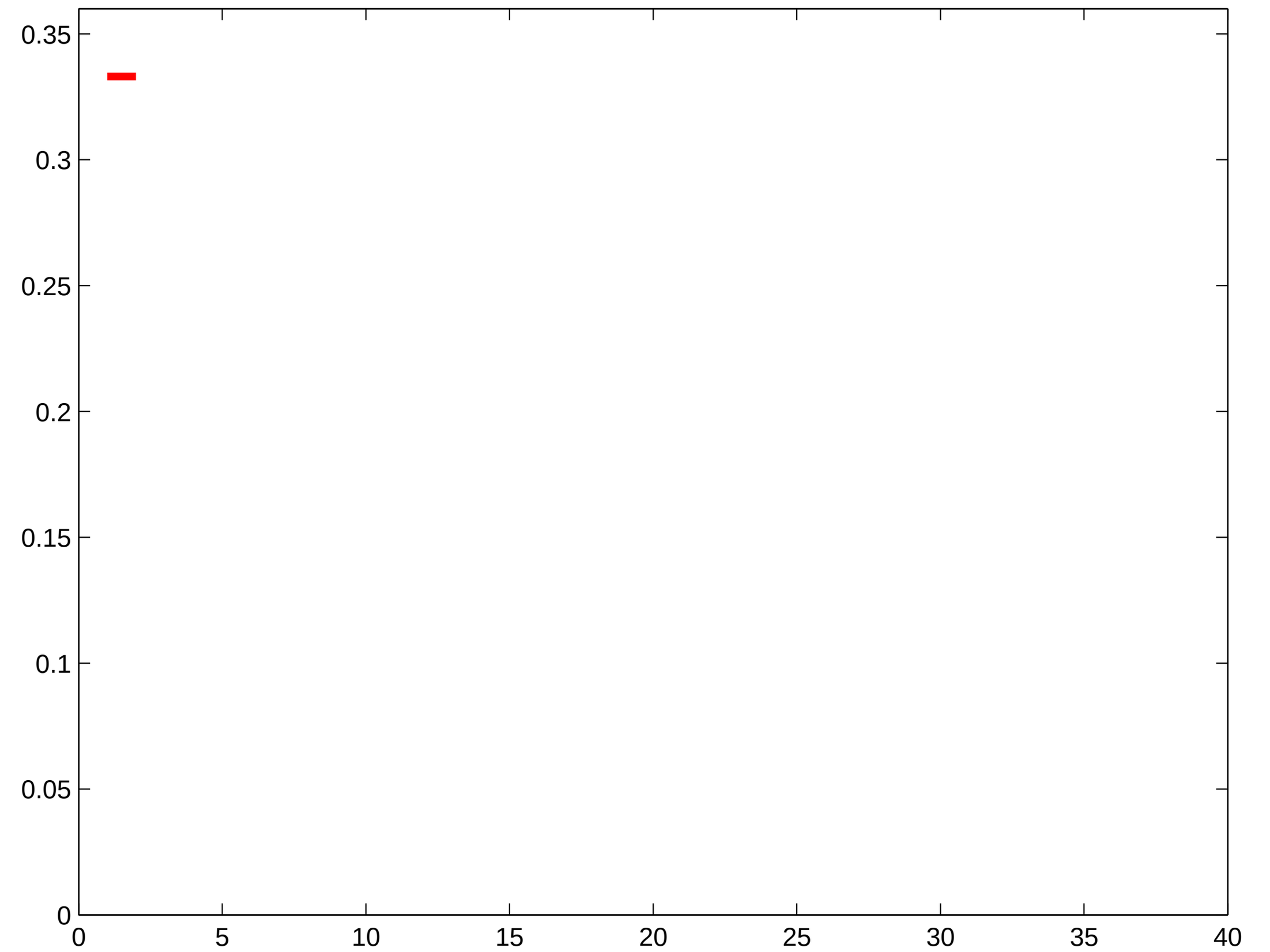
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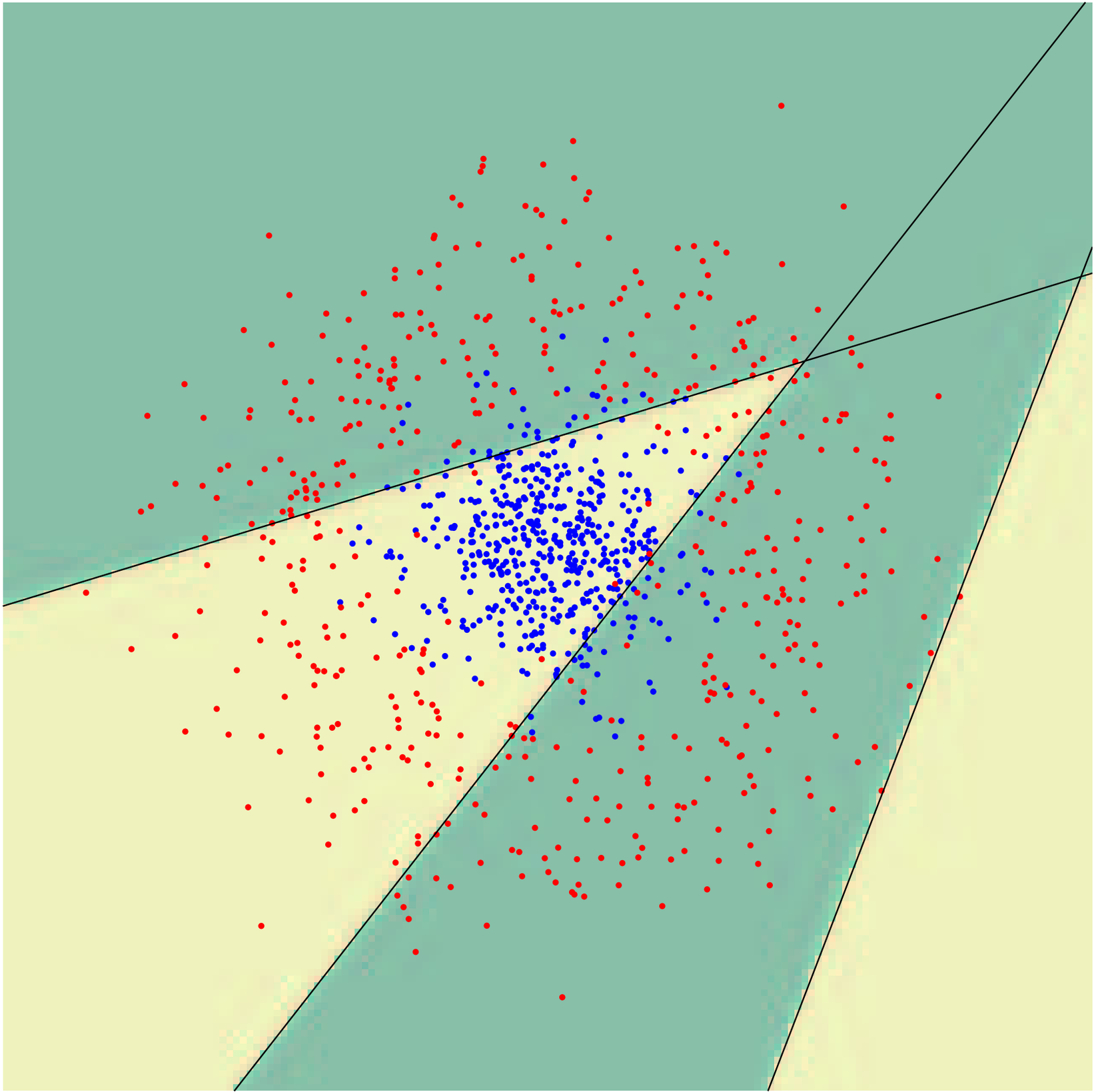

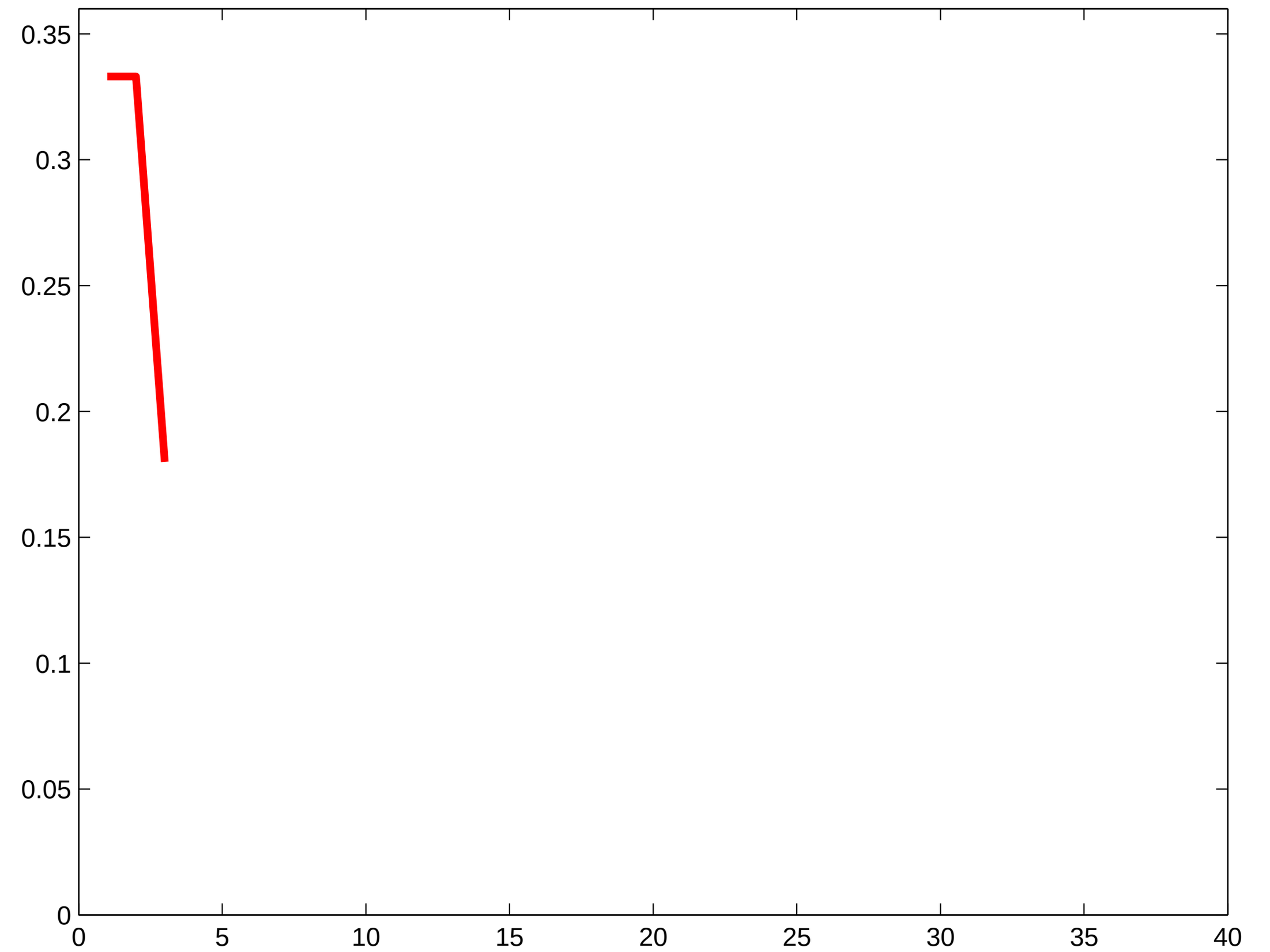
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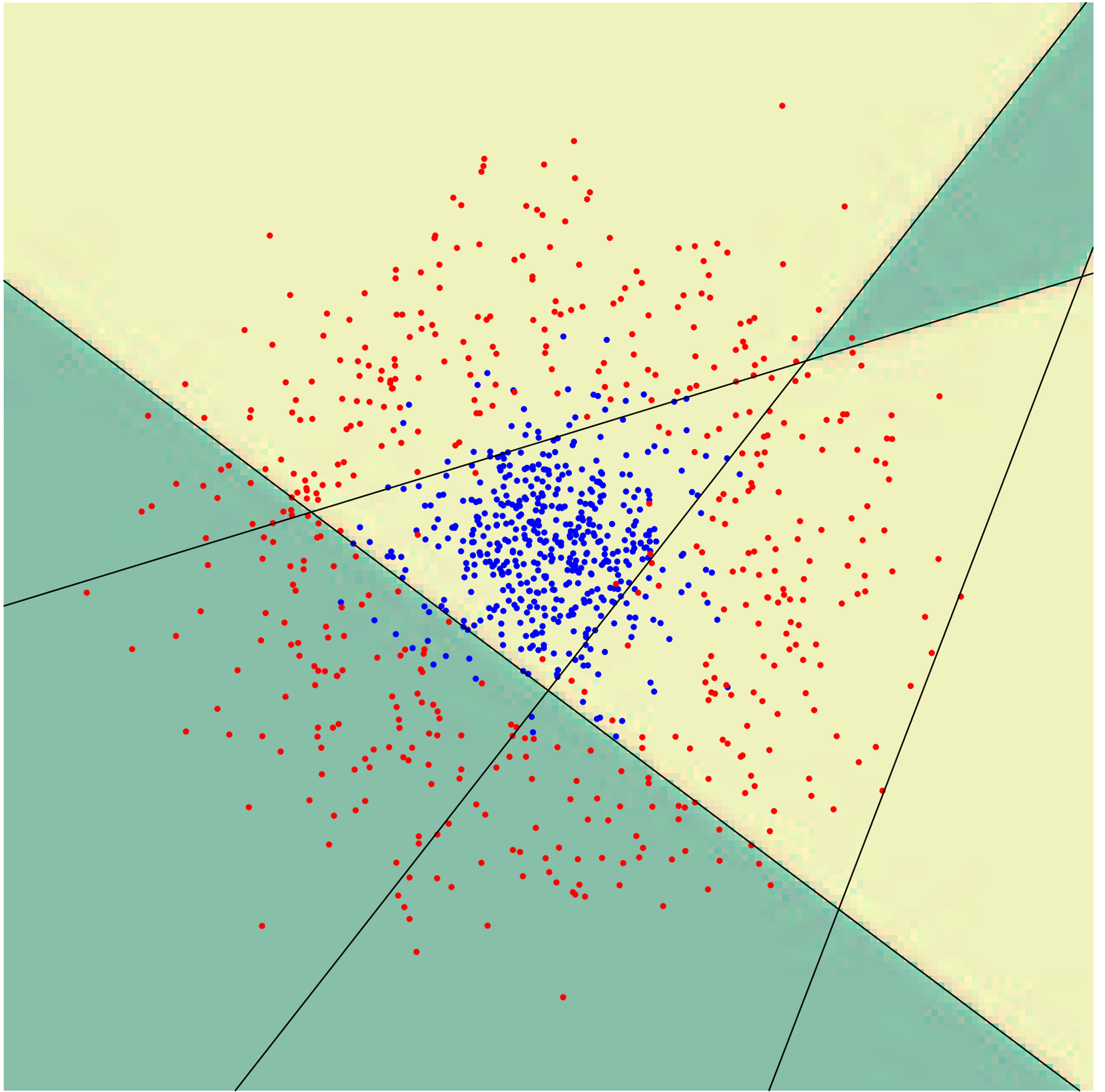

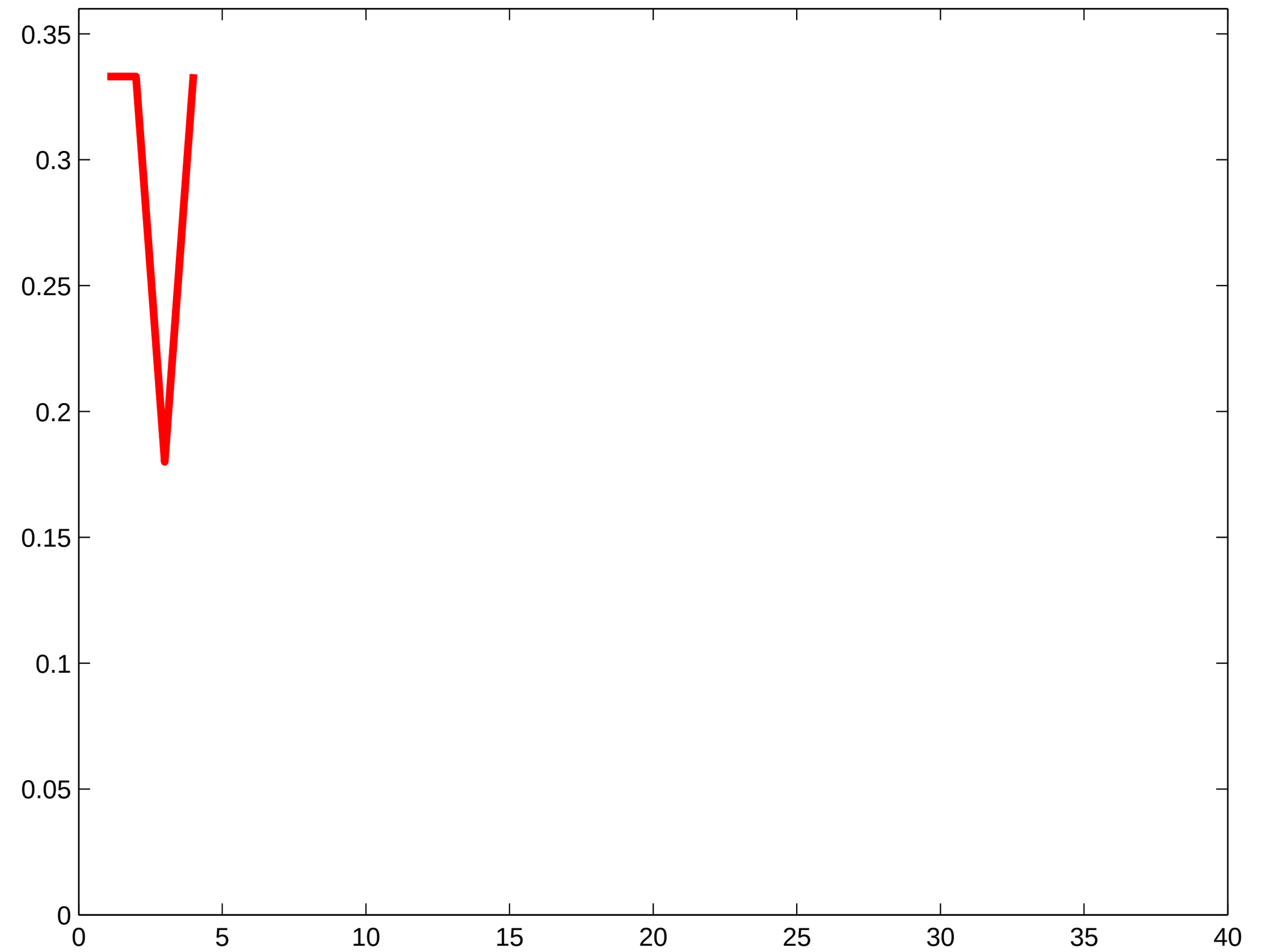
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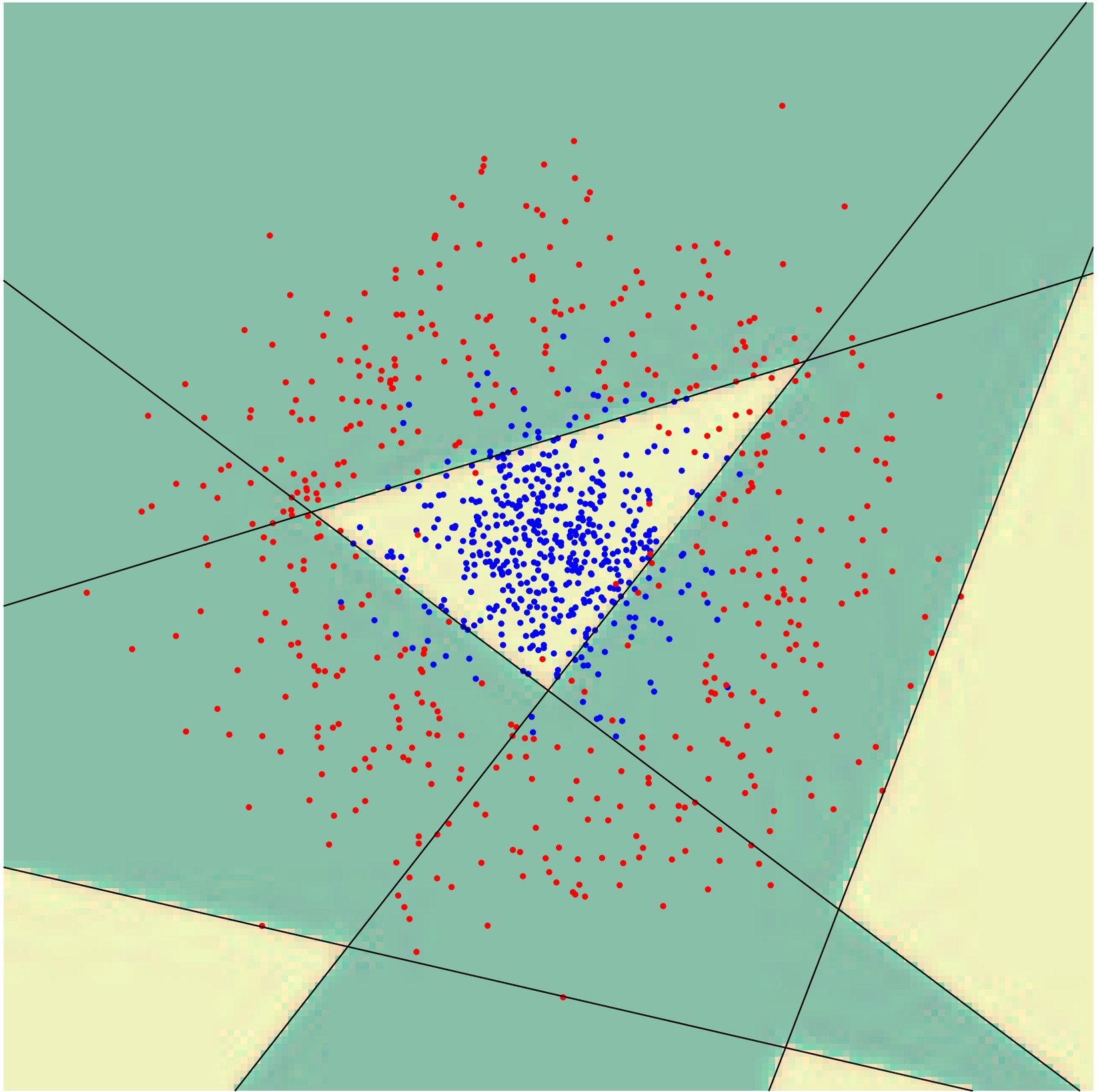

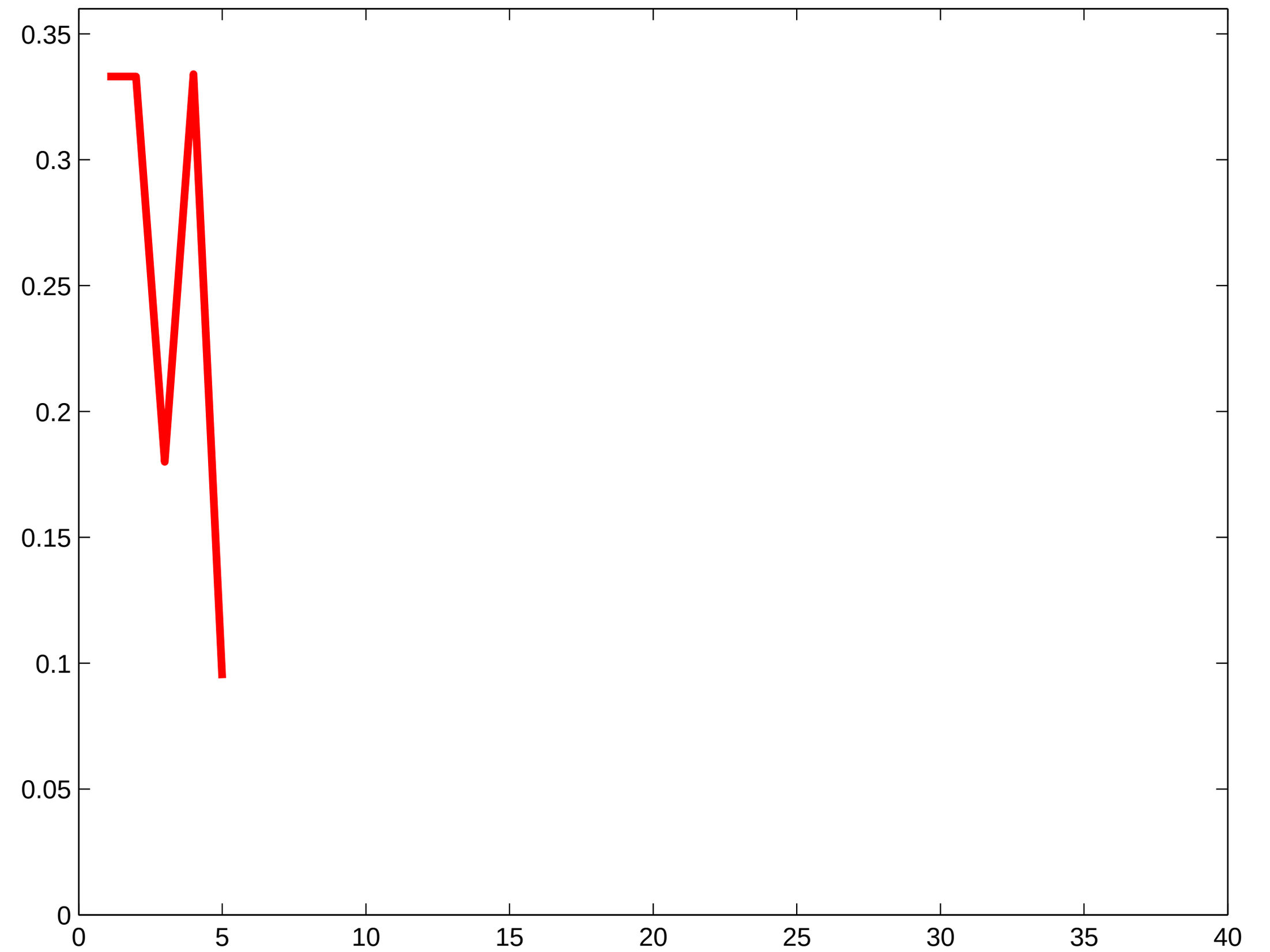
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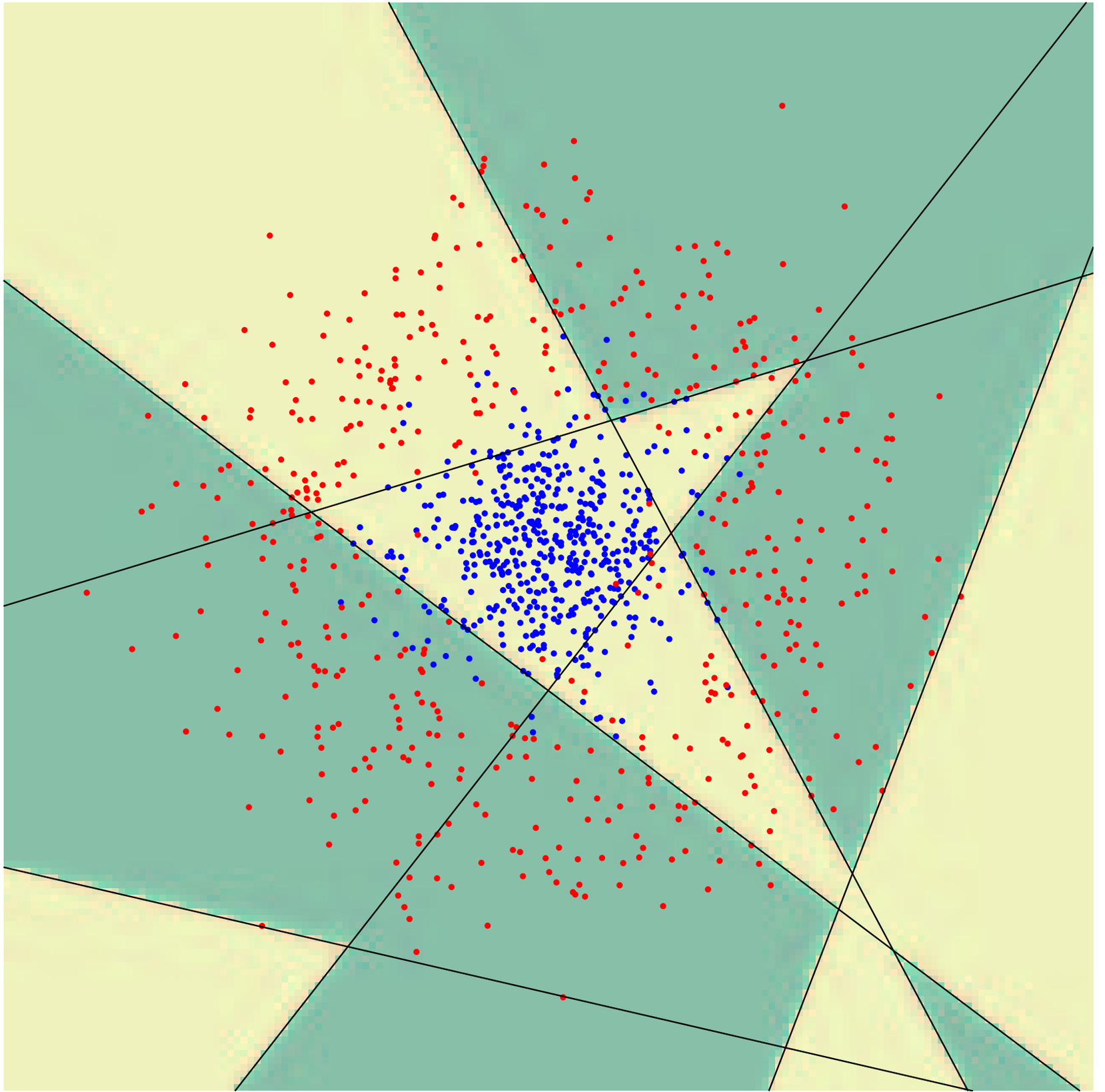

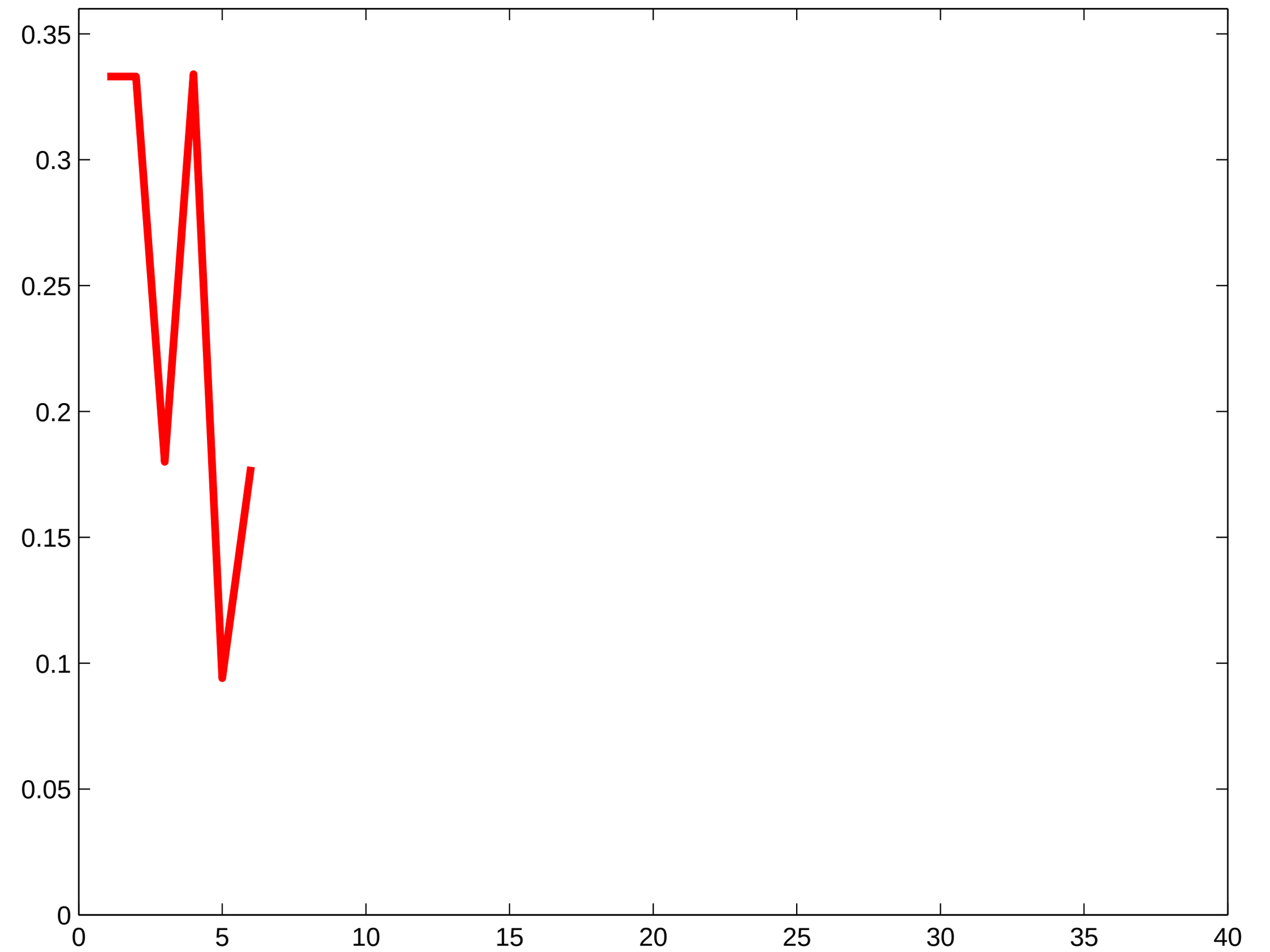
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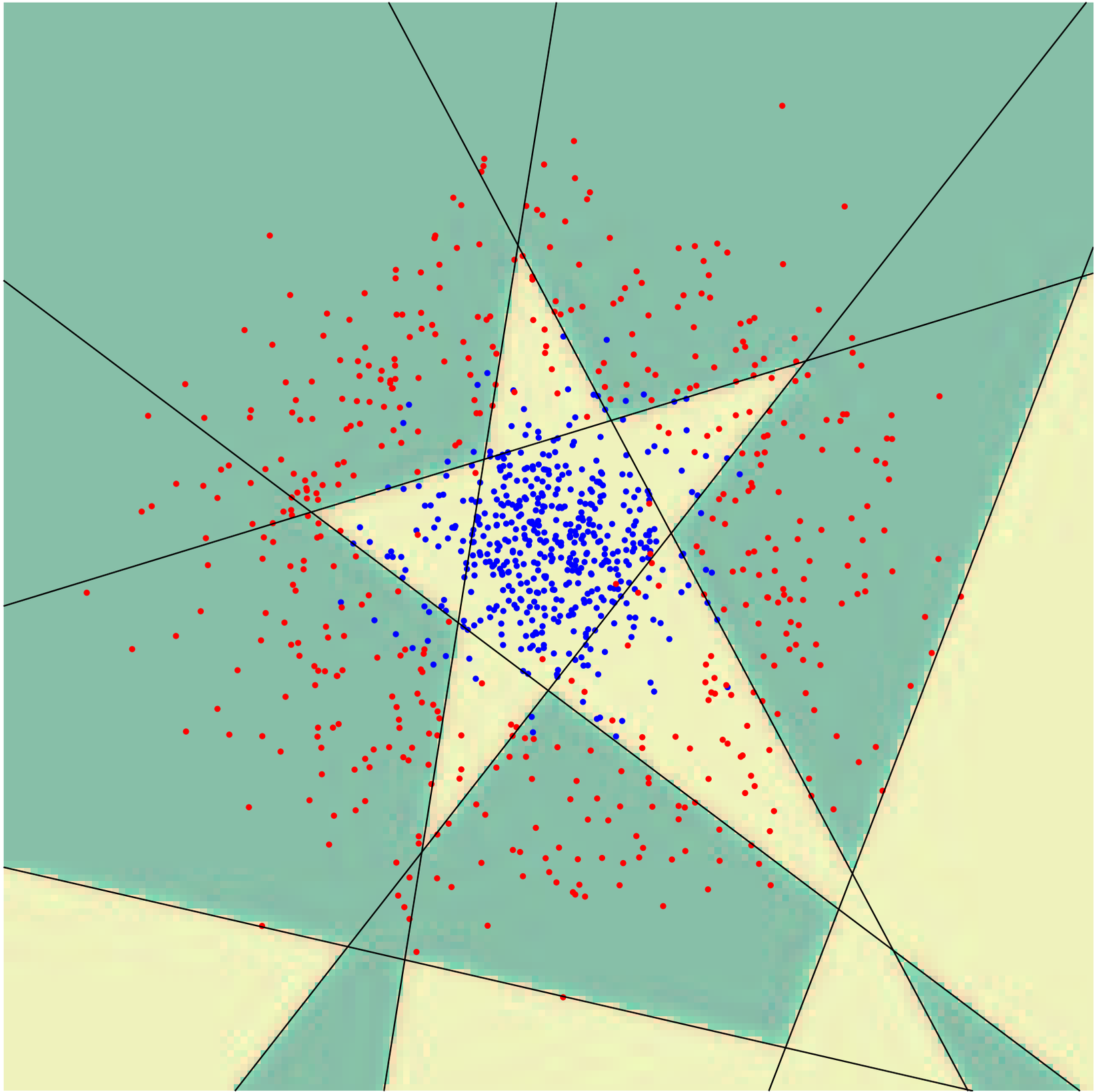

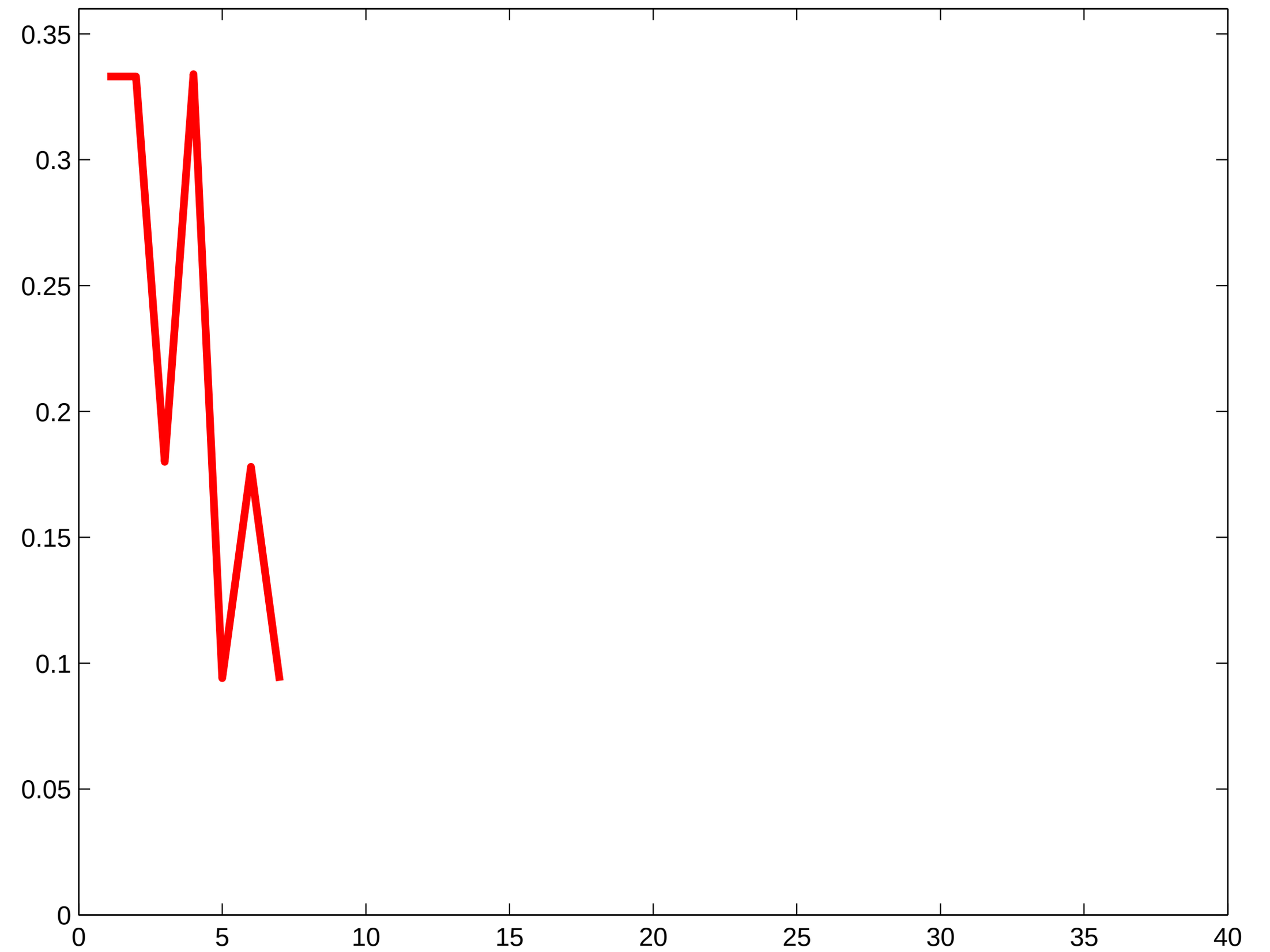
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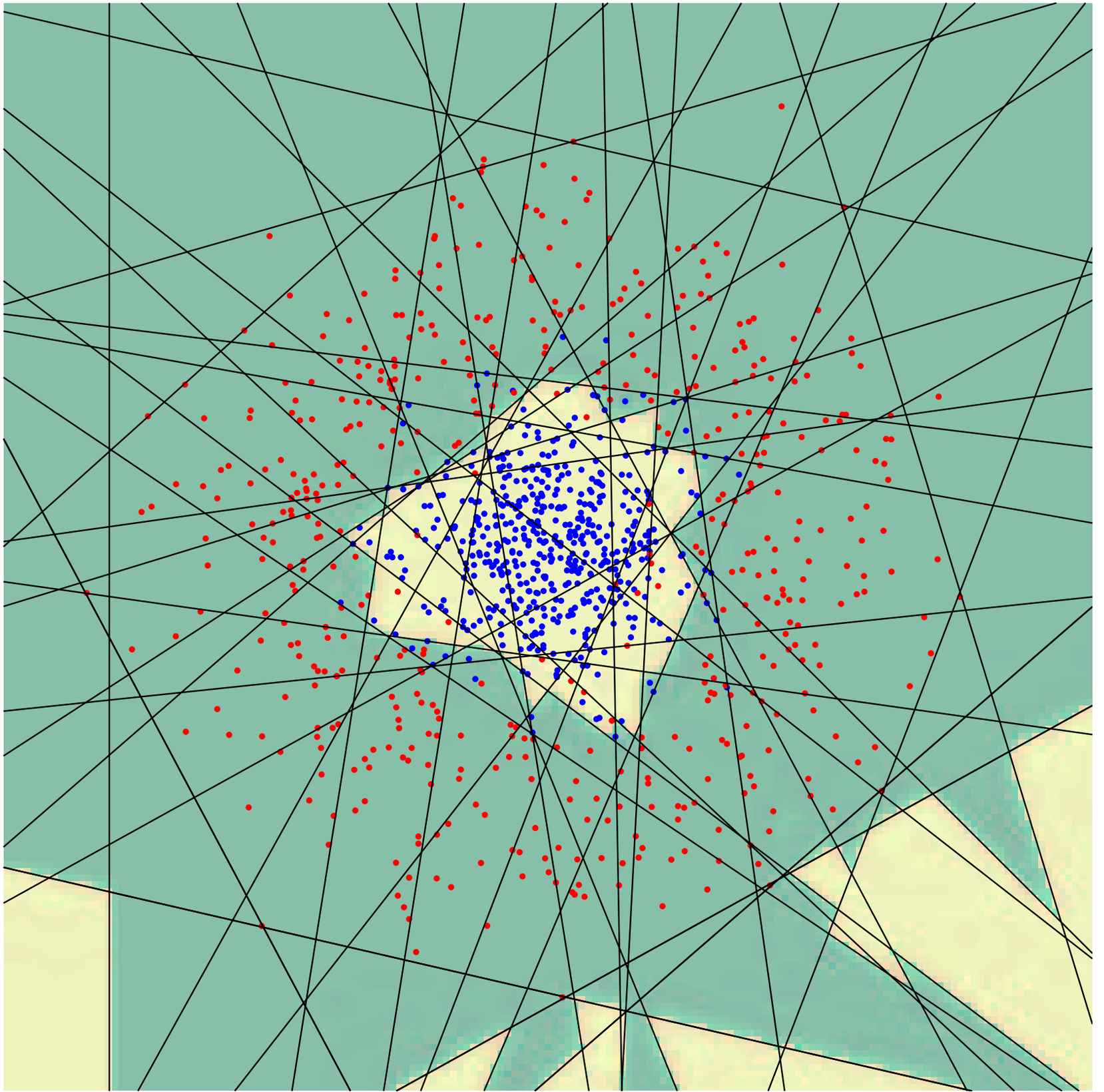

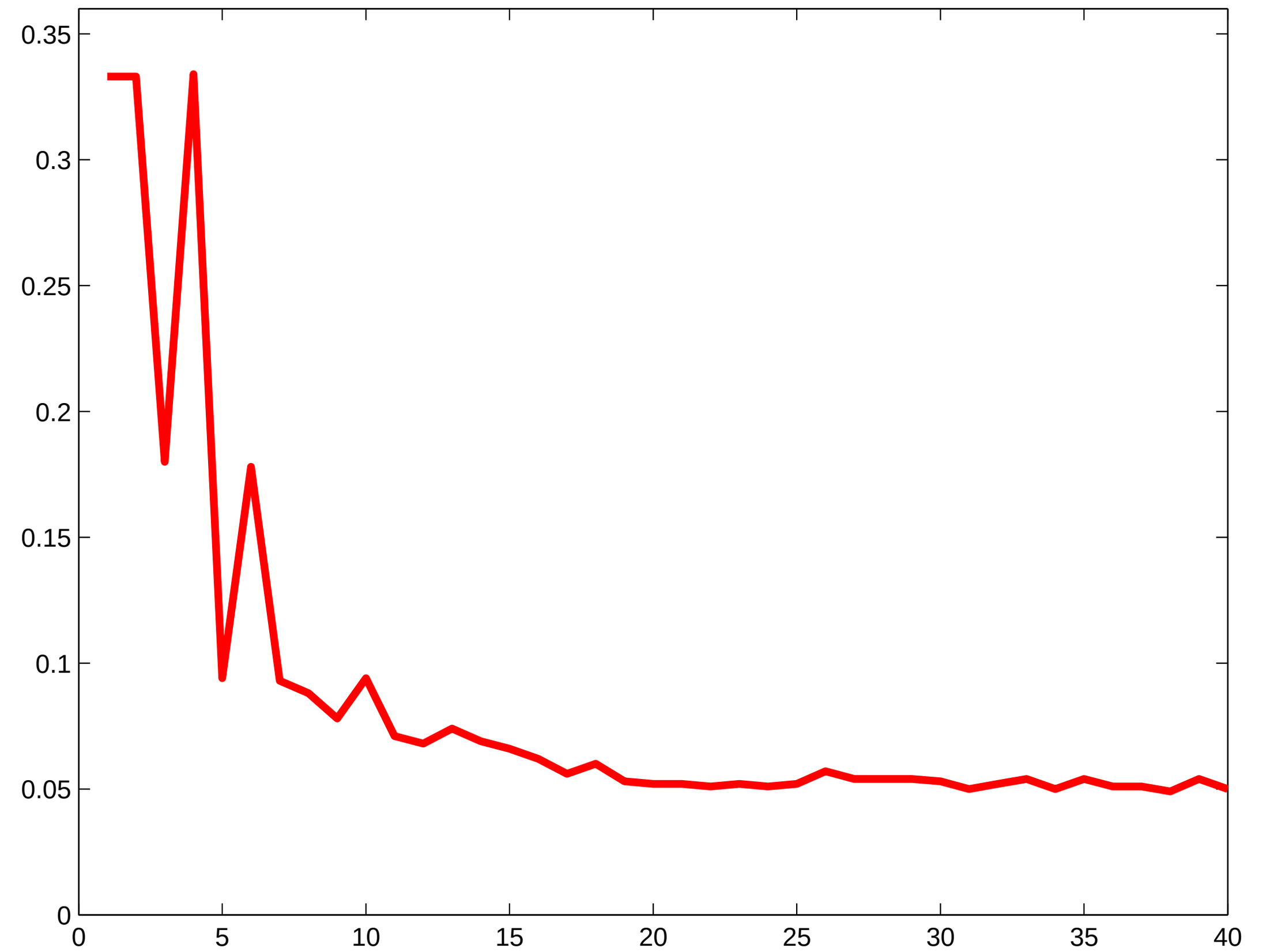
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